Chapter Two

Simple Sorting and Searching Algorithms

1. Simple Searching Algorithms

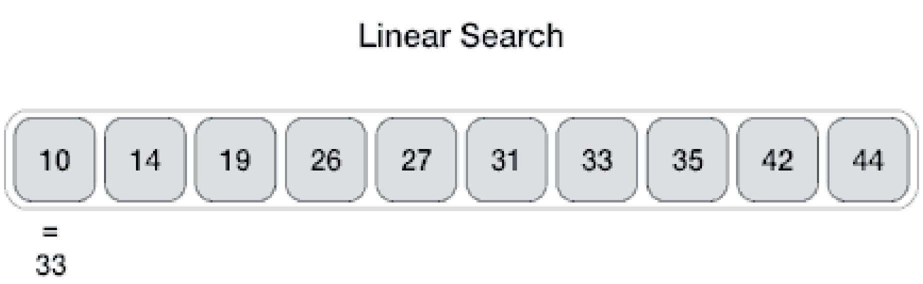
- Searching is a process of finding an element in a list of items or determining that the item is not in the list.
- The process of finding the location of an element within the data structure is called Searching.
  - Locate an element x in a list of distinct elements a1,a2,...an or determine that it is not in the list.
  - The solution to this search problem is the location of the item in the list that equals x and is 0 if x is not in the list.
- There are two simple Searching algorithms:

  A. Linear (Sequential) Search

  B. Binary Search

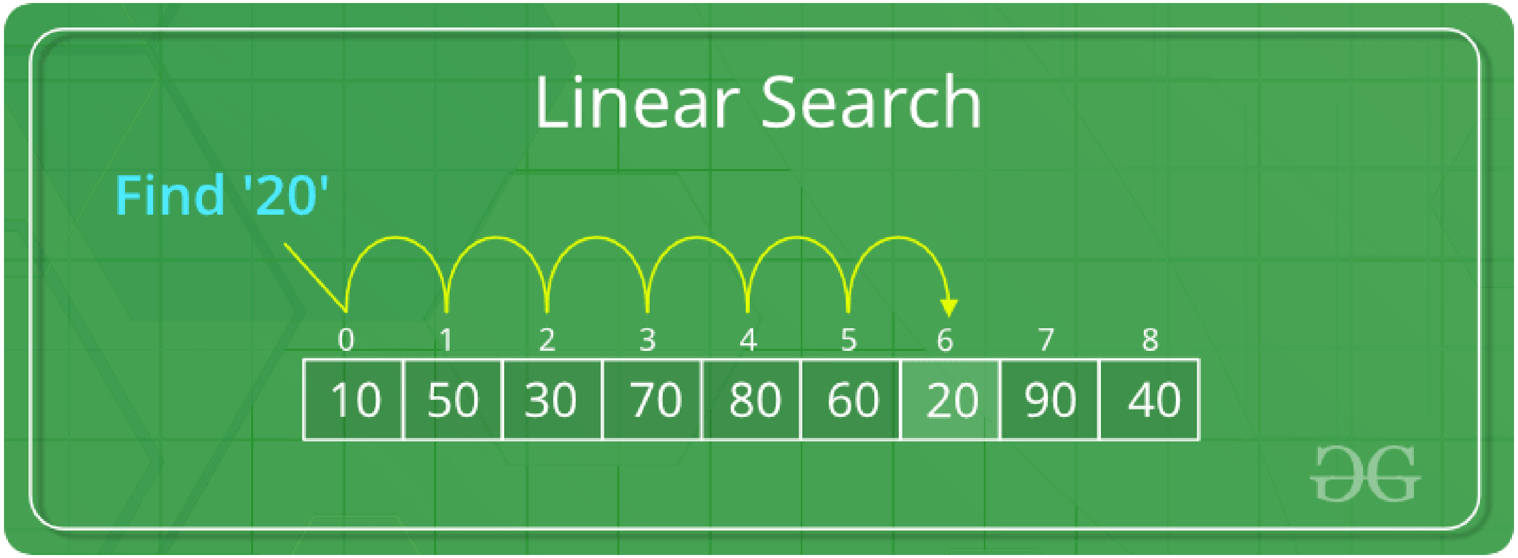
A. Linear Searching

- Linear search is a very simple search algorithm.
- This algorithm can be implemented on the unsorted list.
- In this type of search, a sequential search is made over all items one by one.
- Every item is checked and if a match is found then that particular item is returned, otherwise the search continues till the end of the data collection.
  - It compares the element to be searched with all the elements in an array, if the match is found, then it returns the index of the element else it returns -1.



A. Linear Searching: How it works:

- In a linear search, we start with top (beginning) of the list and compare the element at top with the key.
- If we have a match, the search terminates, and the index number is returned.
- If not, we go on the next element in the list.
- If we reach the end of the list without finding a match, we return -1.



A. Linear Searching: Algorithm:

Linear Search (Array A, Value x)

Step 1: Set i to 1

Step 2: if i > n then go to step 7

Step 3: if A[i] = x then go to step 6

Step 4: Set i to i + 1

Step 5: Go to Step 2

Step 6: Print Element x Found at index i and go to step 8

Step 7: Print element not found

Step 8: Exit

A. Linear Searching: Pseudocode:

```
procedure linear_search (list, value)
   for each item in the list
      if match item == value
         return the item's location
     else
         return the item is not found
      end if
   end for
end procedure
```

A. Linear Searching: Implementation

```
int linearSearch(int a[], int n, int val)
{
 for (int i = 0; i < n; i++)
    {
        if (a[i] == val)
        return i;
    }
  return -1;
}
```

A. Linear Searching: Implementation

```
int main() {
  int a[] = {69, 39, 29, 10, 56, 40, 24, 13, 51}; // given array
  int val = 56; // value to be searched
  int n = sizeof(a) / sizeof(a[0]); // size of array
  int res = linearSearch(a, n, val); // Store result
  cout<<"The elements of the array are - ";
  for (int i = 0; i < n; i++)
    cout<<a[i]<<" ";
  cout<<"\nElement to be searched is - "<<val;
  if (res == -1)
    cout<<"\nElement is not present in the array";
  else
    cout<<"\nElement is present at "<<res<<" position of array";
  return 0; }
```

A. Linear Searching: Complexity Analysis

- In Linear search,
  - the best case occurs when the element we are looking is located at the first position of the array.
  - the worst case occurs when the element we are looking is present at the end of the array.
  - The worst-case in linear search could be when the target element is not present in the given array, and we must traverse the entire array.

