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- Conversion (1) between RE and NFA; (2) from NFA to DFA; (3) between CFG and PDA.
- From CFG to PDA: Given a CFG $G = \langle \Sigma, V, R, S \rangle$, the equivalent PDA $A = \langle \Sigma, \Gamma, Q, q_0, F, \delta \rangle$, where $\Gamma = \Sigma \cup V \cup \{\bot\}$, $Q = \{p, q, r\}$, $q_0 = p$, $F = \{r\}$, and δ is the set of:
  - $(p, \epsilon, \text{pop}(\epsilon)) \to (q, \text{push}(S\,\bot))$
  - $(q, a, \text{pop}(a)) \to (q, \text{push}(\epsilon))\ \forall a \in \Sigma$
  - $(q, \epsilon, \text{pop}(A)) \to (q, \text{push}(w))\ \forall A \to w \in R$
  - $(q, \epsilon, \text{pop}(\bot)) \to (r, \text{push}(\epsilon))$
- From PDA to CFG: Given a PDA $A = \langle \Sigma, \Gamma, Q, q_0, F, \delta \rangle$, the equivalent CFG $G = \langle \Sigma, V, R, S \rangle$, where $V = \{A_{p,q} | p, q \in Q\}$, and R is the set of:
  - $A_{p,q} \to aA_{r,s}b$ for every pair of $(p, a, \text{pop}(\epsilon)) \to (r, \text{push}(z))$ and $(s, b, \text{pop}(z)) \to (q, \text{push}(\epsilon))$
  - $A_{p,r} \to aA_{q,r}$ for every $(p, a, \text{pop}(\epsilon)) \to (q, \text{push}(\epsilon))$
  - $A_{p,q} \to A_{p,r}A_{r,q}$
  - $A_{p,p} \to \epsilon$
- Construction of union, concatenation, Kleene star from two CFGs: Given two CFGs, $G_1 = \langle \Sigma, V_1, R_1, S_1 \rangle$ and $G_2 = \langle \Sigma, V_2, R_2, S_2 \rangle$,
  - $G = G_1 \cup G_2$ has $\Sigma = \Sigma, V = V_1 \cup V_2 \cup \{S\}, R = R_1 \cup R_2 \cup \{S \to S_1, S \to S_2\}, S = S$.
  - $G = G_1G_2$ has $\Sigma = \Sigma, V = V_1 \cup V_2 \cup \{S\}, R = R_1 \cup R_2 \cup \{S \to S_1S_2\}, S = S$.
  - $G = G_1^*$ has $\Sigma = \Sigma, V = V_1 \cup \{S\}, R = R_1 \cup \{S \to SS_1, S \to \varepsilon\}, S = S$.
- **Pumping lemma** for RL: If A is a regular language, then there is a number p (the **pumping length**) where if $s \in A$ and $|s| \geq p$, then s may be divided into three pieces, $s = xyz$, satisfying the following conditions: (1) $\forall i \geq 0, xy^iz \in A$; (2) $|y| > 0$; (3) $|xy| \leq p$.
- **Pumping lemma** for CFL: If A is a context-free language, then there is a number p (the **pumping length**) where if $s \in A$ and $|s| \geq p$, then s may be divided into five pieces, $s = uvxyz$, satisfying the following conditions: (1) $\forall i \geq 0, uv^ixy^iz \in A$; (2) $|vy| > 0$; (3) $|vxy| \leq p$.
- Conversion (1) from multi-tape TM to single tape TM; (2) from NTM to DTM.
- **Closure**:
  - RL: complement, union, intersection, concatenation, Kleene star.
  - CFL: union, concatenation, Kleene star.
  - Decidable language: complement, union, intersection, concatenation, Kleene star.
  - Recognizable language: union, intersection, concatenation, Kleene star.
- Suppose $A \leq_M B$ holds:
  - $A \leq_T B$ holds.
  - If B is recognizable, then A is recognizable.
  - If A is not recognizable, then B is not recognizable.
- Suppose $A \leq_T B$ holds:
  - $A \leq_M B$ does not necessarily hold.
  - If B is decidable, then A is decidable.

- If A is undecidable, then B is undecidable.

- **Theorem**: $A \leq_M B$ if and only if $A' \leq_M B'$.
- **Theorem**: A language L is decidable if and only if L is recognizable and co-recognizable.
- **Theorem**: $A_{\text{TM}} = \{(M, w) | w \in L(M)\}$ is undecidable, recognizable, not co-recognizable.
- **Theorem**: HALT = $\{\lfloor M \rfloor | M$ halts on $\lfloor M \rfloor\}$ is undecidable, recognizable, not co-recognizable.
- **Theorem**: $\text{HALT}_0 = \{\lfloor M \rfloor | \lfloor M \rfloor \in L(M)\}$ is undecidable, recognizable, not co-recognizable.
- **Theorem**: $\text{EMPTY}_{\text{TM}} = \{\lfloor M \rfloor | L(M) = \emptyset\}$ is undecidable.
  - Construct $M'(M, w)$: On input x: Accept if M accepts w. Reject, otherwise.
- **Theorem**: $\text{ALL}_{\text{TM}} = \{\lfloor M \rfloor | L(M) = \Sigma^*\}$ is undecidable.
  - Construct $M'(M, w)$: On input x: Accept if M accepts w. Reject, otherwise.
- **Theorem**: $L = \{\lfloor M \rfloor | L(M) = s\}$ and $L = \{\lfloor M \rfloor | L(M) \ni s\}$ are undecidable.
  - Construct $M'(M, w)$: On input x: Accept if $x = s$ and M accepts w. Reject, otherwise.
- **Theorem**: $\text{RL}_{\text{TM}} = \{\lfloor M \rfloor | L(M)$ is regular$\}$ is undecidable.
  - Construct $M'(M, w)$: On input x: Accept if $x = 0^n 1^n$ or M accepts w. Reject, otherwise.
- **Theorem**: $\text{CFL}_{\text{TM}} = \{\lfloor M \rfloor | L(M)$ is context free$\}$ undecidable.
  - Construct $M'(M, w)$: On input x: Accept if $x = 0^n 1^n 0^n$ or M accepts w. Reject, otherwise.