Report Project 2 Signal Processing

Course code: II1303
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Summery

Part 1

Consider the system function of an IIR filter

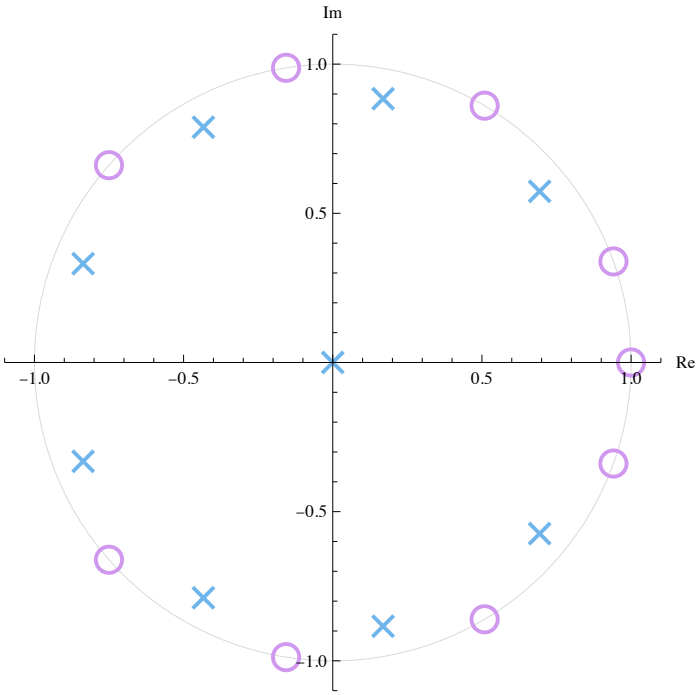
$$H(z) = \frac{(1-z^{-1})\times\prod_{k=1}^{4}(1-z^{-1}\,e^{-j\,0.11\,\pi\,(2\,k-1)})\times(1-z^{-1}\,e^{j\,0.11\,\pi\,(2\,k-1)})}{100\times\prod_{k=1}^{4}(1-0.9\,z^{-1}\,e^{-j\,0.22\,\pi\,k})\times(1-0.9\,z^{-1}\,e^{j\,0.22\,\pi\,k})}$$

a) Make a pole-zero plot of H(z). Is the system stable?

A system is stable if the poles of H(z)lie strictly inside the unit circle.

X = poles
O = zeros

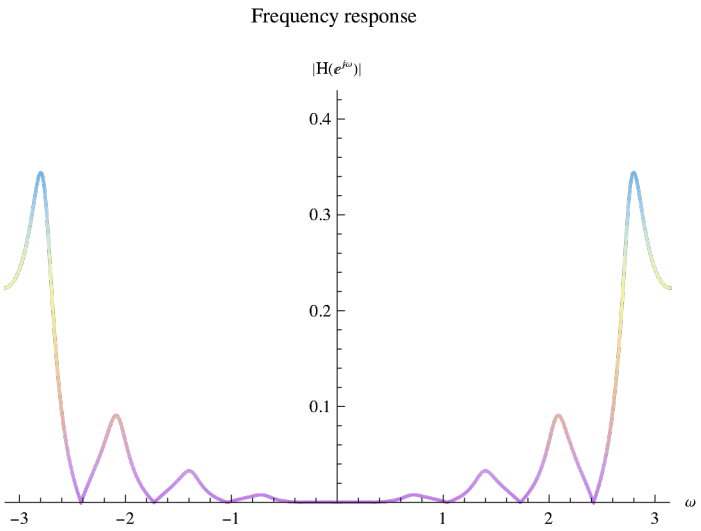
As we can see in the plot all poles lies inside the unit circle, which makes the system stable.



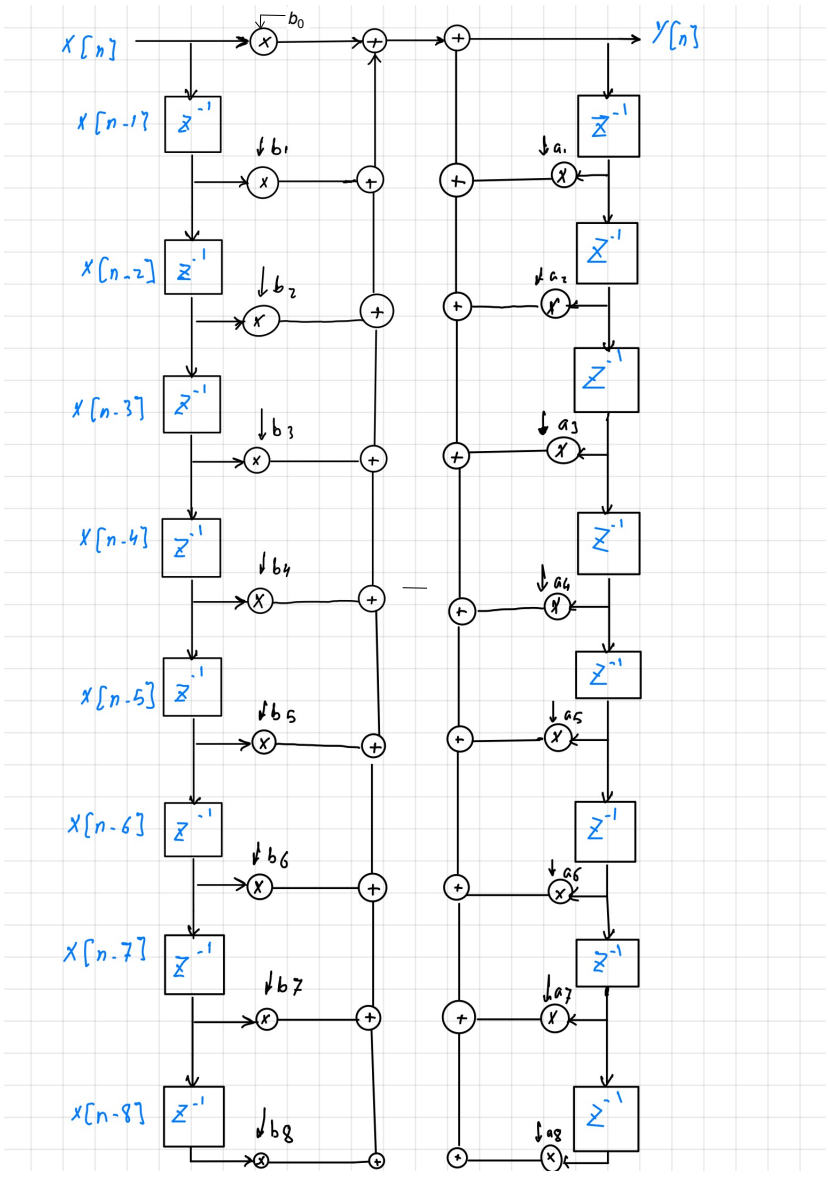
b) Discuss how the filter works, what happens with certain frequencies?