Case	Time Complexity
Best Case	O(1)
Average Case	O(n)
Worst Case	O(n)

- However, The time complexity of linear search is O(n) because every element in the array is compared only once.

B. Binary Searching

- A Binary search algorithm is the simplest algorithm that searches the element very quickly.
- This search algorithm works on the principle of divide and conquer approach.
- It is used to search the element from the sorted list.
  - The elements must be stored in sequential order or the sorted manner to implement the binary algorithm.
  - Binary search cannot be implemented if the elements are stored in a random manner.
- It is used to find the middle element of the list.

B. Binary Searching: How it works?

- In a binary search, we look for the key in the middle of the list. If we get a match, the search is over.
- If the key is greater than the element in the middle of the list, we make the top (upper) half the list to search.
- If the key is smaller, we make the bottom (lower) half the list to search.
- Repeat the above three steps until one element remains.
- If this element matches return the index of the element, else return -1 index. (-1 shows that the key is not in the list).

B. Binary Searching: Algorithm:

```
Binary_Search(a, lower_bound, upper_bound, val)
Step 1: set beg = lower_bound, end = upper_bound, pos = - 1
Step 2: repeat steps 3 and 4 while beg <=end
Step 3: set mid = (beg + end)/2
Step 4: if a[mid] = val
            set pos = mid
            print pos
            go to step 6
        else if a[mid] > val
            set end = mid - 1
        else
            set beg = mid + 1
       [end of if]
   [end of loop]
Step 5: if pos = -1
            print "value is not present in the array"
        [end of if]
Step 6: exit
```

- 'a' is the given array,
- 'lower_bound' is the index of the first ar ray element,
- 'upper_bound' is the index of the last ar ray element,
- 'val' is the value to search

B. Binary Searching: Example 1

- Let the elements of array are

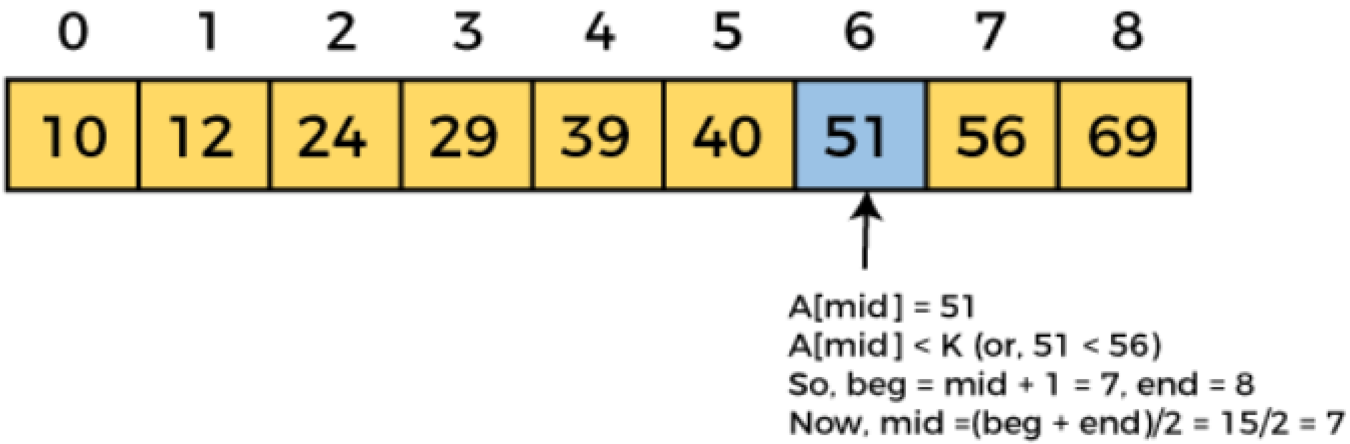
0	1	2	3	4	5	6	7	8
10	12	24	29	39	40	51	56	69

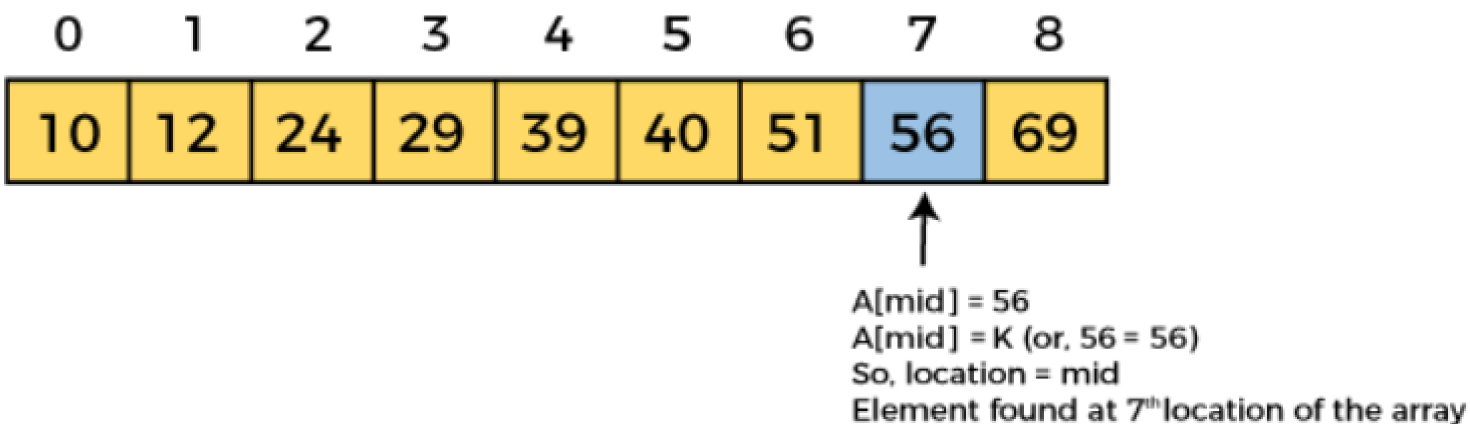
- Let the element to search is, K = 56
- We have to use the below formula to calculate the mid of the array :- mid = (beg + end)/2
- mid = (0 + 8)/2 = 4. So, 4 is the mid of the array.

