King Fahd University For Petroleum and Minerals

PHYS410 Project

The Quantum Measurement Problem And The Collapse of The Wavefunction

---

Authors:
Mohammed Alsadah

Submitted to:
Dr. Ayman El-Said

Date:
December 20, 2024

Contents

The Quantum Measurement Problem And The Collapse of The Wavefunction

Mohammed Alsadah
King Fahd University For Petroleum and Minerals

December 20, 2024

Abstract

The description of the wavefunction offers a probabilistic point of view for predicting observables in quantum mechanics. The Schrodinger equation is deterministic, for if we know the initial state of the system we can predict how it will evolve over time. When the system is observed, the evolution of the wavefunction stops and the superposition state collapses into one outcome. Different interpretations for what a measurement is, are being put fourth by many physicists, but no answer is yet found. In this report we will look at projective measurements and we will examine one of the interpretations that was shown to be indefensible when tested experimentally.

1 Introduction

The theory of quantum mechanics is considered to be one of the most successful theories in the entirety of sciences. At a practical level, quantum mechanics yields excellent predictions that are consistent with experiments, but at the theoretical or philosophical level, there is a problem with the theory at its foundations. Quantum mechanics poses challenges on how to reinterpret measurements in a way different from the interpretation of measurements made on a classical system. One can define a measurement to be the process by which probing the state of the system yields a numerical quantity. Now, any measurement involves some interaction between the measuring device or apparatus and the system to be measured. For example to observe an object that is away from you, you have to shine light on it and from the reflected beam you will be able to observe it. Another example is when you try to measure the electric field generated by some charged object. The way to measure such a field is to put a test charge that can feel that electric field, but that will mean that the test charge itself will create its own field and affect that charged object. In these classical examples, we can see that gauging the state of the system requires altering its state a little bit. In the classical world, such alteration is often negligible and the state of the system is preserved after the measurement. On the other hand, quantum systems are affected by the apparatus-system interaction and the state of the system is usually altered after the measurement and hence we cannot speak of its state before observing it. Let ψ be some state of the system before the measurement (which we are ignorant of what it is) and let ϕ be some other possible configuration of the system, then the probability to find the system in state ϕ is

$$P = |\langle\phi|\psi\rangle|^2 \tag{1}$$

This is what quantum mechanics predicts for a system to transition from one state ψ to another ϕ. The transition happens when all the possibilities in which the system exhibits collapse into one outcome. This is the so called **The collapse of the wavefunction**. This report will be entirely devoted to discussing the interpretations of quantum measurements and the idea of the collapse of the wavefunction.

2 What is a measurement and what is an observer?

There are mainly two types of measurements in quantum mechanics. *The projective* or *Von Neumann measurement* and *Generalized measurements.* We will discuss only the former. Consider an observable A that is Hermitian or self-adjoint. This operator acts on the space $\mathcal{H}$ consisting of the states of a quantum system. If we assume that the operator A has an orthonormal eigenvectors $\{\varphi_n\}$ such that $A|\varphi_n\rangle = a_n|\varphi_n\rangle$ where $\{a_n\}$ is the spectrum (eigenvalues) of A, then we can write A using the **spectral decomposition** as $A = \sum_n a_n |\varphi_n\rangle\langle\varphi_n|$ where $P_n = |\varphi_n\rangle\langle\varphi_n|$ is the projection operator. We will assume that before the measurement, the system is in state $|\psi\rangle$. The probability that the observable A is measured to be a_n is

$$P(a_n) = \langle\psi| P_n |\psi\rangle = \langle\psi|\varphi_n\rangle\langle\varphi_n|\psi\rangle = |\langle\varphi_n|\psi\rangle|^2 \tag{2}$$

After the measurement the system will be in a new state ψ_{new}

$$|\psi_{new}\rangle = \frac{P_n|\psi\rangle}{\langle\psi| P_n |\psi\rangle^2} = \frac{\langle\varphi_n|\psi\rangle}{|\langle\varphi_n|\psi\rangle|}|\varphi_n\rangle \tag{3}$$

equation 2 and 3 are the so called **Born rule**[1].

An immediate measurement (before the system is able to interact with the environment and change its state) will certainly yield a_n to see this we calculate $P(a_n)$ again

$$P(a_n) = \langle\varphi_n| P_n |\varphi_n\rangle = \langle\varphi_n|\varphi_n\rangle\langle\varphi_n|\varphi_n\rangle = 1 \tag{4}$$

We can see that a projective measurement is one where the interaction of the apparatus with the system yield an outcome that is the eigenvalue of a hermitian observable and the new state of the system is a projection of the old state on the corresponding eigenvector of the observable.