The filter works as a high pass filter and blocks low frequency.

c) Illustrate the frequency response of the IIR filter!



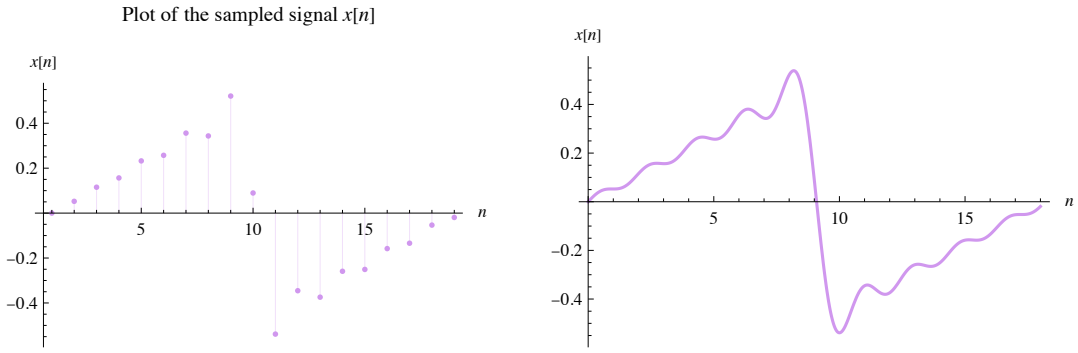
d) Sketch a direct form structure to illustrate the IIR filter!



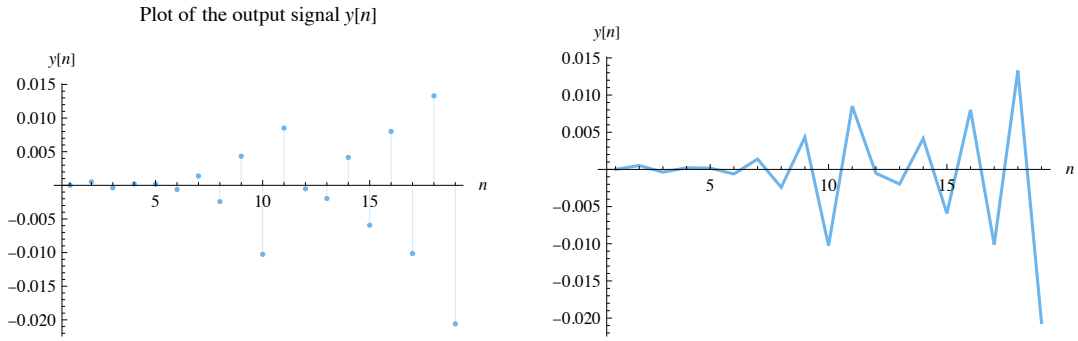
e) Consider the continuous signal x(t) defined as

