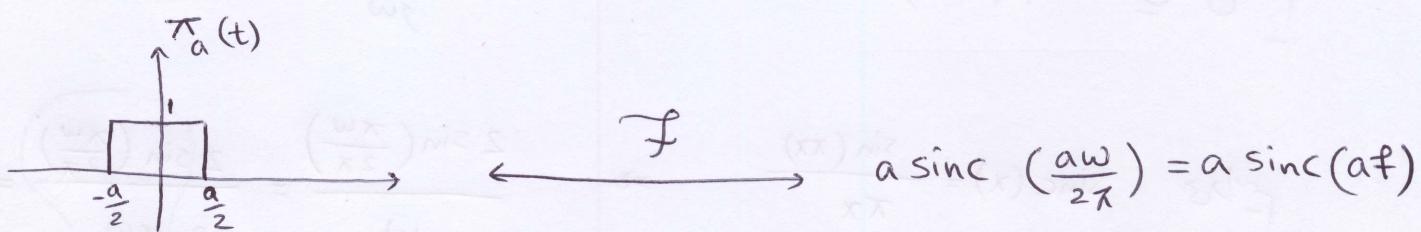


Page 1

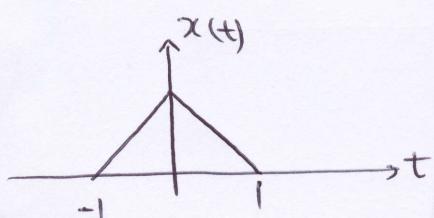
حل صيغة انتقال فوري

سنة: ١٣٩٩ / ٢٠ ، ٢٠

$$x(t) = \delta(t) \xleftrightarrow{\mathcal{F}} X(\omega) = 1$$

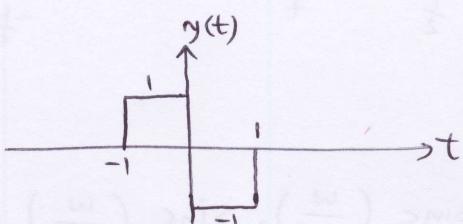


$$e^{-at} \cdot u(t) ; a > 0 \xleftrightarrow{\mathcal{F}} \frac{1}{j\omega + a}$$



مثال: انتقال فوري سكالر حسابه كالت

$$z(t) \xrightarrow{\mathcal{F}} z(\omega) = \operatorname{sinc}\left(\frac{\omega}{2\pi}\right) = \operatorname{sinc}(f)$$



شن اول: مخصوص جمل

$$\Rightarrow y(t) = (t + \frac{1}{2}) - (t - \frac{1}{2}) \Rightarrow Y(j\omega) = e^{j\frac{\omega}{2}} z(\omega) - e^{-j\frac{\omega}{2}} z(\omega)$$

$$\Rightarrow Y(\omega) = \left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}} \right) z(\omega) = 2j \sin\left(\frac{\omega}{2}\right) \cdot \operatorname{sinc}\left(\frac{\omega}{2\pi}\right)$$

مدعى: $y(t) = x'(t) \rightarrow Y(\omega) = j\omega X(\omega) \rightarrow X(\omega) = \frac{Y(\omega)}{j\omega} ; \omega \neq 0$

!!??

page 2

کار در پری جوان کرد

$$x(t) = \int_{-\infty}^t y(\alpha) \cdot d\alpha \Rightarrow X(\omega) = \frac{Y(\omega)}{j\omega} + \pi Y(0) \cdot \delta(\omega)$$

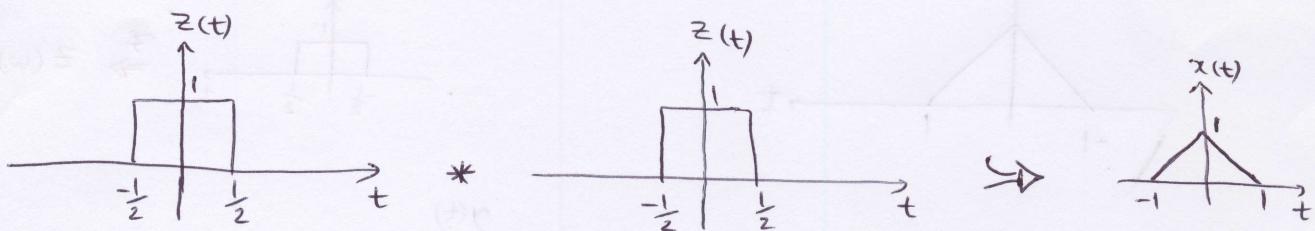
درین مدل دارم $y(\omega=0)=0 \Rightarrow X(\omega) = \frac{Y(\omega)}{j\omega} = \frac{2 \sin(\frac{\omega}{2})}{\omega} \cdot \text{sinc}(\frac{\omega}{2\pi})$

