in the name of God

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در این پروژه قصد دارم قیمت دو سهام اس اند پی و دیزنی را با مدل گارچ پیش بینی کنم

In [786...

```
import pandas_datareader.data as web
import yfinance as yf
from datetime import datetime, timedelta
import pandas as pd
import matplotlib.pyplot as plt
from arch import arch_model
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.api import VAR
from scipy.stats import pearsonr
from statsmodels.tools.eval_measures import rmse, aic, bic
import numpy as np
```

پس از فراخوانی کتابخانه های مورد نیاز از سایت یاهو فایننس اطلاعات دو شرکت را برای بازه زمانی مد نظرمان بارگیری میکنیم

In [787...

```
start = datetime(2017, 10, 1)
end = datetime(2024, 10, 1)
```

In [788...

```
dis = yf.download('DIS', start = '2017-10-1', end = '2024-10-1')
dis.head()
```

```
[*********************100%***********************]  1 of 1 completed
```

Out[788...

Price	Close	High	Low	Open	Volume
Ticker	DIS	DIS	DIS	DIS	DIS
Date					
2017-10-02	95.163742	95.716467	94.449013	94.639605	6923300
2017-10-03	96.050003	96.107179	95.211391	95.344806	5447700
2017-10-04	95.821281	96.183408	95.487736	96.002341	5126100
2017-10-05	95.401978	96.126238	95.316206	95.907051	4736700
2017-10-06	95.363846	95.554437	94.668174	95.249487	4360200

از آنجایی که نوسان قیمت در طول روز نسبت به قیمت بسیار کم است و از طرفی هدف نهایی ما پیش بینی سود و زیان سرمایه می باشد بنابراین از درصد بازده سهام بجای خود قیمت استفاده میکنیم که حول صفر نوسان دارد

In [789...

```
ts = (dis['Close'].squeeze()).dropna()
returns0 = 100 * ts.pct_change().dropna()
returns = returns0 - returns0.mean()
returns.mean()
```

Out[789...

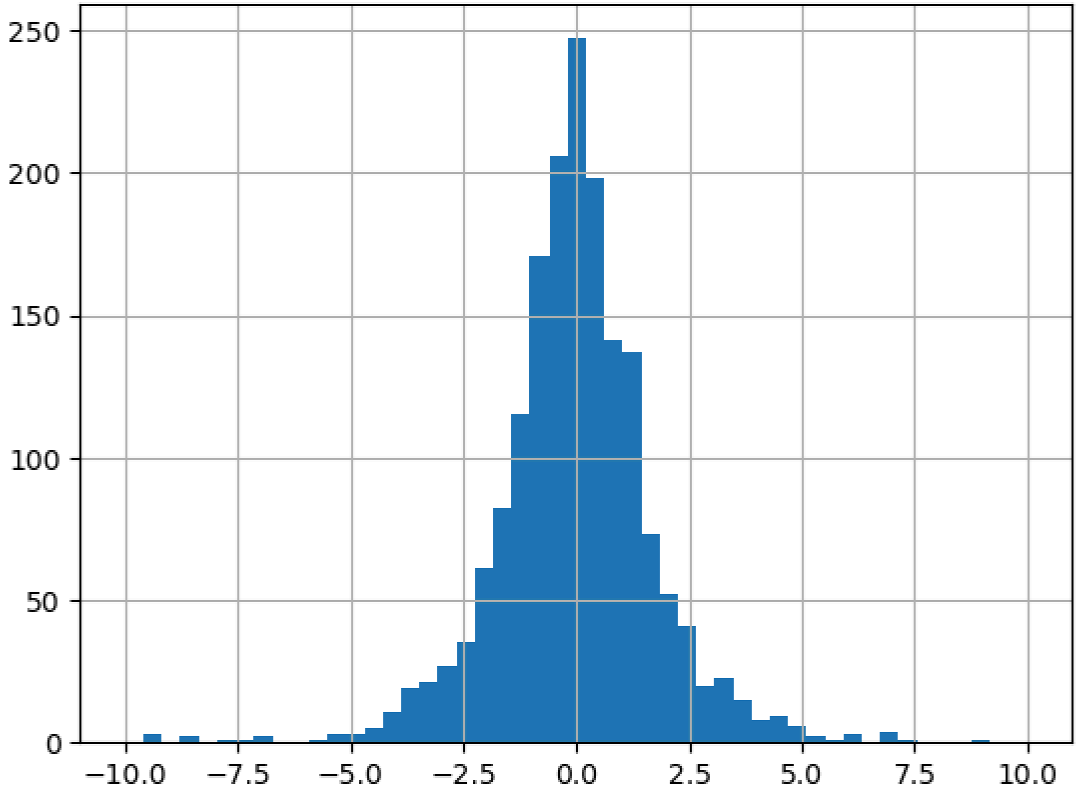
```
8.078939576578874e-18
```

In [790...

```
returns.hist(bins = np.linspace(-10, 10, 50))
```

Out[790...

```
<Axes: >
```



In [791…

```
print(returns)
```

```
Date
2017-10-03    0.912105
2017-10-04   -0.257324
2017-10-05   -0.456786
2017-10-06   -0.059166
2017-10-09   -0.518831
                ...
2024-09-24    0.755251
2024-09-25    0.226289
2024-09-26    1.343665
2024-09-27    0.831644
2024-09-30    0.168285
Name: DIS, Length: 1759, dtype: float64
```

در این مرحله برای اطمینان از ایستایی سری زمانی از آزمایش دیکی فولر افزوده استفاده میکنیم

In [792…
```
def perform_adf_test(series):
    result = adfuller(series)
    print('ADF Statistic: %f' % result[0])
    print('p-value: %f' % result[1])
```

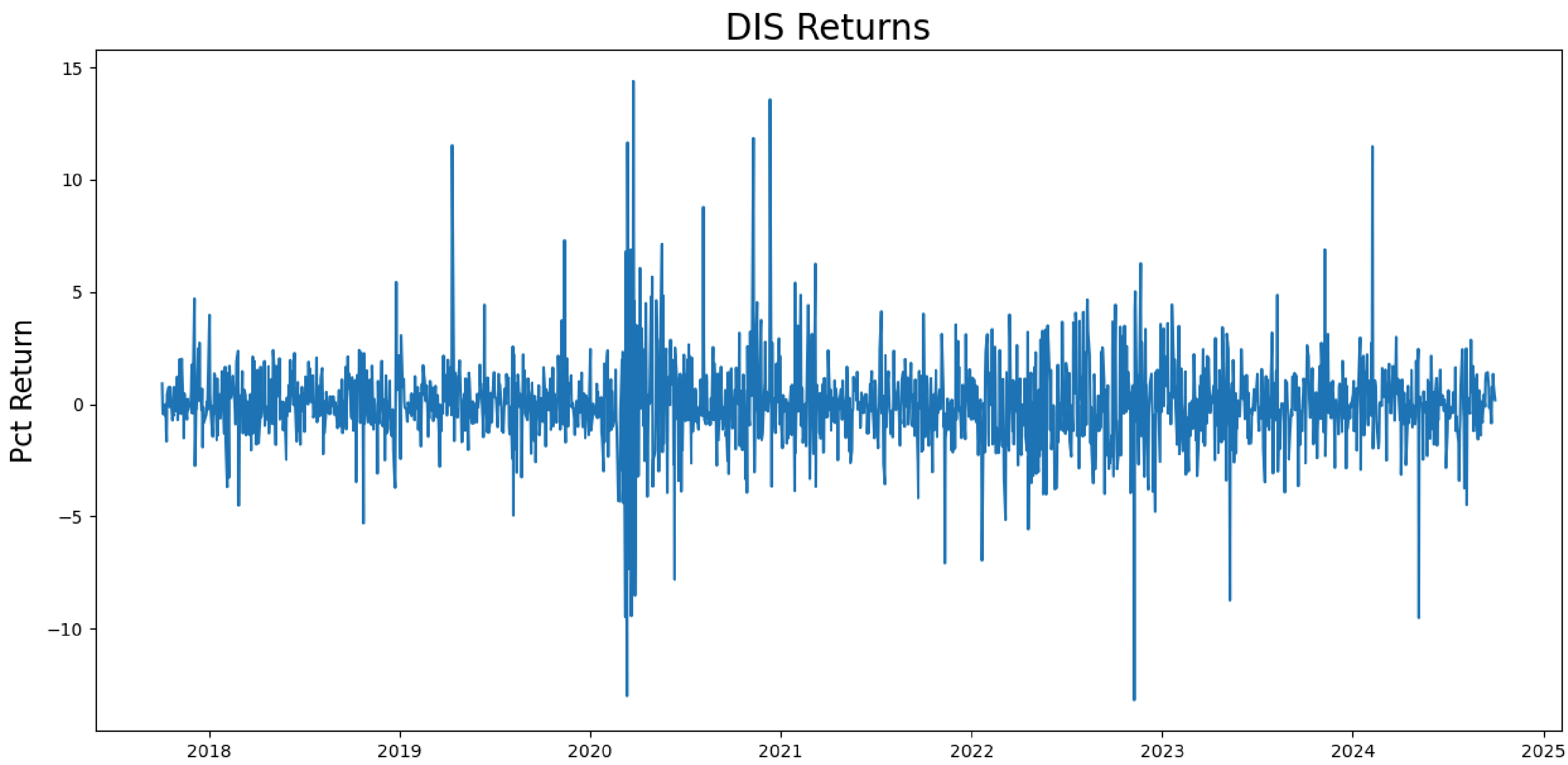
In [793…
```
perform_adf_test(returns)
```

```
ADF Statistic: -13.542531
p-value: 0.000000
```

نمودار سهام دیزنی را رسم میکنیم

In [794…
```
plt.figure(figsize=(15,7))
plt.plot(returns)
plt.ylabel('Pct Return', fontsize=16)
plt.title('DIS Returns', fontsize=20)
```

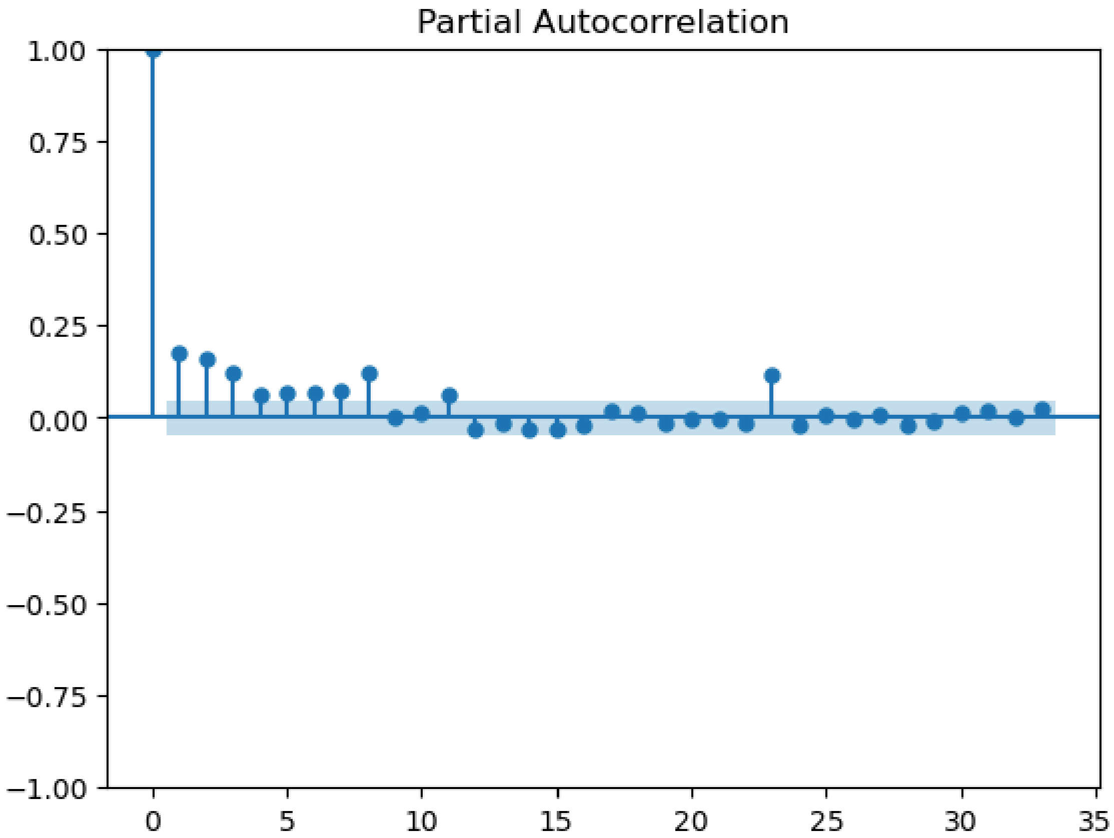
Out[794…
```
Text(0.5, 1.0, 'DIS Returns')
```



با توجه به نوسانات ناهمگون در سال های 2020 و 2023 تا 2025 مدل گارچ را برای این سهم پیشنهاد میکنیم و برای بدست آوردن مقدار پارامترها از نمودار ضریب همبستگی جزئی کمک میگیریم

In [795…

```
plot_pacf(returns**2)
plt.show()
```



In [796…
```
model = arch_model(returns, p=3, q=3)
```

با توجه به نمودار بالا سه لگ اول در قیمت سهام نقش دارند، وابستگی تا سه روز قبل، حال مدل را با پارامترهای بدست آمده برازش میکنیم

In [797…
```
model_fit = model.fit()
```

```
Iteration:      1,   Func. Count:     10,   Neg. LLF: 7826.857287344332
Iteration:      2,   Func. Count:     21,   Neg. LLF: 1111743.3659645568
Iteration:      3,   Func. Count:     32,   Neg. LLF: 3842.05260026816
Iteration:      4,   Func. Count:     42,   Neg. LLF: 3539.731315724344
Iteration:      5,   Func. Count:     52,   Neg. LLF: 3540.0099410426315
Iteration:      6,   Func. Count:     62,   Neg. LLF: 3749.63894115862
Iteration:      7,   Func. Count:     73,   Neg. LLF: 3516.347727983085
Iteration:      8,   Func. Count:     82,   Neg. LLF: 3572.878698161722
Iteration:      9,   Func. Count:     92,   Neg. LLF: 3537.272957330095
Iteration:     10,   Func. Count:    102,   Neg. LLF: 3515.5572707969654
Iteration:     11,   Func. Count:    111,   Neg. LLF: 3515.93039199154
Iteration:     12,   Func. Count:    121,   Neg. LLF: 3515.815931815696
Iteration:     13,   Func. Count:    131,   Neg. LLF: 3515.516675750767
Iteration:     14,   Func. Count:    140,   Neg. LLF: 3515.8586650768752
Iteration:     15,   Func. Count:    151,   Neg. LLF: 3515.5147013062538
Iteration:     16,   Func. Count:    160,   Neg. LLF: 3515.514456726759
Iteration:     17,   Func. Count:    169,   Neg. LLF: 3515.514415389454
Iteration:     18,   Func. Count:    178,   Neg. LLF: 3515.51441448191
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3515.51441448191
            Iterations: 18
            Function evaluations: 178
            Gradient evaluations: 18
```

