Perception in Robotics, T3 2021
Midterm (70 points)

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In this *online* exam you require paper, pen, and correctly set your camera pointing at your desktop. You may not use the textbook, lecture slides, or notes, except for one double-sided hand-written A4 cheat sheet. Calculators may be used only for arithmetic calculations. The use of the keyboard, smartphone or any other electronic devices is not allowed.
At 11:55, you will stop writing and take pictures of your exam, picture per problem. Then, you will submit your results to canvas.

By submitting your exam, you also accept implicitly that you have kept the honor code pledge:

"I have neither given nor received aid on this examination"

1. Expectation (5 points)

Write the definition of expectation for the case below, and further derive that expression up to a simplified form for any two random variables x and y.

$E\{x + y\} =$

2. Mean and Covariance Propagation (10 points)

Let $y = A \cdot (x + \epsilon)$, where $x \sim \mathcal{N}(\mu_x, \Sigma_x)$ and $\epsilon \sim \mathcal{N}(b_\epsilon, M)$ are Gaussian random variables. The term b_ϵ is a bias term. Given that $E\{(x - \mu_x)(\epsilon - \mu_\epsilon)^\top\} = E\{(\epsilon - \mu_\epsilon)(x - \mu_x)^\top\} = L$, derive the mean and covariance of y.

3. Iso-Contours (10 points)

Sketch the 1-sigma iso-contours for the following Gaussian distributions, indicating the approximate value of some point in the ellipsoid:

(A) $\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}\right)$, (B) $\mathcal{N}\left(\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}9 & -2\\-2 & 3\end{bmatrix}\right)$.

4. EKF (15 points)

Consider the following system:

$$x_t = x_{t-1} + 4 \cdot (u_t + \epsilon_t)^3, \tag{1}$$

where $\epsilon_t \sim \mathcal{N}(0, M)$, and $M = 2$.
A) Calculate 1 step of the EKF given that the prior distribution of $x_0 \sim \mathcal{N}(3, 4)$ and $u_1 = 0.5$.
B) Calculate the correction step, knowing that the observation function is $z_t = x_t + \delta_t$, where $\delta_t \sim \mathcal{N}(0, Q), \quad Q = 3$ and the observation is $z = 4$.

5. Wrapping angles (10 points)

Consider the EKF, where x_t is a 2D pose and the observation z_t is a single dimensional angle $z_t = h(x_t) = arctan(dy, dx) - \theta_t$.
A) Indicate in which parts of the filter is strictly necessary to wrap angles for its correct functioning.
B) Indicate in which parts it is strictly necessary to wrap angles to obtain a belief of the angle in $(-\pi, \pi]$.
Justify your answers.

1. $\bar{\mu}_t = g(\mu_{t-1}, u_t)$
2. $\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$
3. $K_t = \bar{\Sigma}_t H_t^T \left(H_t \bar{\Sigma}_t H_t^T + Q_t \right)^{-1}$
4. $\mu_t = \bar{\mu}_t + K_t \left(z_t - h(\bar{\mu}_t) \right)$
5. $\Sigma_t = \left(I - K_t H_t \right) \bar{\Sigma}_t$

6. Data Association SLAM (10 points)

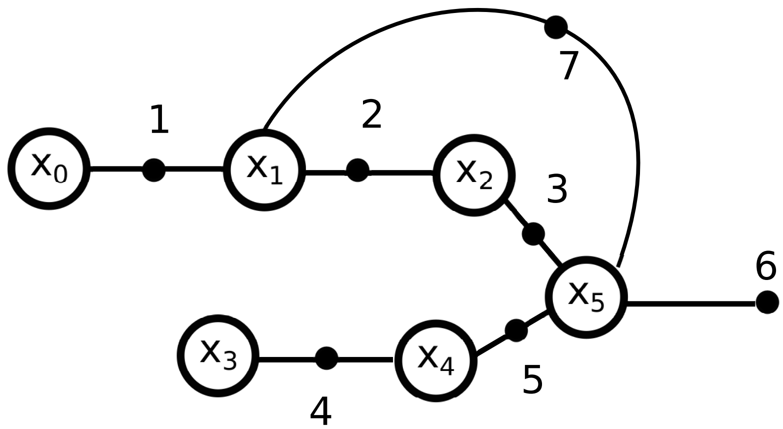
Consider the following current estimate of the state variable $y = [x_t, l_1, l_2, l_3, l_4]^\top$ in a 1 dimensional EKF SLAM problem:

$$y \sim \mathcal{N}(\mu_y, \Sigma_y) = \mathcal{N}\left(\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 & 2.5 & 1.5 & 0.5 & 1 \\ 2.5 & 4 & 0 & 0 & 0 \\ 1.5 & 0 & 2 & 0 & 0 \\ 0.5 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix} \right)$$

Calculate which is the landmark corresponding to the Maximum Likelihood Data Association, given the observation function $h(x, l_i) = x - l_i$ and knowing that $z = 2.5$.

7. SAM: Adjacency Matrix (10 points)

A) Sketch the adjacency matrix A for the following graph.
B) Propose a node to be removed, aiming to minimize the complexity of the equivalent resultant graph (in addition to the complexity of the optimization process).



8. Particle filter (10 points)

Explain briefly why *Resampling* is required in PFs. (do not explain resampling algorithms).