2017-10-25

- Let ϕ be a predicate logic formula, l and l' two environments that agree on free variables of ϕ. Then, $M \vDash_l \phi$ holds iff $M \vDash_{l'} \phi$.
- A **sentence** is a predicate logic formula without free variables.
- Let ϕ be a sentence. Either $M \vDash_l \phi$ holds for every environment l, or $M \vDash_l \phi$ does not hold for every environment l. Hence, we write $M \vDash \phi$ or $M \nvDash \phi$.
- Let Γ be a set of predicate logic formulae and ψ a predicate logic formula.
  - $\Gamma \vDash \psi$ **holds** (or Γ **semantically entails** ψ) if for every model M and environment l, $M \vDash_l \psi$ holds whenever $M \vDash_l \phi$ holds for every $\phi \in \Gamma$.
  - ψ is **satisfiable** if there is a model M and an environment l such that $M \vDash_l \psi$ holds.
  - ψ is **valid** if $M \vDash_l \psi$ holds for every model M and environment l where we can compute ψ.
  - Γ is **consistent** or **satisfiable** if there is a model M and an environment l such that $M \vDash_l \phi$ for every $\phi \in \Gamma$.
- **Validity problem**: Given a predicate logic formula ϕ, check whether $\vDash \phi$ holds or not.
  - The validity problem for propositional logic is *NP-complete*.
  - The validity problem for predicate logic is *undecidable*.
- **Post correspondence problem (PCP)**: Given $C = ((s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k))$ where s_i, t_i are non-empty binary strings for every $1 \leq i \leq k$. Check whether there are $1 \leq i_1, i_2, \ldots, i_n \leq k$ s.t. $s_{i_1} s_{i_2} \ldots s_{i_n} = t_{i_1} t_{i_2} \ldots t_{i_n}$.
- $\vDash \phi$ holds iff $\neg\phi$ is not satisfiable.
  - Theorem: The validity problem for predicate logic is *undecidable*.
  - Corollary: The satisfiability problem for predicate logic is *undecidable*.
- **Soundness** and **completeness**: For any predicate logic sentence ϕ, $\vdash \phi$ iff $\vDash \phi$.
- Lemma: Let Γ be a set of predicate logic formulae. $\Gamma \vDash \phi$ implies $\Gamma \vdash \phi$ is equivalent to $\Gamma \vDash \bot$ implies $\Gamma \vdash \bot$.
- **Compactness theorem**: Let Γ be a set of predicate logic sentences. If all finite subset of Γ is satisfiable, Γ is satisfiable.
- **Lowenhein-Skolem Theorem**: Let ψ be a predicate logic sentence. If ψ has a model with at least n elements for every $n \geq 1$, ψ has a model with infinitely many elements.
- **Reachability**: Given a directed graph G and nodes n, n' in G, the reachability problem for G is to check whether there is a path of transition from n to n'.
- Reachability in predicate logic: There is no predicate logic formula ϕ with exactly two free variables u, v and exactly one binary predicate R such that ϕ holds in directed graphs iff there is a

path in the graph from the node associated with u to the node associated with v.