Chapter 3: Local Search-Based Algorithms
CS-616: Optimization Algorithms

Dr. Mahdi A. Khemakhem & Dr. Esraa M. Eldesouky

Department of Computer Science
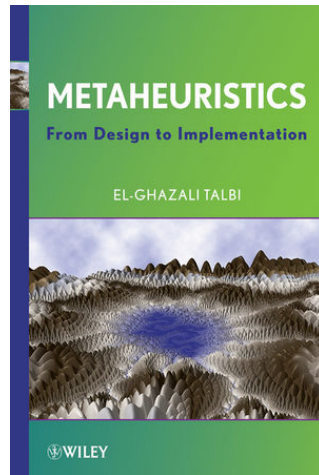*College of Computer Engineering and Science*
Prince Sattam bin Abdulaziz University

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Materials

- **Textbook:** "Metaheuristics: From Design to Implementation" by *El-Ghazali Talbi*
- ➡Read Chapter 2 (Sections 2.3, 2.6, 2.7, and 2.8) for this chapter's material

Outline

Outline

Local Search - Principle

Local Search (LS)[1] is likely the oldest and simplest S-Metaheuristic method.

- ▶ **It starts at a given initial solution.**
- ▶ **At each iteration, the heuristic replaces the current solution by a neighbor that improves the objective function.**
- ▶ **The search stops when all candidate neighbors are worse than the current solution, meaning a local optimum is reached.**
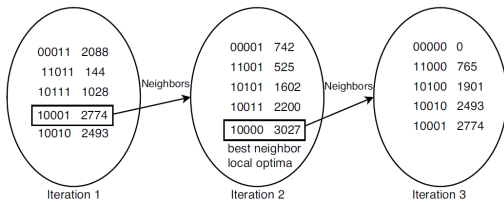
Local search process using a binary representation of solutions, a flip move operator, and the best neighbor selection strategy. The objective function is Max $Z = x^3 - 60x^2 + 900x$. The global optimal solution is $f(01010) = f(10) = 4000$, while the final local optima found is $s = (10000)$, starting from the solution $s_0 = (10001)$.

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[1] Also referred as hill climbing, descent, iterative improvement, and so on. In some literature, local search also refers to general S-metaheuristics.

Local Search - Behavior

- **For large neighborhoods**, the **candidate solutions may be a subset of the** **neighborhood.**
  - **The main objective** of this **restricted neighborhood strategy** is to **speed up** **the search.**
- **Variants of LS** may be distinguished **according** to the **order in which the** **neighboring solutions are generated** (*deterministic/stochastic*) and the **selection strategy** (*selection of the neighboring solution*).
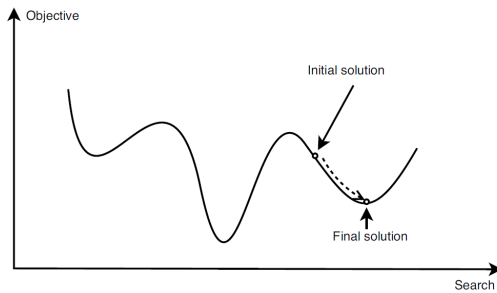
Local Search - General Schema (1/3)

▶ **Local Search** may be seen as a **descent walk** in the **graph** representing the search space.

▶ This **graph** may be defined by $G = (S, V)$, where S represents the **set of all feasible solutions of the search space** and V represents the **neighborhood relation**.
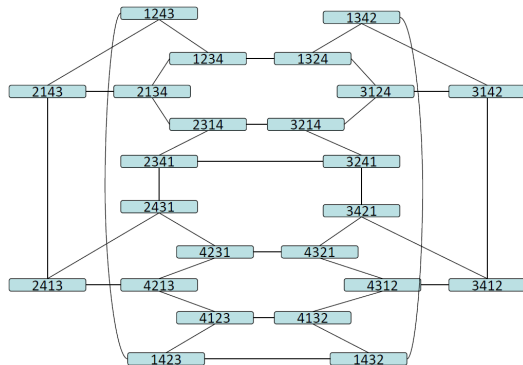
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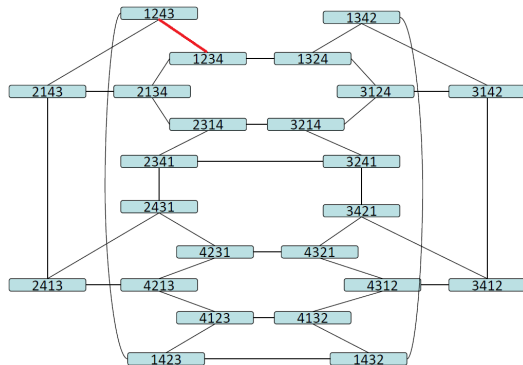- In the graph G, an edge (i, j) **will connect to any neighboring solutions** s_i and s_j.

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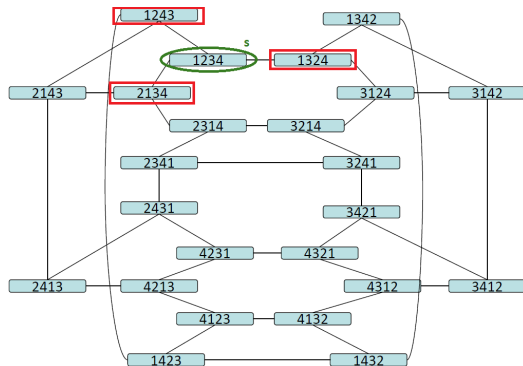
- **In the graph G, an edge (i, j) will connect to any neighboring solutions s_i and s_j.**

- **For a given solution s, the number of associated edges will be $|N(s)|$ (number of neighbors).**

Local Search - General Schema (2/3)

Algorithm 1 shows the general schema of a Local Search Metaheuristic.