0	1	2	3	4	5	6	7	8
10	12	24	29	39	40	51	56	69

A[mid] = 39
A[mid] < K (or,39 < 56)
So, beg = mid + 1 = 5, end = 8
Now, mid =(beg + end)/2 = 13/2 = 6

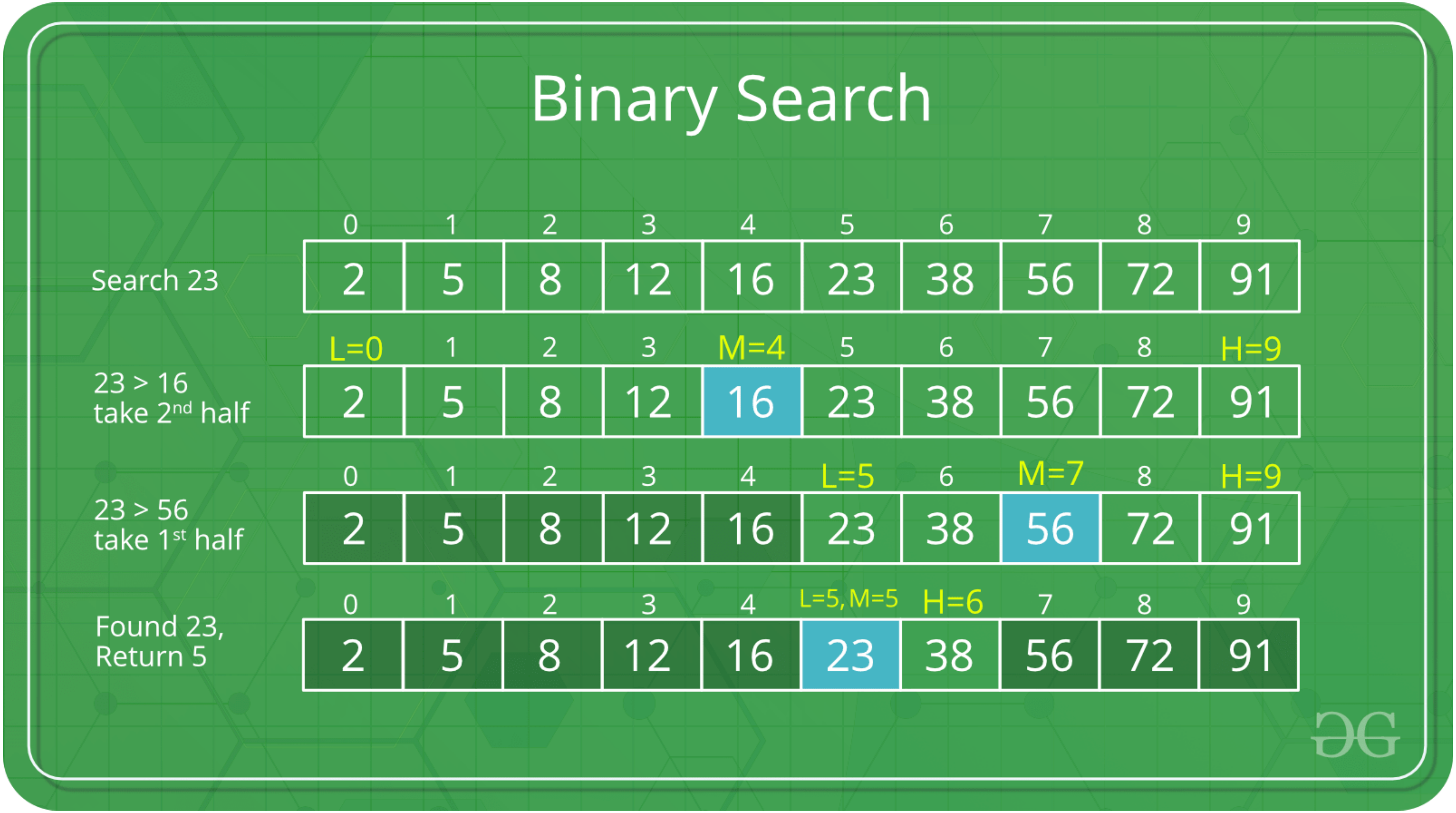
B. Binary Searching: Example 1





- Now, the element to search is found.
- So algorithm will return the index of the element matched.

B. Binary Searching: Example 2



B. Binary Searching: Implementation:

```
int binarySearch(int a[], int beg, int end, int val)   {
    int mid;
    if(end >= beg) {
        mid = (beg + end)/2;
/* if the item to be searched is present at middle */
        if(a[mid] == val) {
            return mid;
        }
/* if the item to be searched is smaller than middle, then it can only be in left subarray */
        else if(a[mid] < val) {
            return binarySearch(a, mid+1, end, val);
        }
  /* if the item to be searched is greater than middle, then it can only be in right subarray */
    else {
            return binarySearch(a, beg, mid-1, val);
        }
    }
    return -1;
}
```

B. Binary Searching: Implementation:

```
int main()
 {
  int a[] = {10, 12, 24, 29, 39, 40, 51, 56, 70}; // given array
  int val = 51; // value to be searched
  int n = sizeof(a) / sizeof(a[0]); // size of array
  int res = binarySearch(a, 0, n-1, val); // Store result
  cout<<"The elements of the array are - ";
  for (int i = 0; i < n; i++)
          cout<<a[i]<<" ";
  cout<<"\nElement to be searched is - "<<val;
  if (res == -1)
          cout<<"\nElement is not present in the array";
  else
          cout<<"\nElement is present at "<<res<<" position of array";
  return 0;
}
```

B. Binary Searching: Complexity Analysis

- In Binary search,
  - the best case occurs when the element to search is found in first comparison,
    - i.e., when the first middle element itself is the element to be searched.
  - the worst case occurs, when we must keep reducing the search space till it has only one element.