$$x(t) = \frac{1}{\pi} \sum_{k=1}^{9} \frac{(-1)^{k-1}}{k} \sin(2\pi f_0 k t)$$

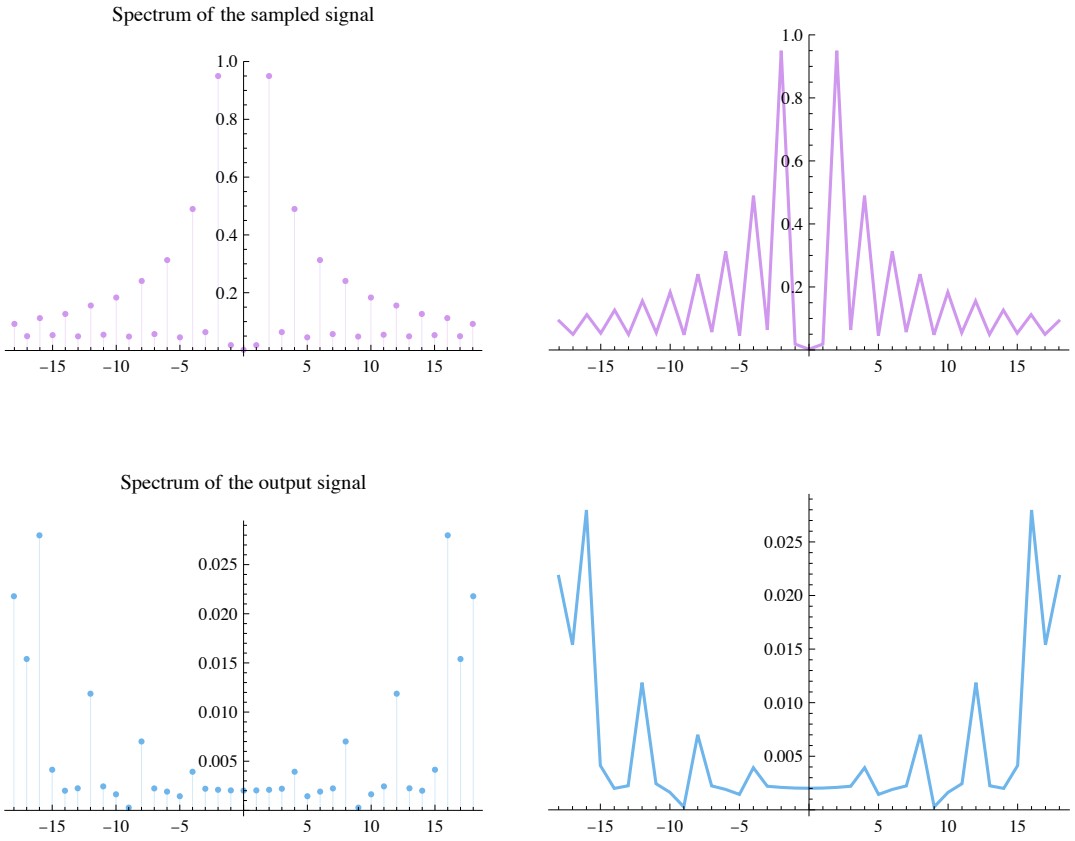
with $f_0 = 220$ Hz. This signal is sampled with the frequency $f_s = 4$ kHz. Plot the sampled signal $x[n]$!



f) Now, the sampled signal $x[n]$ is input to the IIR filter above. Compute and plot the output signal $y[n]$!



g) Draw and compare the two-sided spectrum of $x[n]$ and that of $y[n]$!

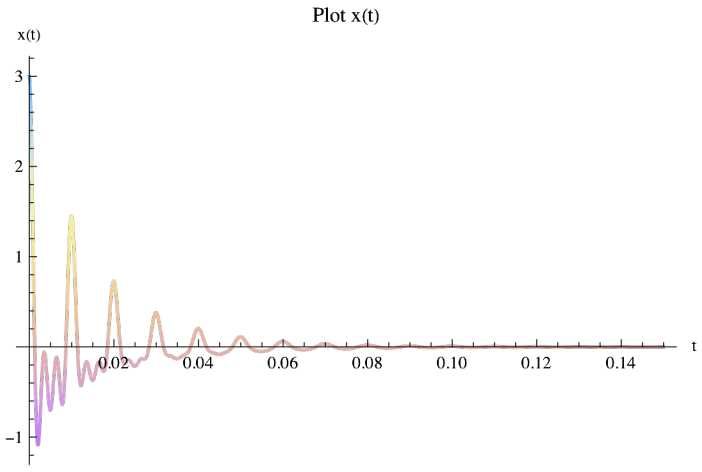


Part 2

Consider the following signal

$$x(t) = \left[e^{-50\,t}\cos(2\,\pi 100\,t) + e^{-75\,t}\cos(2\,\pi 200\,t) + e^{-100\,t}\cos(2\,\pi 300\,t)\right] u(t)$$

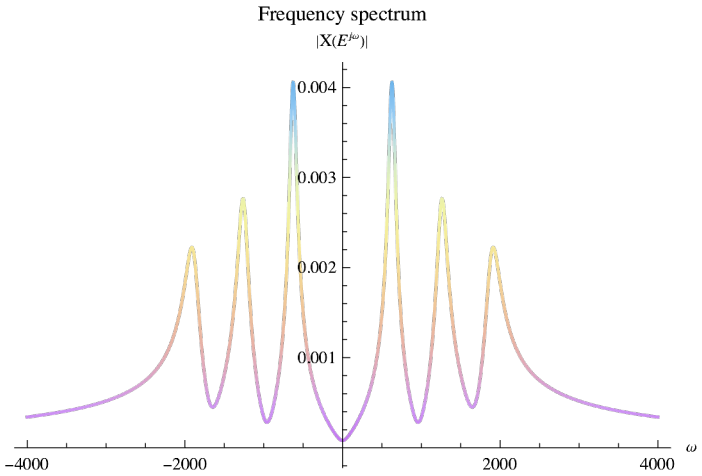
a) Make a plot of $x(t)$ as function of time!

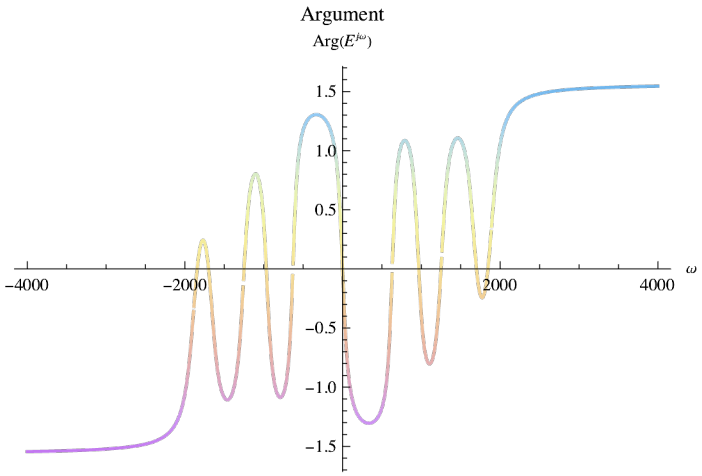


b) Compute the spectrum $X(j\omega)$!

