Maxwell-Boltzmann Statistics and Bose-Einstein Statistics

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Introduction

This report give a brief review about the Maxwell-Boltzmann statistics and Bose-Einstein statistics. First we define statistical mechanics and then give some history or background about the subject then we talk about each statistics individually.

Statistical Mechanics is the study of the laws of mechanics using probability theory and statistical methods. This branch of physics was first applied to classical thermodynamics which we now call statistical thermodynamics. There are two important concepts in this field which are the *macrostates* and the *microstates*. Macrostates refer to the macroscopic properties of the system such as pressure, volume and temperature while microstates contains all the information about the microscopic properties of the system such as position and momentum. The connection between the two is that for every macrostate, there exist one or more corresponding microstate (or arrangement of particles).

History

The field of statistical mechanics was initiated by many physicists, but we will only talk about three of them.The Scottish physicist James Clerk Maxwell who was influenced by the work of Rudolf Clausius derived the well-known Maxwell speed distribution (which we display later on). Having read Maxwell's work, Boltzmann provided the first statistical definition of entropy which is given by $S = k \log W$ where k is the Boltzmann constant and W is the number of microstates. Afterwards, the American physicist Josiah Willard Gibbs formalized the subject and named it *statistical mechanics*.

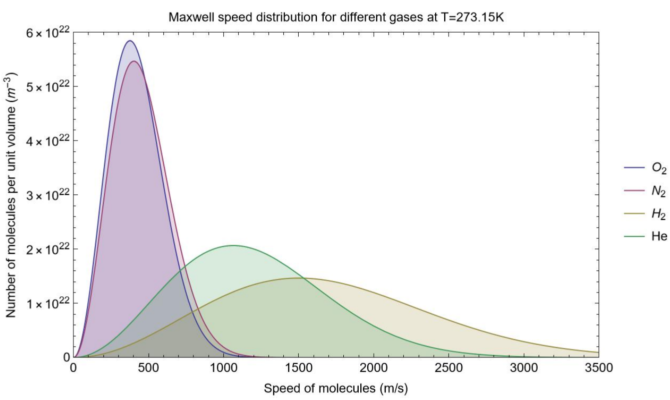
Maxwell-Boltzmann Statistics

We now state (without proof) the Maxwell-Boltzmann probability and Maxwell speed distribution respectively.

$$f_{MB} = Ae^{-E_i/k_BT} \tag{1}$$

$$n(v) = \frac{4\pi N}{V}\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 e^{-mv^2/2k_BT} \tag{2}$$

where in (1), f_{MB} refers to the probability of finding a particle with E_i energy level at temperature T in kelvin and k_B is Boltzmann's constant and A is a normalization constant. for equation (2), $n(v)$ is the number of particles per unit volume with speed v, m is the mass of the particle and N is the total number of particles in the system and V is the volume of the system.



The Maxwell speed distribution is plotted on the left for four different gases and these are oxygen, nitrogen , hydrogen and helium all plotted at standard pressure and temperature with volume $V = 22.7L$ and 1 mol for each gas. There are three important speed in the plot which are the most probable speed v_p and the mean speed $\bar{v}$ and the root mean square speed $\sqrt{\bar{v^2}} = v_{rms}$. The formula for all speeds is

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}, \quad \bar{v} = \sqrt{\frac{8k_BT}{\pi m}}, \quad v_p = \sqrt{\frac{2k_BT}{m}}$$

When Does Maxwell-Boltzmann Statistics Fails?

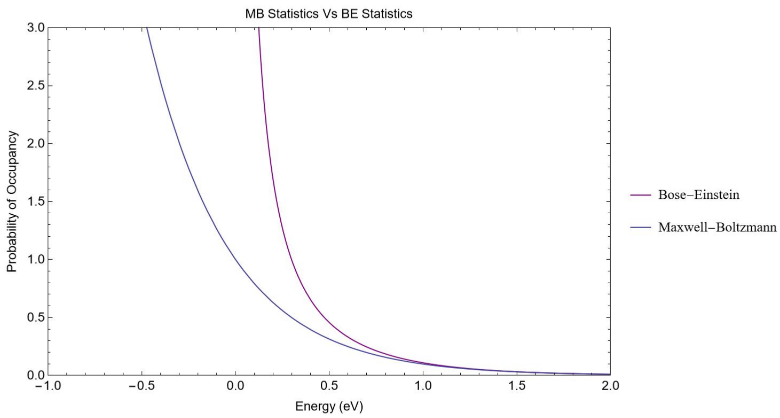
The validity of MB statistics depend on the average distances between the particles d such that $d \gg \Delta x$. This means that the average distance between the particles should be much greater than the quantum uncertainty. The MB statistics is used for identical distinguishable particles, but at such average distance d the wavefunctions of particles overlap and they become indistinguishable.

Bose-Einstein Statistics

When classical statistical mechanics fails, one needs to find a quantum statistical mechanics. Quantum statistics started when the Indian physicist Satyendra Nath Bose derived Planck's law by applying statistical mechanics on light quanta (namely, photons). Bose wrote his paper in English and asked Einstein in a letter to translate it to German and publish it. One can arrive at BE statistics by maximizing the multiplicity (or the number of ways to distribute the particles among the allowed energy states) under the two conditions that the total energy and total number of particles of the system is constant which gives:

$$f_{BE}(E) = \frac{1}{Be^{E/k_BT} - 1} \tag{3}$$

Where $f_{BE}(E)$ is the probability of finding a particle at temperature T in kelvin in an energy state of energy E and B is a parameter to be determined from the total number of particles and for which $B = 1$ in the case of photons and phonons.



The figure above display a comparison between the Bose-Einstein and Maxwell-Boltzmann statistics at a temperature of $T = 5000K$. A final note before ending the section: Bose arrived at quantum statistics before Pauli derived his famous exclusion principle which classified particles into bosons and fermions and justified the existence of Bose-Einstein condensate which we talk about in the next section

Experimental Evidence for the Bose-Einstein Condensate

In 1925, Einstein used Bose's work to predict new state of matter called now *Bose-Einstein condensate*. This state is achieved by cooling a gas of bosons (a gas whose individual constituents sums to an integer spin number) to temperatures near absolute zero. since the gas atoms are bosons this means that they can occupy the lowest energy state at the same time, which then their wavefunction will interfere and form one wavefunction. The experimental verification of this fact came after 70 years at the JILA laboratory by two physicists Carl Wieman and Eric Cornell when they cooled 2000 Rubidium-87 atoms to a temperature in the range of the nano kelvin using laser cooling and evaporative cooling. At MIT, Wolfgang Ketterle cooled a sodium-23 atoms to reach Bose-Einstein condensate. For this achievement, the three were awarded the 2001 Nobel prize in physics.

References

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