Mini Homework 10

1. **push**(): $1 + \text{cap} = \Theta(\text{cap})$.
   pop(): $1 + \text{cap} / 4 = \Theta(\text{cap})$.
2. Define $\Phi(D_i)$ after the i-th operation to be $\begin{cases} (2 \times \text{size} - \text{cap}) \text{ when size} \geq \text{cap} / 2 \\ (\text{cap} / 2 - \text{size}) \text{ when size} \leq \text{cap} / 2 \end{cases}$
   - Validity of $\Phi(D_i)$:
     - $\Phi(D_0) = 0$.
     - $\Phi(D_i) \geq 0$:
       - When $\text{size} \geq \text{cap} / 2$, $(2 \times \text{size} - \text{cap}) \geq 0$.
       - When $\text{size} \leq \text{cap} / 2$, $(\text{cap} / 2 - \text{size}) \geq 0$.
   - The amortized cost of **push**():
     - When pushing an element causes a resize:
       $[1 + \text{cap}] + [2 \times (\text{size} + 1) - 2 \times \text{cap}] - [2 \times \text{size} - \text{cap}] = 3$.
     - When pushing an element does **not** cause a resize:
       $1 + [2 \times (\text{size} + 1) - \text{cap}] - [2 \times \text{size} - \text{cap}] = 3$.
       $1 + [\text{cap} / 2 - (\text{size} + 1)] - [\text{cap} / 2 - \text{size}] = 0$.
     - All operations have O(1) amortized cost.
   - The amortized cost of **pop**():
     - When popping an element causes a resize:
       $[1 + \text{cap} / 4] + [\text{cap} / 4 - (\text{size} - 1)] - [\text{cap} / 2 - \text{size}] = 2$.
     - When popping an element does **not** cause a resize:
       $1 + [2 \times (\text{size} - 1) - \text{cap}] - [2 \times \text{size} - \text{cap}] = -1$.
       $1 + [\text{cap} / 2 - (\text{size} - 1)] - [\text{cap} / 2 - \text{size}] = 2$.
     - All operations have O(1) amortized cost.