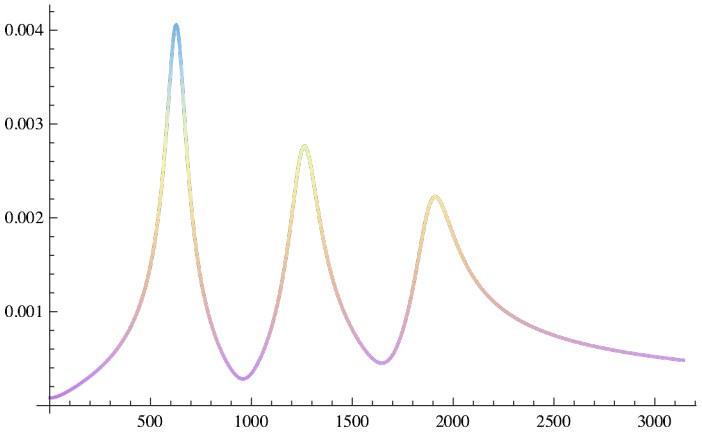
$$\frac{\frac{-50+i\,\omega}{-40\,000\,\pi^2+(50\,i+\omega)^2}+\frac{-75+i\,\omega}{-160\,000\,\pi^2+(75\,i+\omega)^2}+\frac{-100+i\,\omega}{-360\,000\,\pi^2+(100\,i+\omega)^2}}{\sqrt{2\,\pi}}$$

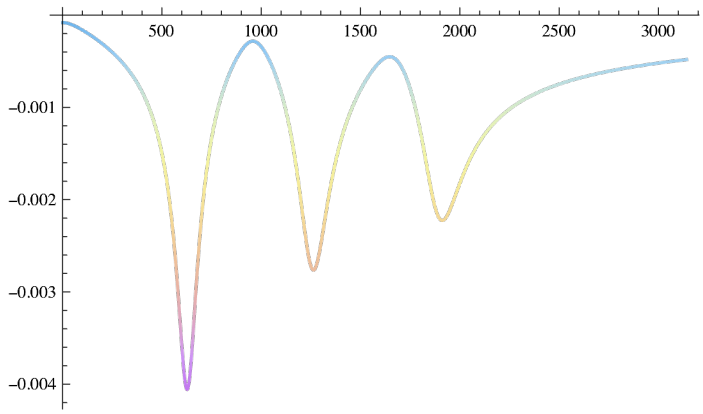
c) Plot $|X(j\omega)|$ and arg $(X(j\omega))$!

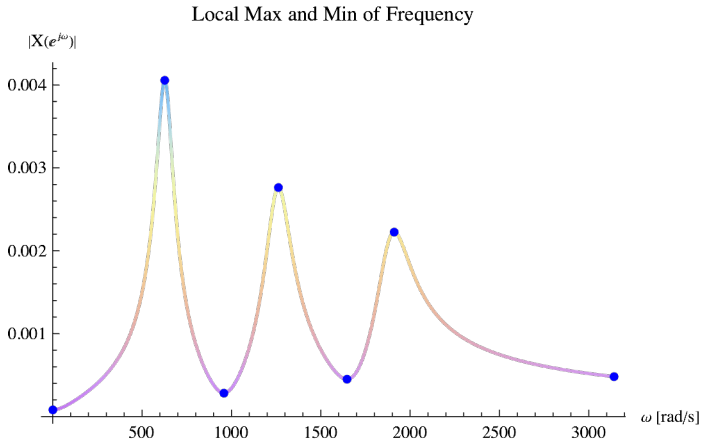




d) There are some distinct local maxima and minima in the spectrum. Solve for these frequencies!







e) Can you motivate the local maxima from the expression of $x(t)$?

The local maxima can be find from the expression of x(t). The omega values will be the max x coordinate. From the expression we obtain 3 omega values: $2\pi 100$, $2\pi 200$, $2\pi 300$. This correspond to the x values we get from the FindPeaks function. {(627,0.00405668),(1263,0.00276316),(1911,0.00222517)}

Code

In[•]:= `ClearAll["Global`*"]`

Package `II1303`

The following package introduces the methods

Reconstruct, Afϕ, A$\omega\phi$,
ToExp, Poles, Zeros, PoleZeroPlot,
ZTransform2, InverseZTransform2, ZApart,
LaplaceTransform2, InverseLaplaceTransform2