Case	Time Complexity
Best Case	O(1)
Average Case	O(logn)
Worst Case	O(logn)

- Therefore, the time complexity of binary search algorithm is O(logn).

2. Simple Sorting Algorithms

- Sorting algorithms are used to rearrange the elements in an array or a given data structure either in an ascending or descending order.
  - It is a process of reordering a list of items in either increasing or decreasing order.
- The comparison operator decides the new order of the elements.

2. Simple Sorting Algorithms

- Why do we need a sorting algorithm?
  - Sorting is the most important operation performed by computers.
  - Sorting is the first step in more complex algorithms.
  - An efficient sorting algorithm is required for optimizing the efficiency of other algorithms like binary search algorithm as a binary search algorithm requires an array to be sorted in a particular order, mainly in ascending order.
  - It produces information in a sorted order, which is a human-readable format.
  - Searching a particular element in a sorted list is faster than the unsorted list.

2. Simple Sorting Algorithms

- Following are some of the examples of sorting in real-life scenarios –
  - Telephone Directory – The telephone directory stores the telephone numbers of people sorted by their names, so that the names can be searched easily.
  - Dictionary – The dictionary stores words in an alphabetical order so that searching of any word becomes easy.

2. Simple Sorting Algorithms

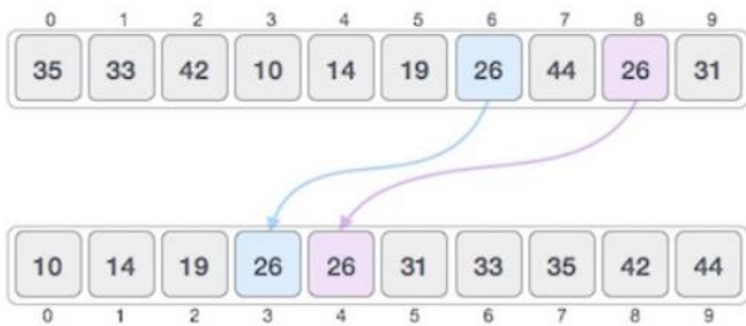
- Properties of sorting algorithms:
  A. In-place Sorting and Not-in-place Sorting
  B. Stable and Not Stable Sorting
  C. Adaptive and Non-Adaptive Sorting Algorithm

A. In-place Sorting vs Not-in-place Sorting

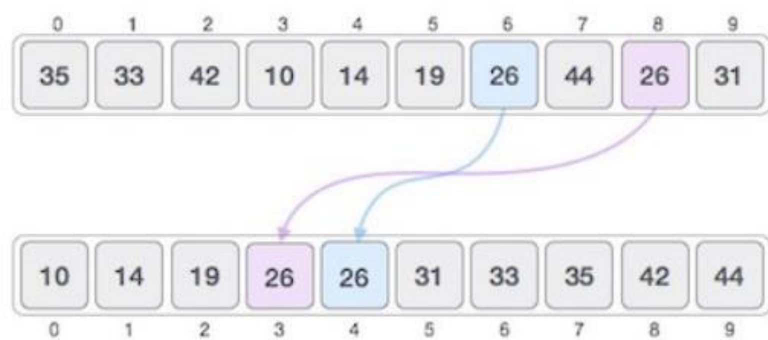
- Sorting algorithms may require some extra space for comparison and temporary storage of few data elements.
- In-place Sorting:
  - These algorithms <u>do not require any extra space</u> and sorting is said to happen in-place, or for example, within the array itself.
  - Example: Bubble sort algorithm
- Not-in-place Sorting:
  - These algorithms <u>require extra space</u> which is more than or equal to the elements being sorted.
  - Example: Merge-sort algorithm

B. Stable vs Not Stable Sorting

- **Stable Sorting:**
  - If a sorting algorithm, after sorting the contents, does not change the sequence of similar content in which they appear, it is called stable sorting.



- **Not-Stable Sorting:**
  - If a sorting algorithm, after sorting the contents, changes the sequence of similar content in which they appear, it is called unstable sorting.



C. Adaptive vs Non-Adaptive Sorting

- **Adaptive Sorting:**
  - A sorting algorithm is said to be adaptive, <u>if it takes advantage of already 'sorted' elements</u> in the list that is to be sorted.
    - i.e., while sorting if the source list has some element already sorted, adaptive algorithms will <u>take this into account and will try not to re-order them.</u>
- **Non-Adaptive Sorting:**
  - A non-adaptive algorithm is one which <u>does not take into account the elements which are already sorted.</u>
  - They try to force every single element to be re-ordered to confirm their sortedness.

