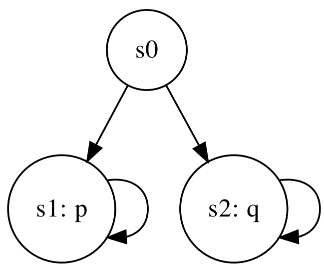
Homework 6

1. Consider any path π.
   - Suppose $\pi \vDash \varphi U \psi$. Let n be the smallest number that $\pi^n \vDash \psi$. $\pi \vDash F\psi$ holds. For every $0 \le i < n$ we have $\pi^i \vDash \varphi$. Hence, for every $0 \le j \le n$ we have $\pi^j \vDash \varphi \vee \psi$. Hence, $\pi \vDash \psi R(\varphi \vee \psi)$ holds.
   - Suppose $\pi \vDash \psi R(\varphi \vee \psi) \wedge F\psi$. Since $\pi \vDash F\psi$, let n be the smallest number that $\pi^n \vDash \psi$. For every $0 \le i < n$ we have $\pi^i \vDash \neg\psi$. Since $\pi \vDash \psi R(\varphi \vee \psi)$, for every $0 \le j \le n$ we have $\pi^j \vDash \varphi \vee \psi$. Hence, for all $0 \le k < n$ we have $\pi^k \vDash \varphi$. Hence, $\pi \vDash \varphi U \psi$ holds.
2. Consider any path π.
   - Suppose $\pi \vDash \varphi W \psi$.
     - If there is some $i \ge 0$ such that $\pi^i \vDash \psi$ and for every $0 \le j < i$ we have $\pi^j \vDash \varphi$, then for every $0 \le k \le i$, we have $\pi^k \vDash \varphi \vee \psi$. Hence, $\pi \vDash \psi R(\varphi \vee \psi)$ holds.
     - If for every $i \ge 0$ we have $\pi^i \vDash \varphi$, then for every $i \ge 0$ we have $\pi^i \vDash \varphi \vee \psi$. Hence, $\pi \vDash \psi R(\varphi \vee \psi)$ also holds.
   - Suppose $\pi \vDash \psi R(\varphi \vee \psi)$.
     - If there is some $i \ge 0$ such that $\pi^i \vDash \psi$ and for every $0 \le j \le i$ we have $\pi^j \vDash \varphi \vee \psi$. Let n be the smallest of such i. For every $0 \le j < n$ we have $\pi^j \vDash \neg\psi$. Hence, for every $0 \le k < n$ we have $\pi^k \vDash \varphi$. Hence, $\pi \vDash \varphi W \psi$ holds.
     - If for every $i \ge 0$ we have $\pi^i \vDash \varphi \vee \psi$ and there is no such $j \ge 0$ such that $\pi^j \vDash \psi$, then for every $k \ge 0$ we have $\pi^k \vDash \varphi$. Hence, $\pi \vDash \varphi W \psi$ holds.
3. Consider $M = (S, \rightarrow, L)$ designed as below. Let $\varphi = p$ and $\psi = q$. Consider any path that starts with s_0 . We have $M, s_0 \vDash AF(p \vee q)$ but $M, s_0 \nvDash AFp \vee AFq$.



4. $[AG(q \Rightarrow \neg EF\ (p\ \wedge EF\ r))] \wedge [AG(r \Rightarrow \neg EF\ (p\ \wedge EF\ q))]$
5. Consider $M = (S, \rightarrow, L)$ designed as below. We have $M, s_0 \vDash AGEFp$ but not $M, s_0 \vDash AGFp$.