Algorithm 1: Template of a Local Search Algorithm

Input: Initial solution s_0; /* Initial solution generated by a Greedy Algorithm   */
Output: Final solution found s; /* Local optima   */

1   $s = s_0$; /* s is the current solution that represents always the best-known solution   */
2   **while** *not termination_Criterion* **do**
3     $N(s) = $ **Generate_Neighbors**(s) ; /* Generation of candidate neighbors   */
4     $s' = $ **Select_Best**$(N(s))$; /* Select the best neighbor in $N(s)$ */
5   **if** $f(s')$ *is <u>not better</u> that* $f(s)$ **then** ; /* Stop the search if a local optima is found */
6
7   │   **Stop**;
8   **end**
9     $s = s'$; /* Update the current solution by the better neighbor   */
10 **end**

Local Search - General Schema (3/3)

From an **initial solution** s_0, the algorithm will **generate a sequence** $s_1, s_2, \ldots, s_k$ **of solutions** with the following characteristics:

- ▶ The size of the sequence k is unknown a priori.
- ▶ $s_{i+1} \in N(s_i), \forall i \in [0, k-1]$.
- ▶ $f(s_{i+1}) < f(s_i), \forall i \in [0, k-1]$.[2]
- ▶ s_k is a local optimum: $f(s_k) \leq f(s), \forall s \in N(s_k)$.

In addition to the definition of the initial solution and the neighborhood, **designing a local search algorithm** **has to address the selection strategy of the neighbor** that will determine the next current solution.

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[2]The problem is supposed to be a minimization problem.

Selection of the Neighbor (1/3)

Many strategies can be applied in the selection of a better neighbor:

1. **Best improvement (steepest descent):** the best neighbor is selected.
   - ▶ The neighborhood is evaluated in a fully deterministic manner.
   - ▶ Hence, the exploration of the neighborhood is exhaustive, and all possible moves are tried for a solution to select the best neighboring solution.
   - ▶ This type of exploration may be time-consuming for large neighborhoods.
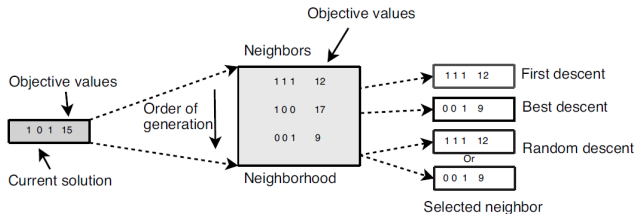
Selection of the Neighbor (2/3)

2 **First improvement:** chooses the first improving neighbor that is better than the current solution.

- ▶ Involves a **partial evaluation of the neighborhood**.
- ▶ In a cyclic exploration, the **neighborhood is evaluated in a deterministic way following a given order of generating the neighbors**.
- ▶ **In the worst case** (*i.e., when no improvement is found*), **a complete evaluation of the neighborhood is performed**.

3 **Random selection:** a random selection is applied to those neighbors improving the current solution.
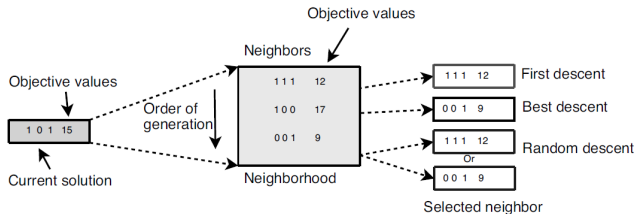
Selection of the Neighbor (3/3)

- ▶ A **compromise** in terms of **quality of solutions** and **search time** may consist in using:
  - ▶ the **first improvement** strategy *when the initial solution is randomly generated*,
  - ▶ the **best improvement** strategy *when the initial solution is generated using a greedy procedure.*
- ▶ **In practice**, on many applications, it has been observed that the *first improving strategy* **leads to the same quality of solutions** as the *best improving strategy* **while using a smaller computational time**.
  - ▶ Moreover, the *probability of premature convergence to a local optima* is **less important in the first improvement strategy**.
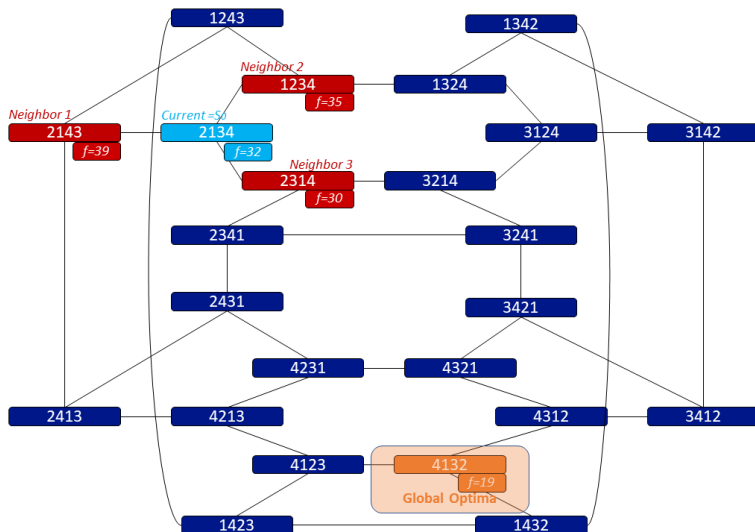
Local Optima Disadvantage (1/2)

