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- Data collection
  - Sample surveys
  - Designed experiments
  - Randomization
  - Observational studies
  - Censoring and truncation

# Chapter 14: Introduction to regression models

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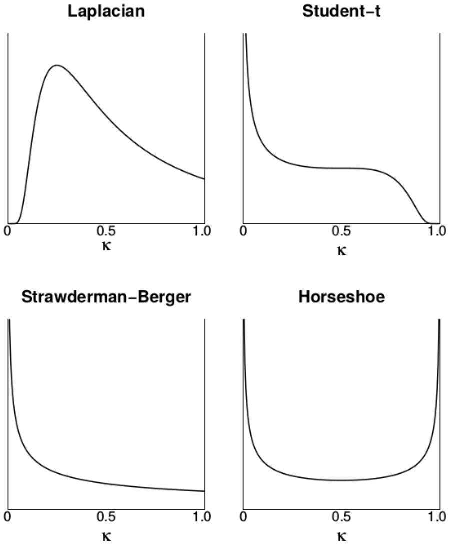
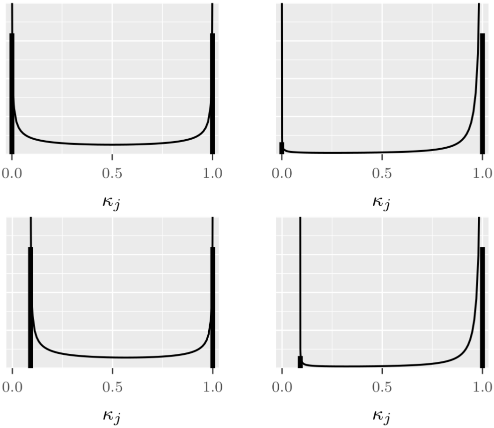
  - empirically better results obtained with more sparse priors
  - it's best to separate selection of sensible prior, good posterior inference, and the decision analysis of which variables are important

# Sparse priors




from Carvalho, Polson, Scott (2009).

# Regularized horseshoe




for more see

- Piironen and Vehtari (2017). Sparsity information and regularization in the horseshoe and other shrinkage priors. In Electronic Journal of Statistics, 11(2):5018-5051. Online
- https://betanalpha.github.io/assets/case_studies/bayes_sparse_regression.html

# Projpred selection vs. Lasso

See projpred in lecture 9.3

Same simulated regression data as in lecture 9,3,
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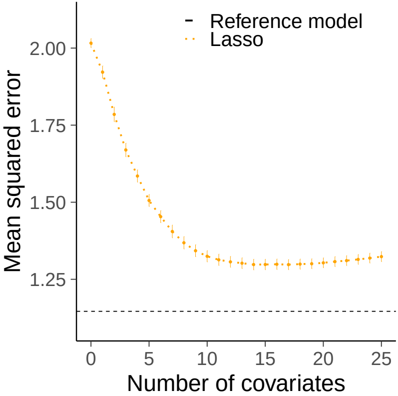

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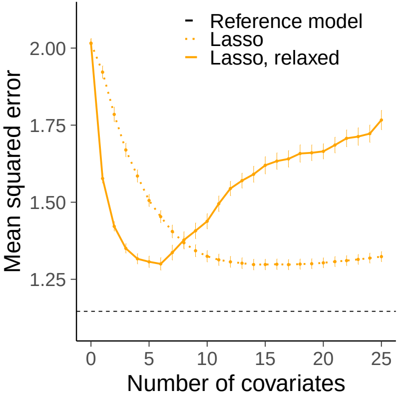

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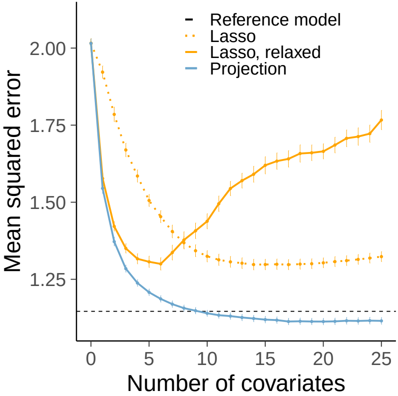

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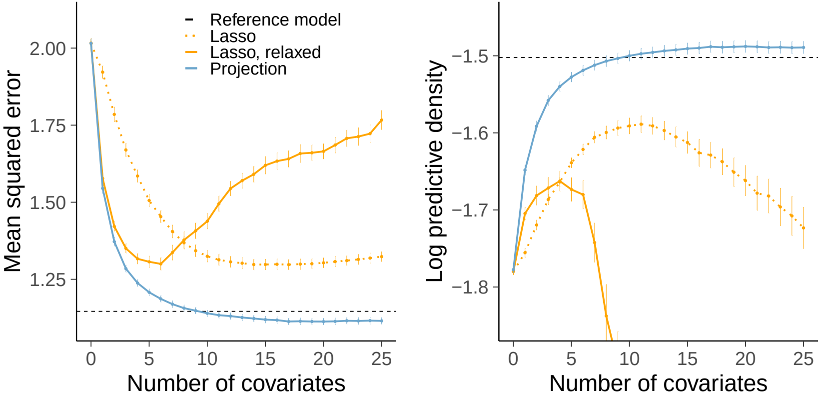

## Chapter 15: Hierarchical linear models

- Since you know hierarchical models, theory is easy
- With probabilistic programming computation is also easy
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  - section on transformations for HMC is relevant
    (see also Stan user guide 21.7 Reparameterization)

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| | |
|---|---|
| `y ~ 1 + x` | fixed / population effect; pooled model |
| `y ~ 1 + (0 + x | g)` | random / group effects |
| `y ~ 1 + x + (1 + x | g)` | mixed effects; hierarchical model |

- ANOVA in section 15.6 (see also `stan_aov`)