با توجه به پی مقدارهای بدست آمده برای آلفا یک تا سه و بتا یک تا سه از ضرایب دو و دو برای برازش استفاده میکنیم

In [798…

```
model_fit.summary()
```

Out[798…

Constant Mean - GARCH Model Results

Dep. Variable:	DIS	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-3515.51
Distribution:	Normal	**AIC:**	7047.03
Method:	Maximum Likelihood	**BIC:**	7090.81
		No. Observations:	1759
Date:	Sun, Feb 02 2025	**Df Residuals:**	1758
Time:	19:52:36	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	-0.0105	4.193e-02	-0.251	0.801	[-9.272e-02,7.164e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.3436	0.294	1.170	0.242	[-0.232, 0.919]
alpha[1]	0.0870	7.331e-02	1.187	0.235	[-5.667e-02, 0.231]
alpha[2]	0.0631	0.103	0.611	0.542	[-0.139, 0.266]
alpha[3]	4.9718e-04	3.969e-02	1.253e-02	0.990	[-7.730e-02,7.829e-02]
beta[1]	0.0515	0.143	0.361	0.718	[-0.228, 0.331]
beta[2]	1.6468e-11	0.156	1.058e-10	1.000	[-0.305, 0.305]
beta[3]	0.7066	9.502e-02	7.437	1.033e-13	[0.520, 0.893]

Covariance estimator: robust

In [799…

```
model = arch_model(returns, p=2, q=2)
```

In [800…

```
model_fit = model.fit()
```

```
Iteration:      1,   Func. Count:      8,   Neg. LLF: 10611.395343505483
Iteration:      2,   Func. Count:     18,   Neg. LLF: 1578373.5364840839
Iteration:      3,   Func. Count:     27,   Neg. LLF: 3611.90588808058
Iteration:      4,   Func. Count:     35,   Neg. LLF: 3571.783540890254
Iteration:      5,   Func. Count:     43,   Neg. LLF: 3544.263632218609
Iteration:      6,   Func. Count:     51,   Neg. LLF: 4979.674869965363
Iteration:      7,   Func. Count:     59,   Neg. LLF: 3560.387240937158
Iteration:      8,   Func. Count:     67,   Neg. LLF: 3536.2079143773117
Iteration:      9,   Func. Count:     75,   Neg. LLF: 3519.265925109554
Iteration:     10,   Func. Count:     83,   Neg. LLF: 3518.4379569613543
Iteration:     11,   Func. Count:     90,   Neg. LLF: 3518.3207334817125
Iteration:     12,   Func. Count:     97,   Neg. LLF: 3518.312972805782
Iteration:     13,   Func. Count:    104,   Neg. LLF: 3518.3083657549023
Iteration:     14,   Func. Count:    111,   Neg. LLF: 3518.3060667601503
Iteration:     15,   Func. Count:    118,   Neg. LLF: 3518.3060225703653
Iteration:     16,   Func. Count:    125,   Neg. LLF: 3518.306021427302
Iteration:     17,   Func. Count:    131,   Neg. LLF: 3518.306021425787
Optimization terminated successfully    (Exit mode 0)
            Current function value: 3518.306021427302
            Iterations: 17
            Function evaluations: 131
            Gradient evaluations: 17
```

In [801…

```
model_fit.summary()
```

Out[801…

Constant Mean - GARCH Model Results

Dep. Variable:	DIS	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-3518.31
Distribution:	Normal	**AIC:**	7048.61
Method:	Maximum Likelihood	**BIC:**	7081.45
		No. Observations:	1759
Date:	Sun, Feb 02 2025	**Df Residuals:**	1758
Time:	19:52:36	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	-0.0150	0.133	-0.113	0.910	[-0.275, 0.246]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.1697	2.802	6.056e-02	0.952	[-5.323, 5.662]
alpha[1]	0.0851	0.525	0.162	0.871	[-0.943, 1.113]
alpha[2]	0.0000	1.958	0.000	1.000	[-3.838, 3.838]
beta[1]	0.4669	19.264	2.424e-02	0.981	[-37.290, 38.224]
beta[2]	0.4039	17.124	2.359e-02	0.981	[-33.159, 33.967]

Covariance estimator: robust

حال برای پیش بینی دقیقتر مدل از تکنیک رولینگ استفاده میکنیم

In [802…

```
rolling_predictions = []
test_size = 365

for i in range(test_size):
    train = returns[:-(test_size-i)]
    model = arch_model(train, p=2, q=2)
    model_fit = model.fit(disp='off')
    pred = model_fit.forecast(horizon=1)
    rolling_predictions.append(np.sqrt(pred.variance.values[-1,:][0]))
```

In [803…

```
rolling_predictions = pd.Series(rolling_predictions, index=returns.index[-365:])
```

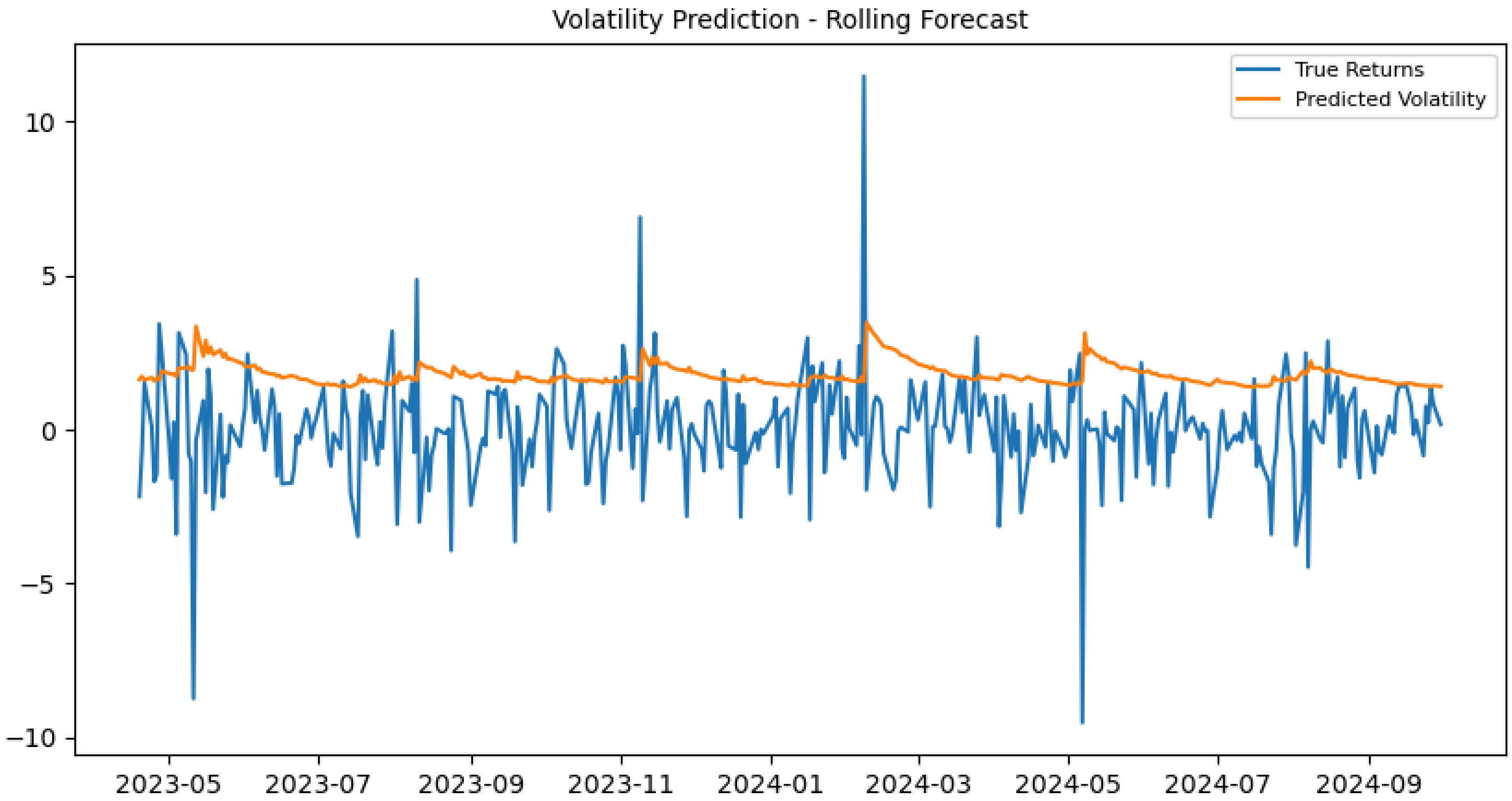
مدل آموزش یافته بازده و مقدار اصلی را رسم میکیم

In [804…

```
plt.figure(figsize=(10,5))
true, = plt.plot(returns[-365:])
preds, = plt.plot(rolling_predictions)
plt.title('Volatility Prediction - Rolling Forecast', fontsize=10)
plt.legend(['True Returns', 'Predicted Volatility'], fontsize=8)
```

Out[804…

```
<matplotlib.legend.Legend at 0x212c75ec610>
```



حال برای هفت روز آینده بازده سهم را پیش بینی میکنیم

In [805…

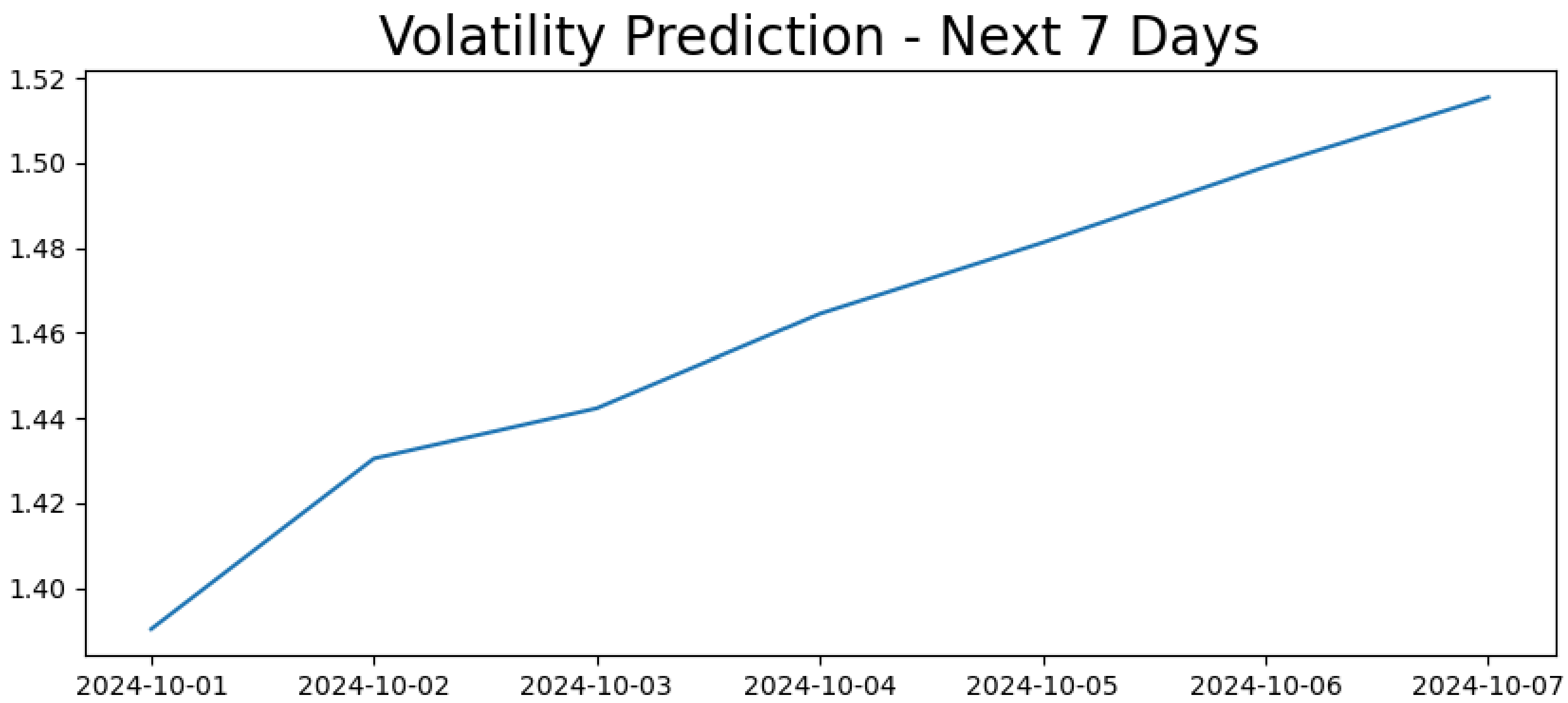
```
pred = model_fit.forecast(horizon=7)
future_dates = [returns.index[-1] + timedelta(days=i) for i in range(1,8)]
pred = pd.Series(np.sqrt(pred.variance.values[-1,:]), index=future_dates)
```

In [806…

```
plt.figure(figsize=(10,4))
plt.plot(pred)
plt.title('Volatility Prediction - Next 7 Days', fontsize=20)
```

