Homework 3

1. $E_1\begin{bmatrix}1&0&0&0&0\\-2&1&0&0&0\\-3&0&1&0&0\\-1&0&0&1&0\\-1&0&0&0&1\end{bmatrix}, E_2\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&-2&1&0&0\\0&-2&0&1&0\\0&-1&0&0&1\end{bmatrix}, E_3=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&-3&1&0\\0&0&-1&0&1\end{bmatrix}, E_4=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&-1&1\end{bmatrix}$

$$\begin{bmatrix}1.5\\7.5\\15.5\\14.0\\4.0\end{bmatrix}\xrightarrow{E_1}\begin{bmatrix}1.5\\4.5\\11.0\\12.5\\2.5\end{bmatrix}\xrightarrow{E_2}\begin{bmatrix}1.5\\4.5\\2.0\\3.5\\-2.0\end{bmatrix}\xrightarrow{E_3}\begin{bmatrix}1.5\\4.5\\2.0\\-2.5\\-4.0\end{bmatrix}\xrightarrow{E_4}\begin{bmatrix}1.5\\4.5\\2.0\\-2.5\\-1.5\end{bmatrix}=\begin{bmatrix}1&0&1&0&1\\0&1&2&1&1\\0&0&1&1&0\\0&0&0&2&1\\0&0&0&0&1\end{bmatrix}x \implies x=\begin{bmatrix}0.5\\1.5\\2.5\\-0.5\\-1.5\end{bmatrix}$$

2. (a) The sum of a sequence and its 0-1 complement has no zero $\Rightarrow$ *not* a subspace. (c) A decreasing sequence multiplied by -1 becomes increasing $\Rightarrow$ *not* a subspace. (f) The sum of two geometric sequence is not geometric $\Rightarrow$ *not* a subspace. Subspaces of $\mathbb{R}^\infty$ are (b), (d), (e).

3. (a) False. $\oplus$ is neither associative nor commutative. (b) True.

4. $C(A)=\{\begin{bmatrix}1\\0\end{bmatrix}t \mid t\in\mathbb{R}\}$. $N(A)=\{\begin{bmatrix}1\\1\end{bmatrix}t \mid t\in\mathbb{R}\}$. $C(B)=\mathbb{R}^2$. $N(B)=\{\begin{bmatrix}0\\0\\0\end{bmatrix}\}$.

5. (c), (d), (e).

6. $C(A)$ is a line spanned by $\begin{bmatrix}1\\0\\0\end{bmatrix}$. $C(B)$ is a plane spanned by $\begin{bmatrix}1\\0\\0\end{bmatrix}$ & $\begin{bmatrix}0\\2\\0\end{bmatrix}$. $C(C)$ is a line spanned by $\begin{bmatrix}1\\2\\0\end{bmatrix}$

7. (a) False. The space without zero vector is not subspace. (b) True. (c) True. (d) False. Let $A=I$, then $C(A-I)=0\neq C(A)=\mathbb{R}^n$.

8. (a) $b\in\mathbb{R}^3$. (b) $b\in\{\begin{bmatrix}t_1\\t_2\\0\end{bmatrix} \mid t_1,t_2\in\mathbb{R}\}$

9. (a) $\begin{bmatrix}1&1&1\\1&0&0\\0&1&1\end{bmatrix}$ (b) $\begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}$