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- Given two DFAs:
  - $A_1 =< \Sigma, Q_1, q_1, F_1, \delta_1 >$ defined as follows: $\Sigma = \{a, b\}$, $Q_1 = \{q_{10}, q_{11}\}$, $q_1 = q_{10}$, $F_1 = \{q_{11}\}$, $\delta_1 : Q_1 \times \Sigma \to Q_1$.
  - $A_2 =< \Sigma, Q_2, q_2, F_2, \delta_2 >$ defined as follows: $\Sigma = \{a, b\}$, $Q_2 = \{q_{20}, q_{21}, q_{22}\}$, $q_2 = q_{20}$, $F_2 = \{q_{20}\}$, $\delta_2 : Q_2 \times \Sigma \to Q_2$.
- The language of all words accepted by A is denoted by $L(A)$
  - $L(A_1) = \{w|w \text{ has odd number of } a\}$
  - $L(A_2) = \{w|w \text{ has the number of } b \text{ divisible by } 3\}$
- **Theorem**: For every DFA $A = \langle \Sigma, Q, q, F, \delta \rangle$, there is a DFA A' s.t. $L(A') = \Sigma^* - L(A)$.
  - This is an example of *closure under complement*.
  - $A' = \langle \Sigma', Q', q', F', \delta' \rangle$ is defined as follows: $\Sigma' = \Sigma$, $Q' = Q$, $q' = q$, $F' = Q - F$, $\delta' = \delta$.
  - $L(A'_1) = \{w|w \text{ has even number of } a\}$
  - $L(A'_2) = \{w|w \text{ has the number of } b \text{ not divisible by } 3\}$
- **Theorem**: For every DFA A_1 and A_2, there is a DFA A s.t. $L(A') = L(A_1) \cap L(A_2)$.
  - This is an example of *closure under intersection*.
  - $A' = \langle \Sigma', Q', q', F', \delta' \rangle$ is defined as follows: $Q = Q_1 \times Q_2$, $q = (q_1, q_2)$, $F = F_1 \times F_2$, $\delta : \delta((p_1, p_2), a) = (\delta(p_1, a), \delta(p_2, a))$
  - Proof: (1) $L(A) \supseteq L(A_1) \cap L(A_2)$ and (2) $L(A) \subseteq L(A_1) \cap L(A_2)$
  - $L(A) = \{w|w \text{ has odd number of } a \text{ and the number of } b \text{ divisible by } 3\}$
- **Theorem**: For every DFA A_1 and A_2, there is a DFA A s.t. $L(A') = L(A_1) \cup L(A_2)$.
  - This is an example of *closure under union*.
  - $A' = \langle \Sigma', Q', q', F', \delta' \rangle$ is defined as follows: $Q = Q_1 \times Q_2$, $q = (q_1, q_2)$, $F = F_1 \times Q_2 \cup Q_1 \times F_2$, $\delta : \delta((p_1, p_2), a) = (\delta(p_1, a), \delta(p_2, a))$
  - Proof: (1) $L(A) \supseteq L(A_1) \cap L(A_2)$ and (2) $L(A) \subseteq L(A_1) \cap L(A_2)$
  - $L(A) = \{w|w \text{ has odd number of } a \text{ or the number of } b \text{ divisible by } 3\}$
- **Regular language**:
  - Definition: A language L is a **regular language** iff there is a DFA A s.t. $L(A) = L$
  - Closure under *complement*, *intersection*, and *union*.
- Conclusions:
  - The number of DFAs is countably infinite.
  - The number of languages is uncountably infinite.
  - Therefore, there is one language that cannot be represented by DFA, i.e. nonregular.