Out[806…

```
Text(0.5, 1.0, 'Volatility Prediction - Next 7 Days')
```

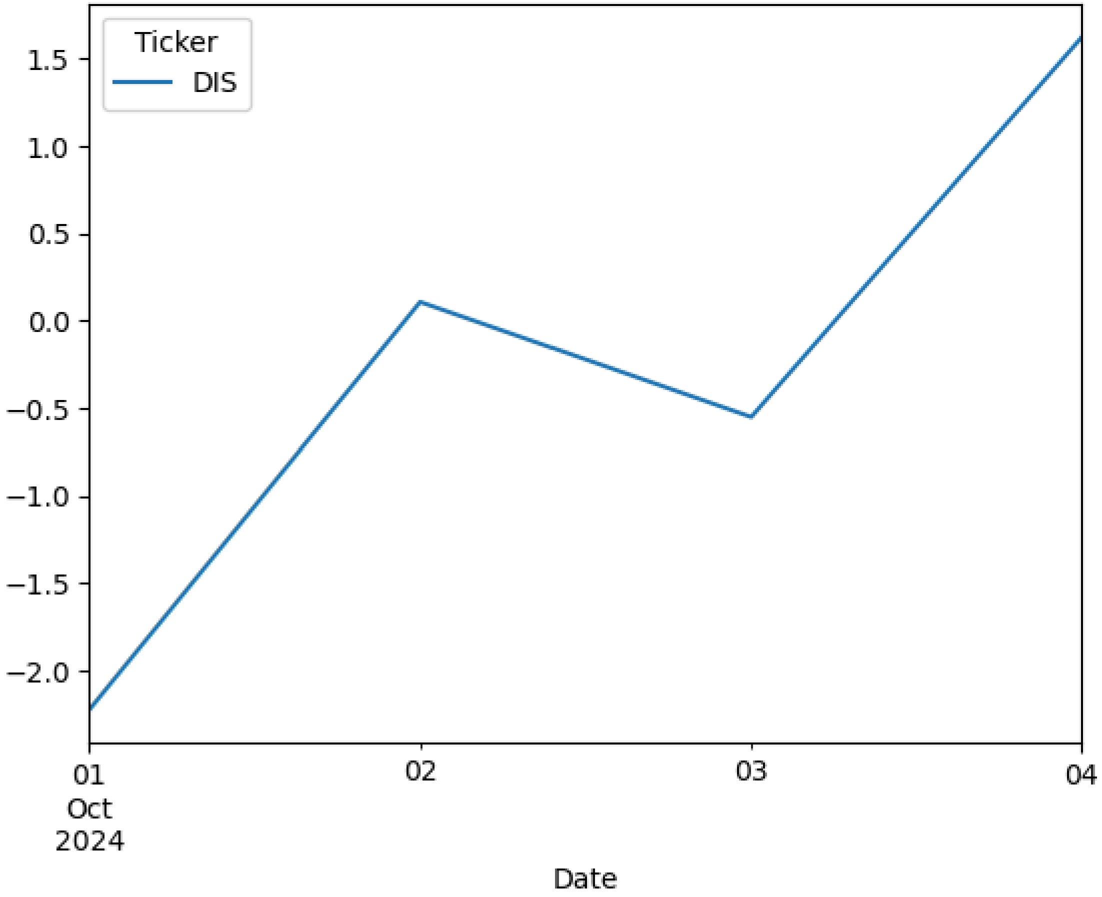


مقدار پیش بینی شده را با مقدار بازده اصلی مقایسه میکنیم

In [807…
```
dis = yf.download('DIS', start = '2024-9-29', end = '2024-10-7')
returns0 = 100 * dis.Close.pct_change().dropna()
returns0.plot()
```

```
[*********************100%***********************]  1 of 1 completed
```

Out[807…
```
<Axes: xlabel='Date'>
```



تمام مراحل بالا را برای سهام اس اند پی 500 انجام میدهیم

In [808…
```
start = datetime(2017, 10, 1)
end = datetime(2024, 10, 1)
```

In [809…
```
spy = yf.download('SPY', start = '2017-10-1', end = '2024-10-1')
spy
```

```
[*********************100%***********************]  1 of 1 completed
```

Out[809…

Price	Close	High	Low	Open	Volume
Ticker	SPY	SPY	SPY	SPY	SPY
Date					
2017-10-02	224.159256	224.159256	223.244199	223.421888	59023000
2017-10-03	224.638992	224.665643	224.079301	224.159266	66810200
2017-10-04	224.905548	225.154297	224.372507	224.488002	55953600
2017-10-05	226.238113	226.255871	224.941054	225.243104	63522800
2017-10-06	225.980484	226.273655	225.518529	225.785036	80646000
...	...	...	...	...	...
2024-09-24	569.383606	569.443402	565.696005	568.566349	46805700
2024-09-25	568.127869	569.971700	567.001654	569.224215	38428600
2024-09-26	570.380310	572.782260	567.988397	572.453350	48336000
2024-09-27	569.553040	572.293815	568.506574	571.466643	42100900
2024-09-30	571.835388	572.453304	566.174448	568.506565	63557400

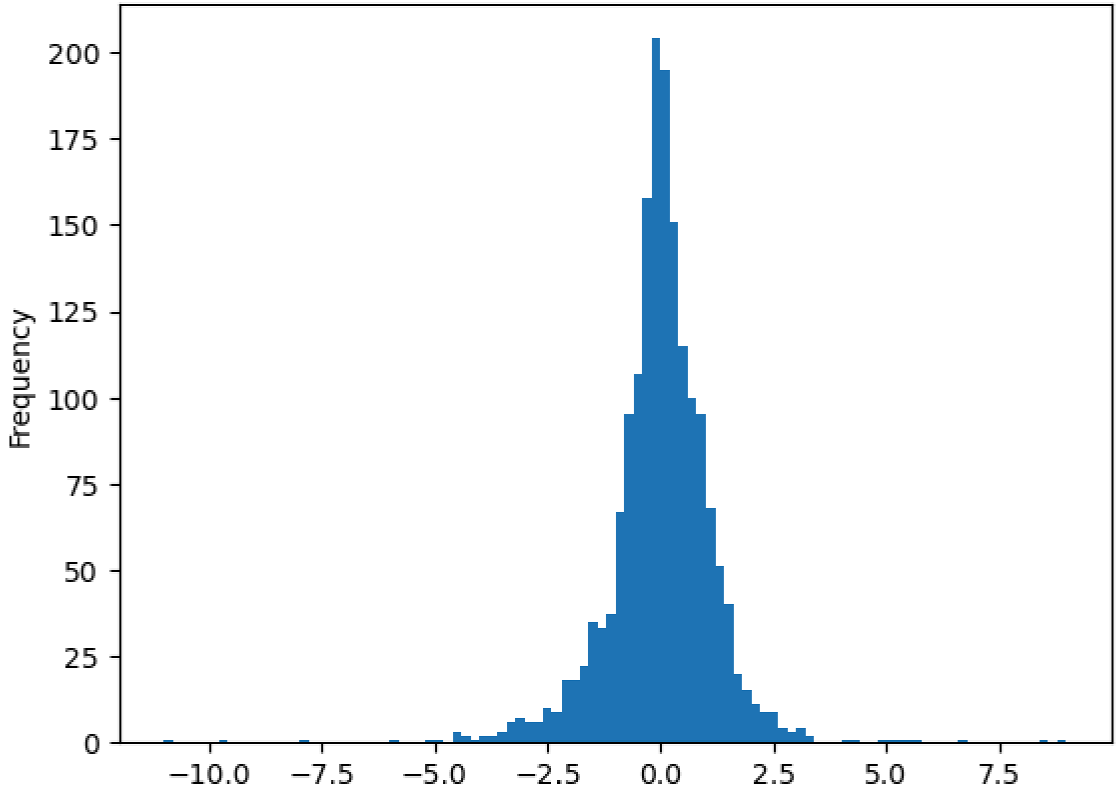
1760 rows × 5 columns

In [810…

```
ts1 = (spy['Close'].squeeze()).dropna()
print(type(ts1))
return2 = 100 * ts1.pct_change().dropna()
returns3 = return2 - return2.mean()
print(returns3.mean())
returns3.plot(kind='hist', bins = 100)
```

```
<class 'pandas.core.series.Series'>
1.9187481494374506e-17
```

Out[810… <Axes: ylabel='Frequency'>



In [811…

```
perform_adf_test(returns3)
```

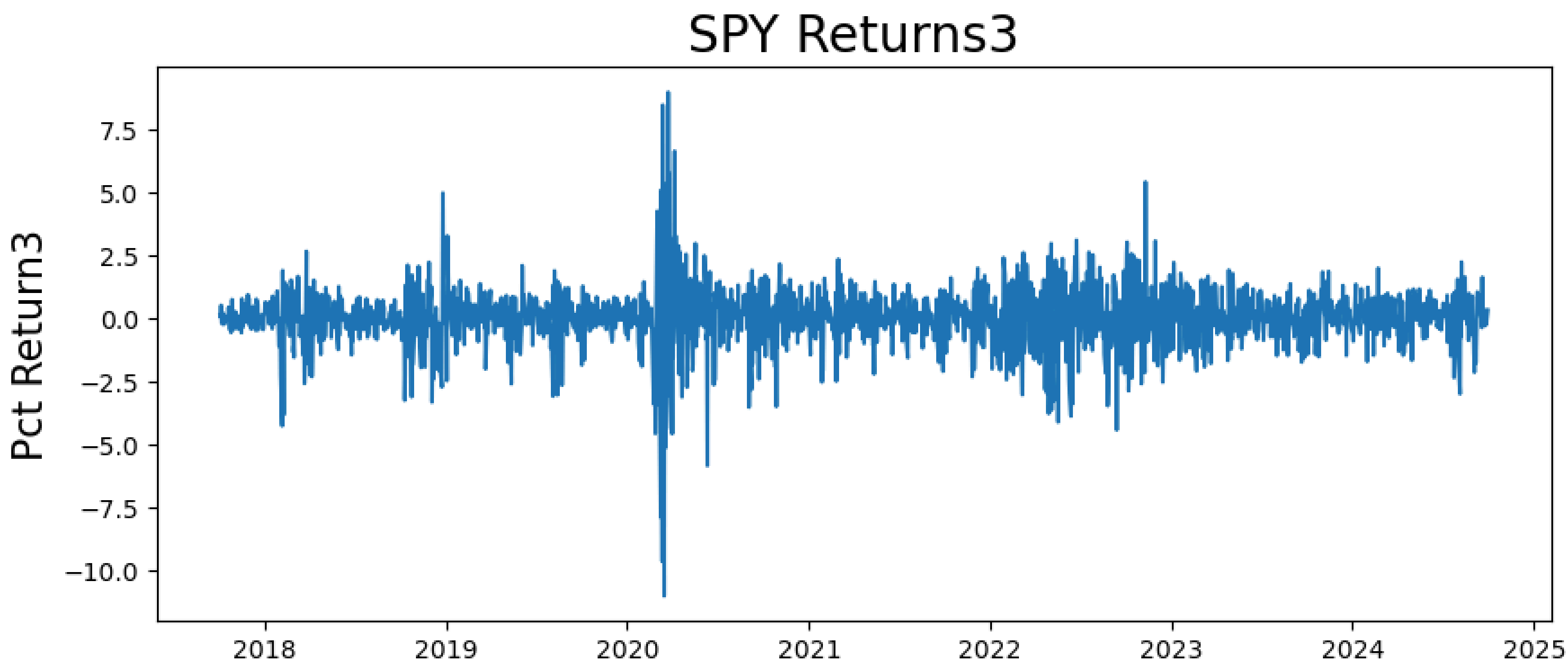
```
ADF Statistic: -12.833365
p-value: 0.000000
```

In [812…

```
plt.figure(figsize=(10,4))
plt.plot(returns3)
plt.ylabel('Pct Return3', fontsize=16)
plt.title('SPY Returns3', fontsize=20)
```

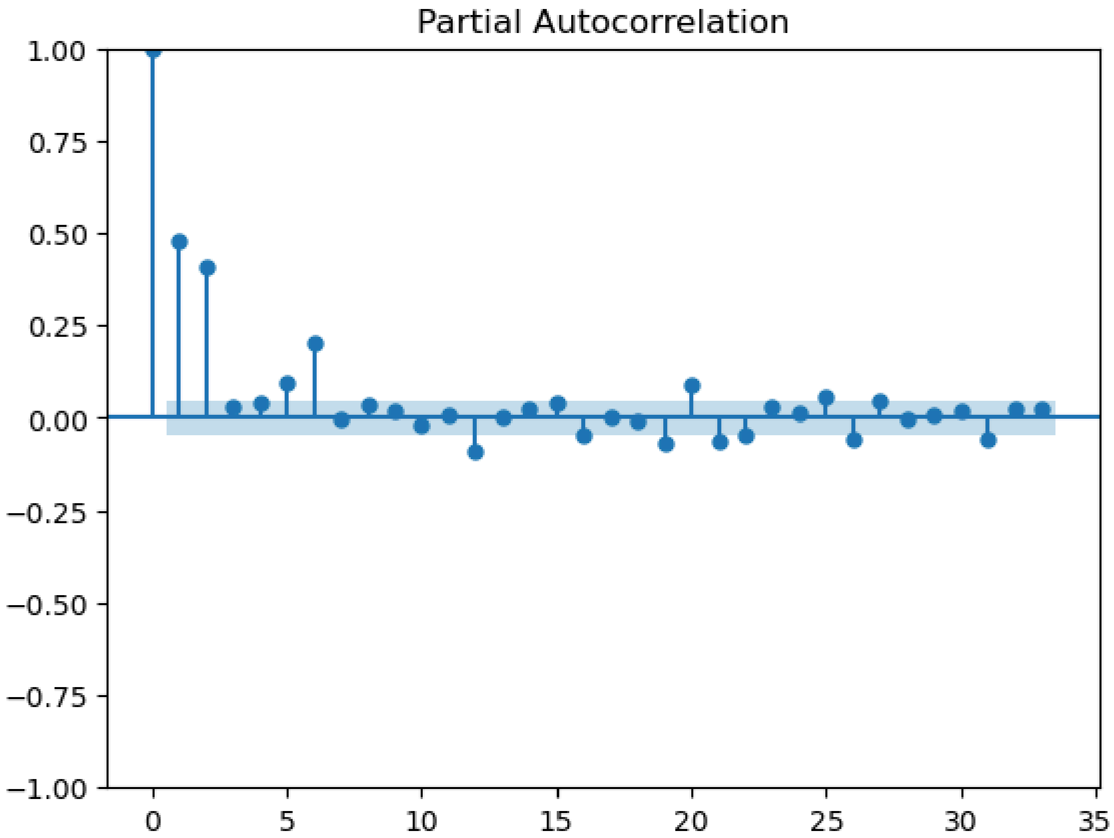
Out[812…

```
Text(0.5, 1.0, 'SPY Returns3')
```