Code

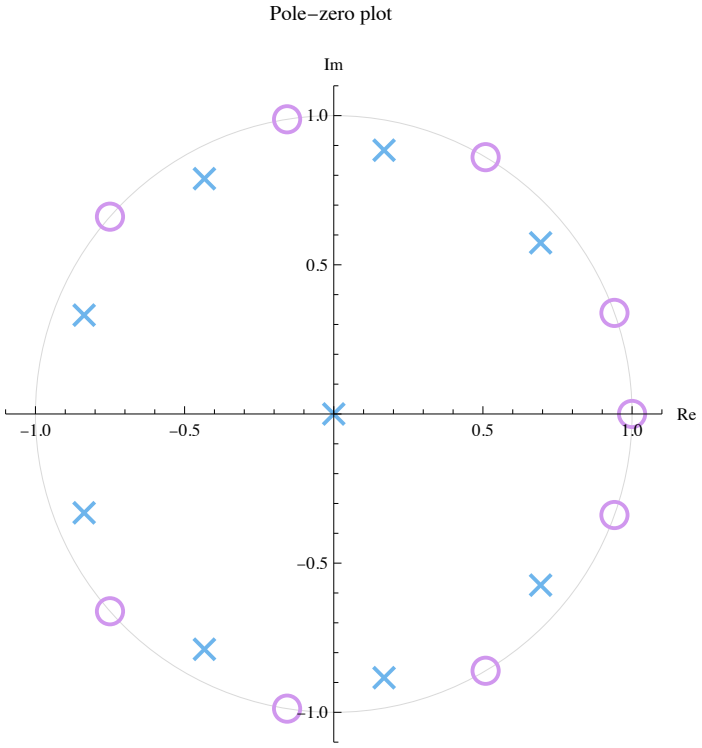
Part 1

In[123]:=
$$H[z\_] := \frac{(1 - z^{-1}) \times \prod_{k=1}^{4} (1 - z^{-1}\, e^{-j\, 0.11\, \pi\, (2k-1)}) \times (1 - z^{-1}\, e^{j\, 0.11\, \pi\, (2k-1)})}{100 \times \prod_{k=1}^{4} (1 - 0.9\, z^{-1}\, e^{-j\, 0.22\, \pi\, k}) \times (1 - 0.9\, z^{-1}\, e^{j\, 0.22\, \pi\, k})}$$

```
Hz1_tfm = TransferFunctionModel [H[z], z^-1, SamplingPeriod -> 1];
Hz_tfm = TransferFunctionModel [H[z], z, SamplingPeriod -> 1];
```

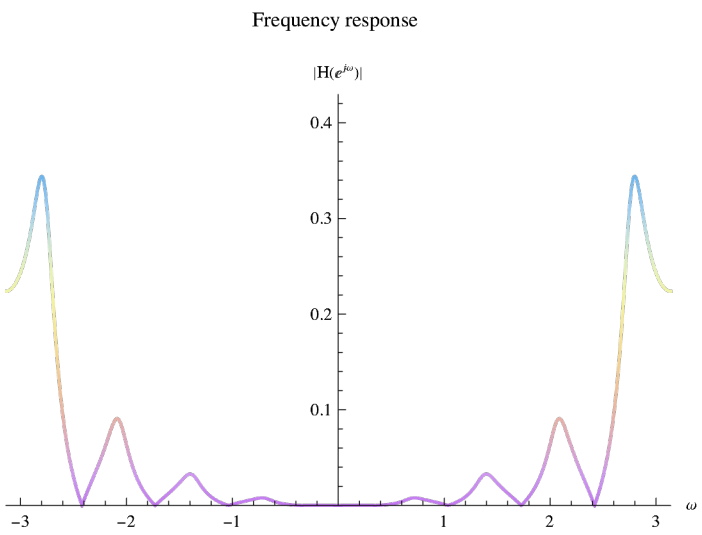
In[126]:=
```
PoleZeroPlot[H[z], z,
 AxesLabel → {"Re", "Im"},
 Prolog → {LightGray, Circle[]},
 MarkerScale → Scaled[0.02], PlotLabel → "Pole-zero plot \n"
]
```

Out[126]=



In[544]:=
```
Plot[Abs[H[e^(j ω)]], {ω, -Pi, Pi},
 PlotRange → {{-Pi, Pi}, {0, 0.43}},
 AxesLabel → {"ω", "|H(e^(jω))|"},
 PlotLabel → "Frequency response \n",
 ColorFunction → Function[{x, y}, ColorData["Pastel"][y]],
 PlotStyle → Medium
]
```