If we look at an experiment where we make a projective measurement, we can take the Stern-Gerlach experiment in which we want to measure the spin of an electron. This quantum system has two states only, either spin up or spin down. Let our hilbert space $\mathcal{H} = \mathbb{C}^2$, then any state can be represented by

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \tag{5}$$

where

$$|\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad |\downarrow\rangle = \begin{pmatrix}1\\0\end{pmatrix} \tag{6}$$

and $\alpha, \beta \in \mathbb{C}$, $|\alpha|^2 + |\beta|^2 = 1$. The observable $\hat{S}_z$ is the spin angular momentum in the z-axis. It has eigenvalues $\hbar/2$ and $-\hbar/2$ corresponding to spin up and spin down respectively. From the spectral decomposition of $\hat{S}_z$ we can write in the following way

$$\hat{S}_z = \frac{\hbar}{2}|\uparrow\rangle\langle\uparrow| - \frac{\hbar}{2}|\downarrow\rangle\langle\downarrow| = \begin{pmatrix}\frac{\hbar}{2} & 0\\ 0 & -\frac{\hbar}{2}\end{pmatrix} \tag{7}$$

Now, the probability to get spin up or $\hbar/2$ is

$$P\left(\frac{\hbar}{2}\right) = |\langle\uparrow|\psi\rangle|^2 = |\alpha|^2 \tag{8}$$

and the new state if the outcome is $\hbar/2$ will be

$$|\phi\rangle = \frac{\langle\uparrow|\psi\rangle}{|\langle\uparrow|\psi\rangle|} = \frac{\alpha}{|\alpha|}|\uparrow\rangle \tag{9}$$

We conclude this section by stating what a measurement is and what is the role of an observer. A measurement is the process in which an interaction of the measuring device and the measured system yields a pre-existing quantity[2]. This definition is somewhat challenged in quantum mechanics since we do not know if our measurements themselves are creating these values or attributes because quantum systems are too sensitive that the act of measurement itself may change its state. The role of an observer in quantum mechanics is to record or register the outcomes of an experiment, thereby collapsing the wavefunction. An observer can be any object that collapses the wavefunction (and it does not matter whether the observer is a conscious being or not like some physicists thought). This means that a Geiger counter or a photomultiplier can be considered as observers that destroy the interference pattern in a double-slit experiment and thus collapse the superposition of the wavefunction.

3 The measurement problem and the wavefunction collapse

The problem with measurements in quantum mechanics is the discrepancy they suffer between the many possibilities predicted by the theory and the random indeterministic outcome found upon a measurement. This is the so-called *quantum measurement problem.* Many theories or *interpretations* appeared to explain the peculiarities of quantum mechanics and tried to get rid of some unappealing aspects to the theory like the collapse of the wavefunction or the uncertainty principle for non-commuting observables. The most famous out of all of these interpretations is the *Copenhagen interpretation* which holds that the wavefunction collapse is an essential aspect of the quantum theory. This view is held by most physicists. Other interpretations like *local or nonlocal hidden-variable theories* which suggest that the description of the wavefunction is incomplete and attempts to add some variables to the theory to fix it. It turns out that local hidden-variable theories are not compatible with quantum mechanics is we will see later in this article. There are actually many interpretations of quantum mechanics to this day (e.g. the many-worlds interpretation and objective-collapse theory) and none of them were validated experimentally and henceforth the problem of quantum measurements remains *open.*

4 The EPR paradox, locality and hidden-variables

In quantum mechanics, non-commuting operators are called **incompatible**. The incompatible operators always satisfy an *uncertainty relation.* For instance, the most famous example is the position and momentum operators. Since these operators do not commute $[\hat{x}, \hat{p}] = i\hbar$, then

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \tag{10}$$

This means that we cannot know both the position and momentum of a system to any degree of precision that we please, instead there is a lower bound on the standard deviation of the distribution of the product of each variable. This is in contrast to classical theory where the uncertainty is mainly due to measuring device, and improving the equipment reduces the error in the measurement. In addition, measurement of an observable in quantum mechanics, destroys or corrupt the measurement of another observable if the two are incompatible.