- **In general, local search is a very easy method to design and implement and gives fairly good solutions very quickly.**
  - **This is why it is a widely used optimization method in practice.**
- **One of the main disadvantages of LS is that it converges toward local optima.**
  - **There is no means to estimate the relative error from the global optimum and the number of iterations performed may not be known in advance.**
- **Local search works well if there are not too many local optima in the search space or the quality of the different local optima is more or less similar.**
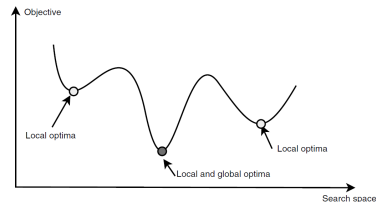
Local Optima Disadvantage (2/2)
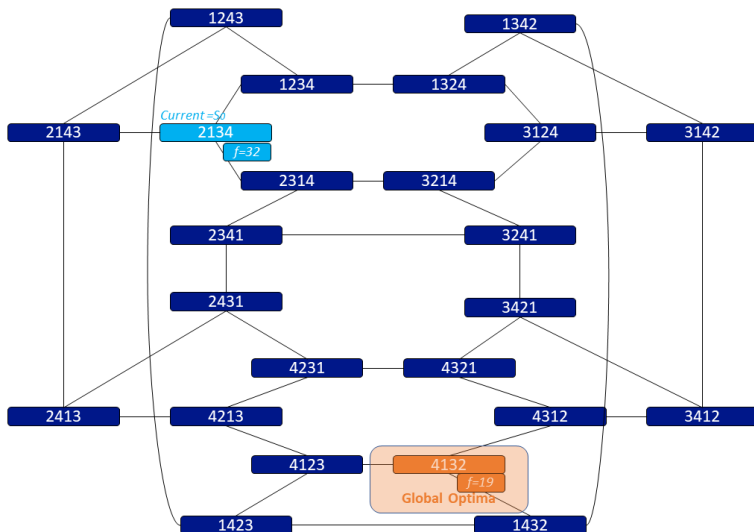
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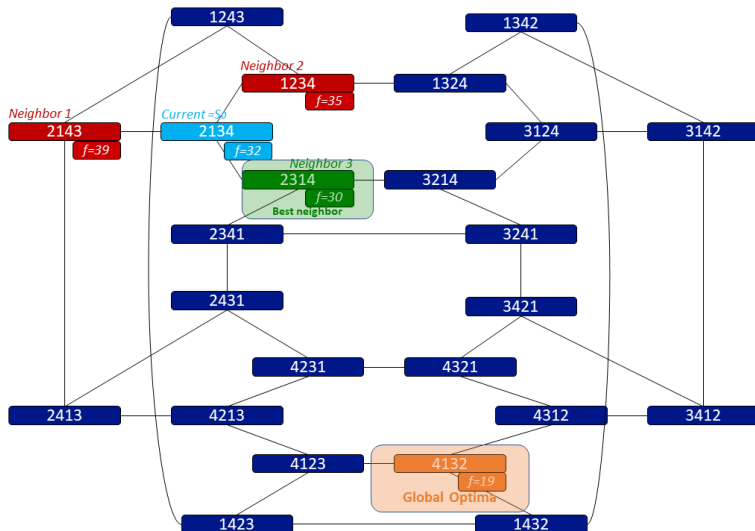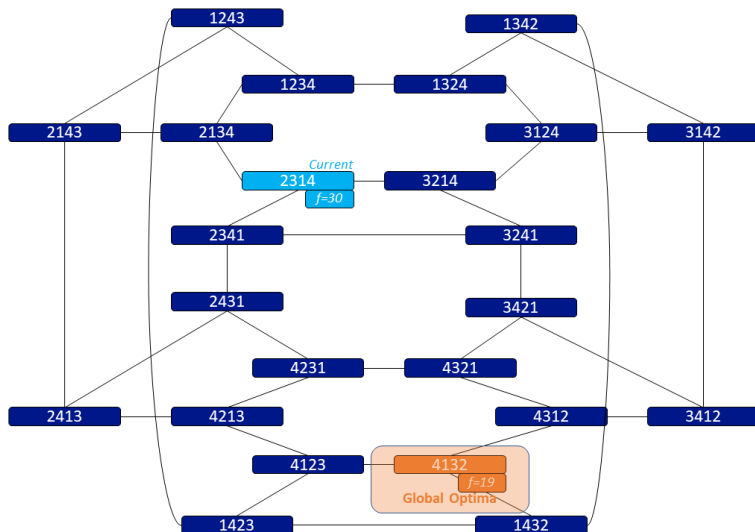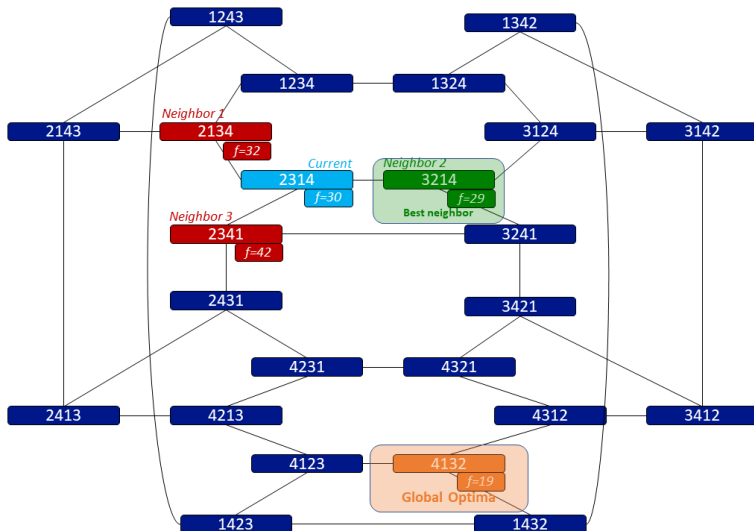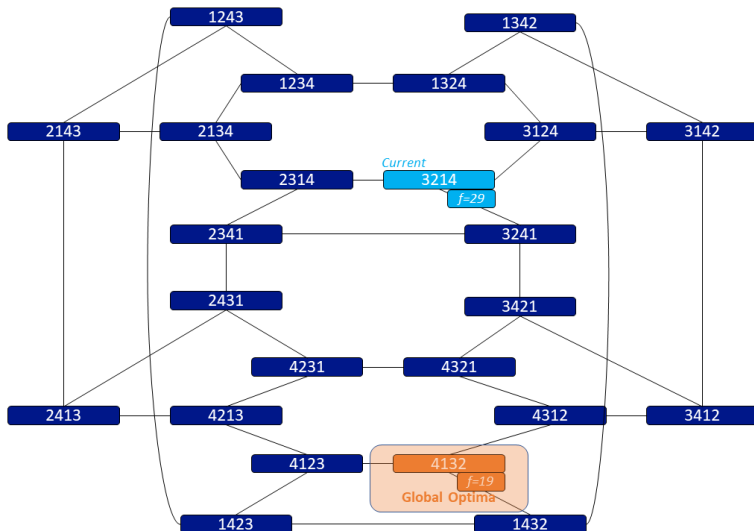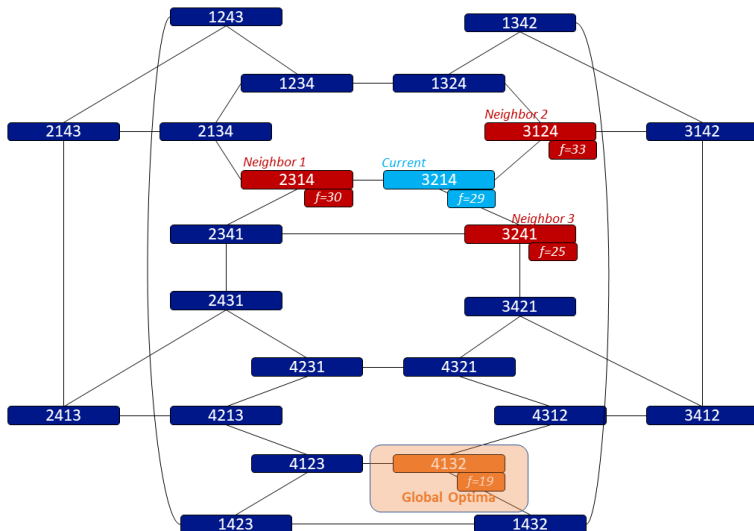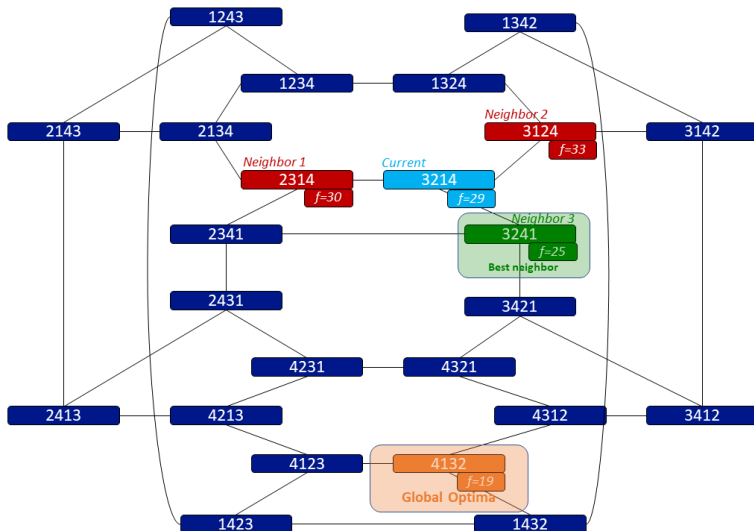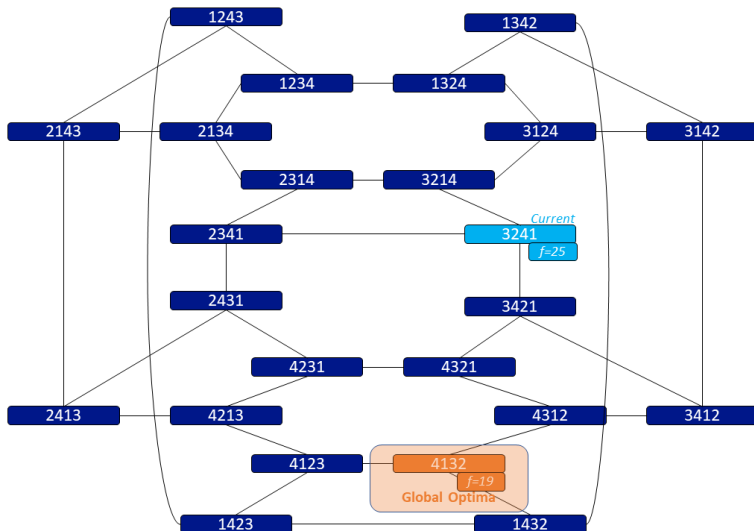
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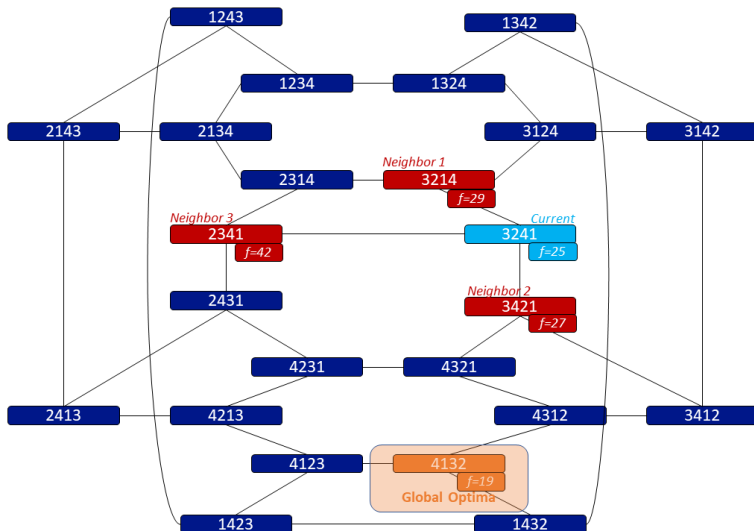
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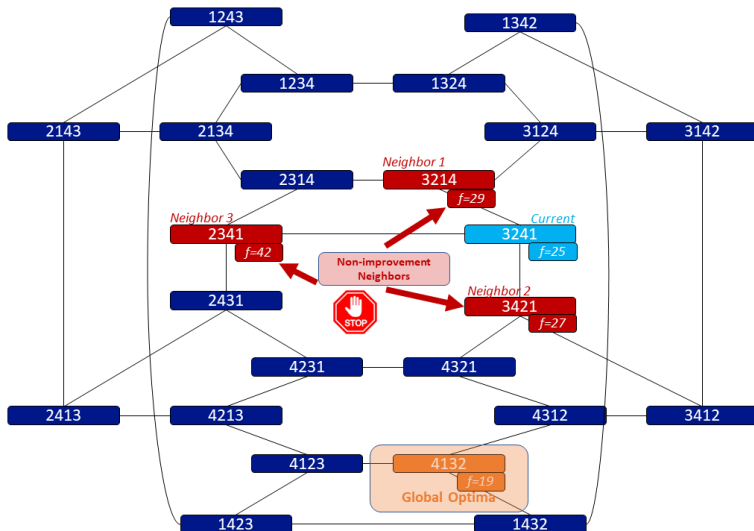
Escaping from Local Optima (1/2)

