# Solutions to Exercise #6

(範圍: Boolean Algebra, Rings)

1. Let ($K$, ·, +) be a Boolean algebra. A proof of $a\cdot(a+b)=a$ for every $a$, $b\in K$ was given on page 65 of lecture notes. Please prove $a+(a\cdot b)=a$ for every $a$, $b\in K$ by the principle of duality. (10%)

Sol: $a+(a\cdot b)=(a+a)\cdot(a+b)=a\cdot(a+b)=(a+0)\cdot(a+b)=a+0\cdot b=a+0=a$.

2. P. 741: 4. (20%)

Sol: (a) $x+y=x\cdot y+y=(x+1)\cdot y=1\cdot y=y$.

(b) $x\le y \Rightarrow x+y=y \Rightarrow \overline{x+y}=\overline{y} \Rightarrow \overline{y}\cdot\overline{x}=\overline{y} \Rightarrow \overline{y}\le\overline{x}$.

3. Prove Theorem 14.5 on page 681 of Grimaldi's book. (20%)

Sol: (a) Suppose that $u$ and $u'$ are two unities of $R$. Then, $u=u\cdot u'=u'$.

(b) Suppose that $b$ and $b'$ are two multiplicative inverses of $x$, i.e.,

$x\cdot b=b\cdot x=u=b'\cdot x=x\cdot b'$.

Then, $b=b\cdot u=b\cdot(x\cdot b')=(b\cdot x)\cdot b'=u\cdot b'=b'$.

4. P. 678: 2 (only for (b) and (c)). (10%)

Sol: (b) Yes.

(c) Yes.

5. P. 678: 8. (30%)

Sol: (a) $x$.

(b) The additive inverses of $s$, $t$, $x$ and $y$ are $t$, $s$, $x$ and $y$, respectively.

(c) $t\cdot(s+xy)=t\cdot(s+x)=t\cdot s=y$.

(d) Yes (because Table 14.4(b) are symmetric).

(e) No.

(f) ($s$, $y$) or ($t$, $y$).

6. P. 684: 4. (10%)

Sol: Suppose that $a$ is a unit of $R$, i.e., $a \cdot b = b \cdot a = u$ for some $b \in R$.
If $a \cdot c = z$, then $b \cdot (a \cdot c) = b \cdot z = z$, which implies $c = z$ because
$b \cdot (a \cdot c) = (b \cdot a) \cdot c = u \cdot c = c$.
Therefore, $a$ is not a proper zero divisor of $R$.