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- A CFG that accepts $w \in \{a^n b^n | n \in \mathbb{N}\}$: $G = \langle \Sigma, V, R, S \rangle$, where $\sigma = \{a, b\}$, $V = \{S\}$, $R = \{S \to aSb, S \to \epsilon\}$, and $S = S$.
- If a grammar can never be free from variables, then the language it generates is $\emptyset$.
- Theorem: CFLs are closed under union, concatenation, and Kleene star, but *not* closed under intersection and complement.
- Given $G_1 \langle \Sigma, V_1, R_1, S_1 \rangle$, and $G_2 \langle \Sigma, V_2, R_2, S_2 \rangle$.
  - Construct the union of $G = G_1 \cup G_2$ as follows: $\Sigma = \Sigma$, $V = V_1 \cup V_2 \cup \{S\}$, $R = R_1 \cup R_2 \cup \{S \to S_1, S \to S_2\}$, $S = S$.
  - Construct the concatenation of $G = G_1 G_2$ as follows: $\Sigma = \Sigma$, $V = V_1 \cup V_2 \cup \{S\}$, $R = R_1 \cup R_2 \cup \{S \to S_1 S_2\}$, $S = S$.
  - Construct the Kleene star of $G = G_1^*$ as follows: $\Sigma = \Sigma$, $V = V_1 \cup V_2 \cup \{S\}$, $R = R_1 \cup R_2 \cup \{S \to SS_1, S \to \epsilon\}$, $S = S$.
- Suppose $L(G_1) = \{a^m b^m c^n | m, n \in \mathbb{N}\}$ and $L(G_2) = \{a^m b^n c^n | m, n \in \mathbb{N}\}$. The intersection of two languages $L(G_1 \cap G_2) = \{a^m b^m c^m | m \in \mathbb{N}\}$, which is not context-free.
- Theorem: Every regular language is also CFL.
  - Given an NFA $A = \langle \Sigma, Q, q_0, F, \delta \rangle$, where $Q = \{q_0, q_1, q_2, \ldots, q_m\}$. The equivalent CFG $G = \langle \Sigma, V, R, S \rangle$ is constructed as follows: $V = \{A_0, A_2, \ldots, A_m\}$, $R = \{A_i \to aA_j | (q_i, a, a_j) \in \delta\} \cup \{A_i \to \epsilon | q_i \in F\}$, and $S = A_1$.
- Lemma: $L(A) \subseteq L(G)$ proved by induction.
  - *Hypothesis*: For every run of A on $w = a_1 a_2 \ldots a_n$ from p_0 to p_n: $p_0 a_1 p_1 \ldots a_n p_n$, there is a derivation from A_{p_0} to $a_1 a_2 \ldots a_n A_{p_n}$. Therefore, if it is an accepting run, i.e. $p_n \in F$, then A_{p_n} can be replaced with ϵ, i.e. w is derivable from A_{p_0}.
  - *Induction* on n.
- Lemma: $L(G) \subseteq L(A)$ proved by induction.
  - *Hypothesis*: For every deviravation of G from A_{p_0} to $a_1 a_2 \ldots a_n A_{p_n}$, there is a run of A on $w = a_1 a_2 \ldots a_n$ from p_0 to p_n: $p_0 a_1 p_1 \ldots a_n p_n$. Therefore, if w is derivable from A_{p_0}, i.e. A_{p_n} can be replaced with ϵ, then $p_n \in F$, i.e. it is an accepting run.
  - *Induction* on n.