As the **main disadvantage of local search algorithms is the convergence toward local optima,** many **alternatives algorithms have been proposed to avoid becoming stuck at local optima. Four different families of approaches can be used to avoid local optima:**

1. **Iterating from different initial solutions:** applied in **Multi-start Local Search**, **Iterated Local Search** (see section 2), **GRASP**, and so forth.

2. **Accepting non-improving neighbors:** enable moves that degrade the current solution (it is possible to move out the basin of attraction of a given local optimum). **Simulated Annealing** and **Tabu Search** (see Chapter 3) are popular representative of this class of algorithms.

3. **Changing the neighborhood:** it consists in changing the neighborhood structure during the search. For instance **Variable Neighborhood Search** (see Section 3) uses these strategies.

4. **Changing the objective function or the input data of the problem:** perturb the input-data or the objective function or the constraints of the problem in order to diversify search space. For instance **Guided Local Search** (see Section 4).

Escaping from Local Optima (2/2)
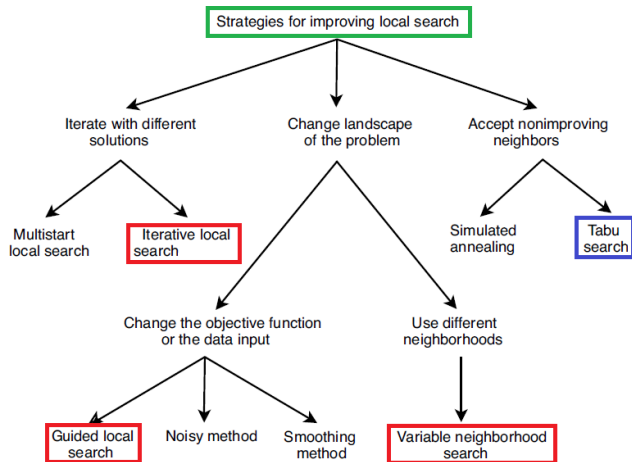
Outline

Iterated Local Search - Principle (1/2)

- ▶ The quality of the local optima obtained by a Local Search method depends on the initial solution.

