Probability Homework 3 B00401062 羅文斌

3.2-3

The CDF of exponential distribution is $F(x) = 1 - e^{-\lambda x}$, so $P(X > x) = 1 - F(x) = e^{-\lambda x}$.

$P(X > x + y | X > x)$ = $P(X > x + y)/P(X > x)$ = $e^{-\lambda(x+y)}/e^{-\lambda x}$ = $e^{-\lambda(x+y-x)}$ = $e^{-\lambda y}$ = $P(X > y)$

3.2-8

The CDF of gamma distribution is $F(x) = 1 - \sum_{i=0}^{\alpha-1} \frac{(\beta x)^i}{i!} e^{-\beta x}$, where $\beta = \theta^{-1}$. Plugging $\alpha = 2$ and $\beta = 1/4$ into the formula, we get $F(x) = 1 - \sum_{i=0}^{1} \frac{(x/4)^i}{i!} e^{-x/4}$

$P(X < 5)$ = $F(5)$ = $1 - \sum_{i=0}^{1} \frac{(5/4)^i}{i!} e^{-5/4}$ = $1 - \frac{(5/4)^0}{0!} e^{-5/4} - \frac{(5/4)^1}{1!} e^{-5/4}$ = $1 - \frac{9}{4} e^{-5/4}$ $\simeq$ 0.35536420706457222

3.2-11

$\chi^2(17)$ implies the degree of freedom $r = 17$. The CDF of chi-squared distribution is $F(x) = \Gamma(\frac{r}{2})^{-1}\gamma(\frac{r}{2}, \frac{x}{2})$, where Γ is the gamma function, and γ is the lower incomplete gamma function, which is $\gamma(\alpha, \beta) = \int_0^{\beta} t^{\alpha-1} e^{-t} dt$. Plugging $r = 17$ into the formula, we get $F(x)$ = $\Gamma(\frac{17}{2})^{-1}\gamma(\frac{17}{2}, \frac{x}{2})$. Also, we can refer to the Table IV for answers.

a. $P(X < 7.564)$ = $F(7.564)$ = $\Gamma(\frac{17}{2})^{-1}\gamma(\frac{17}{2}, \frac{7.564}{2})$ $\simeq$ 0.025
b. $P(X > 27.59)$ = $1 - F(27.59)$ = $1 - \Gamma(\frac{17}{2})^{-1}\gamma(\frac{17}{2}, \frac{27.59}{2})$ $\simeq$ 1 - 0.950 = 0.050
c. $P(6.408 < X < 27.59)$ = $F(27.59) - F(6.408)$ = $\Gamma(\frac{17}{2})^{-1}\gamma(\frac{17}{2}, \frac{27.59}{2}) - \Gamma(\frac{17}{2})^{-1}\gamma(\frac{17}{2}, \frac{6.408}{2})$ $\simeq$ 0.950 - 0.010 = 0.940
d. $\chi^2_{0.95}(17)$ = 8.672
e. $\chi^2_{0.025}(17)$ = 30.19

3.3-5

X can be transformed to a normalized value $Z = \frac{X-\mu}{\sigma} = \frac{X-6}{5}$. We can refer to Table Va and Vb for values.

a. $P(6 \le X \le 12)$ = $P(0 \le Z \le \frac{6}{5})$ = $P(Z \le \frac{6}{5}) - P(Z \le 0)$ = 0.8849 - 0.5 = 0.3849
b. $P(0 \le X \le 8)$ = $P(\frac{-6}{5} \le Z \le \frac{2}{5})$ = $P(Z \le \frac{2}{5}) - P(Z \le \frac{-6}{5})$ = 0.6554 - 0.1151 = 0.5403
c. $P(-2 < X \le 0)$ = $P(\frac{-8}{5} \le Z \le \frac{-6}{5})$ = $P(Z \le \frac{-6}{5}) - P(Z \le \frac{-8}{5})$ = 0.1151 - 0.0548 = 0.0603
d. $P(X > 21)$ = $P(Z > 3)$ = 0.0013
e. $P(|X-6| < 5)$ = $P(|Z| < 1)$ = 1 - 0.1587 · 2 = 0.6826
f. $P(|X-6| < 10)$ = $P(|Z| < 2)$ = 1 - 0.0228 · 2 = 0.9544
g. $P(|X-6| < 15)$ = $P(|Z| < 3)$ = 1 - 0.0013 · 2 = 0.9974
h. $P(|X-6| < 12.41)$ = $P(|Z| < 2.482)$ = $1 - (0.0066 + (0.0064 - 0.0066) \cdot \frac{0.002}{0.01}) \cdot 2$
   = 1 - 0.00656 · 2 ≃ 0.9869

3.3-16

The MGF of normal distribution is $M(t) = \exp(\mu t + \frac{1}{2}\sigma^2 t^2)$
$N(0,1)$: $M(t) = \exp(\frac{1}{2}t^2)$ is curve (a)
$N(-1,1)$: $M(t) = \exp(-t + \frac{1}{2}t^2)$ is curve (b)
$N(2,1)$: $M(t) = \exp(2t + \frac{1}{2}t^2)$ is curve (c)