$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

$$\frac{2 \sin(\frac{\pi\omega}{2\pi})}{\omega} = \frac{2 \sin(\frac{\pi\omega}{2\pi})}{\frac{2\pi\omega}{2\pi}} = \text{sinc} \frac{\omega}{2\pi}$$

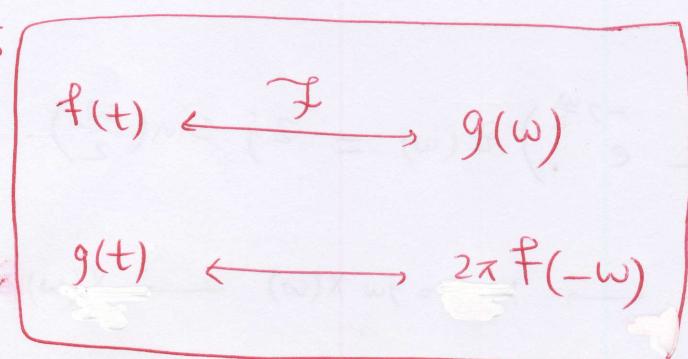
$$\Rightarrow X(\omega) = \left[\text{sinc} \left(\frac{\omega}{2\pi} \right) \right]^2 = \left[\text{sinc}(f) \right]^2 = \text{sinc}^2 \left(\frac{\omega}{2\pi} \right) = \text{sinc}^2(f)$$

راهنمہ: کاٹلوشن \rightarrow $X(t)$ کے تسلی شکل است، درام کاٹلوشن دو تایس است۔



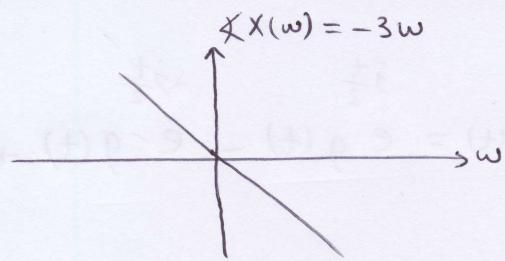
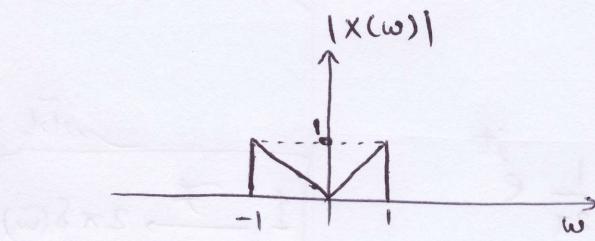
$$\Rightarrow X(\omega) = \text{sinc} \left(\frac{\omega}{2\pi} \right) \cdot \text{sinc} \left(\frac{\omega}{2\pi} \right) = \text{sinc}^2 \left(\frac{\omega}{2\pi} \right)$$

دuality
دوسرا



$$\frac{1}{2\pi} g(t) \xleftrightarrow{} f(-\omega)$$

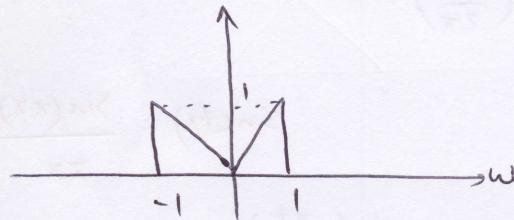
مثال ١: $X(\omega)$ به صورت زیر داده شد. سکنیل $x(t)$ رسم کنیم.



$$X(\omega) = |X(\omega)| \cdot e^{j\frac{X(\omega)}{Y(\omega)}} = |X(\omega)| \cdot e^{-j3\omega} = Y(\omega) \cdot e^{-j3\omega}$$

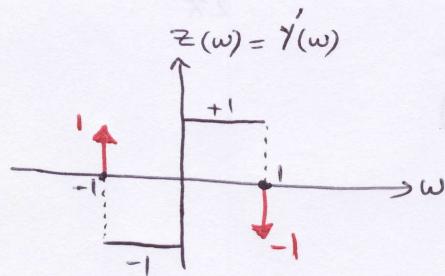
$$\left\{ \begin{array}{l} y(t) \xleftarrow{\mathcal{F}} Y(\omega) \\ y(t-3) \xrightarrow{-j3\omega} Y(\omega) \cdot e^{-j3\omega} \end{array} \right.$$

$$\Rightarrow Y(\omega) = |X(\omega)|$$

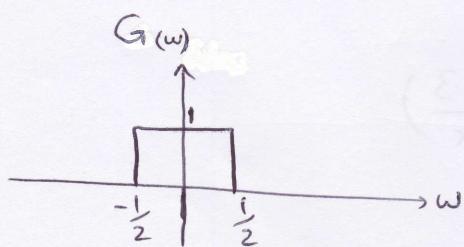


$$\Rightarrow y(t) = ?$$

روش اول: حستق بگیرم.