2. Simple Sorting Algorithms

Simple sorting algorithms include:

A. Bubble Sorting
B. Selection Sorting
C. Insertion Sorting

A. Bubble Sort

- Bubble sort is a simple sorting algorithm that compares two adjacent elements and swaps them if they are not in the intended order.
  - It works on the repeatedly swapping of adjacent elements if they are not in the intended order.
- It is not suitable for large data sets as its average and worst-case complexity are of $O(n^2)$, where n is a number of items.
- Bubble sort is majorly used where -
  - complexity does not matter
  - simple and shortcode is preferred

A. Bubble Sort: Algorithm

- In the algorithm given below, suppose arr is an array of n elements. The assumed swap function in the algorithm will swap the values of given array elements.

```
begin BubbleSort(arr)
   for all array elements
      if arr[i] > arr[i+1]
         swap(arr[i], arr[i+1])
      end if
   end for
   return arr
end BubbleSort
```

A. Bubble Sort: How it works

I. Compare each element (except the last one) with its neighbor to the right.
   - If they are out of order, swap them
   - This puts the largest element at the very end
   - The last element is now in the correct and final place

II. Compare each element (except the last two) with its neighbor to the right.
   - If they are out of order, swap them
   - This puts the second largest element before last
   - The last two elements are now in their correct and final places

III. Continue as above until you have no unsorted elements on the left.

A. Bubble Sort: Example 1

- Let the elements of array are -

13	32	26	35	10

First Iteration:

- Sorting will start from the initial two elements. Let compare them to check which is greater.

13	32	26	35	10

- Here, 32 is greater than 13 (32 > 13), so it is already sorted. Now, compare 32 with 26.

13	32	26	35	10

- Here, 26 is smaller than 36. So, swapping is required. After swapping new array will look like –

13	26	32	35	10

- Now, compare 32 and 35.

13	26	32	35	10

A. Bubble Sort: Example 1

- Here, 35 is greater than 32. So, there is no swapping required as they are already sorted.
- Now, the comparison will be in between 35 and 10.

13	26	32	35	10

- Here, 10 is smaller than 35 that are not sorted. So, swapping is required. Now, we reach at the end of the array. After first pass, the array will be –

13	26	32	10	35

- Now, move to the second iteration.

A. Bubble Sort: Example 1

Second Iteration:

- The same process will be followed for second iteration.

13	26	32	10	35

13	26	32	10	35

13	26	32	10	35

- Here, 10 is smaller than 32. So, swapping is required. After swapping, the array will be -

13	26	10	32	35

13	26	10	32	35

- Now, move to the third iteration.

A. Bubble Sort: Example 1

Third Iteration:

- The same process will be followed for third iteration.

13	26	10	32	35

13	26	10	32	35

- Here, 10 is smaller than 26. So, swapping is required. After swapping, the array will be -

13	10	26	32	35

13	10	26	32	35

13	10	26	32	35

- Now, move to the fourth iteration.

A. Bubble Sort: Example 1

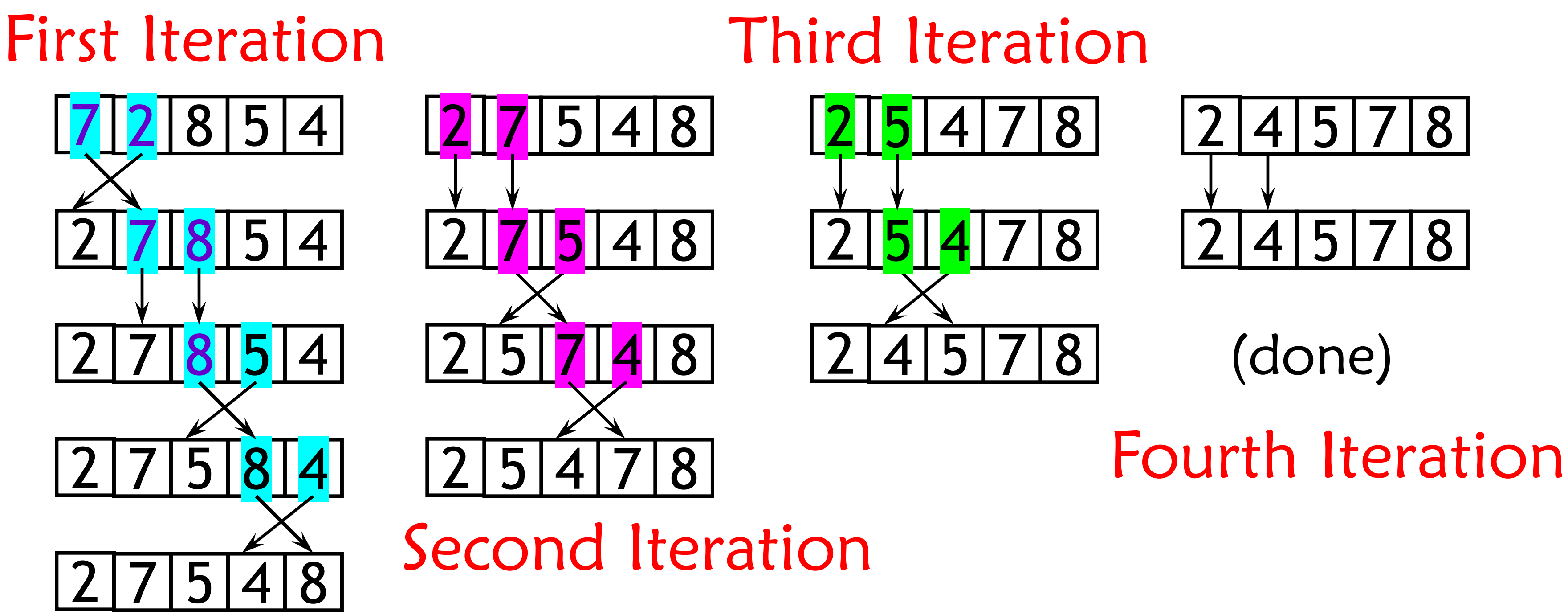
Fourth Iteration:

- Similarly, after the fourth iteration, the array will be -

10	13	26	32	35

- Hence, there is no swapping required, so the array is completely sorted.