In 1935, Einstein, Podolsky and Rosen published an article called "*can quantum mechanical description of physical reality be considered complete*"[3]. The article was published when Einstein was at the Institute for Advanced Study. In it, they argued that a **complete** theory is one where measurable quantities can be predicted with certainty i.e. With probability equal to 1 without disturbing the system (that is without performing any measurement). In their paper, they discussed the example of two parts of a system that is entangled. An *entangled system* is a system in which its parts (for instance the particles constituting the system) had interacted for some time and then they were separated and there is no interaction between them any longer. The terminology of *quantum entanglement* was not actually coined at the time when the paper was published. Einstein and his colleagues concluded in this article that measuring the momentum of one particle will enable us to know the momentum of the other particle with certainty and without performing any measurement. On the other hand, measuring the position of one particle will allows to predict the position of the other particle without performing any measurement of its position. For our practical purposes, we will discuss a variant of their thought experiment due to *David Bohm*[4]. Bohm's variant of the EPR paradox was to consider the spin of the particles instead of their positions or their momenta.

To begin with, consider the decay of the pion π^0 into an electron and a positron

$$\pi^0 \to e^- + e^+ \tag{11}$$

consider that this pion is at rest and carries no orbital angular momentum. This is an entangled system and the wavefunction of this system is either in the triplet state or the singlet state. Because the pion has spin 0 then from the conservation of spin we can conclude that the system is in the singlet state. Therefore, the wvaefunction is given by

$$\psi = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \tag{12}$$

In this scenario, we can see that there are two possibilities for the spin of the electron and the positron. Either the positron has spin up and the electron has spin down or the electron has spin up and the positron has spin down. Both states have equal probabilities to be observed which equal to $\frac{1}{2}$. The situation is illustrated in Figure 1.

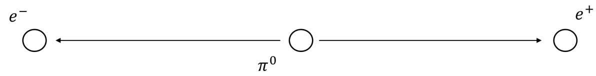

Figure 1: This is an illustration of the decay of the pion into an electron and a positron.

The stance of Einstein about the outcomes of this thought experiment is labeled *local realism.* For Einstein, the particles have certain spin values and by performing a measurement we will get to know these values. Meanwhile, the view held by the so called *Copenhagen interpretation* is that we cannot speak of the state of a system until a measurement is made. Notice however that when we measure one part of the system (that is when we determine the spin of one of the particles) we can speak of the state of the other particle without performing any measurement on it, which mean that we can predict an outcome of an experiment with certainty and without making any measurement. This led Einstein and his colleagues to conclude that quantum mechanics is an *incomplete* theory. This opened up the question about the possibility of supplementing other variables or parameters to the theory together with the description of the wavefunction Ψ so that by determining the wavefunction and the other additional parameters we can predict physical quantities with certainty like in classical mechanics. Such an idea is labeled *hidden variable theories.*

5 Bell's Inequality

In 1964, a physicist working at CERN called John Stewart Bell published in article by the name *On The Einstein Podolsky Rosen Paradox*[5] in that article he commented on the ideal of local hidden-variable theories by saying

"*These additional variables were to restore to the theory causality and locality. In this note that ideal will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics.*"

Here we want to explain the argument that Bell advanced against local hidden variable theories. Consider again the same thought experiment that we considered before due to Bohm. We will change this time the configuration of the experiment by adding a rotating detectors at the location of detection for each particle as shown in Figure 2.

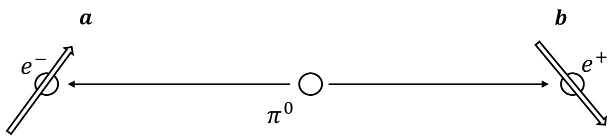

Figure 2: This is the same configuration but with detectors being able to rotate. The detectors have direction $\mathbf{a}$,$\mathbf{b}$.

If the positron spin has a projected component in the same direction as the detector then $\langle S_{\mathbf{b}} \rangle = \hbar/2$ and if the projected component of the spin of the positron have a direction opposite to the detector then $\langle S_{\mathbf{b}} \rangle = -\hbar/2$ and the same goes for the electron. Another thing to be clear about is that we will make measurements in $\hbar/2$ units and hence it is enough to indicate the outcomes by $+1$ or -1. Notice that this new experiment is a generalized version of the one before, in the sense that the previously discussed experiment were considering both detectors to be in the +z-axis, but here we are projecting on general vectors. Let's denote the product of the outcome of the two detectors by $P(\mathbf{a}, \mathbf{b})$. Therefore we can either have one of the four possibilities $-1 \times 1, -1 \times (-1), 1 \times (-1), 1 \times 1$ and therefore the results are two outcomes $+1, -1$. Notice that if we are in the previous experiment were both detectors are in the same direction $\mathbf{a} = \mathbf{b}$ the result is $P(\mathbf{a}, \mathbf{a}) = -1$ and if they are in opposite directions $P(\mathbf{a}, \textbf{-a}) = 1$ and $\mathbf{a}, \mathbf{b}$ are unit vectors. The outcomes of this experiment as predicted by quantum mechanics is [6]

