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- **Theorem**: A language is Turing-recognizable if and only if some multitape Turing machine recognizes it.
- How to convert a k-tape TM M to a single tape TM S?
  - Put its tape into the format that represents all k tapes of M s.t.
    - The content the k tapes are concatenated and delimited by #.
    - Virtual head positions are marked with *.
  - S scans its tape from the first #, which marks the left-hand end, to the $(k+1)$-st #, which marks the right-hand end, in order to determine the symbols under the virtual heads.
  - S makes a second pass to update the tapes according to the way that M's transition function dictates.
  - If at any point S moves one of the virtual heads to the right onto a #, S writes a blank symbol on this tape cell and shifts the tape contents, from this cell until the rightmost #, one unit to the right.
- Synonyms:
  - Decidable = Recursive.
  - Recognizable = Semi-decidable = Recursive enumerable.
- **Theorem**: Decidable languages are closed under complement, union, intersection, concatenation, Kleene star.
  - Complement L': On input w, run M on w, and accept iff M rejects.
  - Union $L_1 \cup L_2$: On input w, run M_1 and M_2 on w, and accept iff either accepts.
  - Intersection $L_1 \cap L_2$: On input w, run M_1 and M_2 on w, and accept iff both accepts.
  - Concatenation $L_1 L_2$: On input w, for each of the $|w|+1$ ways to divide w as $w_1 w_2$: run M_1 on w_1 and M_2 on w_2, and accept if both accept. Else reject.
  - Kleene star L^*: On input w, if $w = \varepsilon$ accept. Else, for each of the $2^{|w|-1}$ ways to divide w as $w_1, \ldots, w_k$ ($w_i \neq \varepsilon$): run M on each w_i and accept if M accepts all. Else reject.
- **Theorem**: Recognizable languages are closed under union, intersection, concatenation, Kleene star.
- **Theorem**: If a language L is recognizable and its complement $L' = \Sigma^* - L$ is also recognizable, then L is decidable.
- An NTM M **accepts** w if there is an accepting run, and **rejects** w if every run is rejecting.
- **Theorem**: A language is Turing-recognizable if and only if some nondeterministic Turing machine recognizes it.