Mohammed Derfallah 1st Year DS Term3

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Perception in Robotics Midterm

1) Expectation: $E\{x\} = \int x\, p(x)\, dx$ definition

$$E\{x+y\} = \iint (x+y)\, p(x,y)\, dx\, dy$$

$$= \iint x\, p(x,y)\, dx\, dy + \iint y\, p(x,y)\, dx\, dy$$

$$= \int x \underbrace{\left[\int p(x,y)\, dy\right]}_{\text{total prob.}} dx + \int y \underbrace{\left[\int p(x,y)\, dx\right]}_{\text{total prob}} dy$$

$$= \int x\, p(x)\, dx + \int y\, p(y)\, dy$$

$$= E\{x\} + E\{y\}$$

$$\therefore \boxed{E\{x+y\} = E\{x\} + E\{y\}}$$

2) Mean & Covariance Propagation:

$y = A(x+\epsilon)$, $x \sim N(\mu_x, \Sigma_x)$, $\epsilon \sim N(b_\epsilon, M)$

$$E\{(x-\mu_x)(\epsilon-\mu_\epsilon)^T\} = E\{(\epsilon-\mu_\epsilon)(x-\mu_x)\} = L$$

Assumption: $\mu_\epsilon = b_\epsilon$ by definition of Gaussian Dist. $N(\mu, \Sigma)$

I Mean of y $\qquad E\{\epsilon\} = \mu_\epsilon \pm b_\epsilon$

$$E\{y\} = E\{A(x+\epsilon)\} = A.E\{x+\epsilon\} = A(E\{x\} + E\{\epsilon\})$$
$$= A(\mu_x + b_\epsilon)$$

II Covariance of y:

$$\Sigma_y = E\{(y-\mu_y)(y-\mu_y)^T\} = E\{[A(x+\epsilon) - A(\mu_x+b_\epsilon)][A(x+\epsilon) - A(\mu_x+b_\epsilon)]^T\}$$

$$= E\{[A(x-\mu_x) + A(\epsilon-b_\epsilon)][A(x-\mu_x) + A(\epsilon-b_\epsilon)]^T\}$$

$$= E\{A(x-\mu_x)(x-\mu_x)^T A^T + A(x-\mu_x)(\epsilon-b_\epsilon)^T A^T + A(\epsilon-b_\epsilon)(x-\mu_x)^T A^T$$
$$+ A(\epsilon-b_\epsilon)(\epsilon-b_\epsilon)^T A^T\}$$

$$= A\Sigma_x A^T + ALA^T + ALA^T + AMA^T$$

$$\therefore \boxed{y \sim N(A(\mu_x+b_\epsilon),\ A(\Sigma_x + 2L + M)A^T)}$$

Note: For bias $\mu_\epsilon = E\{x\} + b_\epsilon$, $b_\epsilon \neq 0$

~~but that~~ isn't clearly stated so assume $\mu_\epsilon = b_\epsilon$

3) Iso-Contours:

(A) $\mathcal{N}\left(\begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}2 & 0\\0 & 3\end{bmatrix}\right)$

Cholesky decomposition: $\Sigma = LL^T$, substitute with $x_1, x_2 \in \{0,1\}$

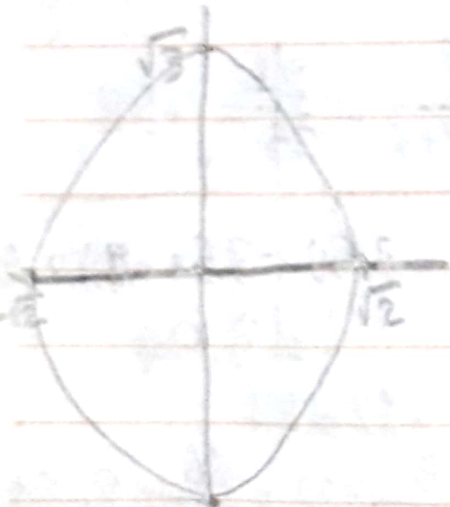
$$\begin{bmatrix}2 & 0\\0 & 3\end{bmatrix} = \begin{bmatrix}a & 0\\b & c\end{bmatrix}\begin{bmatrix}a & b\\0 & c\end{bmatrix}$$