In [813…

```
plot_pacf(returns3**2)
plt.show()
```



In [814…
```
model = arch_model(returns3, p=2, q=2)
```

In [815…
```
model_fit = model.fit()
```

```
Iteration:      1,   Func. Count:      8,   Neg. LLF: 11467.814133056016
Iteration:      2,   Func. Count:     20,   Neg. LLF: 679766.1015725434
Iteration:      3,   Func. Count:     29,   Neg. LLF: 3103.087367223201
Iteration:      4,   Func. Count:     38,   Neg. LLF: 4055.862504453258
Iteration:      5,   Func. Count:     47,   Neg. LLF: 2835.462147392988
Iteration:      6,   Func. Count:     56,   Neg. LLF: 2395.1642958085195
Iteration:      7,   Func. Count:     65,   Neg. LLF: 2385.313492015127
Iteration:      8,   Func. Count:     73,   Neg. LLF: 2385.2035814413885
Iteration:      9,   Func. Count:     81,   Neg. LLF: 2385.220764470892
Iteration:     10,   Func. Count:     89,   Neg. LLF: 2385.0434530316393
Iteration:     11,   Func. Count:     96,   Neg. LLF: 2385.779481492519
Iteration:     12,   Func. Count:    104,   Neg. LLF: 2385.0127286570605
Iteration:     13,   Func. Count:    111,   Neg. LLF: 2385.0113908440817
Iteration:     14,   Func. Count:    118,   Neg. LLF: 2385.0112590150397
Iteration:     15,   Func. Count:    125,   Neg. LLF: 2385.0112519424183
Iteration:     16,   Func. Count:    132,   Neg. LLF: 2385.011251190761
Optimization terminated successfully    (Exit mode 0)
            Current function value: 2385.011251190761
            Iterations: 16
            Function evaluations: 132
            Gradient evaluations: 16
```

In [816…

```
model_fit.summary()
```

Out[816…

Constant Mean - GARCH Model Results

Dep. Variable:	SPY	**R-squared:**	0.000
Mean Model:	Constant Mean	**Adj. R-squared:**	0.000
Vol Model:	GARCH	**Log-Likelihood:**	-2385.01
Distribution:	Normal	**AIC:**	4782.02
Method:	Maximum Likelihood	**BIC:**	4814.86
		No. Observations:	1759
Date:	Sun, Feb 02 2025	**Df Residuals:**	1758
Time:	19:52:49	**Df Model:**	1

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
mu	0.0469	1.872e-02	2.508	1.215e-02	[1.026e-02,8.363e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	0.0605	1.951e-02	3.101	1.931e-03	[2.225e-02,9.872e-02]
alpha[1]	0.1705	4.070e-02	4.191	2.782e-05	[9.078e-02, 0.250]
alpha[2]	0.1603	4.378e-02	3.661	2.516e-04	[7.446e-02, 0.246]
beta[1]	0.0401	0.127	0.316	0.752	[-0.208, 0.288]
beta[2]	0.5990	0.104	5.767	8.080e-09	[0.395, 0.803]

Covariance estimator: robust

In [817…

```
rolling_predictions = []
test_size = 365

for i in range(test_size):
    train = returns3[:-(test_size-i)]
    model = arch_model(train, p=2, q=2)
    model_fit = model.fit(disp='off')
    pred = model_fit.forecast(horizon=1)
    rolling_predictions.append(np.sqrt(pred.variance.values[-1,:][0]))
```

In [818…

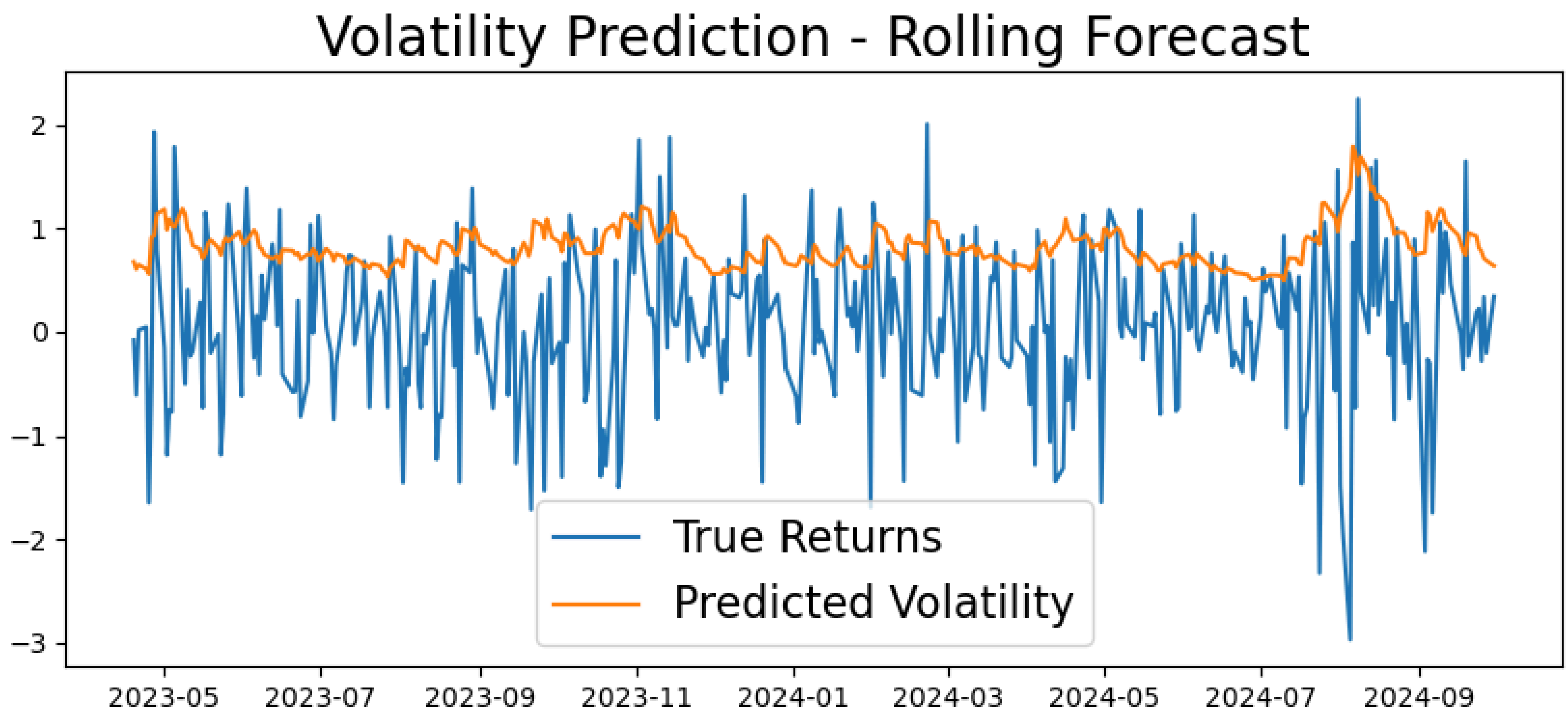
```
rolling_predictions = pd.Series(rolling_predictions, index=returns3.index[-365:])
```

In [819…

```
plt.figure(figsize=(10,4))
true, = plt.plot(returns3[-365:])
preds, = plt.plot(rolling_predictions)
plt.title('Volatility Prediction - Rolling Forecast', fontsize=20)
plt.legend(['True Returns', 'Predicted Volatility'], fontsize=16)
```

Out[819…

```
<matplotlib.legend.Legend at 0x212c638b580>
```



In [820…

```
train = returns3
model = arch_model(train, p=2, q=2)

model_fit = model.fit(disp='off')
```

In [821…

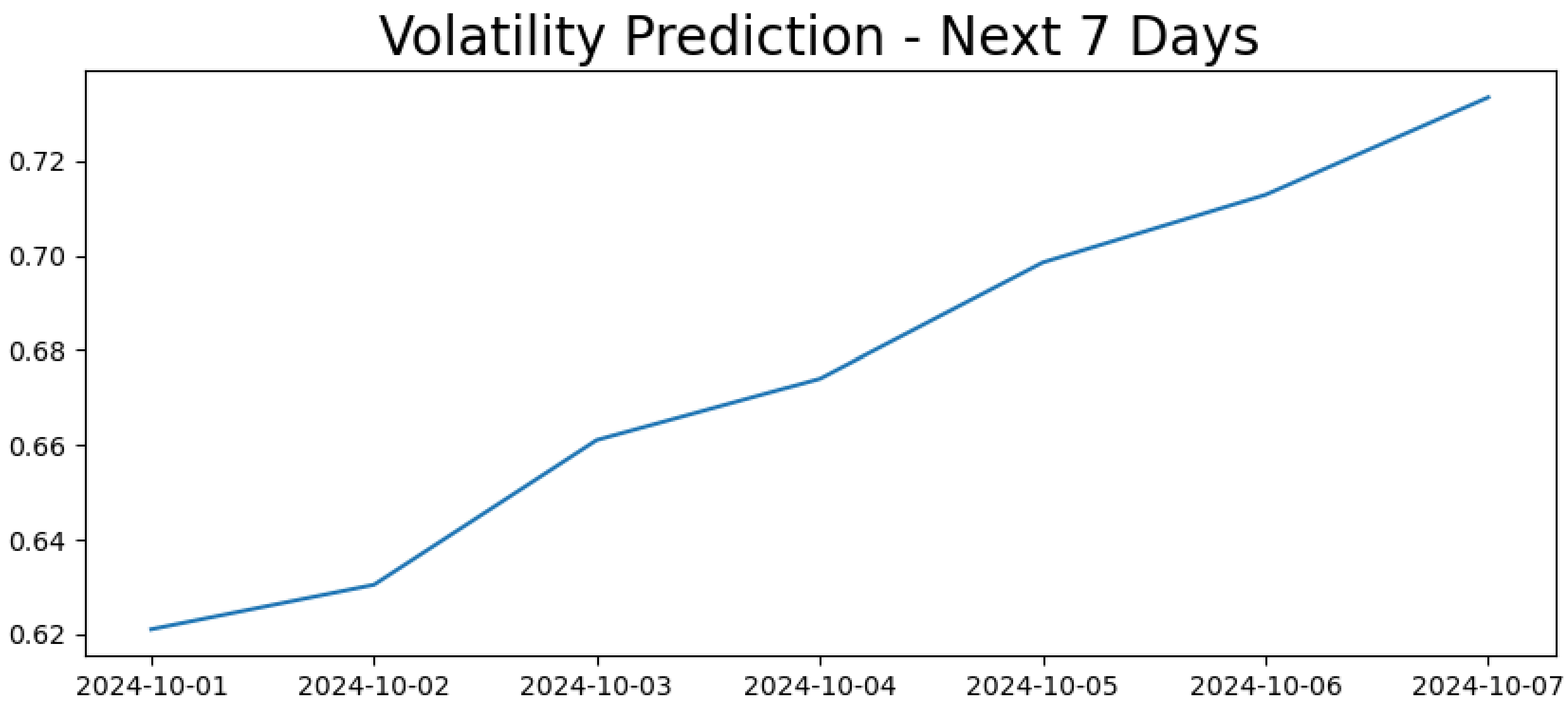
```
pred = model_fit.forecast(horizon=7)
future_dates = [returns3.index[-1] + timedelta(days=i) for i in range(1,8)]
pred = pd.Series(np.sqrt(pred.variance.values[-1,:]), index=future_dates)
```

In [822…

```
plt.figure(figsize=(10,4))
plt.plot(pred)
plt.title('Volatility Prediction - Next 7 Days', fontsize=20)
```

Out[822…

```
Text(0.5, 1.0, 'Volatility Prediction - Next 7 Days')
```



In [823…

```
spy1 = yf.download('SPY', start = '2024-9-29', end = '2024-10-07')
```

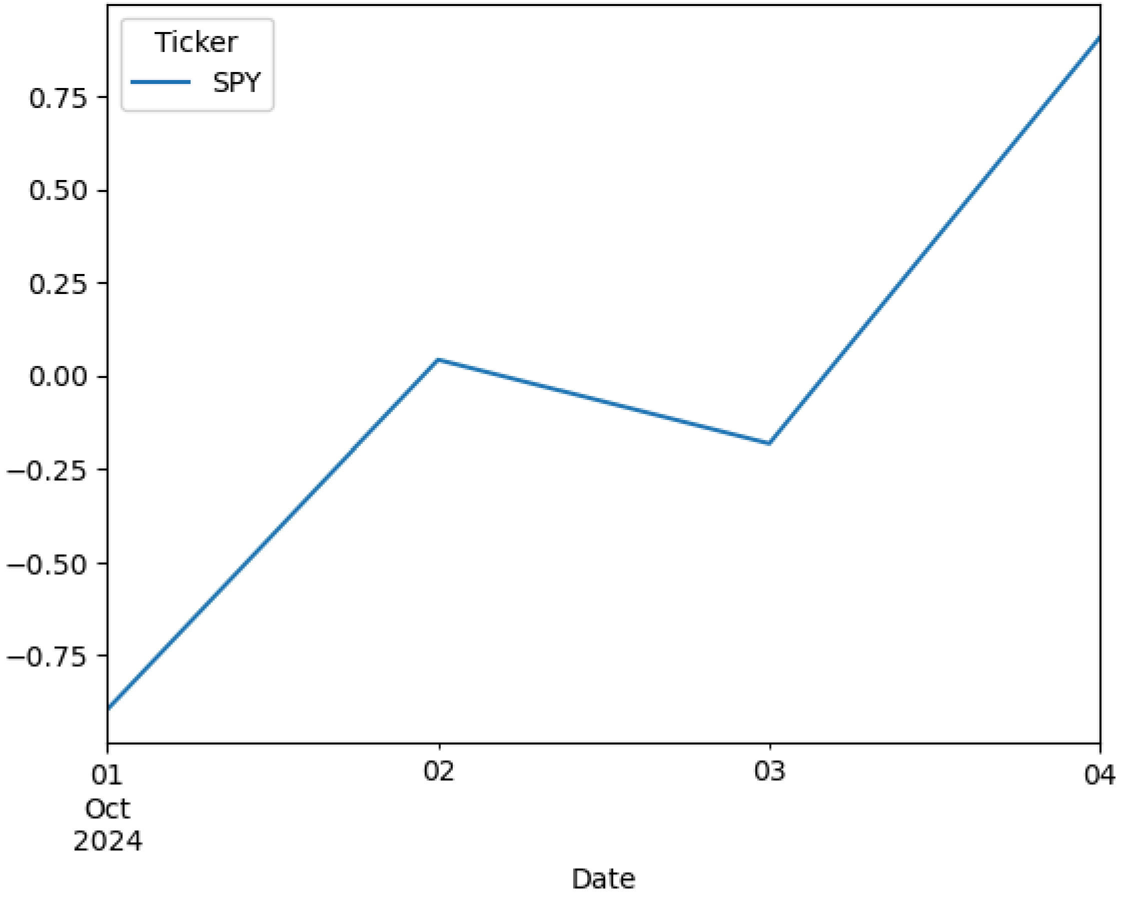
```
[*********************100%***********************]  1 of 1 completed
```

