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- **Knapsack problem**:
  - Definition: $\max\{\sum_{i=1}^{n} p_i x_i \mid \sum_{i=1}^{n} w_i x_i \le M, x_i \in \mathbb{N}, i = 1, 2, \ldots, n\}$.
  - Let $f(k, g)$ be the maximal value of the objective function using only the first k ($1 \le k \le n$) items with capacity limitation g ($0 \le g \le M$).
  - The optimal value is denoted as $f(n, M)$.
  - Recursion 1: $f(k, g) = \max\{p_k x_k + f(k-1, g - w_k x_k) \mid x_k \in \{0, 1, \ldots, \lfloor g/w_k \rfloor\}\} \Rightarrow O(nM + \frac{M(M+1)}{2} \sum_{k=1}^{n} \frac{1}{w_k})$
  - Recursion 2: $f(k, g) = \max\{f(k-1, g), p_k + f(k, g - w_k)\} \Rightarrow O(nM)$.
- **Traveling salesman problem**:
  - Let $L(i, S)$ be the length of a *shortest* tour starting at city i, where $i \notin S$, going through all cities in S exactly once, and ending at city 1.
  - The optimal value is denoted as $L(1, \{2, 3, \ldots, n\})$.
  - Recursion: $L(i, S) = \min_{j \in S}\{d_{ij} + L(j, S - \{j\})\}$.
- Maximum weight independent set in a tree T:
  - **Independent set**: A vertex subset I of T is an *independent set* if no two vertices of I are adjacent.
  - Let $w(I)$ be the sum of weight of all vertices in I, i.e. $w(I) = \sum_{i \in I} w(i)$.
  - Let T_i denote the subtree of T rooted at the vertex i.
    - Let $M(i)$ be $\max\{w(I_i) \mid I_i \text{ is an independent set of } T_i \text{ and } i \in I_i\}$.
    - Let $M'(i)$ be $\max\{w(I_i) \mid I_i \text{ is an independent set of } T_i \text{ and } i \notin I_i\}$.
  - The optimal value is denoted as $\max\{M(0), M'(0)\}$.
  - Recursion: $M(i) = w(i) + \sum_j M'(j)$, $M'(i) = \sum_j \max\{M(j), M'(j)\}$, where j is a child of i.
- **Selection problem**:
  - Definition: Given n distinct numbers $a_1, a_2, \ldots, a_n$, determine the kth smallest one.
  - Divide the n numbers into n/r groups each of r numbers.
  - Sort every group and let m_i be the median of group $i \Rightarrow O(n)$.
    - Choose the median of m_i's $\Rightarrow T(n/r)$.
    - At least one fourth of the solution space is discarded after each iteration $\Rightarrow T(3n/4)$.
  - $T(n) = T(n/r) + T(3n/4) + O(n) \Rightarrow T(n) = O(n)$.