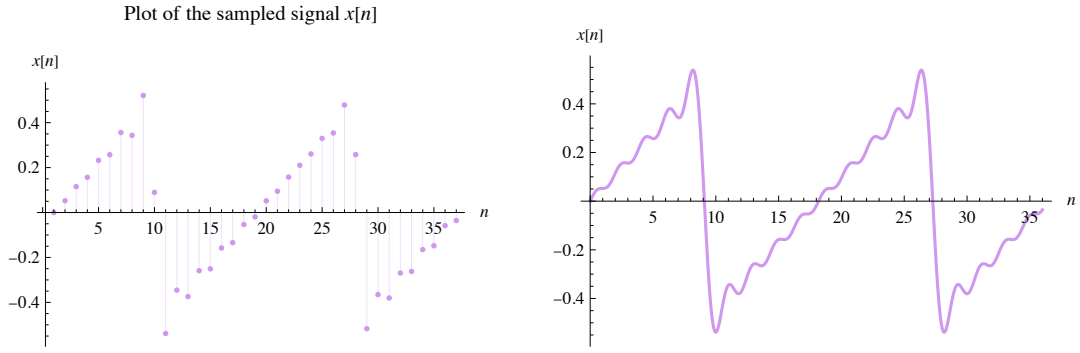
Out[544]=



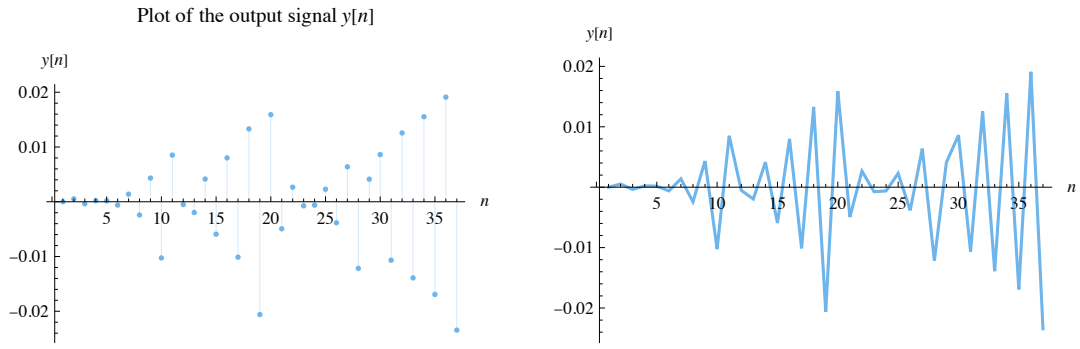
In[534]:=
```
f0 = 220; T0 = 1/f0; fs = 1/T; T = 1/4000; omega = 2 π f0; omegaHat = omega/fs;
x[n_] = 1/π Sum[(-1)^(k-1)/k Sin[omegaHat k n], {k, 1, 9}];
xn = Table[x[n], {n, 0, 36}];
xn_listplot = ListPlot[xn,
   PlotRange → All,
   PlotStyle → ColorData["Pastel"][0.1] ,
   Filling → Axis,
   PlotLabel →
    StringForm["`1`[`2`]", Style["x", Italic], Style["n", Italic]] ×
     "Plot of the sampled signal",
   AxesLabel → {
     Style["n", Italic],
     StringForm["`1`[`2`]", Style["x", Italic], Style["n", Italic]]
    }
  ];
xn_lineplot = Plot[x[n], {n, 0, 36},
   PlotRange → All,
   PlotStyle → ColorData["Pastel"][0.1] ,
   AxesLabel → {
     Style["n", Italic],
     StringForm["`1`[`2`]", Style["x", Italic], Style["n", Italic]]
    }
  ];
```

```
yn = OutputResponse[Hz_tfm, xn];
yn_listplot = ListPlot[yn,
    PlotRange → All,
    PlotStyle → ColorData["Pastel"][1] ,
    Filling → Axis,
    PlotLabel →
     StringForm["`1`[`2`]", Style["y", Italic], Style["n", Italic]] ×
      "Plot of the output signal",
    AxesLabel → {
      Style["n", Italic],
      StringForm["`1`[`2`]", Style["y", Italic], Style["n", Italic]]
     }
   ];
yn_lineplot = ListLinePlot[yn,
    PlotRange → All,
    PlotStyle → ColorData["Pastel"][1] ,
    AxesLabel → {
      Style["n", Italic],
      StringForm["`1`[`2`]", Style["y", Italic], Style["n", Italic]]
     }
   ];
GraphicsRow[{xn_listplot, xn_lineplot}]
GraphicsRow[{yn_listplot, yn_lineplot}]
```

Out[542]=



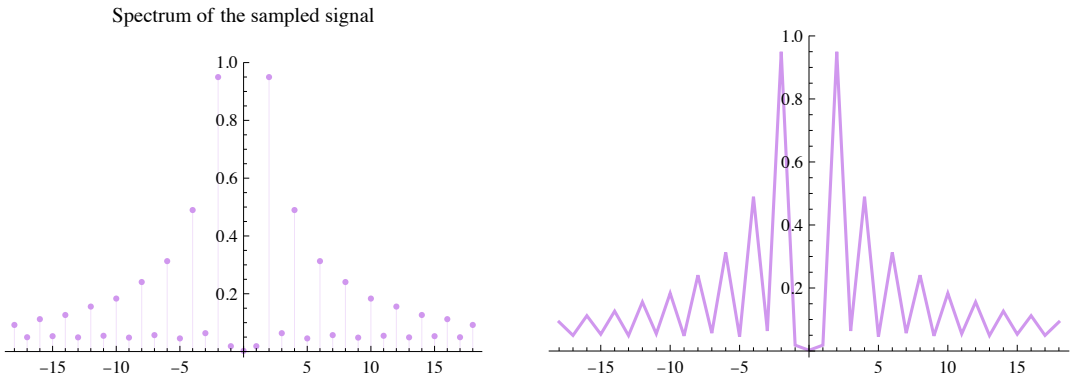
Out[543]=



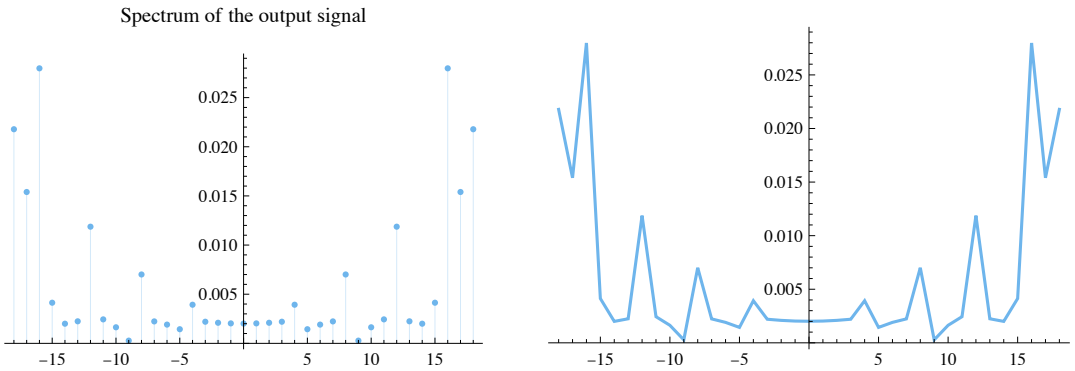
In[667]:=
```
xnF = Fourier[xn]; ynF = Fourier[yn];
FourierShift[a_List] :=
  Module[{d = Dimensions[a]}, RotateRight[a, Quotient[d, 2]]];
InverseFourierShift[a_List] :=
  Module[{d = Dimensions[a]}, RotateLeft[a, Quotient[d, 2]]];
xnFs = FourierShift[xnF]; ynFs = FourierShift[ynF];
```