- ▶ As we can generate local optima with high variability, Iterated Local Search (ILS)[3] may be used to improve the quality of successive local optima.

- ▶ In multi-start Local Search, the initial solution is always chosen randomly and then is unrelated to the generated local optima.

- ▶ ILS improves the classical multi-start Local Search by perturbing the local optima and reconsidering them as initial solutions.

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[3]Also known as Iterated Descent or Chained Local Optimization.

Iterated Local Search - Principle (2/2)

ILS is based on a simple principle described as follows:

▶ First, a Local Search is applied to an initial solution.

▶ Then, at each iteration, a perturbation of the obtained local optima is carried out.

▶ Finally, a Local Search is applied to the perturbed solution.

▶ The generated solution is accepted as the new current solution under some conditions.

▶ This process iterates until a given stopping criterion.
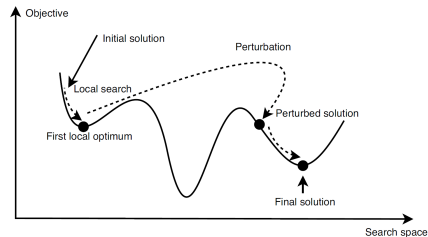
Algorithm 2 describes the ILS algorithm.

Iterated Local Search - General Schema

Algorithm 2 shows the general schema of a Iterated Local Search Metaheuristic.

Algorithm 2: Template of the Iterated Local Search Algorithm

Input: Initial solution s_0; /* Initial solution generated by a Greedy Algorithm */
Output: Best solution found s^*; /* Local optima */
1 $s^* = \textbf{Local\_Search}(s_0)$; /* Apply a given local search algorithm */
2 **repeat**
3     $s' = \textbf{Perturb}(s^*, \text{search history})$; /* Perturb the obtained local optima */
4     $s'_* = \textbf{Local\_Search}(s')$; /* Apply local search on the perturbed solution */
5     $\textbf{Accept}(s^*, s'_*, \text{search memory})$; /* Accepting criteria */
6 **until** *Stopping criteria staisfied*;

Iterated Local Search - Basic Elements

Three basic elements compose an Iterated Local Search:

► **Local_Search** : Any S-metaheuristic (deterministic or stochastic) can be used in the ILS framework such as a simple descent algorithm, Tabu Search, or Simulated Annealing. The search procedure is treated as a black box .

► **Perturb** : The perturbation operator may be seen as a large random move of the current solution. It should keep some part of the solution and perturb strongly another part of the solution.

► **Accept** : The acceptance criterion defines the conditions the new local optima must satisfy to replace the current solution.
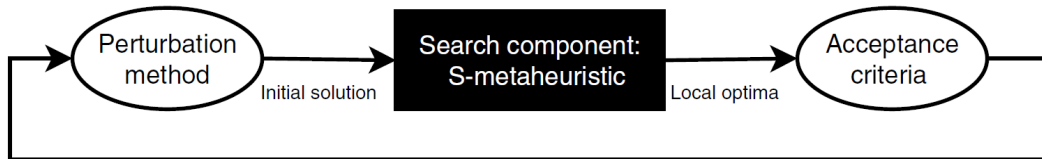
Perturbation Method (1/2)

▶ The first motivation of the ILS algorithm is based on the fact that the perturbation method must be more effective than a random restart approach.

▶ The length of a perturbation may be related to the neighborhood associated with the encoding or with the number of modified components.

  ▶ Too small perturbation may generate cycles in the search and no gain is obtained.

  ▶ Too large perturbation will erase the information about the search memory, and then the good properties of the local optima are skipped.

  ▶ The optimal length depends mainly on the landscape structure of the optimization problem.

▶ The move operator used in the perturbation may be of different nature from the neighborhood relation used in the local search procedure.

Perturbation Method (2/2)

Many biased **perturbation methods** can be designed according to the following criteria:

- ▶ **Fixed or variable perturbations:** The length of the perturbations applied to a local optima may be defined as follows:
  - ▶ **Static:** The length is fixed a priori before the beginning of the ILS search.
  - ▶ **Dynamic:** The length of the perturbation is defined dynamically without taking into account the search memory.
  - ▶ **Adaptive:** The length of the perturbation is adapted during the search according to some information about the search memory. Indeed, the optimal length will depend on the input instance and its structure. More information about the characteristics of the landscape may be extracted during the search.

- ▶ **Random or semi-deterministic perturbation:** The perturbation carried out on a solution may be a random one (memoryless) in which each move is generated randomly in the neighborhood. In semi-deterministic perturbations, the move is biased according to the memory of the search.

Acceptance Criteria

- The role of the acceptance criterion combined with the perturbation method is to control the classical trade-off between the intensification and the diversification tasks.
  - The extreme solution in terms of intensification is to accept only improving solutions in terms of the objective function (*strong selection*).
  - The extreme solution in terms of diversification is to accept any solution without any regard to its quality (*weak selection*).
- Many acceptance criteria that balance the two goals may be applied namely:
  - Probabilistic acceptance criteria
  - Deterministic acceptance criteria

Outline

Variable Neighborhood Search - Principle

▶ **The basic idea of the** Variable Neighborhood Search (VNS) **is** to successively explore a set of predefined neighborhoods **to** provide a better solution.

▶ VNS **explores** a set of neighborhoods **to get** different local optima **to** escape from local optima.

