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- **Function**: for every $x \in X$, there is exactly one $y \in Y$ such that $(x, y) \in R$.
  - **Injective** (**one-to-one**; $|X| \leq |Y|$): for every $y \in Y$, there is at most one $x \in X$ such that $f(x) = y$.
  - **Surjective** (**onto**; $|X| \geq |Y|$): for every $y \in Y$, there is at least one $x \in X$ such that $f(x) = y$.
  - **Bijective** (**one-to-one and onto**; $|X| = |Y|$): both injective and surjective.
- **Countable**: there is an injective function from X to $\mathbb{N}$.
  - Examples of countably infinite sets: $\mathbb{Q}, \mathbb{Z}, \mathbb{N}$.
  - Examples of uncountably infinite sets: $\mathbb{C}, \mathbb{R}, 2^{\mathbb{N}}$.
  - Proof of uncountability: **Cantor's diagonalization**.
- **Alphabet** (Σ): a finite set of symbols.
- **Word** (w): a finite sequence of symbols over Σ.
  - ϵ (**empty word**): the word of length 0.
  - Σ^n: the set of all words of length n over Σ.
  - $\Sigma^* = \cup_{n \geq 0} \Sigma^n$: the set of all finite words over Σ.
  - $\Sigma^+ = \cup_{n \geq 1} \Sigma^n = \Sigma^* \setminus \{\epsilon\}$
- **Language** (L): a subset of Σ^*, i.e. $L \subseteq \Sigma^*$.
  - Given two languages L_1 and L_2 over Σ, $L_1 L_2 = \{w_1 w_2 | w_1 \in L_1, w_2 \in L_2\}$ (similar to Cartesian product).
- Other examples of countably infinite sets:
  - The set of all finite sequences over Σ, i.e. Σ^*.
  - The set of all computer programs. $\because$ computer programs have (1) finite symbols, and (2) finite length.
- Other examples of uncountably infinite sets:
  - The set of all languages over Σ, i.e. 2^{Σ^*}.
- Question: For every language, we can write a computer program to detect it. (*False*)