Homework 2

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- 2.46
  1. This is not a good design choice because

     $500 * 1 + 300 * 10 + 100 * 3 = 3800 \leq (500 * 0.75 * 1 + 300 * 10 + 100 * 3) * 1.1 = 4042.5$
  2. $3800 \div (500 * 1 * 0.5 + 300 * 10 + 100 * 3) = 107.04\%$

     $3800 \div (500 * 1 * 0.1 + 300 * 10 + 100 * 3) = 113.43\%$
- 2.47
  1. $0.7 * 2 + 0.1 * 6 + 0.2 * 3 = 2.6$
  2. $\frac{1}{0.7*x+0.1*6+0.2*3} = \frac{1}{2.6} * 1.25 \implies x = 1.26$
  3. $\frac{1}{0.7*x+0.1*6+0.2*3} = \frac{1}{2.6} * 1.50 \implies x = 0.76$
- 3.12

  $62_8 * 12_8 = 110010_2 * 1010_2$

	Step	Multiplier	Multiplicand	Product
0	Initial value	00101**0**	000000110010	000000000000
1	1: 0 ⇒ No operation	00101**0**	000000110010	000000000000
1	2: Shift left Multiplicand	00101**0**	000001100100	000000000000
1	3: Shift right Multiplier	00010**1**	000001100100	000000000000
2	1a: 1 ⇒ Prod = Prod + Mcand	00010**1**	000001100100	000001100100
2	2: Shift left Multiplicand	00010**1**	000011001000	000001100100
2	3: Shift right Multiplier	00001**0**	000011001000	000001100100
3	1: 0 ⇒ No operation	00001**0**	000011001000	000001100100
3	2: Shift left Multiplicand	00001**0**	000110010000	000001100100
3	3: Shift right Multiplier	00000**1**	000110010000	000001100100
4	1a: 1 ⇒ Prod = Prod + Mcand	00000**1**	000110010000	000111110100
4	2: Shift left Multiplicand	00000**1**	001100100000	000111110100
4	3: Shift right Multiplier	00000**0**	001100100000	000111110100
5	1: 0 ⇒ No operation	00000**0**	001100100000	000111110100
5	2: Shift left Multiplicand	00000**0**	011001000000	000111110100
5	3: Shift right Multiplier	00000**0**	011001000000	000111110100
6	1: 0 ⇒ No operation	00000**0**	011001000000	000111110100
6	2: Shift left Multiplicand	00000**0**	110010000000	000111110100
6	3: Shift right Multiplier	00000**0**	110010000000	000111110100

- 3.14
  - Hardware: There are 3 cycles in each iteration: add (step 1a), shift (step 2 & 3), check stopping criterion. Therefore, total time units = $3 * 8 * 4 = 96$.
  - Software: There are 5 cycles in each iteration: decide to add or not (step 1), add (step 1a), shift left multiplicand (step 2), shift right multiplier (step 3), check stopping criterion. Therefore, total time units = $5 * 8 * 4 = 160$.
- 3.18

  $74_8 \div 21_8 = 111100_2 \div 10001_2$

	Step	Quotient	Divisor	Remainder
0	Initial value	000000	010001000000	000000111100
1	1: Rem = Rem – Div	000000	010001000000	101111111100
1	2b: Rem < 0 ⇒ +Div, sll Q, Q0 = 0	000000	010001000000	000000111100
1	3: Shift Div right	000000	001000100000	000000111100
2	1: Rem = Rem – Div	000000	001000100000	111000011100
2	2b: Rem < 0 ⇒ +Div, sll Q, Q0 = 0	000000	001000100000	000000111100
2	3: Shift Div right	000000	000100010000	000000111100
3	1: Rem = Rem – Div	000000	000100010000	111100101100
3	2b: Rem < 0 ⇒ +Div, sll Q, Q0 = 0	000000	000100010000	000000111100
3	3: Shift Div right	000000	000010001000	000000111100
4	1: Rem = Rem – Div	000000	000010001000	111110110100
4	2b: Rem < 0 ⇒ +Div, sll Q, Q0 = 0	000000	000010001000	000000111100
4	3: Shift Div right	000000	000001000100	000000111100
5	1: Rem = Rem – Div	000000	000001000100	111111111000
5	2b: Rem < 0 ⇒ +Div, sll Q, Q0 = 0	000000	000001000100	000000111100
5	3: Shift Div right	000000	000000100010	000000111100
6	1: Rem = Rem – Div	000000	000000100010	000000011010
6	2a: Rem ≥ 0 ⇒ sll Q, Q0 = 1	000001	000000100010	000000011010
6	3: Shift Div right	000001	000000010001	000000011010
7	1: Rem = Rem – Div	000001	000000010001	000000001001
7	2a: Rem ≥ 0 ⇒ sll Q, Q0 = 1	000011	000000010001	000000001001
7	3: Shift Div right	000011	000000001000	000000001001

- 3.27

  Half precision is represented as $x = (-1)^{\text{sign}} \times (1 + \text{fraction}) \times 2^{\text{exponent}-\text{bias}}$. In the case of -1.5625×10^{-1}, $-1.5625_{10} \times 10^{-1} = -0.15625_{10} \times 10^{0} = -0.00101_2 \times 2^0 = -1.01_2 \times 2^{-3}$. Therefore, the sign bit is 1, the fraction bits are 0100000000, and the exponent bits are $(-3 + 15)_{10} = 12_{10} = 01100_2$. Therefore, the half precision representation of -1.5625×10^{-1} is 1011000100000000.