$$P(\mathbf{a},\mathbf{b}) = -\langle \mathbf{a},\mathbf{b}\rangle \tag{13}$$

Bell was able to show that this prediction is not consistent with idea of local hidden-variable. Consider the state of each particle separately and denote the state of the electron by A. the state A can have only two outcomes, either spin up (i.e. it has a component in the same direction of the detector) or spin down. Such a state function can depend on the direction of the detector $\mathbf{a}$ and the supplied hidden-variable λ. In addition, we want the detectors to be unable to know the direction of the other in such a way that if we change one of them say $\mathbf{a}$ too quickly before making the measurement on the particle at $\mathbf{b}$, then $\mathbf{b}$ cannot show any correlation to the direction of $\mathbf{a}$ this is the assumption of *locality*, which states that no influence can travel faster than the speed of light. Therefore A only depends on $\mathbf{a}$ and λ and the same with B hence

$$A(\mathbf{a},\lambda) = \pm 1 \quad B(\mathbf{b},\lambda) = \pm 1 \tag{14}$$

Now, let the probability density of the variable λ be $\rho(\lambda)$ and thus if we want to calculate the average of the product of these measurements

$$P(\mathbf{a},\mathbf{b}) = \int \rho(\lambda)A(\mathbf{a},\lambda)B(\mathbf{b},\lambda)d\lambda \tag{15}$$

But because $A(\mathbf{a},\lambda) = -B(\mathbf{a},\lambda)$ then

$$P(\mathbf{a},\mathbf{b}) = -\int \rho(\lambda)A(\mathbf{a},\lambda)A(\mathbf{b},\lambda)d\lambda \tag{16}$$

Define a new vector $\mathbf{c}$ such that

$$P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c}) = -\int \rho(\lambda)[A(\mathbf{a},\lambda)A(\mathbf{b},\lambda) - A(\mathbf{a},\lambda)A(\mathbf{c},\lambda)]d\lambda \tag{17}$$

Notice also that $A(\mathbf{b},\lambda) = \frac{1}{A(\mathbf{b},\lambda)}$

$$P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c}) = -\int \rho(\lambda)[1 - A(\mathbf{b},\lambda)A(\mathbf{c},\lambda)]A(\mathbf{a},\lambda)A(\mathbf{b},\lambda)d\lambda \tag{18}$$

Using the triangle inequality $|\int f| \leq \int |f|$ and noting that $\int \rho(\lambda) = 1$ we get

$$|P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})| \leq \int \rho(\lambda)[1 - A(\mathbf{a},\lambda)A(\mathbf{c},\lambda)]d\lambda \tag{19}$$

$$|P(\mathbf{a},\mathbf{b}) - P(\mathbf{a},\mathbf{c})| \leq 1 - P(\mathbf{a},\mathbf{c}) \tag{20}$$

inequality 20 is refered to as *Bell's inequality.* If we now consider a specific orientation of the vectors $\mathbf{a},\mathbf{b},\mathbf{c}$ where $\mathbf{a}$ and $\mathbf{b}$ are orthogonal and $\mathbf{c}$ bisect them in the middle, we can substitute the prediction of quantum mechanics from equation 13 and put in the Bell's inequality derived from local hidden-variable considerations. For that matter equation 13 yields $P(\mathbf{a},\mathbf{b}) = 0$ and $P(\mathbf{a},\mathbf{c}) = P(\mathbf{b},\mathbf{c}) = -0.707$

$$0.707 \nleq 1 - 0.707 = 0.293 \tag{21}$$

From this, Bell was able to conclude that quantum mechanics is not consistent or that it is incompatible with local hidden-variable theory. Note that the argument does not depend on how many variables you supply or which form of probability density $\rho(\lambda)$ is being used. Therefore Bell was able to conceive a way in which experimentalist can test the ideas of local hidden variables

6 Experimental Evidence

There are many experiments that were done to show that local hidden-variable theories are untenable. We describe qualitatively only one of them which is due to Alain Aspect and et al[7]. The difficulties in these experiments on Bell's inequality or often labeled *loopholes.* Aspect is considered to be the first to try to close the locality loophole, that is to ensure that the locality assumption is maintained. The setup is illustrated in Figure 3