$a^2 = 2$ $\quad ab = 0$ $\quad b^2 + c^2 = 3$

$\boxed{a = \sqrt{2}}$ $\quad \boxed{b = 0}$ $\quad \boxed{c = \sqrt{3}}$

$\therefore L = \begin{bmatrix}\sqrt{2} & 0\\0 & \sqrt{3}\end{bmatrix}$

$$\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}\sqrt{2} & 0\\0 & \sqrt{3}\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}\sqrt{2}\,x_1\\\sqrt{3}\,x_2\end{bmatrix}$$



B) $\mathcal{N}\left(\begin{bmatrix}1\\-1\end{bmatrix}, \begin{bmatrix}9 & -2\\-2 & 3\end{bmatrix}\right)$

$$\begin{vmatrix}9-\lambda & -2\\-2 & 3-\lambda\end{vmatrix} = (9-\lambda)(3-\lambda) - 4$$

check PSD

$= \lambda^2 - 12\lambda + 27 - 4$

$= \lambda^2 - 12\lambda + 23$

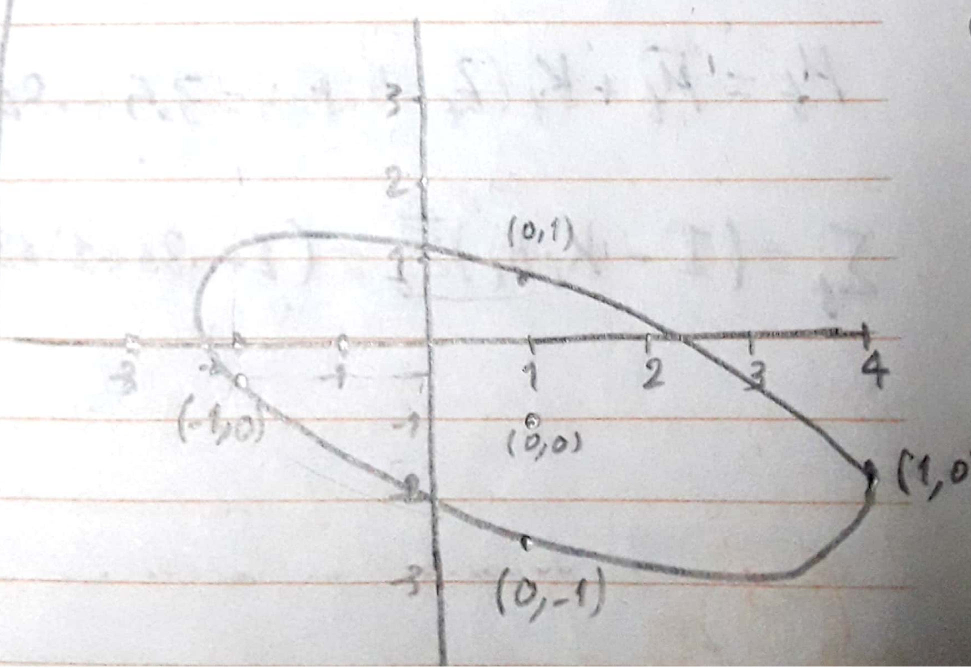
$\lambda_1, \lambda_2 \geq 0$ ✓

$$\begin{bmatrix}9 & -2\\-2 & 3\end{bmatrix} = \begin{bmatrix}a & 0\\b & c\end{bmatrix}\begin{bmatrix}a & b\\0 & c\end{bmatrix}$$

$a^2 = 9$ $\quad ab = -2$ $\quad b^2 + c^2 = 3$

$\boxed{a = 3}$ $\quad \boxed{b = \frac{-2}{3}}$ $\quad \boxed{c \approx 1.6}$

$$\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}3 & 0\\-\frac{2}{3} & 1.6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} + \begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}3x_1 + 1\\-\frac{2}{3}x_1 + 1.6x_2 - 1\end{bmatrix}$$



