2017-10-18

- Equality (=):
  - **Intensional**: two terms are syntactically equal.
  - **Extensional**: two terms denote the same object.
- Proof rules for equality and quantifiers:
  - Equality introduction: $\dfrac{}{t=t}$ =i
  - Equality elimination: $\dfrac{t_1=t_2 \quad \phi[t_1/x]}{\phi[t_2/x]}$ =e
  - Universal quantification introduction: $\dfrac{\boxed{\begin{array}{l} x_0 \\ \vdots \\ \phi[x_0/x] \end{array}}}{\forall x \phi}$ $\forall x$i
  - Universal quantification elimination: $\dfrac{\forall x \phi}{\phi[t/x]}$ $\forall x$e
  - Existential quantification introduction: $\dfrac{\phi[t/x]}{\exists x \phi}$ $\exists x$i
  - Existential quantification elimination: $\dfrac{\exists x \phi \quad \boxed{\begin{array}{l} x_0 \\ \phi[x_0/x] \\ \vdots \\ \chi \end{array}}}{\chi}$ $\exists x$e
- **Subformula property**: an elimination rule that concludes with a subformula of the eliminated formula.
- The semantics of terms and atomic predicates are defined in *models*.
- Let F and P be a set of function and predicate symbols respectively. A **model** M of (F, P) consists of
  - A non-empty set A called the **universe**.
  - For function symbol $f \in F$ with arity $n \geq 0$, a function $f^M : A^n \to A$. Particularly, a constant symbol $c \in F$ is an element $c^M \in A$.
  - For predicate symbol $p \in P$ with arity $n > 0$, a set $p^M \subseteq A^n$.
- **Environment**: An **environment** for a universe A is a function $l : \text{var} \to A$. If l is an **environment**, $x \in \text{var}$, and $a \in A$, the environment $l[x \to a]$ is defined as $l[x \to a](y) = \begin{cases} a \text{ if } x = y \\ l(y) \text{ otherwise} \end{cases}$.