In[741]:=
```
xn_freq_listplot = ListPlot[Abs[xnFs],
    DataRange → {-18, 18},
    PlotRange → All,
    PlotStyle → ColorData["Pastel"][0.1] ,
    Filling → Axis,
    PlotLabel → "Spectrum of the sampled signal"
   ];
xn_freq_lineplot = ListLinePlot[Abs[xnFs],
    DataRange → {-18, 18},
    PlotRange → All,
    PlotStyle → ColorData["Pastel"][0.1]
   ];
yn_freq_listplot = ListPlot[Abs[ynFs],
    DataRange → {-18, 18},
    PlotRange → All,
    PlotStyle → ColorData["Pastel"][1] ,
    Filling → Axis,
    PlotLabel → "Spectrum of the output signal"
   ];
yn_freq_lineplot = ListLinePlot[Abs[ynFs],
    DataRange → {-18, 18},
    PlotRange → All,
    PlotStyle → ColorData["Pastel"][1]
   ];
GraphicsRow[{xn_freq_listplot, xn_freq_lineplot}]
GraphicsRow[{yn_freq_listplot, yn_freq_lineplot}]
```

Out[745]=



Out[746]=



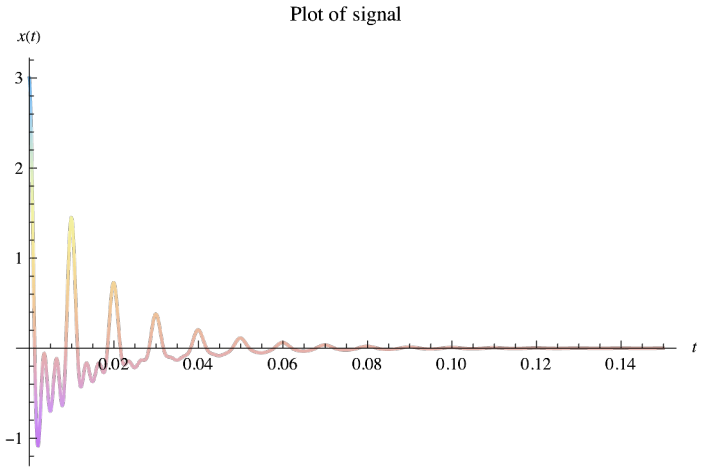
Part 2

In[•]:=
```
ClearAll["`*"]
```

In[•]:=
```
x[t_] = (e^(-50 t) Cos[2 π 100 t] + e^(-75 t) Cos[2 π 200 t] + e^(-100 t) Cos[2 π 300 t]) UnitStep[t];
```

In[•]:=
```
Plot[x[t], {t, 0, 0.15}, PlotRange → All,
  PlotLabel → "Plot of signal", AxesLabel → {Style["t", Italic],
    StringForm["`1`(`2`)", Style["x", Italic], Style["t", Italic]]},
  ColorFunction → Function[{x, y}, ColorData["Pastel"][y]], PlotStyle → Medium
]
```

Out[•]=



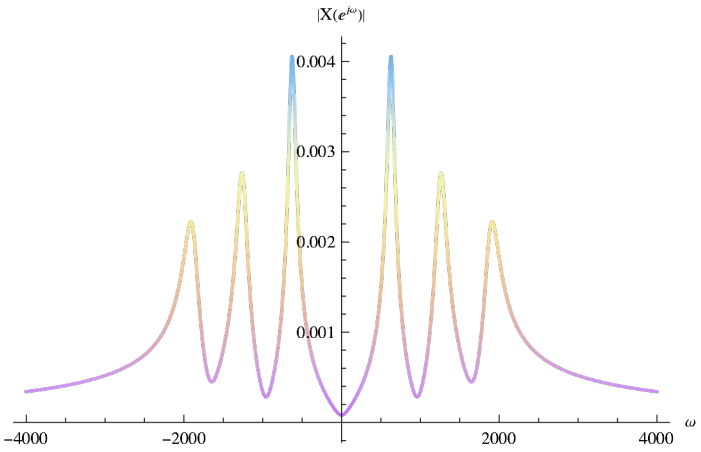
In[•]:=
```
X[ω_] = FourierTransform[x[t], t, ω]
```

Out[•]=

$$\frac{\frac{-50+i\,\omega}{-40\,000\,\pi^2+(50\,i+\omega)^2}+\frac{-75+i\,\omega}{-160\,000\,\pi^2+(75\,i+\omega)^2}+\frac{-100+i\,\omega}{-360\,000\,\pi^2+(100\,i+\omega)^2}}{\sqrt{2\,\pi}}$$

In[•]:=
```
Plot[Abs[X[ω]], {ω, -4000, 4000},
  AxesLabel → {"ω", "|X(e^(jω))|"}, PlotLabel → "Frequency spectrum \n",
  ColorFunction → Function[{x, y}, ColorData["Pastel"][y]], PlotStyle → Medium]
```

