Discrete Mathematics Midterm 1

2017-02-23

- **Principle of inclusion and exclusion**: to count the number of elements in S with t conditions satisfying (1) none of the t conditions, (2) at least 1 condition, (3) exactly m conditions, and (4) at least m conditions.
- **Derangement** is to arrange objects in a order that none of them is at their original place.
- [x] Homework: #1-1, #1-2, #1-3, #1-4
- Solutions: Solutions1.pdf

2017-03-02

- **Rook polynomial** problem is to find the the number of ways of placing k non-taking rooks on a chessboard C.
- **Generating functions** $F(x)$ can be expressed as $F(x) = \sum a_i \mu_i(x)$, where μ is called the **indicator function**, and a is the coefficient of the corresponding indicator function.
  - **Ordinary generating function**: $1/(1-x) = 1 + x + x^2 + ...$
  - **Exponential generating function**: $e^x = 1 + x^1/1! + x^2/2! + ...$
- Indicator functions are chosen in such a way that *no two distinct sequences will yield the same generating function*.
- In the formula $C(n, r)$, n can be any real number.
- [x] Homework: #1-5, #2-1, #2-2, #2-3
- Solutions: Solutions1.pdf, Solutions2.pdf

2017-03-09

- **Partition of integers** is the problem to compute the number of partitions $P(n)$ of a positive integer n into positive summands, disregarding their order.
- Orinary generating functions are used to solve *combination* problems, whereas exponential generating functions are used to solve *permutation* problems.
- $f(n) = \sum_{i=0}^{k} c_{n-i} a_{n-i}$ is a **linear recurrence relations** with constant coefficients of order k. The relation is *homogeneous* if $f(n) = 0$, and *nonhomogeneous* if $f(n) \neq 0$.
- [x] Homework: #2-4, #2-5, #3-1

- Solutions: [Solutions2.pdf](), [Solutions3.pdf]()

2017-03-16

- The **characteristic equation** is the equation which is solved to find a matrix's eigenvalues, also called the **characteristic polynomial**, and its roots are called the **characteristic roots**, or **eigenvalues**.
- The solution of **linear nonhomogeneous recurrence relations** hsa 2 parts:
    - **Homogeneous solution**: satisfies the equation with $f(n) = 0$.
    - **Particular solution**: depends on the family of $f(n)$.
- [x] Homework: #3-2, #3-3, #3-4, #3-5, #3-6, #4-1, #4-2, #4-3, #4-4
- Solutions: [Solutions3.pdf](), [Solutions4.pdf]()

2017-03-23

- **Ordinary generating functions** can help determine the solution a_n of a recurrence relation by looking for the coefficient of x^n.
- There is *no general method* to solve nonlinear recurrence relations. However, some can be simplified to linear recurrence relations.
- [x] Homework: #4-5, #4-6
- Solutions: [Solutions4.pdf]()