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4) EKF: $x_t = x_{t-1} + 4(u_t + \epsilon_t)^3$, $\epsilon_t \sim N(0, M)$ $M \to 2$

$= g(x_{t-1}, u_t, \epsilon_t)$

A) $x_0 \sim N(3, 4)$, $u_1 = .5$

$$G_t = \frac{\partial g}{\partial x_{t-1}}\Big|_{\mu_{t-1}} = 1 \quad , V_t = \frac{\partial g}{\partial u_t}\Big|_{u_t} = 12(u_t + \epsilon_t)^2 = 12(.5+0)^2 = 3$$

$$\bar{\mu}_t = g(\mu_{t-1}, u_t) + \epsilon_t = 3 + 4(.5)^3 + 0 = 3.5$$

$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + V_t M V_t^T = 1*4*1 + 3*2*3 = 4 + 18 = 22$$

B) $z_t = x_t + \delta_t$, $\delta_t \sim N(0, Q)$ $Q \to 3$, $z = 4$

$$H_t = \frac{\partial h}{\partial x} = 1$$

$$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} = \frac{22*1}{1*22*1+3} = \frac{22}{25} = .88$$

$$\mu_t = \bar{\mu}_t + K_t(z_t - h(\bar{\mu}_t)) = 3.5 + .88(4 - 3.5) = 3.5 + .88 * .5$$
$$= 3.94$$

$$\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t = (1 - .88*1)*22 = .12*22$$
$$= \frac{3}{25} * 22 = \frac{66}{25} = 2.64$$

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5) Wrapping angles:

$$z_t = h(x_t) = \arctan(dy, dx) - \theta_t$$

Wrapping angles can be added anywhere of the Particle, however, it's necessary to add it in the fourth line

$$\mu_t = wrap(\bar{\mu}_t + K_t(wrap(z_t - h(\bar{\mu}))))$$

A, B) I Can't understand the difference between the two sub questions.

6) Data Association SLAM:

$$y = [x_t, l_1, l_2, l_3, l_4]^T$$

$$y \sim N(\mu_y, \Sigma_y), \quad h(x, l_i) = x - l_i, \quad z = 2.5$$

$$\arg\max_{c_t} \left(\prod_{i=1}^{K} p(z_t^i \mid c_t^i, y_t) \right) = \prod_{i=1}^{K} \max_{c_t^i} p(z_t^i \mid c_t^i, y_t)$$

$$z_t^i \sim N(z_t^i ; h(\bar{\mu}, c^i), H_t^i \bar{\Sigma}_t (H_t^i)^T + Q_t)$$

$$H_t = \frac{\partial h}{\partial x} = 1 \qquad \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{bmatrix}$$

$$\frac{\partial h}{l} = -1$$

$$H_t^i \bar{\Sigma}_t H_t^{i^T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 6 & & & & \\ & 4 & & & \\ & & 2 & & \\ & & & 1 & \\ & & & & 2 \end{bmatrix} \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & & & & \\ & -4 & & & \\ & & -2 & & \\ & & & -1 & \\ & & & & -2 \end{bmatrix} \begin{bmatrix} 1 & & & & \\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ & & & & -1 \end{bmatrix} = \begin{bmatrix} 6 & & & & \\ & 4 & & & \\ & & 2 & & \\ & & & 1 & \\ & & & & 2 \end{bmatrix}$$

I would go with l_1 because it corresponds to the maximum element.

7) SAM: Adjacency matrix

A)

$$
\begin{array}{c c}
 & \begin{array}{cccccc} x_0 & x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} &
\begin{bmatrix}
\bullet & \bullet & & & & \\
 & \bullet & \bullet & & & \\
 & & \bullet & & & \bullet \\
 & & & \bullet & \bullet & \\
 & & & & \bullet & \bullet \\
 & & & & & \bullet \\
 & \bullet & & & & \bullet
\end{bmatrix}
\end{array}
$$

B) I propose to remove (x_5) node as it has the most number of Connections, in addition that it will only add an edge between x_2 & x_4

8) Particle Filter:

Resampling is required in PF because the filter will suffer from degeneracy over time, such that the particle set will end having many particles with low weights due to propagating and correcting multiple times.