

Homework 1

1.

<u>1 $q \Rightarrow r$ premise</u>	
<u>2 $p \Rightarrow q$ assumption</u>	
<u>3 p assumption</u>	
4 q	\Rightarrow e : 3, 2
5 r	\Rightarrow e : 4, 1
6 $p \Rightarrow r$ \Rightarrow i : 3 – 5	
7 $(p \Rightarrow q) \Rightarrow (p \Rightarrow r)$ \Rightarrow i : 2 – 6	

2.

<u>1 $\neg p$ assumption</u>	
<u>2 p assumption</u>	
<u>3 p assumption</u>	
4 \perp	\neg e : 3, 1
5 q	\perp e : 4
6 $p \Rightarrow q$ \Rightarrow i : 3 – 5	
7 $p \Rightarrow (p \Rightarrow q)$ \Rightarrow i : 2 – 6	
8 $\neg p \Rightarrow (p \Rightarrow (p \Rightarrow q))$ \Rightarrow i : 1 – 7	

3.

<u>1 $p \Rightarrow q$ premise</u>	
<u>2 $\neg(\neg p \vee q)$ assumption</u>	
<u>3 $\neg p$ assumption</u>	
4 $\neg p \vee q$	\vee i ₁ : 3
5 \perp	\neg e : 4, 2
6 p RAA : 3 – 5	
<u>7 q assumption</u>	
8 $\neg p \vee q$	\vee i ₂ : 7
9 \perp	\neg e : 8, 2
10 $\neg q$	\neg i : 7 – 9
11 q	\Rightarrow e : 6, 1
12 \perp	\neg e : 11, 10
13 $\neg p \vee q$ RAA : 2 – 12	

4.

1 $(s \implies p) \vee (t \implies q)$ premise	
2 $s \implies p$ assumption	22 $t \implies q$ assumption
3 $s \vee \neg s$ LEM	23 $t \vee \neg t$ LEM
4 s assumption	24 t assumption
5 $p \implies e : 4, 2$	25 $q \implies e : 24, 22$
6 $\neg s \vee p \vee i_2 : 5$	26 $\neg t \vee q \vee i_2 : 25$
7 $\neg s$ assumption	27 $\neg t$ assumption
8 $\neg s \vee p \vee i_1 : 7$	28 $\neg t \vee q \vee i_1 : 27$
9 $\neg s \vee p \vee e : 3, 4 - 6, 7 - 8$	29 $\neg t \vee q \vee e : 23, 24 - 26, 27 - 28$
10 $\neg s$ assumption	30 $\neg t$ assumption
11 s assumption	31 t assumption
12 $\perp \neg e : 11, 10$	32 $\perp \neg e : 31, 30$
13 $q \perp e : 12$	33 $p \perp e : 32$
14 $s \implies q \implies i : 11 - 13$	34 $t \implies p \implies i : 31 - 33$
15 $(s \implies q) \vee (t \implies p) \vee i_1 : 14$	35 $(s \implies q) \vee (t \implies p) \vee i_2 : 34$
16 p assumption	36 q assumption
17 t assumption	37 s assumption
18 p	38 q
19 $t \implies p \implies i : 17 - 18$	39 $s \implies q \implies i : 37 - 38$
20 $(s \implies q) \vee (t \implies p) \vee i_2 : 19$	40 $(s \implies q) \vee (t \implies p) \vee i_1 : 39$
21 $(s \implies q) \vee (t \implies p) \vee e : 9, 10 - 15, 16 - 20$	41 $(s \implies q) \vee (t \implies p) \vee e : 29, 30 - 35, 36 - 40$
42 $(s \implies q) \vee (t \implies p) \vee e : 1, 2 - 21, 22 - 41$	

5.

1 $(p \wedge q) \implies r$ premise
2 $r \implies s$ premise
3 $q \wedge \neg s$ premise
4 $\neg s \wedge e_2 : 3$
5 $\neg r$ MT : 2, 4
6 $\neg(p \wedge q)$ MT : 1, 5
7 p assumption
8 $q \wedge e_1 : 3$
9 $p \wedge q \wedge i : 7, 8$
10 $\perp \neg e : 9, 6$
11 $\neg p \neg i : 7 - 10$