A. Bubble Sort: Example 2



A. Bubble Sort: Implementation

```
void bubble(int a[], int n) {
int i, j, temp;
  for(i = 0; i < n; i++)
   {
     for(j = i+1; j < n; j++)
       {
          if(a[j] < a[i])
          {
              temp = a[i];
              a[i] = a[j];
              a[j] = temp;
          }
       }
   }
}
```

```
int main()
{
    int i, j,temp;
    int a[5] = {45, 1, 32, 13, 26};
    int n = sizeof(a)/sizeof(a[0]);
    cout<<"Before sorting:- \n";
    print(a, n);
    bubble(a, n);
    cout<<"\nAfter sorting:- \n";
    print(a, n);
return 0;
}
```

A. Bubble Sort: Complexity Analysis

- In bubble sort algorithm,
  - the **best case** time complexity occurs when there is no sorting required,
    - i.e. the array is already sorted.
  - the **worst case** time complexity occurs when the array elements are required to be sorted in reverse order.

Case	Time Complexity
Best Case	$O(n)$
Average Case	$O(n^2)$
Worst Case	$O(n^2)$

- Therefore, the time complexity of a bubble sort algorithm is $O(n^2)$.

B. Selection Sort

- Selection sort is a sorting algorithm that selects the smallest element from an unsorted list in each iteration and places that element at the beginning of the unsorted list.
  - i.e., the smallest value among the unsorted elements of the array is selected in every pass and inserted to its appropriate position into the array.
- It is the simplest algorithm.
- It is an in-place comparison sorting algorithm.

B. Selection Sort: How it works?

- In this algorithm, the array is divided into two parts,
  - first is sorted part, and another one is the unsorted part.
  - Initially, the sorted part of the array is empty, and unsorted part is the given array.
  - Sorted part is placed at the left, while the unsorted part is placed at the right.
- In selection sort, the first smallest element is selected from the unsorted array and placed at the first position.
- After that second smallest element is selected and placed in the second position.
- The process continues until the array is entirely sorted.

B. Selection Sort: Algorithm

SELECTION SORT(arr, n)

Step 1: Repeat Steps 2 and 3 for i = 0 to n-1

Step 2: CALL SMALLEST(arr, i, n, pos)

Step 3: SWAP arr[i] with arr[pos]

[END OF LOOP]

Step 4: EXIT

SMALLEST (arr, i, n, pos)

Step 1: [INITIALIZE] SET SMALL = arr[i]

Step 2: [INITIALIZE] SET pos = i

Step 3: Repeat for j = i+1 to n

if (SMALL > arr[j])

    SET SMALL = arr[j]

SET pos = j

[END OF if]

[END OF LOOP]

Step 4: RETURN pos

B. Selection Sort

- Selection sort is generally used when
  - A small array is to be sorted
  - Swapping cost doesn't matter
  - It is compulsory to check all elements

<u>Example 1:</u>

- Let the elements of array are –

12	29	25	8	32	17	40

- Now, for the first position in the sorted array, the entire array is to be scanned sequentially.
- At present, 12 is stored at the first position, after searching the entire array, it is found that 8 is the smallest value.

12	29	25	8	32	17	40

- So, swap 12 with 8. After the first iteration, 8 will appear at the first position in the sorted array.

8	29	25	12	32	17	40

B. Selection Sort: Example 1

- For the second position, where 29 is stored presently, we again sequentially scan the rest of the items of unsorted array.
- After scanning, we find that 12 is the second lowest element in the array that should be appeared at second position.

8	29	25	12	32	17	40

- Now, swap 29 with 12.
- After the second iteration, 12 will appear at the second position in the sorted array.
- So, after two iterations, the two smallest values are placed at the beginning in a sorted way.

8	12	25	29	32	17	40

B. Selection Sort: Example 1

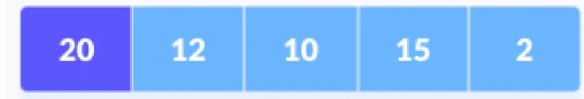
- The same process is applied to the rest of the array elements.
- Now, we are showing a pictorial representation of the entire sorting process.

8	12	25	29	32	17	40
8	12	25	29	32	17	40
8	12	17	29	32	25	40
8	12	17	29	32	25	40
8	12	17	29	32	25	40
8	12	17	25	32	29	40
8	12	17	25	32	29	40
8	12	17	25	32	29	40
8	12	17	25	29	32	40
8	12	17	25	29	32	40

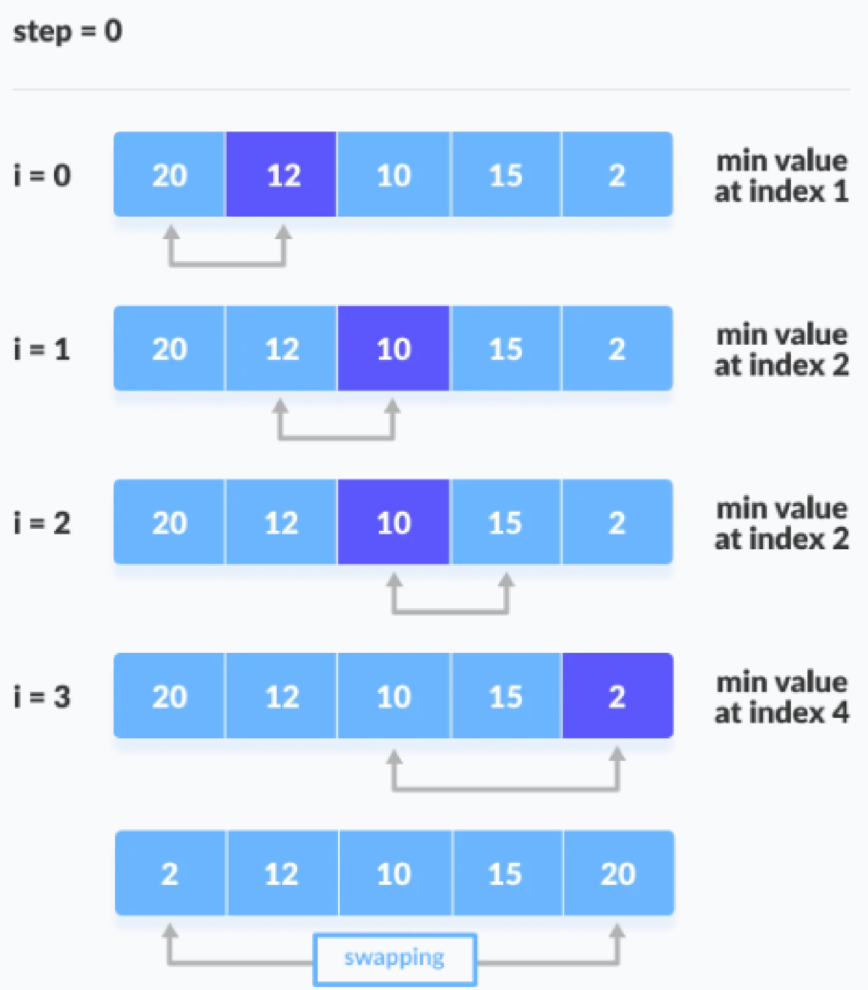
- Now, the array is completely sorted.