In [824…

```
returns2 = 100 * spy1.Close.pct_change().dropna()
returns2.plot()
```

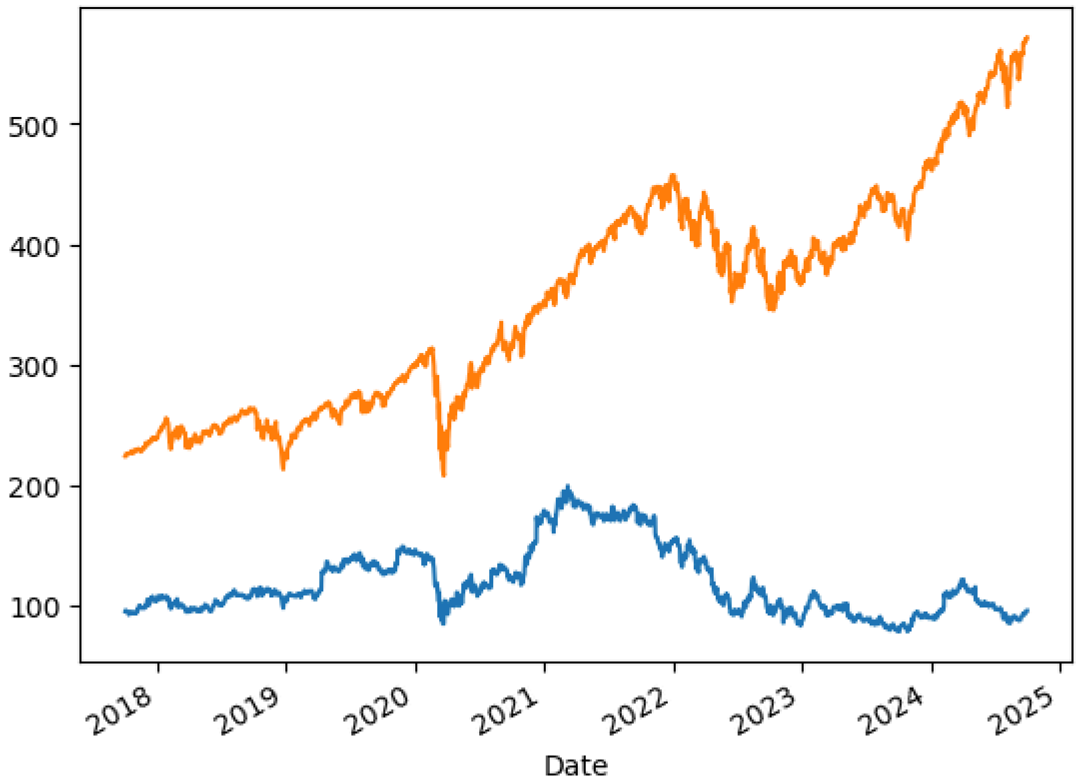
Out[824…

```
<Axes: xlabel='Date'>
```



In [825…
```
ts.plot()
ts1.plot()
```

Out[825…
```
<Axes: xlabel='Date'>
```



از سال 2021 این دو سهم یکی صعودی و دیگری نزولی شد، در دو حالت به بررسی آن میپردازیم، یکی تا سال 2021 و دیگری از سال 2021

In [826…

```
spy0 = yf.download('SPY', start = '2021-12-29', end = '2024-10-07')
dis0 = yf.download('DIS', start = '2021-12-29', end = '2024-10-07')

ts_0 = (dis0['Close'].squeeze()).dropna()
returns_1 = 100 * ts_0.pct_change().dropna()
return_0 = returns_1 - returns_1.mean()

ts_1 = (spy0['Close'].squeeze()).dropna()
returns_2 = 100 * ts_1.pct_change().dropna()
returns_3 = returns_2 - returns_2.mean()
returns_3.mean()
```

```
df_difference.index = pd.DatetimeIndex(df_difference.index).to_period('M')
df_difference= pd.concat([return_0, returns_3], axis=1)
df_difference.replace([np.inf, -np.inf], np.NaN, inplace=True)
df_difference.dropna(axis=0)
df_difference
```

```
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
```

Out[826…

	DIS	SPY
Date		
2021-12-30	0.734538	-0.314835
2021-12-31	-0.616851	-0.290408
2022-01-03	1.257398	0.540608
2022-01-04	-0.606954	-0.071873
2022-01-05	-0.296661	-1.958595
...	...	...
2024-09-30	0.237579	0.362342
2024-10-01	-2.174661	-0.934226
2024-10-02	0.156421	0.003824
2024-10-03	-0.502214	-0.221204
2024-10-04	1.673513	0.870345

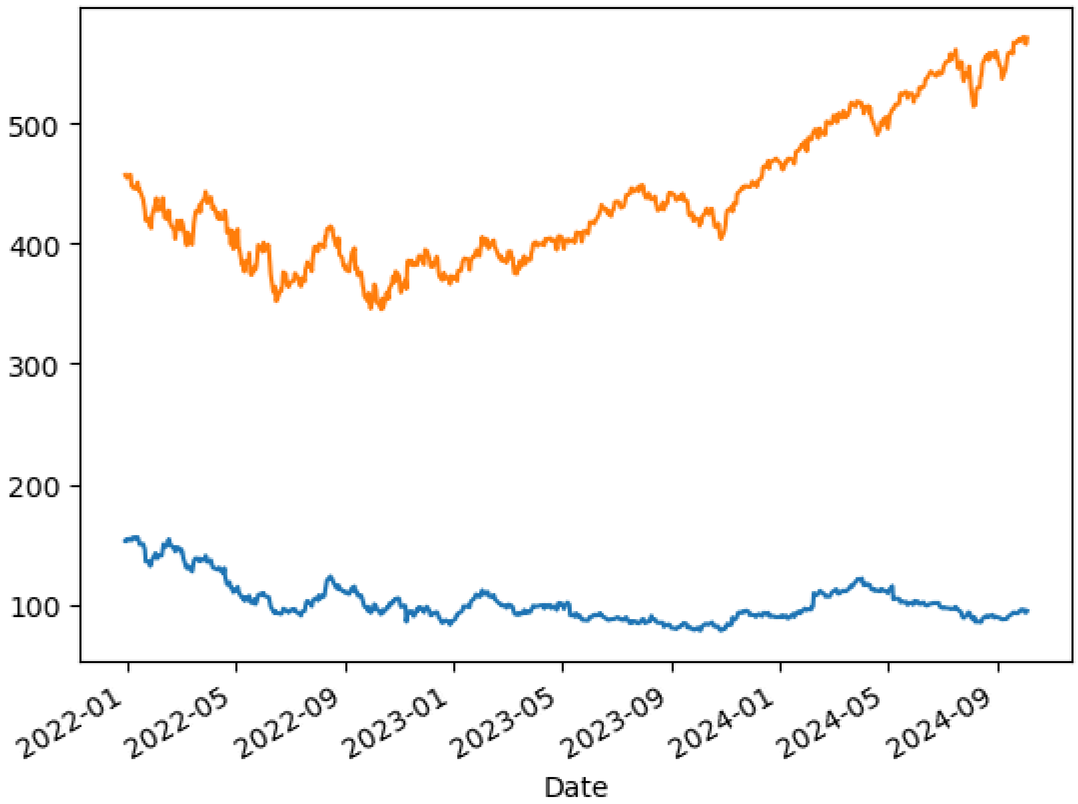
695 rows × 2 columns

In [827…

```
ts_0.plot()
ts_1.plot()
```

Out[827…

```
<Axes: xlabel='Date'>
```



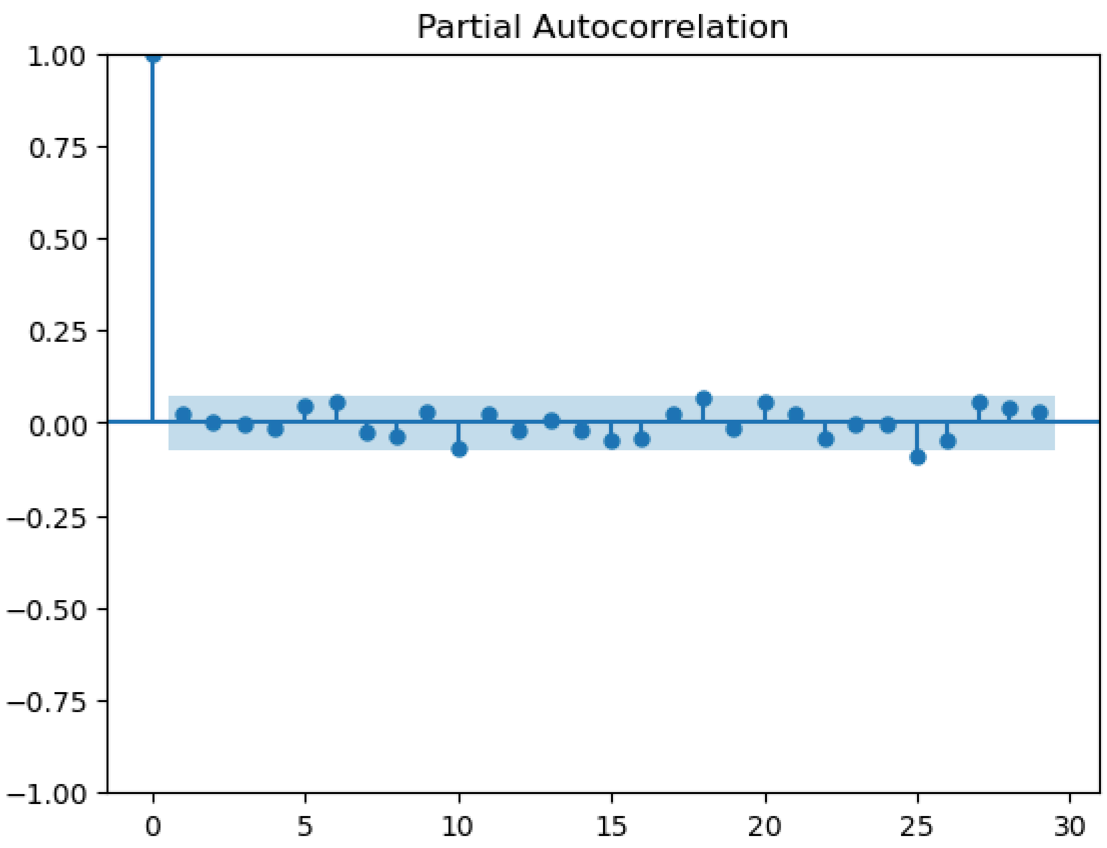
In [828…
```
test_obs = 12
train = df_difference[:-test_obs]
test = df_difference[-test_obs:]
```

In [829…
```
plt.figure(figsize=(12,6))
spy_df, = plt.plot(df_difference['SPY'])
dis_df, = plt.plot(df_difference['DIS'], color='red')

for year in range(2022, 2024):
    plt.axvline(datetime(year,1,1), linestyle='--', color='k', alpha=0.5)

plt.legend(['SPY', 'DIS'], fontsize=16)
```

Out[829…
```
<matplotlib.legend.Legend at 0x212c30992e0>
```

In [830…

```
df_difference0 = (df_difference['DIS'].squeeze()).dropna()
plot_pacf(df_difference0)
plt.show()
```



هم برای DIS و هم برای SPY
به شکل روبرو درآمد اتکا مدل AR
برای این سهام مناسب نیست. (البته در
این بازه زمانی)

اما با توجه به تست علیت گرانجر به همدیگر
وابستگی بالائی دارند.

In [831…

```
from statsmodels.tsa.stattools import grangercausalitytests
gc_res0 = grangercausalitytests(df_difference, 30)
```

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=7.7050  , p=0.0057  , df_denom=691, df_num=1
ssr based chi2 test:   chi2=7.7385  , p=0.0054  , df=1
likelihood ratio test: chi2=7.6957  , p=0.0055  , df=1
parameter F test:         F=7.7050  , p=0.0057  , df_denom=691, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=3.8364  , p=0.0220  , df_denom=688, df_num=2
ssr based chi2 test:   chi2=7.7285  , p=0.0210  , df=2
likelihood ratio test: chi2=7.6857  , p=0.0214  , df=2
parameter F test:         F=3.8364  , p=0.0220  , df_denom=688, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=3.0212  , p=0.0292  , df_denom=685, df_num=3
ssr based chi2 test:   chi2=9.1561  , p=0.0273  , df=3
likelihood ratio test: chi2=9.0961  , p=0.0280  , df=3
parameter F test:         F=3.0212  , p=0.0292  , df_denom=685, df_num=3

Granger Causality
number of lags (no zero) 4
ssr based F test:         F=2.4215  , p=0.0471  , df_denom=682, df_num=4
ssr based chi2 test:   chi2=9.8139  , p=0.0437  , df=4
likelihood ratio test: chi2=9.7449  , p=0.0450  , df=4
parameter F test:         F=2.4215  , p=0.0471  , df_denom=682, df_num=4

Granger Causality
number of lags (no zero) 5
ssr based F test:         F=2.1218  , p=0.0611  , df_denom=679, df_num=5
ssr based chi2 test:   chi2=10.7807 , p=0.0559  , df=5
likelihood ratio test: chi2=10.6973 , p=0.0577  , df=5
parameter F test:         F=2.1218  , p=0.0611  , df_denom=679, df_num=5

Granger Causality
number of lags (no zero) 6
ssr based F test:         F=2.0703  , p=0.0547  , df_denom=676, df_num=6
ssr based chi2 test:   chi2=12.6607 , p=0.0488  , df=6
likelihood ratio test: chi2=12.5458 , p=0.0508  , df=6
parameter F test:         F=2.0703  , p=0.0547  , df_denom=676, df_num=6
```