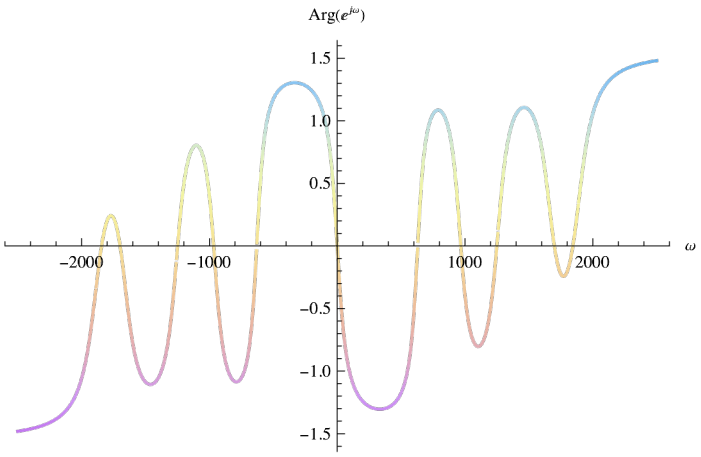
Frequency spectrum



Out[ ]=

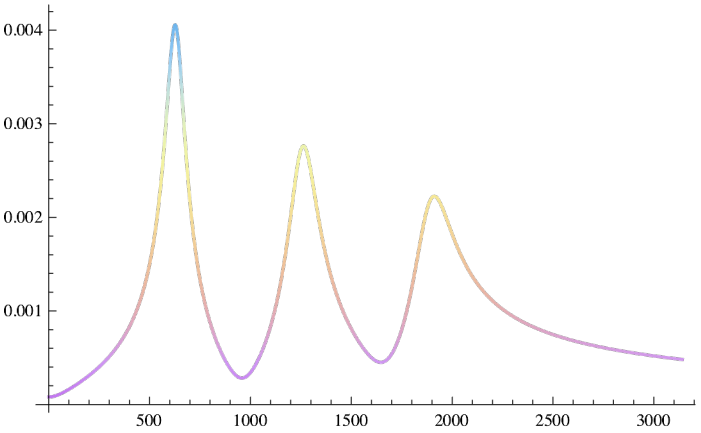
In[ ]:=
```
Plot[Arg[X[ω]], {ω, −2500, 2500},
 AxesLabel → {"ω", "Arg (e^jω)"}, PlotLabel → "Argument \n",
 ColorFunction → Function[{x, y}, ColorData["Pastel"][y]], PlotStyle → Medium]
```

Argument



Out[ ]=

In[ ]:=
```
ListLinePlot[Table[N[Abs[X[ω]]], {ω, 1000 π}],
 ColorFunction → Function[{x, y}, ColorData["Pastel"][y]], PlotStyle → Medium]
```



Out[ ]=

In[ ]:=
```
max = FindPeaks[Table[N[Abs[X[ω]]], {ω, 1000 π}]]
```

Out[ ]=
```
{{627, 0.00405668}, {1263, 0.00276316}, {1911, 0.00222517}}
```

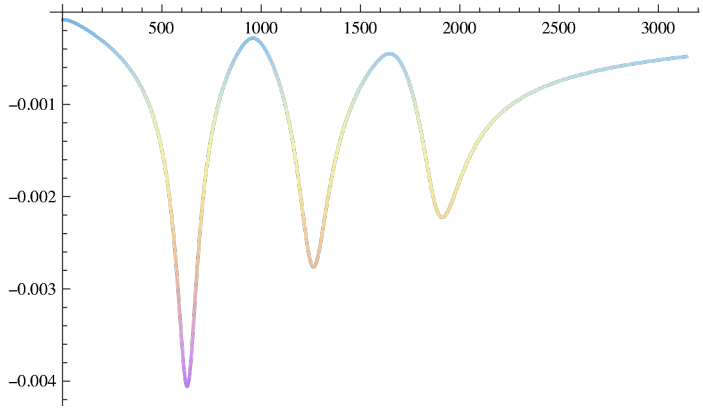
In[ ]:=
```
(* To get the minimum value we shift the graph around the x axis. We
  then use the function FindPeaks to find the maximum values. The
  maximum values will no be under the x axis and the should be over.  *)
```

In[ ]:=
```
min = FindPeaks [Table [N[-Abs[X[ω]]], {ω, 1000 π}]]
```

Out[ ]=
```
{{1, -0.0000802974}, {958, -0.000282227},
 {1647, -0.000450278}, {3141, -0.000481222}}
```

In[ ]:=
```
ListLinePlot [Table [N[-Abs[X[ω]]], {ω, 1000 π}],
  ColorFunction → Function [{x, y}, ColorData ["Pastel" ][y]], PlotStyle → Medium ]
```

Out[ ]=



In[ ]:=
```
(*To get the right minimum values we change the y values from negative to positive.  *)
```

In[ ]:=
```
minR = Abs [min ]
```

Out[ ]=
```
{{1, 0.0000802974}, {958, 0.000282227}, {1647, 0.000450278}, {3141, 0.000481222}}
```

In[ ]:=
```
ListLinePlot [Table [N[Abs[X[ω]]], {ω, 1000 π}],
  Epilog → {Blue, PointSize [0.015 ], Point [Join [minR, max ]]},
  PlotLabel → "Local Max and Min of the frequency",
  AxesLabel → {"ω [rad/s]", "|X(e^jω)|"},
  ColorFunction → Function [{x, y}, ColorData ["Pastel" ][y]],
  PlotStyle → Medium, PlotRange → Automatic ]
```

Out[ ]=