$$\Rightarrow z(t) = ? =$$



$$\Rightarrow g(t) = ? = \frac{1}{2\pi} \operatorname{sinc}\left(\frac{t}{2\pi}\right)$$

$$z(\omega) = G(\omega - \frac{1}{2}) - G(\omega + \frac{1}{2}) + S(\omega+1) - S(\omega-1)$$

$$\Rightarrow z(t) = e^{jt} g(t) - e^{-jt} g(t) + \frac{1}{2\pi} e^{-jt} - \frac{1}{2\pi} e^{jt}$$

وسيط

$1 \xrightarrow{\text{زوج}} 2\pi \delta(\omega)$

$$\Rightarrow z(t) = g(t) \left[e^{jt/2} - e^{-jt/2} \right] + \frac{-1}{2\pi} \left[e^{jt} - e^{-jt} \right]$$

$\underbrace{2j \sin \frac{t}{2}}$ $\underbrace{2j \sin(t)}$

$$\Rightarrow z(t) = \frac{2j}{2\pi} \left[\sin\left(\frac{t}{2}\right) \cdot \text{sinc}\left(\frac{t}{2\pi}\right) - \sin(t) \right] \quad \Rightarrow \quad z(t=0) = z(0) = 0$$

مهمة اسلاك
وصول زكانت

$$Y(\omega) = \int_{-\infty}^{\omega} z(\tau) d\tau \quad \rightarrow \quad y(t) = \frac{z(t)}{-jt} + \pi z(0) \cdot \delta(t)$$

$$\Rightarrow y(t) = \frac{z(t)}{-jt} = \frac{\sin(t)}{\pi t} - \frac{\sin(t/2)}{\pi t} \cdot \text{sinc}\left(\frac{t}{2\pi}\right)$$

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\frac{\sin(t)}{\pi t} = \frac{\sin\left(\frac{\pi t}{\pi}\right)}{\frac{\pi \cdot \pi t}{\pi}} = \frac{1}{\pi} \text{sinc}\left(\frac{t}{\pi}\right)$$

$$\frac{\sin(t/2)}{\pi t} = \frac{\sin\left(\frac{\pi t}{2\pi}\right)}{\frac{2\pi \cdot \pi t}{2\pi}} = \frac{1}{2\pi} \text{sinc}\left(\frac{t}{2\pi}\right)$$

$$\Rightarrow y(t) = \frac{1}{\pi} \text{sinc}\left(\frac{t}{\pi}\right) - \frac{1}{2\pi} \text{sinc}^2\left(\frac{t}{2\pi}\right)$$

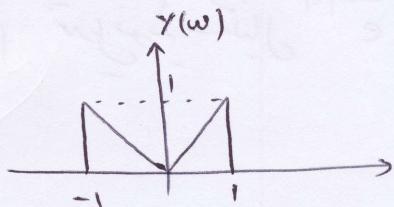
$$\Rightarrow x(t) = y(t-3) \quad \checkmark$$

$$\Rightarrow x(t) = \frac{1}{\pi} \text{sinc}\left(\frac{t-3}{\pi}\right) - \frac{1}{2\pi} \text{sinc}^2\left(\frac{t-3}{2\pi}\right)$$

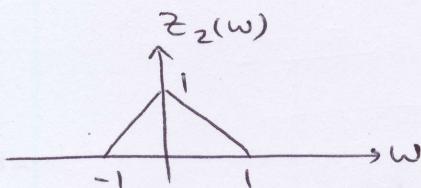
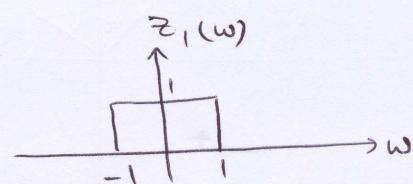
$$\left(\frac{t}{\pi}\right) \rightarrow \text{sinc} \frac{1}{\pi} = 0$$

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وَرْشَ حَوْمَ حَلِّ اِنْسَلِ:



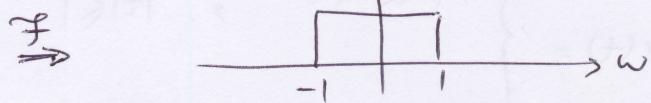
$$y(\omega) = z_1(\omega) - z_2(\omega)$$