```
Granger Causality
number of lags (no zero) 7
ssr based F test:         F=1.8993  , p=0.0671  , df_denom=673, df_num=7
ssr based chi2 test:   chi2=13.5915 , p=0.0589  , df=7
likelihood ratio test: chi2=13.4590 , p=0.0617  , df=7
parameter F test:         F=1.8993  , p=0.0671  , df_denom=673, df_num=7

Granger Causality
number of lags (no zero) 8
ssr based F test:         F=1.7368  , p=0.0867  , df_denom=670, df_num=8
ssr based chi2 test:   chi2=14.2473 , p=0.0755  , df=8
likelihood ratio test: chi2=14.1016 , p=0.0792  , df=8
parameter F test:         F=1.7368  , p=0.0867  , df_denom=670, df_num=8

Granger Causality
number of lags (no zero) 9
ssr based F test:         F=1.7999  , p=0.0650  , df_denom=667, df_num=9
ssr based chi2 test:   chi2=16.6608 , p=0.0543  , df=9
likelihood ratio test: chi2=16.4617 , p=0.0578  , df=9
parameter F test:         F=1.7999  , p=0.0650  , df_denom=667, df_num=9

Granger Causality
number of lags (no zero) 10
ssr based F test:         F=1.6001  , p=0.1023  , df_denom=664, df_num=10
ssr based chi2 test:   chi2=16.5071 , p=0.0860  , df=10
likelihood ratio test: chi2=16.3114 , p=0.0911  , df=10
parameter F test:         F=1.6001  , p=0.1023  , df_denom=664, df_num=10

Granger Causality
number of lags (no zero) 11
ssr based F test:         F=1.5765  , p=0.1011  , df_denom=661, df_num=11
ssr based chi2 test:   chi2=17.9451 , p=0.0829  , df=11
likelihood ratio test: chi2=17.7137 , p=0.0885  , df=11
parameter F test:         F=1.5765  , p=0.1011  , df_denom=661, df_num=11

Granger Causality
number of lags (no zero) 12
ssr based F test:         F=1.5348  , p=0.1068  , df_denom=658, df_num=12
ssr based chi2 test:   chi2=19.1180 , p=0.0857  , df=12
likelihood ratio test: chi2=18.8553 , p=0.0921  , df=12
```

```
parameter F test:         F=1.5348  , p=0.1068  , df_denom=658, df_num=12

Granger Causality
number of lags (no zero) 13
ssr based F test:         F=1.4257  , p=0.1419  , df_denom=655, df_num=13
ssr based chi2 test:   chi2=19.2984 , p=0.1141  , df=13
likelihood ratio test: chi2=19.0304 , p=0.1222  , df=13
parameter F test:         F=1.4257  , p=0.1419  , df_denom=655, df_num=13

Granger Causality
number of lags (no zero) 14
ssr based F test:         F=1.3084  , p=0.1965  , df_denom=652, df_num=14
ssr based chi2 test:   chi2=19.1317 , p=0.1600  , df=14
likelihood ratio test: chi2=18.8679 , p=0.1701  , df=14
parameter F test:         F=1.3084  , p=0.1965  , df_denom=652, df_num=14

Granger Causality
number of lags (no zero) 15
ssr based F test:         F=1.1853  , p=0.2779  , df_denom=649, df_num=15
ssr based chi2 test:   chi2=18.6291 , p=0.2310  , df=15
likelihood ratio test: chi2=18.3785 , p=0.2433  , df=15
parameter F test:         F=1.1853  , p=0.2779  , df_denom=649, df_num=15

Granger Causality
number of lags (no zero) 16
ssr based F test:         F=1.2197  , p=0.2468  , df_denom=646, df_num=16
ssr based chi2 test:   chi2=20.5128 , p=0.1980  , df=16
likelihood ratio test: chi2=20.2091 , p=0.2110  , df=16
parameter F test:         F=1.2197  , p=0.2468  , df_denom=646, df_num=16

Granger Causality
number of lags (no zero) 17
ssr based F test:         F=1.2113  , p=0.2492  , df_denom=643, df_num=17
ssr based chi2 test:   chi2=21.7138 , p=0.1960  , df=17
likelihood ratio test: chi2=21.3733 , p=0.2100  , df=17
parameter F test:         F=1.2113  , p=0.2492  , df_denom=643, df_num=17

Granger Causality
number of lags (no zero) 18
ssr based F test:         F=1.1325  , p=0.3152  , df_denom=640, df_num=18
ssr based chi2 test:   chi2=21.5642 , p=0.2519  , df=18
```

```
likelihood ratio test: chi2=21.2279 , p=0.2681  , df=18
parameter F test:         F=1.1325  , p=0.3152  , df_denom=640, df_num=18

Granger Causality
number of lags (no zero) 19
ssr based F test:         F=1.0671  , p=0.3812  , df_denom=637, df_num=19
ssr based chi2 test:   chi2=21.5166 , p=0.3090  , df=19
likelihood ratio test: chi2=21.1813 , p=0.3269  , df=19
parameter F test:         F=1.0671  , p=0.3812  , df_denom=637, df_num=19

Granger Causality
number of lags (no zero) 20
ssr based F test:         F=0.9832  , p=0.4808  , df_denom=634, df_num=20
ssr based chi2 test:   chi2=20.9348 , p=0.4010  , df=20
likelihood ratio test: chi2=20.6167 , p=0.4200  , df=20
parameter F test:         F=0.9832  , p=0.4808  , df_denom=634, df_num=20

Granger Causality
number of lags (no zero) 21
ssr based F test:         F=0.9475  , p=0.5286  , df_denom=631, df_num=21
ssr based chi2 test:   chi2=21.2540 , p=0.4435  , df=21
likelihood ratio test: chi2=20.9258 , p=0.4635  , df=21
parameter F test:         F=0.9475  , p=0.5286  , df_denom=631, df_num=21

Granger Causality
number of lags (no zero) 22
ssr based F test:         F=1.0085  , p=0.4510  , df_denom=628, df_num=22
ssr based chi2 test:   chi2=23.7775 , p=0.3590  , df=22
likelihood ratio test: chi2=23.3671 , p=0.3813  , df=22
parameter F test:         F=1.0085  , p=0.4510  , df_denom=628, df_num=22

Granger Causality
number of lags (no zero) 23
ssr based F test:         F=0.9576  , p=0.5199  , df_denom=625, df_num=23
ssr based chi2 test:   chi2=23.6804 , p=0.4217  , df=23
likelihood ratio test: chi2=23.2727 , p=0.4449  , df=23
parameter F test:         F=0.9576  , p=0.5199  , df_denom=625, df_num=23

Granger Causality
number of lags (no zero) 24
ssr based F test:         F=0.9248  , p=0.5678  , df_denom=622, df_num=24
```

```
ssr based chi2 test:   chi2=23.9439 , p=0.4648  , df=24
likelihood ratio test: chi2=23.5266 , p=0.4889  , df=24
parameter F test:         F=0.9248  , p=0.5678  , df_denom=622, df_num=24

Granger Causality
number of lags (no zero) 25
ssr based F test:         F=1.0058  , p=0.4567  , df_denom=619, df_num=25
ssr based chi2 test:   chi2=27.2161 , p=0.3452  , df=25
likelihood ratio test: chi2=26.6778 , p=0.3722  , df=25
parameter F test:         F=1.0058  , p=0.4567  , df_denom=619, df_num=25

Granger Causality
number of lags (no zero) 26
ssr based F test:         F=1.0465  , p=0.4021  , df_denom=616, df_num=26
ssr based chi2 test:   chi2=29.5492 , p=0.2867  , df=26
likelihood ratio test: chi2=28.9153 , p=0.3150  , df=26
parameter F test:         F=1.0465  , p=0.4021  , df_denom=616, df_num=26

Granger Causality
number of lags (no zero) 27
ssr based F test:         F=0.9695  , p=0.5102  , df_denom=613, df_num=27
ssr based chi2 test:   chi2=28.5253 , p=0.3843  , df=27
likelihood ratio test: chi2=27.9330 , p=0.4144  , df=27
parameter F test:         F=0.9695  , p=0.5102  , df_denom=613, df_num=27

Granger Causality
number of lags (no zero) 28
ssr based F test:         F=0.9428  , p=0.5516  , df_denom=610, df_num=28
ssr based chi2 test:   chi2=28.8654 , p=0.4194  , df=28
likelihood ratio test: chi2=28.2582 , p=0.4508  , df=28
parameter F test:         F=0.9428  , p=0.5516  , df_denom=610, df_num=28

Granger Causality
number of lags (no zero) 29
ssr based F test:         F=0.9232  , p=0.5837  , df_denom=607, df_num=29
ssr based chi2 test:   chi2=29.3756 , p=0.4456  , df=29
likelihood ratio test: chi2=28.7462 , p=0.4783  , df=29
parameter F test:         F=0.9232  , p=0.5837  , df_denom=607, df_num=29

Granger Causality
number of lags (no zero) 30
```

```
ssr based F test:         F=0.9184  , p=0.5937  , df_denom=604, df_num=30
ssr based chi2 test:   chi2=30.3339 , p=0.4487  , df=30
likelihood ratio test: chi2=29.6624 , p=0.4830  , df=30
parameter F test:         F=0.9184  , p=0.5937  , df_denom=604, df_num=30
```

In [832…

```
for lag in range(1, 14):
    spy0_series = df_difference['SPY'].iloc[lag:]
    lagged_dis0_series = df_difference['DIS'].iloc[:-lag]
    print('Lag: %s'%lag)
    print(pearsonr(spy0_series, lagged_dis0_series))
    print('------')
```

```
Lag: 1
PearsonRResult(statistic=0.0016694992244321565, pvalue=0.9649826525857012)
------
Lag: 2
PearsonRResult(statistic=-0.01815321395321781, pvalue=0.6333207977109512)
------
Lag: 3
PearsonRResult(statistic=-0.0006332092178889664, pvalue=0.9867341673141135)
------
Lag: 4
PearsonRResult(statistic=-0.027325579703068675, pvalue=0.47328917104611556)
------
Lag: 5
PearsonRResult(statistic=-0.004346014218305987, pvalue=0.9092742526560599)
------
Lag: 6
PearsonRResult(statistic=-0.025096704116348956, pvalue=0.5107527361603499)
------
Lag: 7
PearsonRResult(statistic=0.02426182857240103, pvalue=0.5252209496055964)
------
Lag: 8
PearsonRResult(statistic=-0.03976113884922176, pvalue=0.2980238945087466)
------
Lag: 9
PearsonRResult(statistic=0.03595560513758693, pvalue=0.34705266217232034)
------
Lag: 10
PearsonRResult(statistic=-0.06943551999156325, pvalue=0.06934355170744698)
------
Lag: 11
PearsonRResult(statistic=0.017566953364766786, pvalue=0.6464995167892542)
------
Lag: 12
PearsonRResult(statistic=-0.04368115856237018, pvalue=0.2542745185235487)
------
Lag: 13
PearsonRResult(statistic=0.01833080495448437, pvalue=0.6327403453410207)
------
```

کمترین مقدار پی-مقدار برای لگ های 2 و 4 و 5 و 8 و9 میباشد

In [833…

```
df_difference = df_difference[['SPY', 'DIS']]
model = VAR(df_difference)
model0_fit = model.fit(9)
model0_fit.summary()
```

```
C:\Users\Mohammad\anaconda3\envs\timeseries\lib\site-packages\statsmodels\tsa\base\tsa_model.py:473: ValueWarning: A date index has been provided, but it has no associated frequency information and so will be ignored when e.g. forecasting.
  self._init_dates(dates, freq)
```

Out[833…

```
  Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sun, 02, Feb, 2025
Time:                     19:53:01
--------------------------------------------------------------------
No. of Equations:         2.00000    BIC:                    1.38457
Nobs:                     686.000    HQIC:                   1.23070
Log likelihood:          -2297.60    FPE:                    3.10688
AIC:                      1.13359    Det(Omega_mle):         2.94167
--------------------------------------------------------------------
Results for equation SPY
=========================================================================
            coefficient       std. error           t-stat            prob
-------------------------------------------------------------------------
const          0.002981         0.043265            0.069           0.945
L1.SPY         0.020792         0.049575            0.419           0.675
L1.DIS        -0.000518         0.028983           -0.018           0.986
L2.SPY        -0.068859         0.049949           -1.379           0.168
L2.DIS         0.017513         0.029009            0.604           0.546
L3.SPY        -0.037815         0.049955           -0.757           0.449
L3.DIS         0.011185         0.028948            0.386           0.699
L4.SPY         0.037149         0.049824            0.746           0.456
L4.DIS        -0.032730         0.028800           -1.136           0.256
L5.SPY         0.008788         0.049595            0.177           0.859
L5.DIS        -0.002969         0.028702           -0.103           0.918
L6.SPY        -0.041650         0.049614           -0.839           0.401
L6.DIS         0.001130         0.028713            0.039           0.969
L7.SPY         0.012707         0.049653            0.256           0.798
L7.DIS         0.009309         0.028818            0.323           0.747
L8.SPY         0.011867         0.049621            0.239           0.811
L8.DIS        -0.035775         0.028853           -1.240           0.215
L9.SPY         0.085779         0.049529            1.732           0.083
L9.DIS        -0.006785         0.028667           -0.237           0.813
=========================================================================

Results for equation DIS
=========================================================================
            coefficient       std. error           t-stat            prob
```

```
==========================================================================
const          -0.004297         0.074102          -0.058           0.954
L1.SPY          0.236474         0.084909           2.785           0.005
L1.DIS         -0.054573         0.049641          -1.099           0.272
L2.SPY         -0.017598         0.085549          -0.206           0.837
L2.DIS          0.021038         0.049686           0.423           0.672
L3.SPY         -0.092448         0.085559          -1.081           0.280
L3.DIS          0.028059         0.049580           0.566           0.571
L4.SPY          0.078183         0.085335           0.916           0.360
L4.DIS         -0.048273         0.049327          -0.979           0.328
L5.SPY          0.079719         0.084944           0.938           0.348
L5.DIS          0.019628         0.049158           0.399           0.690
L6.SPY         -0.082040         0.084976          -0.965           0.334
L6.DIS          0.088452         0.049178           1.799           0.072
L7.SPY          0.090902         0.085043           1.069           0.285
L7.DIS         -0.047659         0.049358          -0.966           0.334
L8.SPY         -0.027442         0.084988          -0.323           0.747
L8.DIS         -0.042122         0.049418          -0.852           0.394
L9.SPY          0.124377         0.084830           1.466           0.143
L9.DIS         -0.011729         0.049100          -0.239           0.811
==========================================================================

