

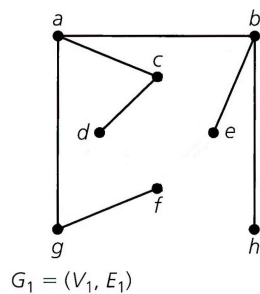
Solutions to Exercise #10

(範圍: Graph Theory)

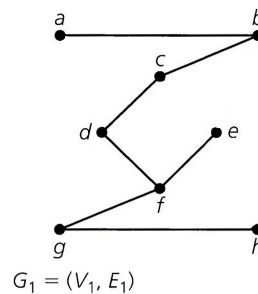
1. Build a BFS spanning tree and a DFS spanning tree of the graph G_1 in Figure 11.42, where it is assumed that vertex a is the root and the priorities of the other vertices to be branched or visited are $b > c > d > e > f > g > h$. (20%)

Sol:

BFS spanning tree:



DFS spanning tree:



2. Suppose that (u, v) is an edge of a graph G and it has the least cost among all edges that are incident to vertex v . Does every minimum spanning tree contain (u, v) (a) when all edges have distinct costs or (b) when multiple edges may have the same cost? Explain your answer. (20%)

Sol:

- (a) Yes.

Suppose that T is a minimum spanning tree of G and T does not contain (u, v) . If (u, v) is added to T , then a cycle is formed. Assume that (w, v) is the other edge in the cycle that is incident to v . Replacing (w, v) with (u, v) in T will induce a spanning tree whose cost is smaller than T , a contradiction.

- (b) No.

If both (w, v) and (u, v) in (a) have the same cost, then T is still a minimum spanning tree of G .

3. Is it possible to obtain a maximum-cost spanning tree of a weighted graph G by modifying Kruskal's algorithm? (10%)

Sol: Yes.

We only need to sort edges of G nonincreasingly according to their costs.

4. Consider the graph of Figure 11.54(a) and assign its edges with costs as follows: $c(a, b)=62$, $c(a, d)=37$, $c(a, h)=45$, $c(b, c)=19$, $c(b, g)=28$, $c(c, d)=70$, $c(c, f)=53$, $c(d, e)=81$, $c(e, f)=15$, $c(e, h)=40$, $c(f, g)=39$, and $c(g, h)=11$. What is the edge sequence obtained by executing Prim's MST algorithm on the weighted graph with starting vertex a ? (10%)

Sol: $(a, d), (a, h), (g, h), (b, g), (b, c), (f, g), (e, f)$.

5. For the graph of Figure 12.39(a), first compute $DFN(i)$ and $L(i)$ for each vertex i with the following assumptions when building a DFS spanning tree: vertex c is the root and the priorities of the other vertices to be visited are $a > b > d > e > f > g > h > i$, and then find all articulation points and bridges accordingly. (20%)

Sol:

vertex i :	a	b	c	d	e	f	g	h	i
$DFN(i)$:	2	3	1	4	5	6	7	8	9
$L(i)$:	1	1	1	1	1	6	6	6	6

articulation points: c, f .

bridges: (c, f) .

6. P. 621: 4 (only for (b)). (10%)

Sol: Suppose that T is a spanning tree of G . There are two or more nodes of T whose degrees are one. According to (a), they are not articulation points of T (and hence G).

7. Let $k_v(G)$ and $k_e(G)$ represent the vertex connectivity and edge connectivity, respectively, of a graph G . Give an example of $k_v(G) < k_e(G)$. (10%)

Sol: Refer to the lower 2-connected graph on page 52 of lecture notes. It has $2 = k_v(G) < k_e(G) = 3$.