4. The Role of the Covariance Kernel

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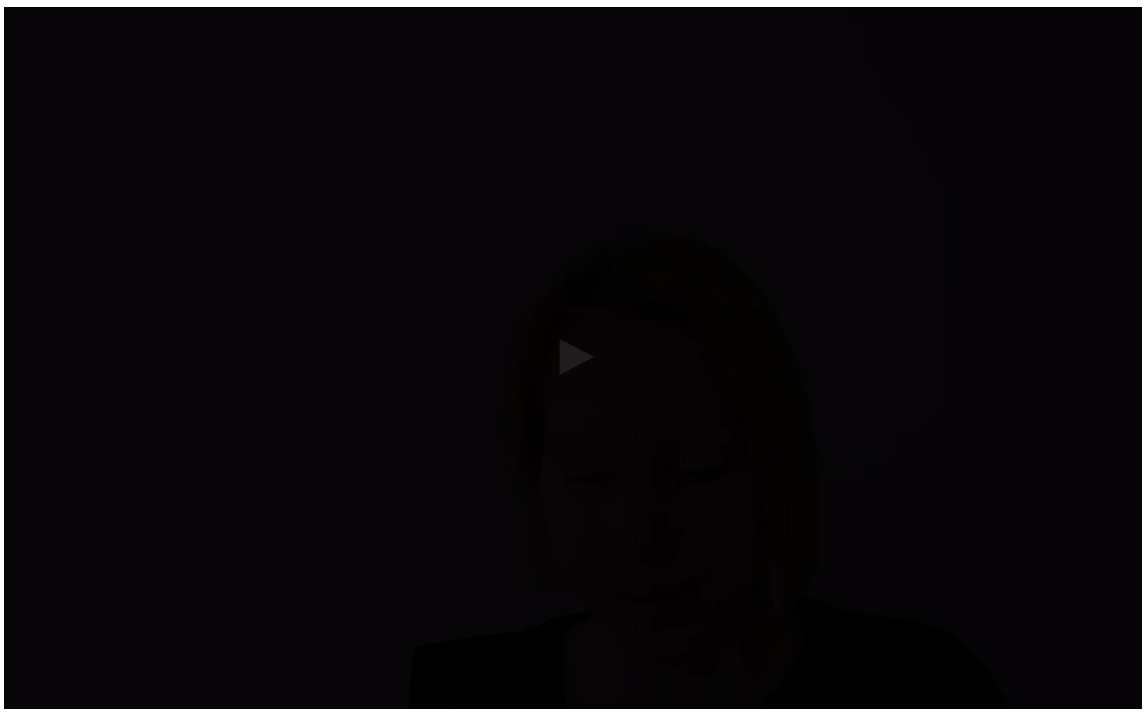
Exercises due May 21, 2021 19:59 EDT

The Role of the Covariance Kernel



These have actually a covariance structure

with this exponential function kernel

that I had in my previous slide.

And as I said in lecture 1, this actually

looks already fairly close to being a slice in this weather

map, potentially.

So next, we look a little bit more at the effect

that various forms of this kernel function could have.

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Previously, we have presented an example of a covariance kernel defined as

$$k(Z_1, Z_2) = \exp\left(-\frac{\|Z_1 - Z_2\|^2}{2\ell^2}\right).$$

However, the obvious questions are

1. Why use such a kernel function?

2. Can I use any function as a kernel?

Why use such kernel function?

The use of this kernel function allows for a relatively easy and computationally efficient way to parametrize the correlations. Since the kernel function might be defined on the whole space, it allows for smooth computations over the support of the variables to be estimated. Such a parametrization allows for the introduction of other information like smoothness and dynamic behavior.

Can I use any function as a kernel?

The short answer is no. In general, any arbitrary function whose arguments are Z_1 and Z_2 will not be a valid covariance function.

Kernel Function

1 point possible (graded)

Definition 4.1 A function k of two arguments Z_1 and Z_2, mapping its inputs to $\mathbb{R}$ is called a *kernel*.

Can a kernel k such that $k(x, y) \neq k(y, x)$ be used to build a covariance function?

- Yes
- No

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Moreover, covariance matrices need to be positive semi-definite. Thus, one needs to guarantee that the kernel function's output creates a positive definite matrix or a square of matrices.

Definition 4.2 If the covariance function is translation invariant, then it is called stationary.

This will happen if the covariance function depends on $Z_1 - Z_2$.

Definition 4.3 If the covariance function depends only on a norm $|Z_1 - Z_2|$, then it is called isotropic.

This means that the covariance function depends only on the distance between Z_1 and Z_2.

Definition 4.4 If the covariance function depends on $Z_1^\intercal Z_2$, then it is called dot product covariance.

Kernel Function

1 point possible (graded)

Is the the following kernel function isotropic?

$$k(Z_1, Z_2) = \exp\left(-\frac{\|Z_1 - Z_2\|^2}{2\ell^2}\right), \tag{7.6}$$

- Yes
- No

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