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- Definition of PSPACE, NPSPACE, coNPSPACE:
 - **PSPACE** $:= \bigcup_{k \geq 1} \text{SPACE}[n^k]$.
 - **NPSPACE** $:= \bigcup_{k \geq 1} \text{NSPACE}[n^k]$.
 - **coNPSPACE** $:= \{L \mid \Sigma^* - L \in \text{NPSPACE}\}$.
- Definition of L, NL, coNL:
 - **L** $:= \bigcup_{k \geq 1} \text{SPACE}[\log n]$.
 - **NL** $:= \bigcup_{k \geq 1} \text{NSPACE}[\log n]$.
 - **coNL** $:= \{L \mid \Sigma^* - L \in \text{NL}\}$.
- Obviously, we have $L \subseteq \text{NL}$, $P \subseteq \text{NP}$, and $\text{PSPACE} \subseteq \text{NPSPACE}$.
- **Savitch's Theorem:** $\text{NSPACE}(f(n)) \subseteq \text{SPACE}(f^2(n))$, where $f(n) \geq n$.
- $L \subseteq \text{NL} \subseteq P \subseteq \text{NP} \subseteq \text{PSPACE} = \text{NPSPACE}$.
- **Theorem: NPSPACE = coNPSPACE.** If $L \in \text{NSPACE}[n^k]$, then $\Sigma^* - L \in \text{NSPACE}[n^k]$.
- **Theorem: NL = coNL.** If $L \in \text{NSPACE}[\log n]$, then $\Sigma^* - L \in \text{NSPACE}[\log n]$.
- Definition of NL-hard, NL-complete:
 - **NL-hard:** $\{L \mid \forall L' \in \text{NL}, L' \leq_{\log} L\}$.
 - **NL-complete:** $\{L \mid L \in \text{NL} \text{ and } L \in \text{NL-hard}\}$.
- **Theorem:** Reachability is NL-complete.
- **Theorem:** True quantified Boolean formula (TQBF) is PSPACE-complete.