PS2

Task A:

$u_1 = [0, 10, 0]^T$, $\mu_1 = [180, 50, 0]^T$

$\beta = 20$, $\alpha_1 = [.05, .001, .05, .01]$

$\downarrow$ square

$[2.5e{-3}, 1e{-6}, 2.5e{-3}, 1e{-4}]$

$* \; Q = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\phi^2 \end{bmatrix}$; $\sigma_s^2 = 0$ (Known Correspondences)

we have one observation, $\sigma_\phi = \beta$ from lecture 6

$\therefore Q = [\sigma_\phi^2] = [\beta^2] = [400]$

$$* \; R_t = \begin{bmatrix} \alpha_1 \delta_{rot1}^2 + \alpha_2 \delta_{trans}^2 & 0 & 0 \\ 0 & \alpha_3 \delta_{trans}^2 + \alpha_4 (\delta_{rot1}^2 + \delta_{rot2}^2) & 0 \\ 0 & 0 & \alpha_1 \delta_{rot2}^2 + \alpha_2 \delta_{trans}^2 \end{bmatrix}$$

$$u = \begin{bmatrix} \delta_{rot_1} \\ \delta_{trans} \\ \delta_{rot_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$\alpha = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]$$

$$= [2.5e\text{-}3, 1e\text{-}6, 2.5e\text{-}3, 1e\text{-}4]$$

Substitute with u & α in R_t

$$R_t = \begin{bmatrix} 1e\text{-}6 * 10^2 & 0 & 0 \\ 0 & 2.5e^{-3} * 10^2 & 0 \\ 0 & 0 & 1e^{-6} * 10^2 \end{bmatrix} = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & .25 & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}$$

$$* \; g(x_{t-1}, u_t) = \begin{bmatrix} x_{t-1} + \delta_{trans} \cos(\theta + \delta_{rot1}) \\ y_{t-1} + \delta_{trans} \sin(\theta + \delta_{rot1}) \\ \theta_{t-1} + \delta_{rot1} + \delta_{rot2} \end{bmatrix}$$

$$G_t = \frac{\partial g}{\partial x_{t-1}}\Big|_{\mu_{t-1}} = \begin{array}{c} \begin{bmatrix} 1 & 0 & -\delta_{trans} \sin(\theta + \delta_{rot1}) \\ 0 & 1 & \delta_{trans} \cos(\theta + \delta_{rot1}) \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccc} \partial/\partial x & \quad\quad \partial/\partial y & \quad\quad\quad \partial/\partial\theta \end{array} \end{array}$$

Substitute with: $\mu_1 = \begin{bmatrix} x_1 \\ y_1 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 180 \\ 50 \\ 0 \end{bmatrix}$

$$u = \begin{bmatrix} \delta_{rot1} \\ \delta_{trans} \\ \delta_{rot2} \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$\therefore G_t = \begin{bmatrix} 1 & 0 & -10 * \sin(0+0) \\ 0 & 1 & 10 * \cos(0+0) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{bmatrix}$$

$$V_t = \left.\frac{\partial g(x_{t-1}, u_t)}{\partial u_t}\right|_{u_t} = \begin{bmatrix} -\delta_{trans} \sin(\theta + \delta_{rot1}) & \cos(\theta + \delta_{rot1}) & 0 \\ \delta_{trans} \cos(\theta + \delta_{rot1}) & \sin(\theta + \delta_{rot1}) & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cccc} & \partial/\partial\delta_{rot1} & \partial/\partial\delta_{trans} & \partial/\partial\delta_{rot2} \end{array}$$

$$\therefore V_t = \begin{bmatrix} -10 \sin(0+0) & \cos(0+0) & 0 \\ 10 \cos(0+0) & \sin(0+0) & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 10 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} r_t^j \\ \phi_t^j \\ s_t^j \end{bmatrix} = \begin{bmatrix} \sqrt{(m_{i,x}-x)^2+(m_{i,y}-y)^2} \\ \text{atan2}(m_{i,y}-y, m_{i,x}-x)-\theta \\ s_i \end{bmatrix} + \begin{bmatrix} \delta_{\sigma_r^2} \\ \delta_{\sigma_\phi^2} \\ \delta_{\sigma_s^2} \end{bmatrix}$$

$$\underbrace{}_{f^j} = \underbrace{}_{h(x_t, m_t)} + \underbrace{}_{\delta_t}$$

For simplicity, assume $q = (m_{i,x}-x)^2 + (m_{i,y}-y)^2$

$$H_t = \left.\frac{\partial h}{\partial x_t}\right|_{\bar{\mu}_t} = \begin{bmatrix} \frac{-\cancel{2}(m_{i,x}-x)}{\cancel{2}\sqrt{q}} & \frac{-\cancel{2}(m_{i,y}-y)}{\cancel{2}\sqrt{q}} & 0 \\ \frac{m_{i,y}-y}{q} & \frac{-(m_{i,x}-x)}{q} & -1 \end{bmatrix}$$

without s_t

$$\begin{array}{ccc} \partial/\partial x & \partial/\partial y & \partial/\partial \theta \end{array}$$

Substitute with $\mu_1 = \begin{bmatrix} 180 \\ 50 \\ 0 \end{bmatrix}$, $m_1 = \begin{bmatrix} 21 \\ 0 \end{bmatrix}$ → from fieldmap.py

$q = (21-180)^2 + (0-50)^2 = 27781$

$\sqrt{q} = \sqrt{27781} = 166.676$

$$\therefore H_t = \begin{bmatrix} \frac{-(21-180)}{166.676} & \frac{-(0-50)}{166.676} & 0 \\ \frac{(0-50)}{27781} & \frac{-(21-180)}{27781} & -1 \end{bmatrix} = \begin{bmatrix} .954 & .3 & 0 \\ 1.8\times10^{-3} & 5.7\times10^{-3} & -1 \end{bmatrix}$$