▶ VNS **exploits** the fact that **using various** neighborhoods in local search **may** generate different local optima **and that** the global optima is a local optima for a given neighborhood.

  ▶ Indeed, different neighborhoods generate different landscapes.

▶ **The VNS algorithm is based on the** Variable Neighborhood Descent (VND), **which is a** deterministic version of VNS.
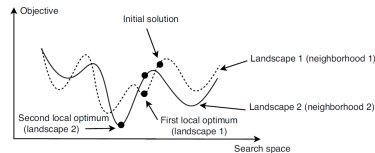
Variable Neighborhood Descent

- ► **VND** uses <u>successive neighborhoods</u> in descent to a local optimum.
  - ► First, one has to define a **set of neighborhood structures** $N_\ell(\ell = 1, \ldots, \ell_{max})$.
  - ► Let N_1 be the **first neighborhood** to use and x the **initial solution**.
    - ► If an **improvement of the solution** x in its **current neighborhood** $N_\ell(x)$ is **not possible**, the **neighborhood structure** is changed from N_ℓ to $N_{\ell+1}$.
    - ► **Otherwise**, **the neighborhood structure returns to the first one** $N_1(x)$ **to restart the search.**
- ► **This strategy will be effective if the different neighborhoods used are complementary in the sense that a local optima for a neighborhood** N_i **will not be a local optima in the neighborhood** N_j.
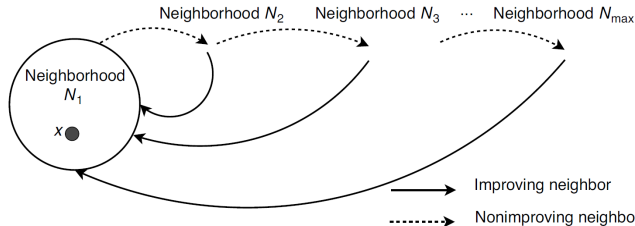
Variable Neighborhood Descent - General Schema

Algorithm 3 shows the general schema of the VND algorithm

Algorithm 3: Template of the Variable Neighborhood Descent Algorithm

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Input: A set of neighborhood structures N_ℓ for $\ell = 1, \ldots, \ell_{max}$.

Output: Best solution found

```
1  x = x_0 ;  /* Generate the initial solution using Greedy Algorithm        */
2  ℓ = 1;
3  while ℓ ≤ ℓ_max do
4  │    Find the best neighbor x' of x in N_ℓ(x);
5  │    if f(x') < f(x) then
6  │    │    x = x';
7  │    │    ℓ = 1;
8  │    else
9  │    │    ℓ = ℓ + 1;
10 │    end
11 end
```

Variable Neighborhood Descent - Features

▶ The **design** of the VND algorithm is **mainly related** to the **selection of neighborhoods** and the order of their application.

▶ The **complexity of the neighborhoods** in terms of their **exploration** and **evaluation** must be taken into account.

▶ The **larger are the neighborhoods**, the **more time consuming is the VND** algorithm.

▶ Concerning the **application order**, the most popular strategy is to rank the neighborhoods following the increasing order of their complexity (e.g., the size of the neighborhoods $|Nl(x)|$).

General Variable Neighborhood Search

VNS is a stochastic algorithm in which:

▶ First, a **set of neighborhood structures** $N_k(k = 1, \ldots, n)$ is defined.

▶ Then, each iteration of the algorithm is composed of three steps: shaking , local search , and move :

   1. At each iteration, an initial solution is shaked from the current neighborhood N_k. For instance, a solution x' is selected/generated randomly in the current neighborhood $N_k(x)$.

   2. A local search procedure is applied to the solution x' to generate the solution x''.

   3. The move step consists to:
      ▶ **replace** the current solution is replaced by the new local optima x'' if and only if a better solution has been found (i.e., $f(x'') < f(x)$). The same search procedure is thus restarted from the solution x'' in the first neighborhood N_1.
      ▶ move to the next neighborhood N_{k+1}, randomly generates a new solution in this neighborhood, and attempts to improve it if no better solution is found (i.e., $f(x'') \geq f(x)$).

Basic Variable Neighborhood Search - General Schema

Algorithm 4 presents the template of the Basic VNS algorithm

Algorithm 4: Template of the Basic Variable Neighborhood Search Algorithm.

Input: A set of neighborhood structures N_k for $k = 1, \ldots, k_{max}$ for **Shaking**.

Output: Best solution found

1   $x = x_0$; /* Generate the initial solution using Greedy Algorithm   */

2   **repeat**

3      $k = 1$;

4   **repeat**

5   **Shaking:** pick a random solution x' from the kth neighborhood $N_k(x)$ of x;

6         $x'' = $ **Local_Search**(x');

7   **if** $f(x'') < f(x)$ **then**

8             $x = x''$;

9   Continue to search with N_1, $k = 1$;

10   **else**

11            $k = k + 1$;

12   **end**

13   **until** $k = k_{max}$;

14 **until** *Stopping criteria statisfied*;

General Variable Neighborhood Search - General Schema

A more General VNS algorithm where the simple local search procedure is replaced by the VND algorithm can be found in Algorithm 5

Algorithm 5: Template of the General Variable Neighborhood Search Algorithm.