B. Selection Sort: Example 2

- Let the elements of array are –

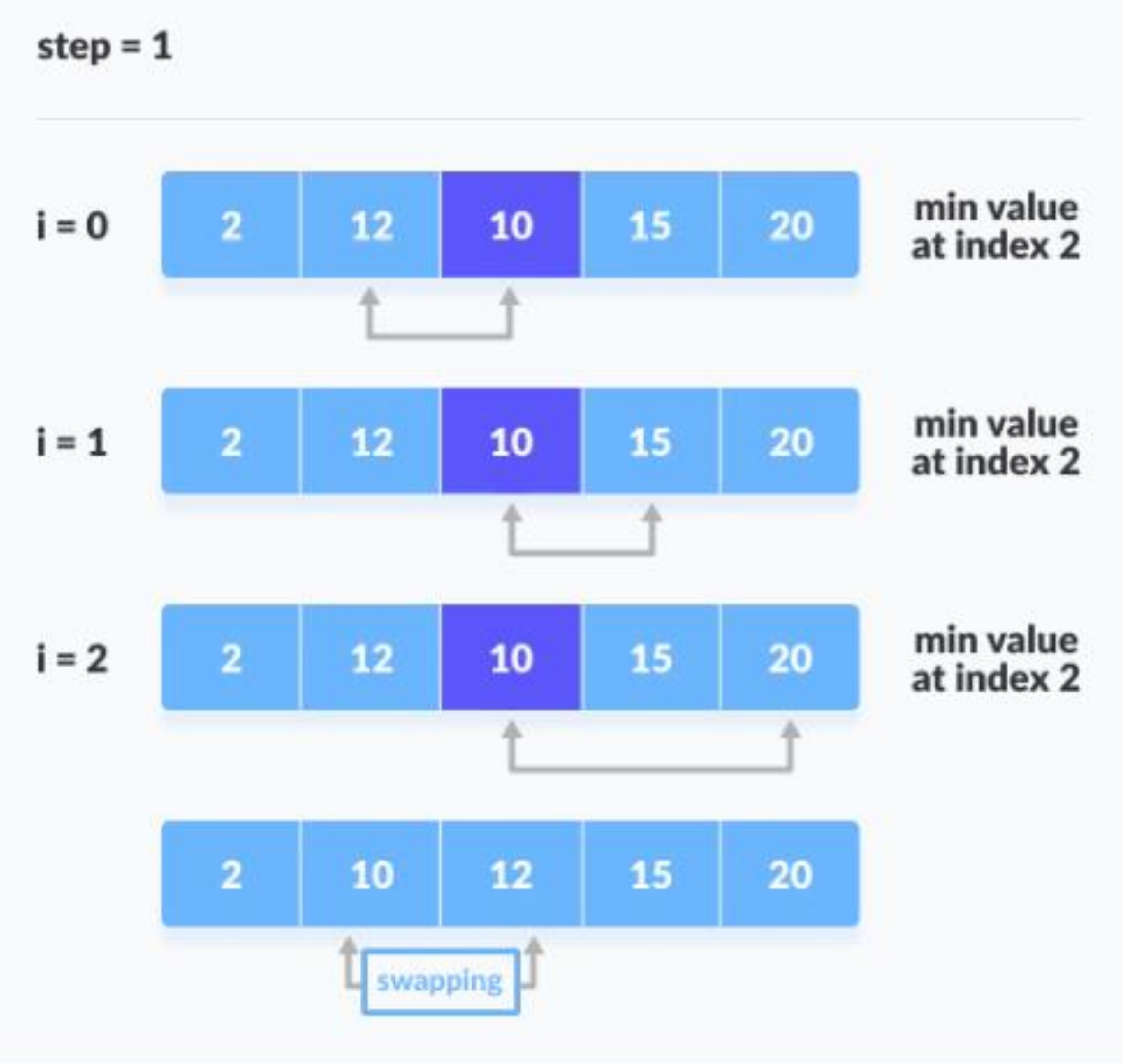


Select first element as minimum



The first iteration

B. Selection Sort: Example 2



The second iteration

B. Selection Sort: Example 2

The third iteration

The fourth iteration

B. Selection Sort: Implementation:

```
void selection(int arr[], int n)
{
    int i, j, small;
    for (i = 0; i < n-1; i++)// One by one move boundary of unsorted subarray
    {
        small = i; //minimum element in unsorted array

        for (j = i+1; j < n; j++)
        if (arr[j] < arr[small])
            small = j;
// Swap the minimum element with the first element
    int temp = arr[small];
    arr[small] = arr[i];
    arr[i] = temp;
    }
}
```

B. Selection Sort: Implementation:

```
int main()
{
    int a[] = { 80, 10, 29, 11, 8, 30, 15 };
    int n = sizeof(a) / sizeof(a[0]);
    cout<< "Before sorting array elements are - "<<endl;
    printArr(a, n);
    selection(a, n);
    cout<< "\nAfter sorting array elements are - "<<endl;
    printArr(a, n);

    return 0;
}
```

B. Selection Sort: Complexity Analysis

- In Selection sort algorithm,
  - the best case time complexity occurs when there is no sorting required,
    - i.e. the array is already sorted.
  - the average case time complexity occurs when the array elements are in jumbled order that is not properly ascending and not properly descending.
  - the worst case time complexity occurs when the array elements are required to be sorted in reverse order.
    - That means suppose you have to sort the array elements in ascending order, but its elements are in descending order.

