6. The Effects of Measurement Noise

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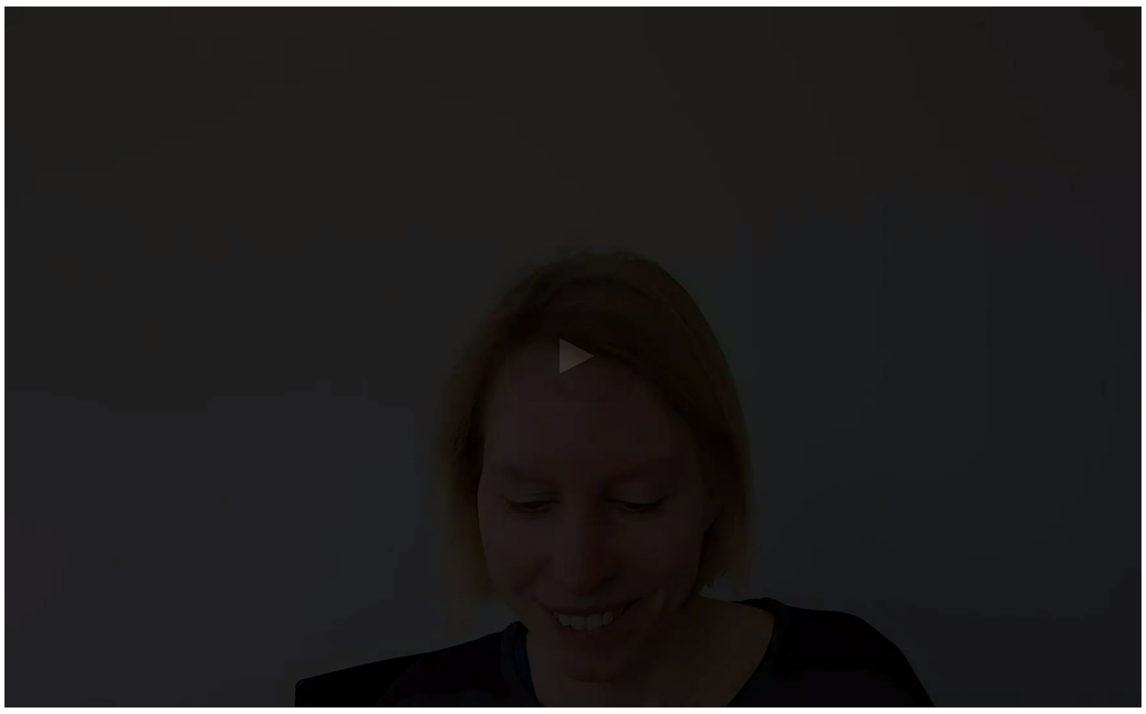
Exercises due May 21, 2021 19:59 EDT

The Effects of Measurement Noise



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For now, I want to stop here and just stress again

that Gaussian processes are a very flexible prediction

method.

We got this closed form for the prediction.

The kernel function plays a key role,

and we can include measurement--

modeling considerations, such as measurement noise,

non-stationarity, and many other things.

Thank you.

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Thus far, we have assumed that the kernel is the only contribution to the covariance matrix of the data. Although is was not stated explicitly, this means we took the data, or the observations, as exact.

However, what happens if our sensors are faulty or imprecise? How can one introduce in the presented framework the uncertainty in the observed values itself?

Let us return to the original example of temperature measurements. Imagine the sensor providing the measurements is not perfect, and each time it takes a measurement, it induces some additional noise due to its inherit limitations, such as thermal noise. One can characterize such noise, for example, as an additional variable ε that is also Normally distributed with mean zero and standard deviation τ. We can denote this as $\varepsilon \sim \mathcal{N}(0, \tau^2)$.

Now, using the same notation as before, instead of directly observing the realizations of the random variable $\mathbf{X}_2$, we observe the realizations $\mathbf{y}_2$ of the random variable $\mathbf{Y}_2 = \mathbf{X}_2 + \varepsilon$. By definition, we will assume the random variable ε is independent of $\mathbf{X}_2$.

Recall that the parameters of the conditional distribution of $\mathbf{X}_1$ given $\mathbf{X}_2$ are

$$\begin{aligned}\mu_{\mathbf{X}_1|\mathbf{X}_2} &= \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(\mathbf{x}_2 - \mu_2) \\ \Sigma_{\mathbf{X}_1|\mathbf{X}_2} &= \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}.\end{aligned}$$

Adding ε to $\mathbf{X}_2$ alters the covariance matrix:

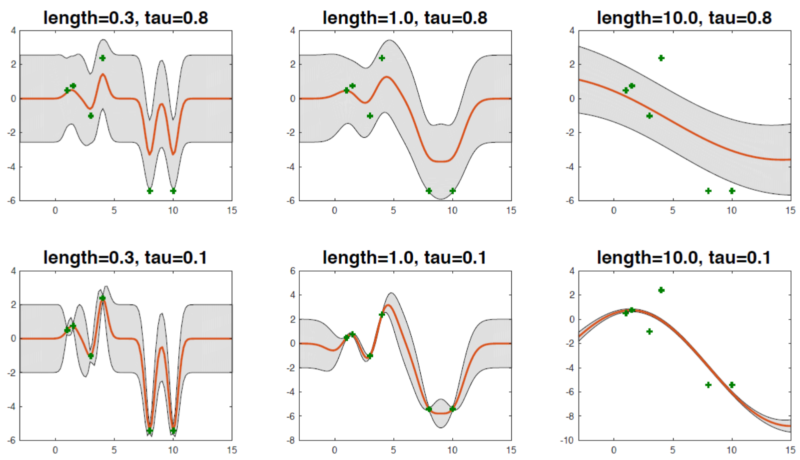
$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_{11} & \mathbf{\Sigma}_{12} \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} + \tau^2 I \end{bmatrix}$$

such that diagonal elements of magnitude τ^2 are added to the $\mathbf{\Sigma}_{22}$ component.

This changes the conditional equations to be

$$\begin{aligned}\mu_{\mathbf{X}_1|\mathbf{Y}_2} &= \mu_1 + \Sigma_{12}(\Sigma_{22} + \tau^2 I)^{-1}(\mathbf{y}_2 - \mu_2) \\ \Sigma_{\mathbf{X}_1|\mathbf{Y}_2} &= \Sigma_{11} - \Sigma_{12}(\Sigma_{22} + \tau^2 I)^{-1}\Sigma_{21}.\end{aligned}$$

The below figure shows a Guassian process for two different values of τ, $\tau = 0.8$ and $\tau = 0.1$.



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We can note that as τ is increases, so too does the variance on the estimate increase.

Observational Noise 1

1 point possible (graded)

What happens in the extreme case where $\tau \to \infty$? What happens to the mean of the estimates compared to the prior assumed mean?

- Mean remains the same as the prior
- Mean decreases compared to the prior
- Mean increases compared to the prior

Submit You have used 0 of 2 attempts

Observational Noise 2

1 point possible (graded)

Consider, again, the extreme case where $\tau \to \infty$. If we increase the number of observations in this limit, do the additional observations reduce the variance on the estimate?

- Variance decreases
- Variance remains the same

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