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    - Hierarchical GLM natural extension
    - 16.3 Weakly informative priors section is excellent although the recommendation on using Cauchy has changed (see https://github.com/stan-dev/stan/wiki/Prior-Choice-Recommendations)

# Chapter 17: Models for robust inference

- For example

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| normal | $\rightarrow$ | $t$-distribution |
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- Computation with MCMC easy
  - posterior can be multimodal
  - rstanarm doesn't have $t$-distribution for outcome, but brms has

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- Extends the data collection modelling from Chapter 8
- Useful terms

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    missingness depends on missing values
- Multiple imputation
  1. make a model predicting missing data
  2. sample repeatedly from the missing data model to generate multiple imputed data sets
  3. make usual inference for each imputed data set
  4. combine results

# Chapter 21: Gaussian process models

- Gaussian process is
  - infinite dimensional extension of normal distribution
  - useful prior for non-linear functions
  - for any finite number of variables, the marginal is multivariate normal $f_1, \ldots, f_n \sim \mathrm{N}\left(\mu(x_1, \ldots, x_n), K(x_1, \ldots, x_n)\right)$

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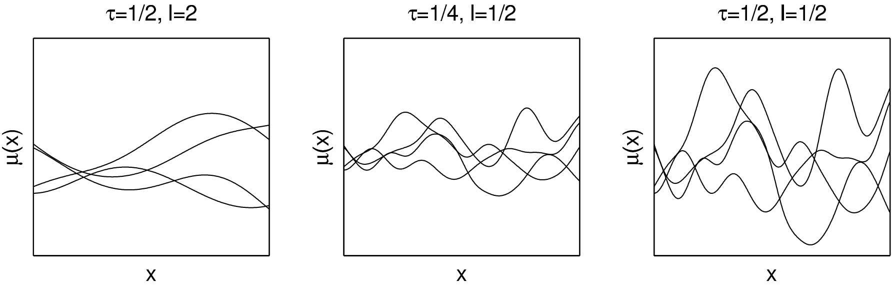

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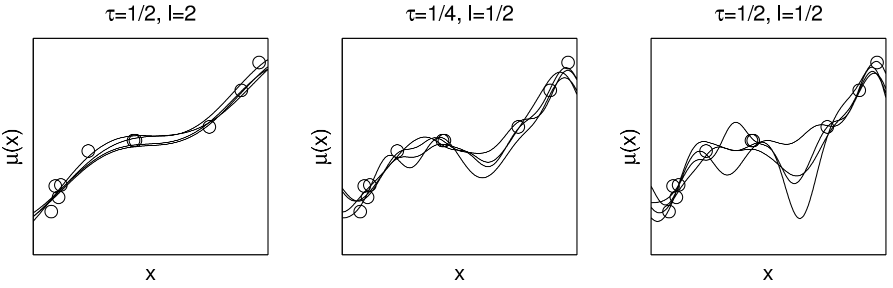
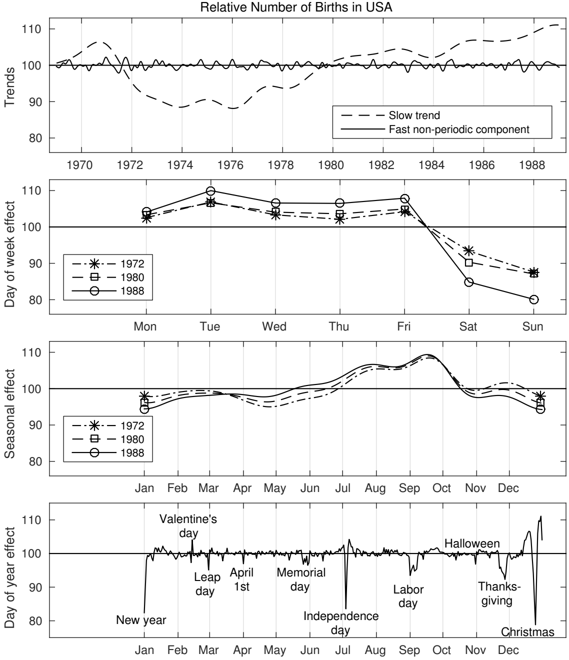


## Chapter 21: Gaussian process models

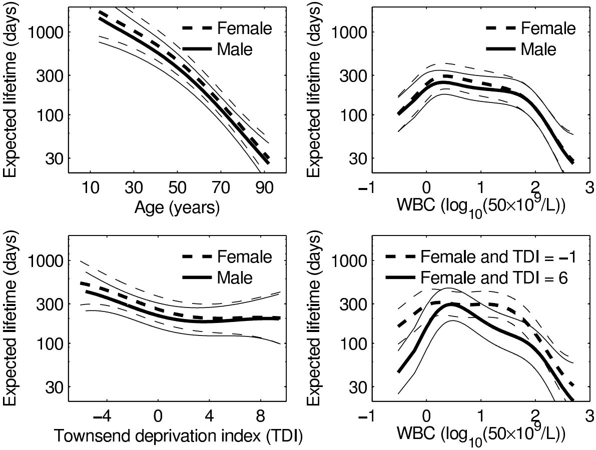
- Conditional on covariance function parameter the posterior is just multivariate normal
  - need to make inference for covariance function parameters given the marginal likelihood
  - the exact computation of the marginal likelihood scales $O(N^3)$

- Easy to make additive models
$y_t(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t) + f_5(t) + \epsilon_t$



# Chapter 21: Gaussian process models

- For non-Gaussian outcome models similar extension as GLMs
- Survival model example:



## GPs in Stan

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- Instead of covariance matrix based approach, for low dimensional cases faster to use basis function representation
    - e.g. `stan_glm(y ~ s(x, bs="gp"))`