Case	Time Complexity
Best Case	$O(n^2)$
Average Case	$O(n^2)$
Worst Case	$O(n^2)$

- Overall algorithm complexity is $O(n^2)$

C. Insertion Sort

- Insertion sort is a sorting algorithm that places an unsorted element at its suitable place in each iteration.
- The idea behind the insertion sort is that first take one element, iterate it through the sorted array.
- It is simple to use,
- It is not appropriate for large data sets as the time complexity of insertion sort in the average case and worst case is O(n2), where n is the number of items.
- Insertion sort is less efficient than the other sorting algorithms like heap sort, quick sort, merge sort, etc.

C. Insertion Sort: How it works?

- Insertion sort algorithm somewhat resembles Selection Sort and Bubble sort.
- Array is imaginary divided into two parts - sorted one and unsorted one.
- At the beginning, sorted part contains first element of the array and unsorted one contains the rest.
- At every step, algorithm takes first element in the unsorted part and inserts it to the right place of the sorted one.
- When unsorted part becomes empty, algorithm stops.

C. Insertion Sort: How it works?

Using binary search

- It is reasonable to use binary search algorithm to find a proper place for insertion.
- This variant of the insertion sort is called binary insertion sort.
- After position for insertion is found, algorithm shifts the part of the array and inserts the element.

- Insertion sort works by inserting item into its proper place in the list.

C. Insertion Sort: How it works?

- Insertion sort works similarly as we sort cards in our hand in a card game.
  - We assume that the first card is already sorted then, we select an unsorted card.
  - If the unsorted card is greater than the card in hand, it is placed on the right otherwise, to the left.
  - In the same way, other unsorted cards are taken and put in their right place.
  - This process is repeated until all the cards are in the correct sequence.
- A similar approach is used by insertion sort.
- Insertion sort is over twice as fast as the bubble sort and is just as easy to implement as the selection sort.

C. Insertion Sort: Algorithm

The simple steps of achieving the insertion sort are listed as follows -

Step 1 - If the element is the first element, assume that it is already sorted. Return 1.

Step 2 - Pick the next element and store it separately in a key.

Step 3 - Now, compare the key with all elements in the sorted array.

Step 4 - If the element in the sorted array is smaller than the current element, then move to the next element. Else, shift greater elements in the array towards the right.

Step 5 - Insert the value.

Step 6 - Repeat until the array is sorted.

C. Insertion Sort: Example 1

Let the elements of array are –

12	31	25	8	32	17

- Initially, the first two elements are compared in insertion sort.

12	31	25	8	32	17

- Here, 31 is greater than 12.
  - That means both elements are already in ascending order.
  - So, for now, 12 is stored in a sorted sub-array.

12	31	25	8	32	17

C. Insertion Sort: Example 1

- Now, two elements in the sorted array are 12 and 25. Move forward to the next elements that are 31 and 8.

12	25	31	8	32	17

12	25	31	8	32	17

- Both 31 and 8 are not sorted. So, swap them.

12	25	8	31	32	17

- After swapping, elements 25 and 8 are unsorted.

12	25	8	31	32	17

- So, swap them.

12	8	25	31	32	17

C. Insertion Sort: Example 1

- Now, elements 12 and 8 are unsorted.

12	8	25	31	32	17

- So, swap them too.

8	12	25	31	32	17

- Now, the sorted array has three items that are 8, 12 and 25. Move to the next items that are 31 and 32.

8	12	25	31	32	17

- Hence, they are already sorted. Now, the sorted array includes 8, 12, 25 and 31.

8	12	25	31	32	17

C. Insertion Sort: Example 1

- Move to the next elements that are 32 and 17.

8	12	25	31	32	17

- 17 is smaller than 32. So, swap them.

8	12	25	31	17	32

8	12	25	31	17	32

- Swapping makes 31 and 17 unsorted. So, swap them too.

8	12	25	17	31	32

8	12	25	17	31	32

- Now, swapping makes 25 and 17 unsorted. So, perform swapping again.

8	12	17	25	31	32

- Now, the array is completely sorted.

C. Insertion Sort: Implementation

```
void insert(int a[], int n) /* function to sort an array with insertion sort */
{
    int i, j, temp;
    for (i = 1; i < n; i++) {
        temp = a[i];
        j = i - 1;

        while(j>=0 && temp <= a[j])  /* Move the elements greater than
temp to one position ahead from their current position*/
        {
            a[j+1] = a[j];
            j = j-1;
        }
        a[j+1] = temp;
    }
}
```

C. Insertion Sort: Implementation

```
int main()
{
    int a[] = { 89, 45, 35, 8, 12, 2 };
    int n = sizeof(a) / sizeof(a[0]);
    cout<<"Before sorting array elements are - "<<endl;
    printArr(a, n);
    insert(a, n);
    cout<<"\nAfter sorting array elements are - "<<endl;
    printArr(a, n);

    return 0;
}
```

B. Insertion Sort: Complexity Analysis

- In Insertion sort algorithm,
  - the best case time complexity occurs when there is no sorting required,
    - i.e. the array is already sorted.
  - the average case time complexity occurs when the array elements are in jumbled order that is not properly ascending and not properly descending.
  - the worst case time complexity occurs when the array elements are required to be sorted in reverse order.
    - That means suppose you have to sort the array elements in ascending order, but its elements are in descending order.

Case	Time Complexity
Best Case	$O(n)$
Average Case	$O(n^2)$
Worst Case	$O(n^2)$

- Overall algorithm complexity is $O(n^2)$

Thank You

Question?