Correlation matrix of residuals
            SPY       DIS
SPY    1.000000  0.625845
DIS    0.625845  1.000000
```

In [834…

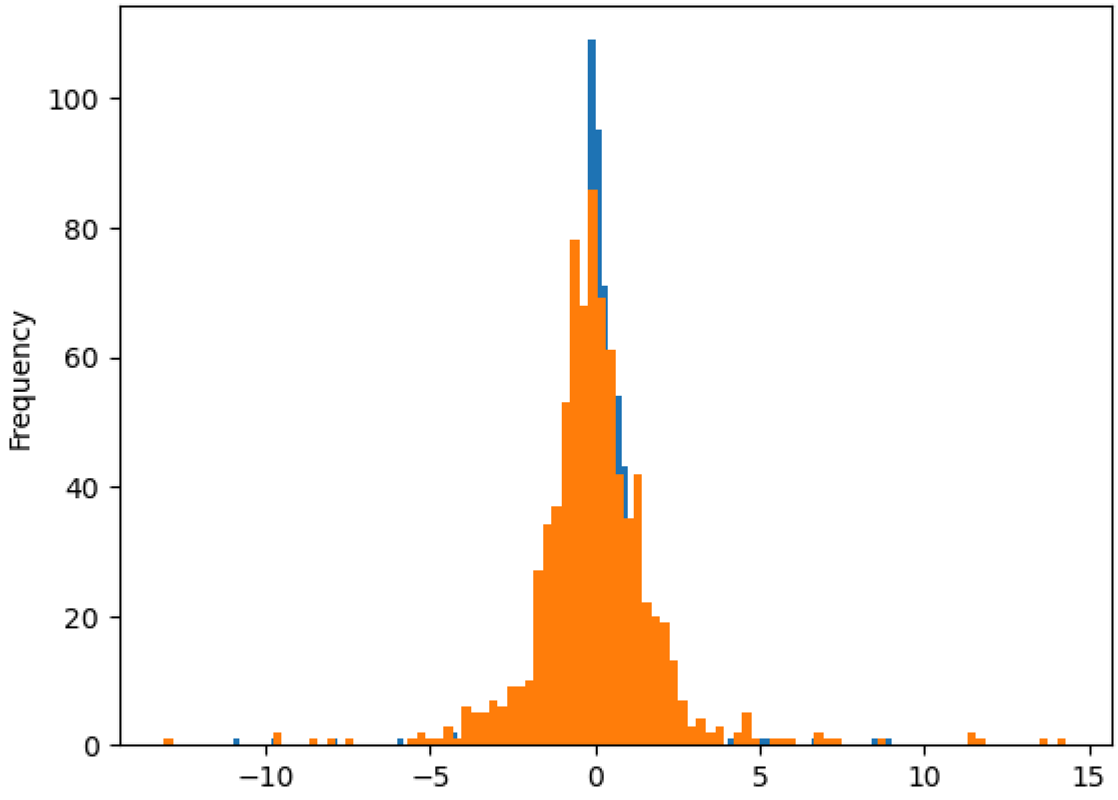
```python
spy_new = yf.download('SPY', start = '2017-10-1', end = '2021-1-1')
dis_new = yf.download('DIS', start = '2017-10-1', end = '2021-1-1')


ts1_new = (spy_new['Close'].squeeze()).dropna()
ts_new = (dis_new['Close'].squeeze()).dropna()


new_returns1 = 100 * ts1_new.pct_change().dropna()
new_return1 = new_returns1 - new_returns1.mean()


new_returns2 = 100 * ts_new.pct_change().dropna()
```

```
new_return2 = new_returns2 - new_returns2.mean()

print(new_return1.mean(), new_return2.mean())


new_return1.plot(kind='hist', bins = 100)
new_return2.plot(kind='hist', bins = 100)
```

```
[*********************100%***********************]  1 of 1 completed
[*********************100%***********************]  1 of 1 completed
1.0857926891199576e-17 2.931640260623885e-17
```

Out[834…

```
<Axes: ylabel='Frequency'>
```

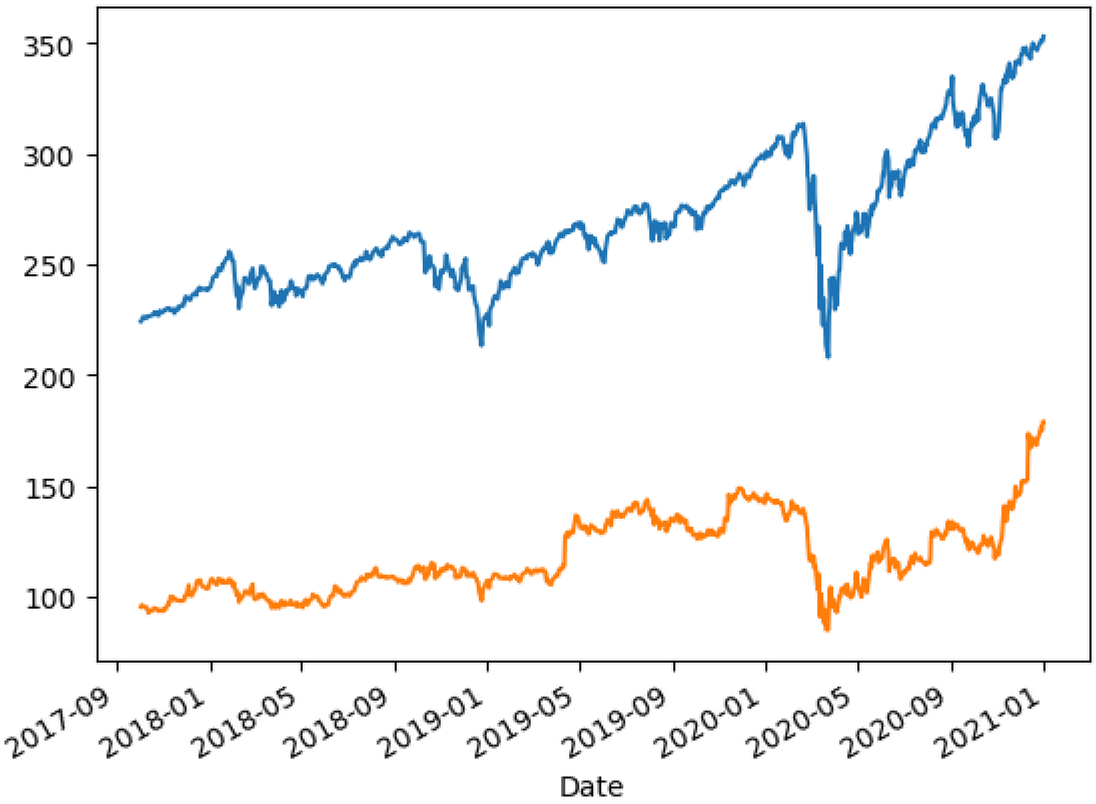


In [835…

```
ts1_new.plot()
ts_new.plot()
```

Out[835…

<Axes: xlabel='Date'>



In [ ]:

In [836…

```
new_difference.index = pd.DatetimeIndex(new_difference.index).to_period('M')
#new_difference = new_difference.asfreq(pd.infer_freq(new_difference.index))
new_difference= pd.concat([new_return1, new_return2], axis=1)
new_difference.replace([np.inf, -np.inf], np.NaN, inplace=True)
new_difference.dropna(axis=0)
print(new_difference.info())
```

```
<class 'pandas.core.frame.DataFrame'>
DatetimeIndex: 818 entries, 2017-10-03 to 2020-12-31
Data columns (total 2 columns):
 #   Column  Non-Null Count  Dtype  
---  ------  --------------  -----  
 0   SPY     818 non-null    float64
 1   DIS     818 non-null    float64
dtypes: float64(2)
memory usage: 19.2 KB
None
```

In [837…

```
new_difference
```

Out[837…

Date	SPY	DIS
2017-10-03	0.148783	0.833314
2017-10-04	0.053413	-0.336123
2017-10-05	0.527281	-0.535585
2017-10-06	-0.179122	-0.137949
2017-10-09	-0.230353	-0.597646
...	...	...
2020-12-24	0.323776	0.005725
2020-12-28	0.793862	2.854862
2020-12-29	-0.256009	-0.970181
2020-12-30	0.077446	2.084733
2020-12-31	0.442850	-0.092470

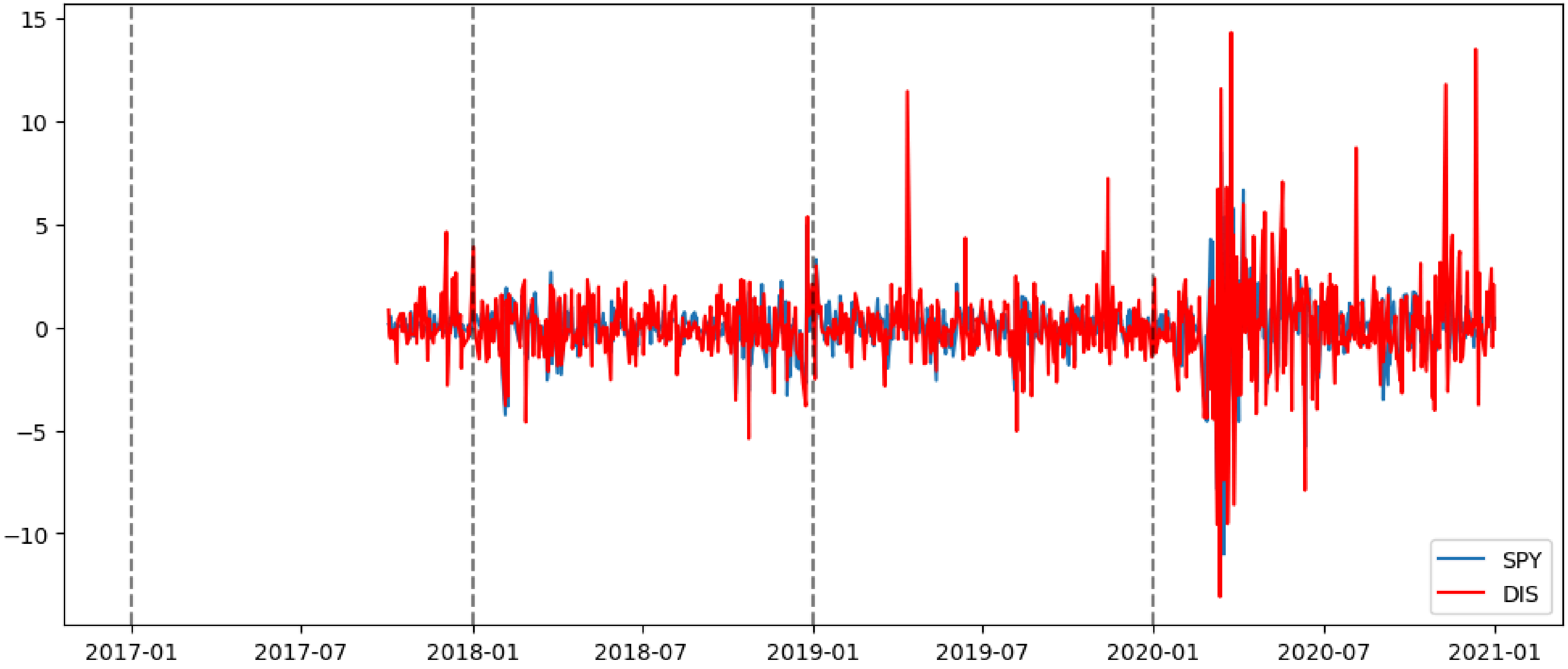
818 rows × 2 columns

In [838…

```
plt.figure(figsize=(12,5))
spy_new, = plt.plot(new_difference['SPY'])
dis_new, = plt.plot(new_difference['DIS'], color='red')

for year in range(2017, 2021):
    plt.axvline(datetime(year,1,1), linestyle='--', color='k', alpha=0.5)

plt.legend(['SPY', 'DIS'], fontsize=10)
```

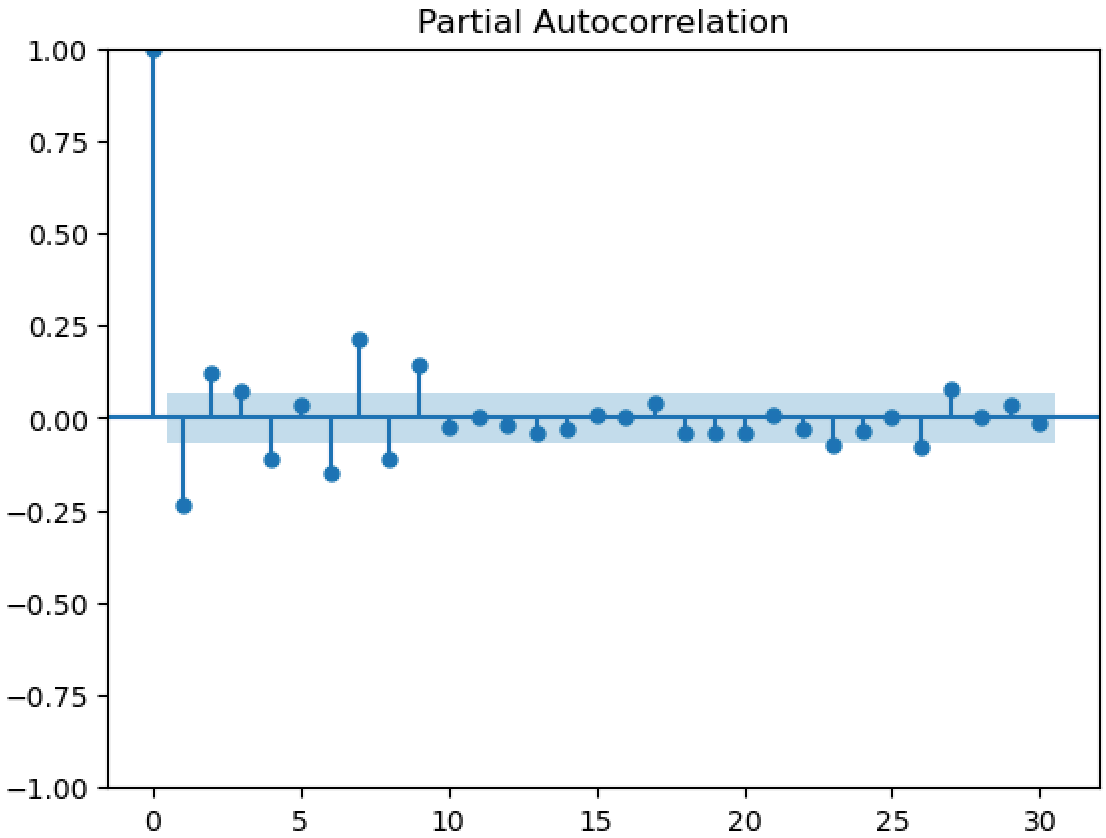
Out[838…

