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- The language of **propositional logic** is based on **propositions**, or **declarative sentences** which one can, in principle, argue as being *true* or *false*.
- The basic blocks of the language are **atomic** (or **indecomposable**) sentences.
- More complex sentences can be constructed with **connectives**: $\neg$ (negation/not), $\land$ (conjunction/and), $\lor$ (disjunction/or), $\Longrightarrow$ (implication).
- **Sequent**: $\phi_1, \phi_2, \ldots, \phi_n \vdash \psi$, where ϕ are **premises**, and ψ is a **conclusion**.
- A sequent is **valid** if a proof (built by the proof rules) can be found.
- For each connective, we have introduction proof rule(s) and also elimination proof rule(s).
    - **Double negation introduction** ($\neg\neg$i): $\frac{p}{\neg\neg p}$ $\neg\neg$i
    - **Double negation elimination** ($\neg\neg$e): $\frac{\neg\neg p}{p}$ $\neg\neg$e
    - **Conjunction introduction** ($\land$i): $\frac{p \quad q}{p \land q}$ $\land$i
    - **Conjunction elimination** ($\land e_1$/$\land e_2$): $\frac{p \land q}{p}$ $\land e_1$ or $\frac{p \land q}{q}$ $\land e_2$
    - **Disjunciton introduction** ($\lor i_1$/$\lor i_2$): $\frac{p}{p \lor q}$ $\lor i_1$ or $\frac{q}{p \lor q}$ $\lor i_2$
    - **Disjunciton elimination** ($\lor$e): $\frac{p \lor q \quad \boxed{\begin{array}{c} p \\ \vdots \\ \chi \end{array}} \quad \boxed{\begin{array}{c} q \\ \vdots \\ \chi \end{array}}}{\chi}$ $\lor$e
    - **Implication introduction** ($\Longrightarrow$i): $\frac{\boxed{\begin{array}{c} \phi \\ \vdots \\ \psi \end{array}}}{\phi \Longrightarrow \psi}$ $\Longrightarrow$i
    - **Implication elimination** ($\Longrightarrow$e) or **Modus ponens (MP)**: $\frac{\phi \quad \phi \Longrightarrow \psi}{\psi}$ $\Longrightarrow$e
    - **Modus tollens (MT)**: $\frac{\phi \Longrightarrow \psi \quad \neg\psi}{\neg\phi}$ MT
- A sentence ϕ such that $\vdash \phi$ is called a **theorem**.