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- **Pumping lemma** for CFL: Suppose G is a CFG, and $w \in L(G)$ s.t. the depth to the derivation tree of G on w is $> |G|$, then w can be partitioned into five parts $w = sxyzt$, where $|x| + |z| \geq 1$, and $sx^n yz^n t \in L(G)$ for every $n \geq 0$.
- **Pumping lemma** for CFL: Suppose M is the maximum of all the $|w|$'s that appear on the right hand side of the rules in R, i.e., $M = \max_{A \to w \in R} |w|$. Then, for every $u \in L(G)$ such that $|u| \geq M^{|R|} + 1$, u can be partitioned into $u = sxyzt$, where $|x| + |z| \geq 1$, and $sx^i yz^i t \in L(G)$ for every $i \geq 0$.
- *Pumping lemma* is used to prove that a language is not context-free.
- Proof $L = \{a^n b^n c^n | n \geq 0\}$ is not context-free:
  - Suppose L is CFL. However, there is no a partition of $u = sxyzt$ s.t. $sx^i yz^i t \in L(G)$ for every $i \geq 0$, which is a contradiction.
  - Remark: $L = \{a^n b^m c^m | n, m \geq 0\}$, $L = \{a^n b^n c^m | n, m \geq 0\}$ are context-free. However, their intersection $L = \{a^n b^n c^n | n \geq 0\}$ is *not*.
- A **push-down automaton (PDA)** is a system $A = \langle \Sigma, \Gamma, Q, q_0, F, \delta \rangle$ defined on Σ the input alphabet, Γ the stack alphabet, Q a finite set of states, $q_0 \in Q$ the initial state, $F \subseteq Q$ the set of final states, δ a set of transition functions: $(p, x, \text{pop}(y)) \to (q, \text{push}(z))$, where $p, q \in Q$, $x \in \Sigma \cup \{\epsilon\}$, $y, z \in \Gamma \cup \{\epsilon\}$.
- Consider a PDA $A = \langle \Sigma, \Gamma, Q, q_0, F, \delta \rangle$, where $\Sigma = \{a, b\}$, $\Gamma = \{a, b\}$, $Q = \{q_0\}$, q_0 is the initial state, $F = \{q_0\}$, and $\delta = \{(q_0, a, \text{pop}(\epsilon)) \to (q_0, \text{push}(a)), (q_0, b, \text{pop}(a)) \to (q_0, \text{push}(b))\}$. Test if *aaabba* and *b* are in $L(A)$.
- A **configuration** of a PDA is a pair $(q, v) \in (Q \times \Gamma^*)$. A **run** of a PDA on w is $(q_0 v_0) \vdash_{c_1} (q_1 v_1) \vdash_{c_2} \ldots \vdash (q_n v_n)$. It is an accepting run if $q_n \in F$.
- Given a PDA A, there is an integer M s.t. for every word w, consider the run of A on w, the stack always contains $\leq M$ symbol. What is $L(A)$?