Input: A set of neighborhood structures N_k for $k = 1, \ldots, k_{max}$ for **shaking**, A set of neighborhood structures N_ℓ for $\ell = 1, \ldots, \ell_{max}$ for **Local_Search**.
Output: Best solution found

1 $x = x_0$; /* Generate the initial solution using Greedy Algorithm   */
2 **repeat**
3   **for** $k = 1$ **to** k_{max} **do**
4   **Shaking:** pick a random solution x' from the kth neighborhood $N_k(x)$ of x;
5   **for** $\ell = 1$ **to** ℓ_{max} **do** ; /* Local search by VND   */
6
7   Find the best neighbor x'' of x' in $N_\ell(x')$;
8   **if** $f(x'') < f(x')$ **then**
9                  $x' = x''$;
10   Continue to search with N_1, $\ell = 1$;
11   **else**
12                 $\ell = \ell + 1$;
13   **if** *local optimum is better than* x, $f(x'') < f(x)$ **then** ; /* Move or not   */
14
15             $x = x''$;
16   Continue to search with N_1, $k = 1$;
17   **else**
18             $k = k + 1$;
19 **until** *Stopping criteria satisfied*;

Variable Neighborhood Search - Features

- In addition to the design of a simple LS or a VND algorithm, the design of the VNS algorithm is mainly related to the selection of neighborhoods for the shaking phase.

- Usually, nested neighborhoods are used, where each neighborhood $N_k(x)$ contains the previous one $N_{k-1}(x)$: $N_1(x) \subset N_2(x) \subset \cdots \subset N_k(x), \forall x \in S$

- A compromise must be found between intensification of the search and its diversification through the distribution of work between the local search phase and the shaking phase.
  - An increased work in the local search phase will generate better local optima (more intensification), whereas an increased work in the shaking phase will lead to potentially better regions of the search space (more diversification).

- As in LS, VNS requires a small number of parameters.
  - For the shaking phase, the single parameter is the number of neighborhood structures k_{max}. If large values of k_{max} are used (i.e., very large neighborhoods are considered), VNS will be similar to a multi-start LS. For small values of k_{max}, VNS will degenerate to a simple LS algorithm.

Outline

Guided Local Search - Principle (1/2)

- **Guided Local Search (GLS)** is a deterministic **S-Metaheuristic** that has been mainly applied to combinatorial optimization problems.

- The **basic principle** of GLS is the **dynamic changing of the objective function** according to the already generated local optima.

- The **features of the obtained local optima** are used to transform the objective function.

- It **allows the modification of the landscape structure** to be explored by an S-metaheuristic **to escape from the obtained local optima.**
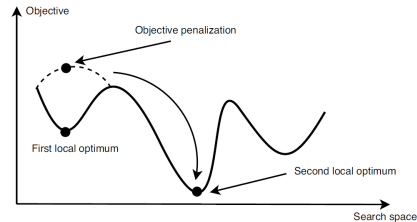
Guided Local Search - Principle (2/2)

- In **GLS**, a set of m features $ft_i (i = 1, \ldots, m)$ of a solution are defined.
- A solution feature defines a given characteristic of a solution regarding the optimization problem to solve. A cost c_i is associated to each feature i.
- When trapped by a local optima, the algorithm will penalize solutions according to some selected features.
  - To each feature i is associated a penalty p_i that represents the importance of the feature. The objective function f associated with a solution s is then penalized as follows:

  $$f'(s) = f(s) + \lambda \sum_{i=1}^{m} p_i \times I_i(s)$$

  where λ represents the weights associated with different penalties and, $I_i(s)$ an indicator function that determines whether the feature ft_i is present in the solution s:

  $$I_i(s) = \begin{cases} 1 & \text{if the feature } ft_i \in s \\ 0 & \text{otherwise.} \end{cases}$$

  All penalties p_i are initialized to 0.

Guided Local Search - General Schema

Algorithm 6 describes the template of the GLS algorithm.

Algorithm 6: Template of the Guided Local Search algorithm.

Input: S-Metaheuristic **LS**, λ, **Features** $ft_i(i = 1, \ldots, m)$, **Costs** $c_i(i = 1, \ldots, m)$.
Output: Best solution found

1   $s = s_0$; /* Generate the initial solution using Greedy Algorithm   */
2   $p_i = 0, \forall i \in \{1, 2, \ldots, m\}$; /* Penalties initialization   */
3   **repeat**
4   **Apply** a S-metaheuristic **LS** ; /* Let $s*$ the final solution obtained   */
5   **for** *each feature i of s* do*
6       $u_i = \frac{c_i}{1+p_i}$; /* Compute its utility   */
7   **end**
8     $j = \max_{i=1,\ldots,m}(u_i)$; /* Compute the maximum utilities   */
9     $p_j = p_{j+1}$; /* Change the objective function by penalizing the feature j */
10 **until** *Stopping criteria satisified*;

Suitable Features Identification

The application of GLS to a given optimization problem requires the identification of suitable features of the solutions.

The choice of features depends mainly on the model associated with the optimization problem.

For example:

▶ **Routing problems:** In routing problems such as the Traveling Salesman Problem and the Vehicle Routing Problem, a feature may be associated with the presence of an edge (a, b) in the solution. The cost corresponds to the distance (or travel time) associated with the edge.

▶ **Assignment problems:** In assignment problems, such as the generalized assignment problem and location facility problems, a solution represents an assignment of a given number of objects to a set of locations. A feature may be represented by the pair (i, k), where i represents an object and k represents a specific location.

The End