# EECS 498/598 Deep Learning - Homework 4

March 26th, 2019

#### Instructions

- This homework is Due April 16th at 11.59pm. Late submission policies apply.
- You will submit a write-up and your code for this homework.

#### 1 [25 points] Deep Q-Network (DQN).

In this question, you will implement DQN algorithm (the detailed algorithm is in the Nature paper https://web.stanford.edu/class/psych209/Readings/MnihEtAlHassibis15NatureControlDeepRL.pdf) in dqn.py and train an agent to play the CartPole task.

- 1. Fill the blank in function select\_action to implement  $\epsilon$ -greedy action selection method.
- 2. Fill the blank in function optimize\_model to train the deep Q network.
- 3. Fill the blank in class DQN.
- 4. Run the script to train the agent playing CartPole task and report the learning curve of the episode duration. You can change the hyper-parameters and network architecture to make the agent perform well.

## 2 [25 points] Policy Gradients.

1. We will derive the REINFORCE algorithm in this question. Consider the agent's objective,

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}[r(\tau)]$$

Show that the gradient of the objective function above can be approximated as following.

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(a_{t}^{i} | s_{t}^{i}) r(\tau^{i})$$

where the outer sum runs over different agent episodes  $\tau^1,...,\tau^N$  and  $\tau^i=((a_1^i,s_1^i),...,(a_T^i,s_T^i))$  represents a single episode.

2. Let  $r(\tau) = \sum_{i=1}^{T} r_i$ . In this case, the above gradient estimator becomes

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(a_t^i | s_t^i) \sum_{t'=1}^{T} r_{t'}^i$$

Show that the following is an unbiased estimate of  $\nabla_{\theta} J(\theta)$ .

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(a_{t}^{i} | s_{t}^{i}) \sum_{t'=t}^{T} r_{t'}^{i}$$

That is, we can omit the rewards collected in the past while keeping the estimator unbiased. The new estimator has the advantage of having lower variance than the original estimator.

#### 3 [25 points] REINFORCE algorithm.

In this problem, you will implement the REINFORCE algorithm and train an agent to play the CartPole task.

- 1. Policy network:
  - Fill in the PolicyNet class whose forward pass returns a probability of taking one out of two actions. We will implement this by a simple sigmoid where 1 means left action and 0 means right action after sampling.

Policy: Linear  $(4 - 24) \rightarrow \texttt{ReLU} \rightarrow \texttt{Linear} (24 - 36) \rightarrow \texttt{ReLU} \rightarrow \texttt{Linear} (36 - 1) \rightarrow \texttt{sigmoid}$ 

- 2. REINFORCE algorithm:
  - Fill in the code inside the simulate function to sample actions given the policy\_network.
  - Fill in the code to compute the discounted rewards used by the policy gradients update and store them in reward\_pool.
  - Fill in the code for the policy gradients update using the elements in reward\_pool.
- 3. Run the script and train the agent to play the CartPole task. Report the learning curve after all training episodes are finished. You should expect the agent to reach a reward of 200.

## 4 [25 points] Actor-Critic algorithm.

You will be implementing an Actor-Critic algorithm for the Cart-pole environment in this problem.

1. Subclass the nn.Module to implement the Policy class whose forward pass returns the action distribution and state value for a given input state.

The actor and critic networks have the following network architectures.

Actor: Linear 
$$(4 - 128) \rightarrow \texttt{ReLU} \rightarrow \texttt{Linear} \ (128 - 2) \rightarrow \texttt{softmax}$$
 Critic: Linear  $(4 - 128) \rightarrow \texttt{ReLU} \rightarrow \texttt{Linear} \ (128 - 1)$ 

where the first linear layer is shared between the two networks.

2. Recall policy gradients,

$$J(\theta) = \mathbb{E}_{\tau \sim p(\tau;\theta)}[r(\tau)]$$

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(a_{t}^{i} | s_{t}^{i}) A^{\pi}(s_{t}^{i}, a_{t}^{i})$$

where N, T represent the number of agent trajectories and episode length, respectively.

In Actor-Critic, the advantage is estimated using a critic network as

$$A^{\pi}(s_t^i, a_t^i) = \left(\sum_{t'=t}^T \gamma^{t'-t} r(s_{t'}^i, a_{t'}^i)\right) - V_{\phi}^{\pi}(s_t^i)$$

Given a single agent episode  $\tau = ((s_1, a_1, r_1), ..., (s_T, a_T, r_T))$  as input, implement the compute\_losses function which computes the actor\_loss and critic\_loss as follows.

• actor\_loss: loss function  $\mathcal{L}_{\theta}$  for the actor network

$$\mathcal{L}_{\theta} = -\sum_{t=1}^{T} \log \pi_{\theta}(a_t|s_t) A^{\pi}(s_t, a_t)$$

• critic\_loss: loss function  $\mathcal{L}_{\phi}$  for the critic network

$$\mathcal{L}_{\phi} = \sum_{t=1}^{T} A^{\pi}(s_t, a_t)^2$$

- 3. Train the model using the provided optimization code.
- 4. Report the curve showing average reward as the training progresses.

### 5 [10 points] Extra credit.

In question 2, show that adding a state dependent baseline does not introduce any bias in the estimator. i.e., Show that the following is an unbiased estimator of the gradient. Adding a baseline can further reduce the variance of the estimator.

$$\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\theta} \log \pi_{\theta}(a_{t}^{i} | s_{t}^{i}) \left( \left[ \sum_{t'=t}^{T} r_{t'}^{i} \right] - b(s_{t}^{i}) \right)$$