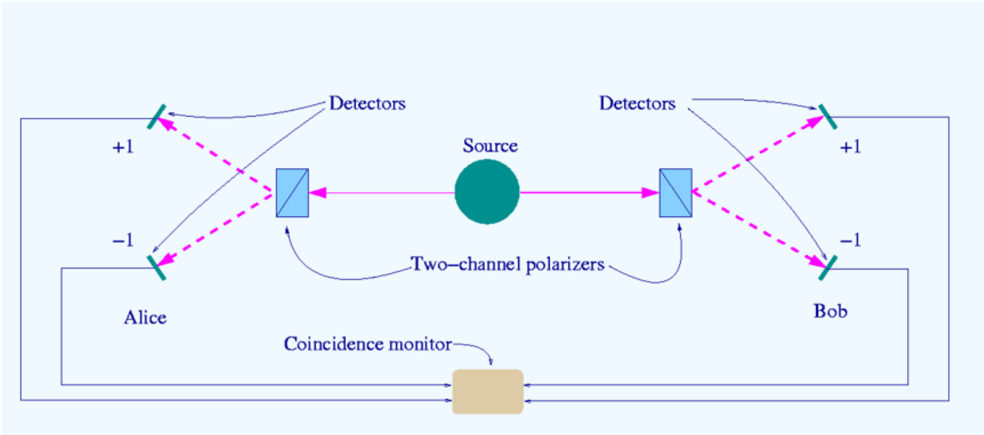

Figure 3: This is the setup that Aspect used in his experiment[8].

Aspect used Calcium source to obtain a pair of entangled photon emitted from a radiative cascade of Calcium. The two-channel polarizers split the beams of photons depending on the polarization or state they have and therefore directed to one of the two detectors. A coincidence monitor is used to check whether the registered photons came from the same atom or not that is it checks if the two photons are entangled. Aspect used acousto-optical switches to change the orientation of the polarizers in a short time compared to the photon transit time, thereby ensuring locality is fulfilled.

The outcome of this experiment and others show clear violation of Bell's inequality and a good agreement with quantum mechanics. Therefore the position of local hidden-variable became untenable.

Because of the importance of such a work, The Nobel prize in physics 2022 was awarded to three physicists, John Clauser who was one of the first to conduct these types of experiments, Alain Aspect who tried to improve on Clauser by closing the locality loophole and Anton Zeilinger who improved on the work of the two even further.

7 Conclusion

The quantum measurement problem is a long standing open problem that tries to find an interpretation to measurements in the quantum realm. The theory is probabilistic and an uncertainty relation is imposed on its outcomes. This made some physicists like Einstein to describe the theory as being incomplete because we cannot predict physical quantities in this theory with certainty. He and his colleagues proposed a thought experiment to show that we can predict physical quantities without even making a measurement. This led people to wonder if it is possible to supply so-called hidden variables to make the theory complete. John Stewart was able to derive an inequality that showed that hidden-variables are incompatible with quantum mechanics. The experiements conducted later by Clauser, Aspect and Zeilinger favored quantum mechanics and showed a wide violation of Bell's inequality. Other interpretations to this day are waiting for experimental validation, but the quantum measurement problem remains open.

References

[1] Nilanjana Datta. Quantum information & computation: Postulate (qm4): Quantum measurements, 2020. Lecture notes for Lent Term 2019–2020.

[2] Asher Peres. *Quantum theory: concepts and methods*, volume 72. Springer, 1997.

[3] Albert Einstein, Boris Podolsky, and Nathan Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical review*, 47(10):777, 1935.

[4] David Bohm and Yakir Aharonov. Discussion of experimental proof for the paradox of einstein, rosen, and podolsky. *Physical Review*, 108(4):1070, 1957.

[5] John S Bell. On the einstein podolsky rosen paradox. *Physics Physique Fizika*, 1(3):195, 1964.

[6] David J Griffiths and Darrell F Schroeter. *Introduction to quantum mechanics.* Cambridge university press, 2019.

[7] Alain Aspect, Jean Dalibard, and Gérard Roger. Experimental test of bell's inequalities using time-varying analyzers. *Physical review letters*, 49(25):1804, 1982.

[8] Wikimedia Commons. File:bell-test-photon-analyer.png — wikimedia commons, the free media repository, 2024. [Online; accessed 20-December-2024].