Probability Homework 1

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## 1.3-13a

S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

## 1.3-13b

Let $A_i$ be the event of rolling $i$, for $i$ = 2, 3, ..., 12.

Rolling a 7 or 11 is the union of $A_7$ and $A_{11}$, i.e. $A_7 \cup A_{11}$.

$A_7 \cup A_{11}$ = { (1,6), (2,5), (3,4), (4,3), (5,2), (5,6), (6,1), (6,5) }. Therefore, P($A_7 \cup A_{11}$) = 8/36.

## 1.3-13c

$A_8$ = { (2,6), (3,5), (4,4), (5,3), (6,2) }. Therefore, P($A_8$) = 5/36.

$A_7$ = { (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) }. Therefore, P($A_7$) = 6/36.

P(8|7 or 8) = P($A_8 | A_8 \cup A_7$) = P($A_8 \cap (A_8 \cup A_7)$)/P($A_8 \cup A_7$) = P($A_8$)/P($A_8 \cup A_7$).

Since $A_8$ and $A_7$ are exclusive events, P($A_8$)/P($A_8 \cup A_7$) = P($A_8$)/(P($A_8$) + P($A_7$)) = (5/36)/(5/36 + 6/36) = 5/11.

## 1.3-13d

Based on the notation used in 1.3-13c, P(8) · P(8|7 or 8) = P($A_8$) · P($A_8 | A_8 \cup A_7$) = (5/36) · (5/11).

## 1.3-13e

The individual probability of the events that end up with a win includes (1) P($A_7 \cup A_{11}$), (2) P($A_i$) · P($A_i | A_i \cup A_7$), where $i$ = 4, 5, 6, 8, 9, 10.

P($A_4$) = 3/36, P($A_5$) = 4/36, P($A_6$) = 5/36, P($A_8$) = 5/36, P($A_9$) = 4/36, P($A_{10}$) = 3/36.

Therefore, the overall probability is P($A_7 \cup A_{11}$) + P($A_4$) · P($A_4 | A_4 \cup A_7$) + P($A_5$) · P($A_5 | A_5 \cup A_7$) + P($A_6$) · P($A_6 | A_6 \cup A_7$) + P($A_8$) · P($A_8 | A_8 \cup A_7$) + P($A_9$) · P($A_9 | A_9 \cup A_7$) + P($A_{10}$) · P($A_{10} | A_{10} \cup A_7$) =

8/36 + (3/36) · (3/36)/(3/36 + 6/36) + (4/36) · (4/36)/(4/36 + 6/36) + (5/36) · (5/36)/(5/36 + 6/36) + (5/36) · (5/36)/(5/36 + 6/36) + (4/36) · (4/36)/(4/36 + 6/36) + (3/36) · (3/36)/(3/36 + 6/36) ≈ 0.49293

## 1.4-15a

Let P($i$) be the probabilities that the fourth white ball is the $i$-th ball drawn **WITH** replacement, i.e. there are $i$-4 red ball(s) and 3 white balls before the 4-th white ball is drawn. The probability for a red or white ball to be drawn is always 1/2. Therefore, P($i$) can be generalized as the form $\binom{i-1}{3} \cdot \left(\frac{1}{2}\right)^i$

P(4) = $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$

P(5) = $\binom{4}{3} \cdot \left(\frac{1}{2}\right)^5 = \frac{1}{8}$

P(6) = $\binom{5}{3} \cdot \left(\frac{1}{2}\right)^6 = \frac{5}{32}$

P(7) = $\binom{6}{3} \cdot \left(\frac{1}{2}\right)^7 = \frac{5}{32}$

## 1.4-15b

Let P($i$) be the probabilities that the fourth white ball is the $i$-th ball drawn **WITHOUT** replacement, i.e. there are $i$-4 red ball(s) and 3 white balls before the 4-th white ball is drawn. The probability for a red or white ball to be drawn is #(red or white balls left)/#(balls left).

P(4) = $\frac{10 \cdot 9 \cdot 8 \cdot 7}{20 \cdot 19 \cdot 18 \cdot 17} = \frac{14}{323}$

P(5) = $\binom{4}{3} \cdot \frac{(10) \cdot (10 \cdot 9 \cdot 8 \cdot 7)}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16} = \frac{35}{323}$

P(6) = $\binom{5}{3} \cdot \frac{(10 \cdot 9) \cdot (10 \cdot 9 \cdot 8 \cdot 7)}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15} = \frac{105}{646}$

P(7) = $\binom{6}{3} \cdot \frac{(10 \cdot 9 \cdot 8) \cdot (10 \cdot 9 \cdot 8 \cdot 7)}{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14} = \frac{60}{323}$

## 1.4-15c

I think the probability for a team to win each game is a constant, i.e. independent of the result of the last game, therefore, it looks more like sampling **WITH** replacement.

## 1.5-4

Let $A$ be the event that the driver has an accident, and $B$ be the event that the driver is in the 16−25 age group. According to **Bayes' theorem**, P(the driver is in the 16−25 age group given the drive has an accident) = P($B|A$) = P($A|B$)·P($B$)/P($A$) = (0.05·0.1)/(0.05·0.1 + 0.02·0.55 + 0.03·0.20 + 0.04·0.15) ≈ 0.17857142857142863 ≈ 0.18