فَهُوَ مُعَادِلٌ لِـ

$$\left\{ \begin{array}{l} z_1(t) \\ \frac{2 \operatorname{sinc}(\frac{t}{\pi})}{2\pi} \end{array} \right. \xrightarrow{\mathcal{F}} 2 \operatorname{sinc}\left(\frac{2\omega}{2\pi}\right) = 2 \operatorname{sinc}\left(\frac{\omega}{\pi}\right)$$

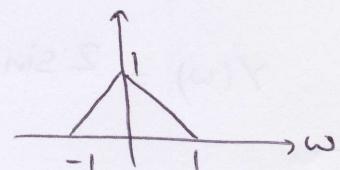
$$\frac{2 \operatorname{sinc}(\frac{t}{\pi})}{2\pi}$$



فَهُوَ مُعَادِلٌ لِـ

$$\left\{ \begin{array}{l} \operatorname{sinc}^2\left(\frac{\omega}{2\pi}\right) \\ \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{t}{2\pi}\right) \end{array} \right. \xrightarrow{\mathcal{F}} \operatorname{sinc}^2\left(\frac{\omega}{2\pi}\right)$$

$$\frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{t}{2\pi}\right)$$



$$\Rightarrow y(t) = \frac{1}{\pi} \operatorname{sinc}\left(\frac{t}{\pi}\right) - \frac{1}{2\pi} \operatorname{sinc}^2\left(\frac{t}{2\pi}\right) \quad \checkmark$$

page 6) حل سؤال:



$$\text{حل سؤال: } x(t) = e^{-at} u(t) \xrightarrow{\mathcal{F}} \frac{1}{a+j\omega}$$

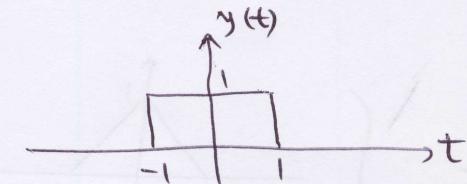
$$\text{حل سؤال: } x(t) = e^{-at} = e^{-a(-t)} + e^{-a(t)} = y(-t) + y(t)$$

$$\Rightarrow X(\omega) = Y(-\omega) + Y(\omega) = \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \boxed{\frac{2a}{a^2 + \omega^2}}$$

$$x(t) = \begin{cases} 1 + \omega \pi t & ; |t| \leq 1 \\ 0 & ; |t| > 1 \end{cases}$$

حل سؤال: $x(t) = 1 + \omega \pi t$

$$\Rightarrow x(t) = (1 + \omega \pi t) \cdot y(t)$$



$$Y(\omega) = 2 \operatorname{sinc}(2\omega) = 2 \operatorname{sinc}\left(\frac{\omega}{\pi}\right)$$

مشكل:

$$x(t) = \underbrace{(1 + \omega \pi t)}_{z(t)} \cdot y(t) \Rightarrow X(\omega) = (Z(\omega) * Y(\omega)) \frac{1}{2\pi}$$

$$Z(\omega) = ? \quad z(t) = 1 + \omega \pi t = \frac{1}{2} e^{j\pi t} + \frac{1}{2} e^{-j\pi t}$$

$$\Rightarrow Z(\omega) = 2\pi \delta(\omega) + \pi \delta(\omega - \pi) + \pi \delta(\omega + \pi)$$

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$$X(\omega) = \frac{1}{2\pi} (Z(\omega) * Y(\omega)) = \left[[2\pi \delta(\omega) + \pi \delta(\omega - \pi) + \pi \delta(\omega + \pi)] * Y(\omega) \right] \frac{1}{2\pi}$$

$$\Rightarrow X(\omega) = \frac{1}{2\pi} \left[2\pi Y(\omega) + \pi Y(\omega - \pi) + \pi Y(\omega + \pi) \right]$$

$$\Rightarrow X(\omega) = Y(\omega) + \frac{1}{2} Y(\omega - \pi) + \frac{1}{2} Y(\omega + \pi)$$

$$x(t) = (1 + \omega_0 \pi t) y(t) = \left(1 + \frac{1}{2} e^{j\pi t} + \frac{1}{2} e^{-j\pi t} \right) y(t)$$

$$\Rightarrow x(t) = y(t) + \frac{1}{2} e^{j\pi t} y(t) + \frac{1}{2} e^{-j\pi t} y(t)$$

$$\Rightarrow X(\omega) = Y(\omega) + \frac{1}{2} Y(\omega - \pi) + \frac{1}{2} Y(\omega + \pi)$$