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- Theorem: For every CFG G, there is a PDA A s.t. $L(A) = L(G)$.
- Given a CFG $G = \langle \Sigma, V, R, S \rangle$, the equivalent PDA $A = \langle \Sigma, \Gamma, Q, q_0, F, \delta \rangle$, where $\Gamma = \Sigma \cup V \cup \{\perp\}$, $Q = \{p, q, r\}$, $q_0 = p$, $F = \{r\}$, and δ is the set of:
  - $(p, \epsilon, \text{pop}(\epsilon)) \rightarrow (q, \text{push}(S \perp))$
  - $(q, a, \text{pop}(a)) \rightarrow (q, \text{push}(\epsilon))\ \forall a \in \Sigma$
  - $(q, \epsilon, \text{pop}(A)) \rightarrow (q, \text{push}(w))\ \forall A \in V, A \rightarrow w \in R$
  - $(q, \epsilon, \text{pop}(\perp)) \rightarrow (r, \text{push}(\epsilon))$
- Recall the CFG $G = \langle \Sigma, V, R, S \rangle$, and its PDA $A = \langle \Sigma, \Gamma, Q, q_0, F, \delta \rangle$, where $\Sigma = \{a, b\}$, $V = \{S\}$, $R = \{S \rightarrow aSb | \epsilon\}$, $\Gamma = \{a, b, S, \perp\}$, and $Q = \{p, q, r\}$, $q_0 = p$, and $F = \{r\}$, and δ is the set of:
  - $(p, \epsilon, \text{pop}(\epsilon)) \rightarrow (q, \text{push}(S \perp))$
  - $(q, a, \text{pop}(a)) \rightarrow (q, \text{push}(\epsilon))$
  - $(q, b, \text{pop}(b)) \rightarrow (q, \text{push}(\epsilon))$
  - $(q, \epsilon, \text{pop}(S)) \rightarrow (q, \text{push}(aSb))$
  - $(q, \epsilon, \text{pop}(S)) \rightarrow (q, \text{push}(\epsilon))$
  - $(q, \epsilon, \text{pop}(\perp)) \rightarrow (r, \text{push}(\epsilon))$
- Claim: For every word $u \in \Sigma \cup V$, for every word $v \in \Sigma$, if there is a derivation $u \rightarrow^* w$, then there is a run $(q, wu) \vdash^* (q, w)$ on v for every $w \in \Gamma^*$.
- Theorem: For every PDA A, there is a CFG G s.t. $L(G) = L(A)$.
- Assumptions for the proof of PDA to CFG:
  - For every $w \in L(A)$, there is an accepting run of A on w that ends with a configuration with empty stack.
  - For every transition, A can only push or pop symbol, but not both.
- Given a PDA $A = \langle \Sigma, \Gamma, Q, q_0, F, \delta \rangle$, the equivalent CFG $G = \langle \Sigma, V, R, S \rangle$, where $V = \{A_{p,q} | p, q \in Q\}$, and R is the set of:
  - $A_{p,q} \rightarrow aA_{r,s}b$ for every pair of transitions $(p, a, \text{pop}(\epsilon)) \rightarrow (r, \text{push}(z))$, and $(s, b, \text{pop}(z)) \rightarrow (q, \text{push}(\epsilon))$.
  - $A_{p,r} \rightarrow aA_{q,r}$ for every transition $(p, a, \text{pop}(\epsilon)) \rightarrow (q, \text{push}(\epsilon))$.
  - $A_{p,q} \rightarrow A_{p,r}A_{r,q}$.
  - $A_{p,p} \rightarrow \epsilon$.
- Claim: For every $w \in \Gamma^*$, for every word $u \in \Sigma \cup V$, for every word $v \in \Sigma$, if there is a run $(q, wu) \vdash^* (q, w)$ on v, then there is a derivation $u \rightarrow^* v$.