```
<matplotlib.legend.Legend at 0x212b428e340>
```



In [839…

```
new_difference0 = (new_difference['SPY'].squeeze()).dropna()
plot_pacf(new_difference0)
plt.show()
```



بهترین مدل از روی تابع همبستگی جزئی مدل خود برگشتی یک است

In [840…

```
from statsmodels.tsa.stattools import grangercausalitytests
gc_res = grangercausalitytests(new_difference, 20)
```

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=0.0007  , p=0.9796  , df_denom=814, df_num=1
ssr based chi2 test:   chi2=0.0007  , p=0.9795  , df=1
likelihood ratio test: chi2=0.0007  , p=0.9795  , df=1
parameter F test:         F=0.0007  , p=0.9796  , df_denom=814, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=0.4381  , p=0.6454  , df_denom=811, df_num=2
ssr based chi2 test:   chi2=0.8817  , p=0.6435  , df=2
likelihood ratio test: chi2=0.8812  , p=0.6436  , df=2
parameter F test:         F=0.4381  , p=0.6454  , df_denom=811, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=0.6247  , p=0.5992  , df_denom=808, df_num=3
ssr based chi2 test:   chi2=1.8904  , p=0.5955  , df=3
likelihood ratio test: chi2=1.8882  , p=0.5959  , df=3
parameter F test:         F=0.6247  , p=0.5992  , df_denom=808, df_num=3

Granger Causality
number of lags (no zero) 4
ssr based F test:         F=0.4075  , p=0.8033  , df_denom=805, df_num=4
ssr based chi2 test:   chi2=1.6482  , p=0.8001  , df=4
likelihood ratio test: chi2=1.6465  , p=0.8004  , df=4
parameter F test:         F=0.4075  , p=0.8033  , df_denom=805, df_num=4

Granger Causality
number of lags (no zero) 5
ssr based F test:         F=0.4495  , p=0.8138  , df_denom=802, df_num=5
ssr based chi2 test:   chi2=2.2785  , p=0.8094  , df=5
likelihood ratio test: chi2=2.2753  , p=0.8099  , df=5
parameter F test:         F=0.4495  , p=0.8138  , df_denom=802, df_num=5

Granger Causality
number of lags (no zero) 6
ssr based F test:         F=1.1718  , p=0.3192  , df_denom=799, df_num=6
ssr based chi2 test:   chi2=7.1451  , p=0.3076  , df=6
likelihood ratio test: chi2=7.1139  , p=0.3104  , df=6
parameter F test:         F=1.1718  , p=0.3192  , df_denom=799, df_num=6
```

```
Granger Causality
number of lags (no zero) 7
ssr based F test:         F=1.1297  , p=0.3421  , df_denom=796, df_num=7
ssr based chi2 test:   chi2=8.0567  , p=0.3276  , df=7
likelihood ratio test: chi2=8.0169  , p=0.3311  , df=7
parameter F test:         F=1.1297  , p=0.3421  , df_denom=796, df_num=7

Granger Causality
number of lags (no zero) 8
ssr based F test:         F=1.7247  , p=0.0891  , df_denom=793, df_num=8
ssr based chi2 test:   chi2=14.0930 , p=0.0794  , df=8
likelihood ratio test: chi2=13.9718 , p=0.0825  , df=8
parameter F test:         F=1.7247  , p=0.0891  , df_denom=793, df_num=8

Granger Causality
number of lags (no zero) 9
ssr based F test:         F=1.6635  , p=0.0938  , df_denom=790, df_num=9
ssr based chi2 test:   chi2=15.3319 , p=0.0822  , df=9
likelihood ratio test: chi2=15.1884 , p=0.0859  , df=9
parameter F test:         F=1.6635  , p=0.0938  , df_denom=790, df_num=9

Granger Causality
number of lags (no zero) 10
ssr based F test:         F=1.6875  , p=0.0794  , df_denom=787, df_num=10
ssr based chi2 test:   chi2=17.3249 , p=0.0675  , df=10
likelihood ratio test: chi2=17.1417 , p=0.0713  , df=10
parameter F test:         F=1.6875  , p=0.0794  , df_denom=787, df_num=10

Granger Causality
number of lags (no zero) 11
ssr based F test:         F=1.6585  , p=0.0784  , df_denom=784, df_num=11
ssr based chi2 test:   chi2=18.7786 , p=0.0652  , df=11
likelihood ratio test: chi2=18.5634 , p=0.0694  , df=11
parameter F test:         F=1.6585  , p=0.0784  , df_denom=784, df_num=11

Granger Causality
number of lags (no zero) 12
ssr based F test:         F=1.5499  , p=0.1014  , df_denom=781, df_num=12
ssr based chi2 test:   chi2=19.1939 , p=0.0840  , df=12
likelihood ratio test: chi2=18.9689 , p=0.0893  , df=12
```

```
parameter F test:         F=1.5499  , p=0.1014  , df_denom=781, df_num=12

Granger Causality
number of lags (no zero) 13
ssr based F test:         F=1.6754  , p=0.0613  , df_denom=778, df_num=13
ssr based chi2 test:   chi2=22.5362 , p=0.0476  , df=13
likelihood ratio test: chi2=22.2265 , p=0.0519  , df=13
parameter F test:         F=1.6754  , p=0.0613  , df_denom=778, df_num=13

Granger Causality
number of lags (no zero) 14
ssr based F test:         F=1.6153  , p=0.0696  , df_denom=775, df_num=14
ssr based chi2 test:   chi2=23.4607 , p=0.0532  , df=14
likelihood ratio test: chi2=23.1249 , p=0.0583  , df=14
parameter F test:         F=1.6153  , p=0.0696  , df_denom=775, df_num=14

Granger Causality
number of lags (no zero) 15
ssr based F test:         F=1.6136  , p=0.0646  , df_denom=772, df_num=15
ssr based chi2 test:   chi2=25.1755 , p=0.0476  , df=15
likelihood ratio test: chi2=24.7889 , p=0.0529  , df=15
parameter F test:         F=1.6136  , p=0.0646  , df_denom=772, df_num=15

Granger Causality
number of lags (no zero) 16
ssr based F test:         F=1.5513  , p=0.0762  , df_denom=769, df_num=16
ssr based chi2 test:   chi2=25.8854 , p=0.0557  , df=16
likelihood ratio test: chi2=25.4765 , p=0.0619  , df=16
parameter F test:         F=1.5513  , p=0.0762  , df_denom=769, df_num=16

Granger Causality
number of lags (no zero) 17
ssr based F test:         F=1.4433  , p=0.1093  , df_denom=766, df_num=17
ssr based chi2 test:   chi2=25.6576 , p=0.0809  , df=17
likelihood ratio test: chi2=25.2552 , p=0.0891  , df=17
parameter F test:         F=1.4433  , p=0.1093  , df_denom=766, df_num=17

Granger Causality
number of lags (no zero) 18
ssr based F test:         F=1.4156  , p=0.1162  , df_denom=763, df_num=18
ssr based chi2 test:   chi2=26.7159 , p=0.0845  , df=18
```

```
likelihood ratio test: chi2=26.2795 , p=0.0935  , df=18
parameter F test:         F=1.4156  , p=0.1162  , df_denom=763, df_num=18

Granger Causality
number of lags (no zero) 19
ssr based F test:         F=1.3408  , p=0.1498  , df_denom=760, df_num=19
ssr based chi2 test:   chi2=26.7829 , p=0.1098  , df=19
likelihood ratio test: chi2=26.3438 , p=0.1209  , df=19
parameter F test:         F=1.3408  , p=0.1498  , df_denom=760, df_num=19

Granger Causality
number of lags (no zero) 20
ssr based F test:         F=1.2818  , p=0.1826  , df_denom=757, df_num=20
ssr based chi2 test:   chi2=27.0251 , p=0.1346  , df=20
likelihood ratio test: chi2=26.5776 , p=0.1476  , df=20
parameter F test:         F=1.2818  , p=0.1826  , df_denom=757, df_num=20
```

In [841…

```
for lag in range(1, 14):
    spy_series = new_difference['SPY'].iloc[lag:]
    lagged_dis_series = new_difference['DIS'].iloc[:-lag]
    print('Lag: %s'%lag)
    print(pearsonr(spy_series, lagged_dis_series))
    print('------')
```

```
Lag: 1
PearsonRResult(statistic=-0.16230082318586259, pvalue=3.117452817849981e-06)
------
Lag: 2
PearsonRResult(statistic=0.14144653102913873, pvalue=5.019228462058869e-05)
------
Lag: 3
PearsonRResult(statistic=0.017672899146515778, pvalue=0.6144057751620212)
------
Lag: 4
PearsonRResult(statistic=-0.0741927061906949, pvalue=0.034307965751801166)
------
Lag: 5
PearsonRResult(statistic=0.07960086433713004, pvalue=0.02322080188896711)
------
Lag: 6
PearsonRResult(statistic=-0.18605111629246632, pvalue=9.286137817796049e-08)
------
Lag: 7
PearsonRResult(statistic=0.18533741701542528, pvalue=1.0605472560715994e-07)
------
Lag: 8
PearsonRResult(statistic=-0.09566475332218319, pvalue=0.006435734178295348)
------
Lag: 9
PearsonRResult(statistic=0.10143104192258734, pvalue=0.00387717143848273)
------
Lag: 10
PearsonRResult(statistic=0.0027340126666399733, pvalue=0.93815016143774)
------
Lag: 11
PearsonRResult(statistic=0.017548369976204427, pvalue=0.6186424691747267)
------
Lag: 12
PearsonRResult(statistic=0.04316189667423406, pvalue=0.2209367785436535)
------
Lag: 13
PearsonRResult(statistic=-0.06835525787522098, pvalue=0.052542107182405345)
------
```

In [842…

```
new_difference = new_difference[['SPY', 'DIS']]
model = VAR(new_difference)
model_fit = model.fit(9)
model_fit.summary()
```

```
C:\Users\Mohammad\anaconda3\envs\timeseries\lib\site-packages\statsmodels\tsa\base\tsa_model.py:473: ValueWarning: A date index
has been provided, but it has no associated frequency information and so will be ignored when e.g. forecasting.
  self._init_dates(dates, freq)
```

Out[842…

```
Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Sun, 02, Feb, 2025
Time:                     19:53:02
--------------------------------------------------------------------
No. of Equations:         2.00000    BIC:                    1.52485
Nobs:                     809.000    HQIC:                   1.38897
Log likelihood:          -2785.42    FPE:                    3.68509
AIC:                      1.30428    Det(Omega_mle):         3.51791
--------------------------------------------------------------------
Results for equation SPY
==========================================================================
            coefficient       std. error           t-stat            prob
--------------------------------------------------------------------------
const         -0.000907         0.044577           -0.020           0.984
L1.SPY        -0.127349         0.046993           -2.710           0.007
L1.DIS         0.006893         0.030483            0.226           0.821
L2.SPY         0.064982         0.047306            1.374           0.170
L2.DIS         0.035802         0.030559            1.172           0.241
L3.SPY         0.054281         0.047072            1.153           0.249
L3.DIS         0.016132         0.030661            0.526           0.599
L4.SPY        -0.090397         0.046738           -1.934           0.053
L4.DIS        -0.002688         0.030655           -0.088           0.930
L5.SPY        -0.016033         0.046842           -0.342           0.732
L5.DIS         0.011827         0.030605            0.386           0.699
L6.SPY        -0.042494         0.046506           -0.914           0.361
L6.DIS        -0.059560         0.030559           -1.949           0.051
L7.SPY         0.184900         0.046444            3.981           0.000
L7.DIS        -0.003769         0.030672           -0.123           0.902
L8.SPY        -0.159572         0.046372           -3.441           0.001
L8.DIS         0.064908         0.030502            2.128           0.033
L9.SPY         0.185971         0.046554            3.995           0.000
L9.DIS        -0.050920         0.030464           -1.671           0.095
==========================================================================

Results for equation DIS
==========================================================================
            coefficient       std. error           t-stat            prob
```

```
-------------------------------------------------------------------------
const          0.004884       0.069204        0.071          0.944
L1.SPY        -0.016538       0.072955       -0.227          0.821
L1.DIS        -0.088068       0.047323       -1.861          0.063
L2.SPY         0.085078       0.073441        1.158          0.247
L2.DIS        -0.057829       0.047441       -1.219          0.223
L3.SPY         0.052456       0.073077        0.718          0.473
L3.DIS         0.006298       0.047599        0.132          0.895
L4.SPY        -0.034591       0.072559       -0.477          0.634
L4.DIS         0.014192       0.047591        0.298          0.766
L5.SPY        -0.017916       0.072720       -0.246          0.805
L5.DIS        -0.031578       0.047512       -0.665          0.506
L6.SPY        -0.084186       0.072199       -1.166          0.244
L6.DIS        -0.001999       0.047442       -0.042          0.966
L7.SPY         0.217107       0.072102        3.011          0.003
L7.DIS         0.025775       0.047617        0.541          0.588
L8.SPY        -0.266795       0.071990       -3.706          0.000
L8.DIS         0.089165       0.047354        1.883          0.060
L9.SPY         0.177476       0.072273        2.456          0.014
L9.DIS        -0.064072       0.047294       -1.355          0.175
=========================================================================

Correlation matrix of residuals
            SPY       DIS
SPY    1.000000  0.659597
DIS    0.659597  1.000000
```

-0.09

Dis = d

$L_1 = t-1$

d_{t-1}

d_hat = -0.09 d(t-1) + 0.22 s(t-7) - 0.27 s(t-8) + 0.1 d(t-8) + 0.09 s(t-2) - 0.06 d(t-2) and ...

In [ ]: