Supplementary Material

for

A Non-Adversarial Approach to Idempotent Generative Modelling

1 NAIGN Torch Pseudocode

```
def train_batch(f, f_copy, model_optim, batch, loss_function, M_imle):
     f.train()
    ####
    f_copy.load_state_dict(f.state_dict())
    x = batch
    loss = 0

    #### Reconstruction loss
    x_rec = x + 0.01*torch.randn_like(x)
    fx_rec = f(x_rec)
    L_rec = loss_function(fx_rec, x_rec)
    loss = loss + L_rec

    #### IMLE
    x_imle = x.detach()
    with torch.no_grad():
        Nx, C, H, W = x_imle.shape
        z_imle = torch.randn(M_imle, C, H, W)
        fz_imle = f_copy(z_imle).detach()
        # Compute the distance matrix between points1 and points2
        dist_matrix = torch.cdist(x_imle.view(Nx, -1), fz_imle.view(M_imle, -1))
        col_ind = torch.argmin(dist_matrix, axis=1)
    fz_star = f(z_imle[col_ind])
    L_imle = loss_function(x, fz_star)
    loss = loss + L_imle

    #### Modified idempotency loss
    z_naidem = torch.randn_like(x)
    fz_naidem = f(z_naidem)
    f_fz_naidem = f_copy(fz_naidem)
    L_naidem_1 = loss_function(fz_naidem, f_fz_naidem)

    q_naidem = fz_naidem.detach()
    fq_naidem = f(q_naidem)
    f_fq_naidem = f_copy(fq_naidem)
    L_naidem_2 = loss_function(fq_naidem, f_fq_naidem)

    L_naidem = 0.5 * (L_naidem_1 + L_naidem_2)
    loss = loss + L_naidem

    #### Backpropagate
    loss.backward()
    model_optim.step()
    model_optim.zero_grad()
```

2 Modified Idempotent Loss

The modified idempotent loss in NAIGNs ensures that the model behaves idempotently on data manifold points, source distribution points, and off-manifold points.

The original idempotent loss, introduced by shocher2023idempotent, enforces idempotency on the source distribution by penalizing discrepancies between the model's output and its second application:

$$\mathcal{L}_{\text{idem}}(\theta) = \mathbb{E}_{\mathbf{z}\sim p_Z}\left[d\left(f_\theta(\mathbf{z}), f_{\theta_\perp}\left(f_\theta(\mathbf{z})\right)\right)\right]$$

where $\mathbf{z}$ is a sample from the source distribution p_Z.

We extend this loss by also applying it to points generated by the model during training, capturing off-manifold behavior. The modified loss is the average of the original idempotent loss and a similar term for model-generated points:

$$\mathcal{L}_{\text{idem}}(\theta) = \frac{1}{2}\left(\mathbb{E}_{\mathbf{z}\sim p_Z}\left[d\left(f_\theta(\mathbf{z}), f_{\theta_\perp}\left(f_\theta(\mathbf{z})\right)\right)\right] + \mathbb{E}_{\mathbf{q}\sim p_\theta}\left[d\left(f_\theta(\mathbf{q}), f_{\theta_\perp}\left(f_\theta(\mathbf{q})\right)\right)\right]\right)$$

where $\mathbf{q}$ represents samples generated by the model with inputs from the source distribution. This modification ensures the model is trained on both source and off-manifold points, encouraging it to project generated points back onto the manifold. By applying the loss to both types of points, the model becomes explicitly exposed to off-manifold behavior and learns how to act idempotently on such points as well.

3 Experimental Details: Degradation and Restoration

We conducted experiments on the MNIST dataset, resized to 32x32 to fit a smaller DCGAN model architecture. We applied four types of degradation: Gaussian blur, Gaussian noise, salt and pepper noise, and random deletion (setting random rows and columns to zero).

- **Gaussian blur:** We used the Torch transform's Gaussian blur function. The amount of blur was controlled by the kernel size and a blur level parameter (sigma), where the kernel size was computed based on the blur level, and the sigma value matched the blur level.
- **Gaussian noise:** We used a sigma value of 1.0, implemented via the Torch transform's Gaussian noise functionality.
- **Random row and column deletions:** Each row and column had a 20% chance of being deleted, with the pixel values set to -1.
- **Salt and pepper noise:** 20% of the pixels were deemed corrupted, and of those, 50% were set to salt (value 1) and 50% to pepper (value -1), as the MNIST dataset was normalized to the range of -1 to 1.

4 Reconstructed, Generated and Restored Sample Images

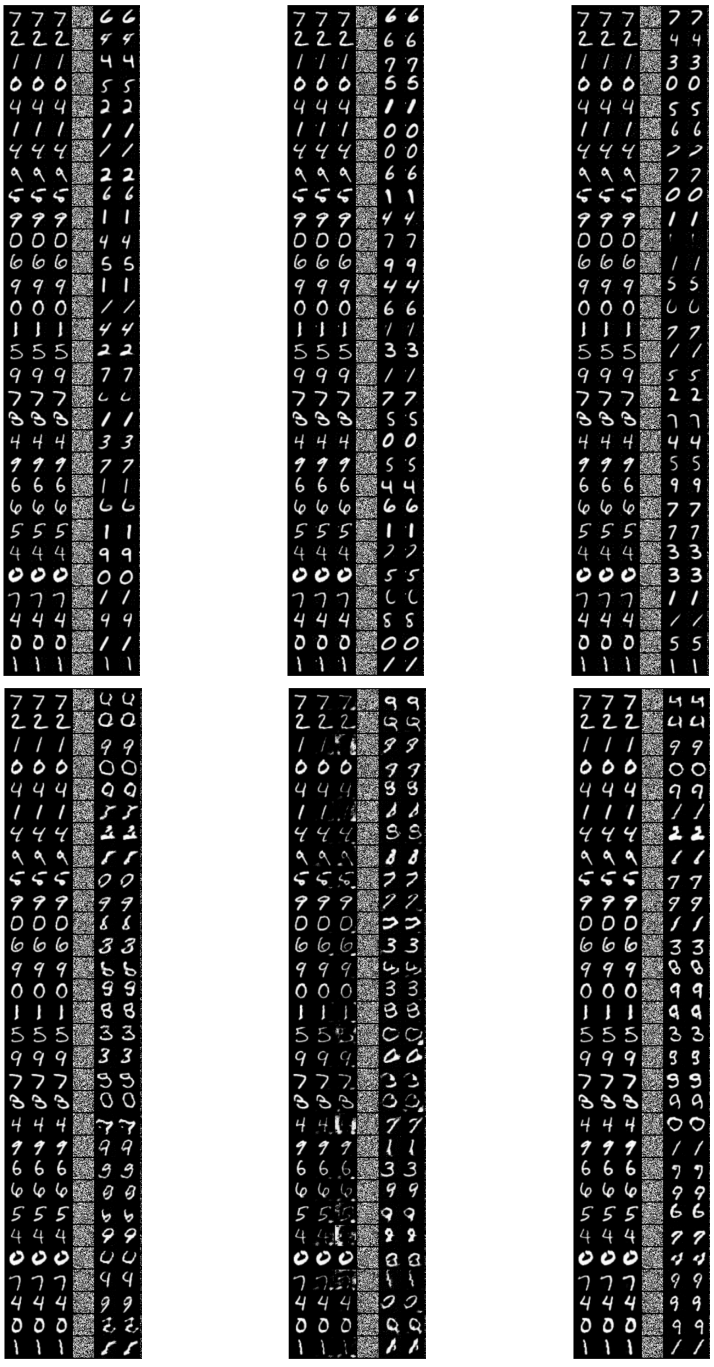
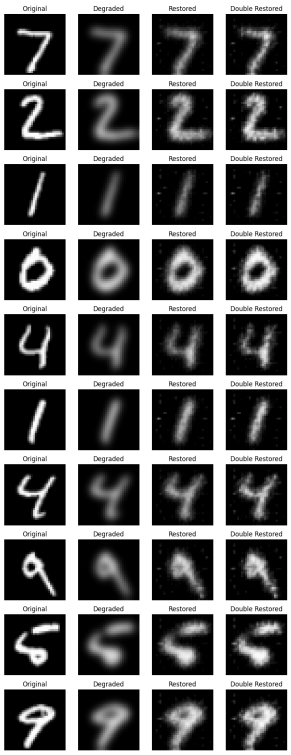
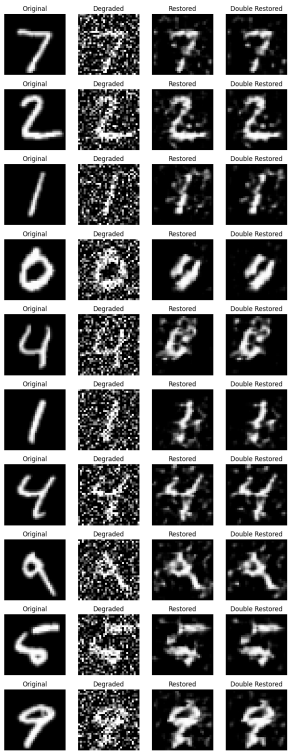


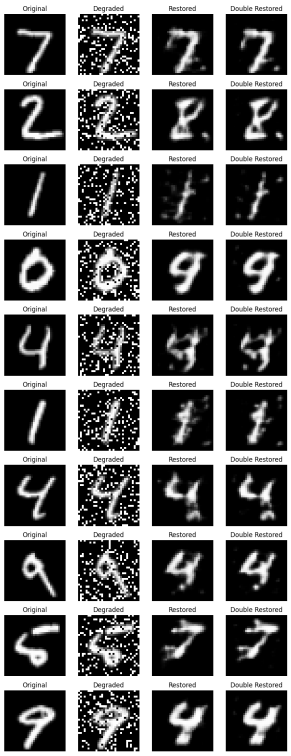
Figure 1: Reconstruction, generation over 3 seeds (one seed per sub-figure) for NAIGN (top) and IGN (bottom). In each sub-figure, the first column depicts images from the test set. Columns 2-3 are the first and second applications of the model on the real images. Columns 3-5 depict noise sampled from the source distribution and the first and second applications of the model, respectively.



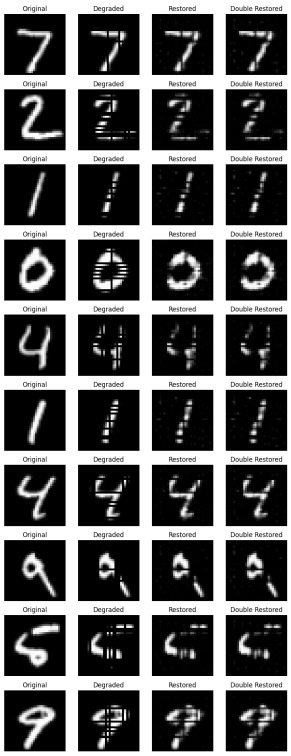
(a) Blur



(b) Gaussian Noise

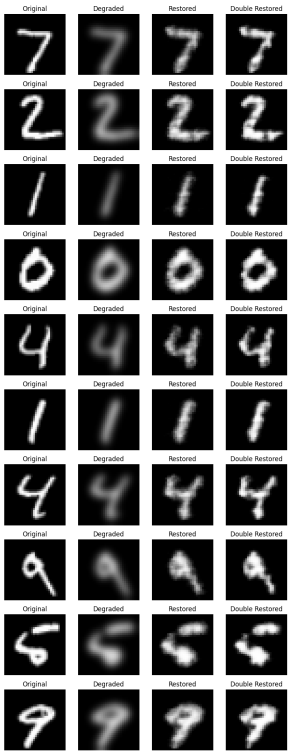


(c) Salt&Pepper

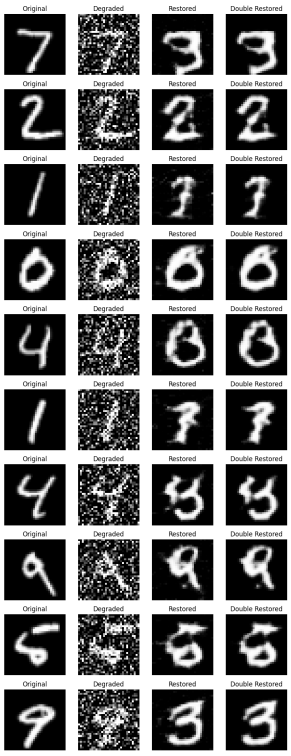


(d) LinesRows

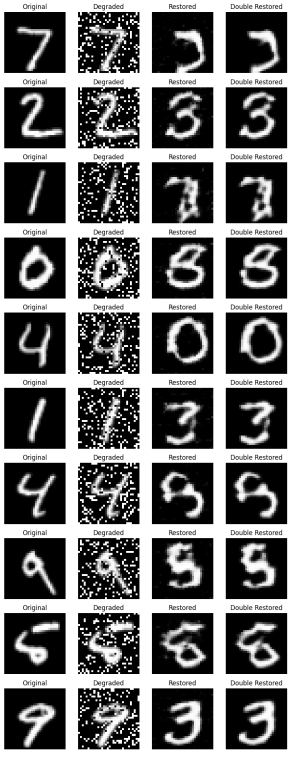
Figure 2: Restoration for NAIGN.



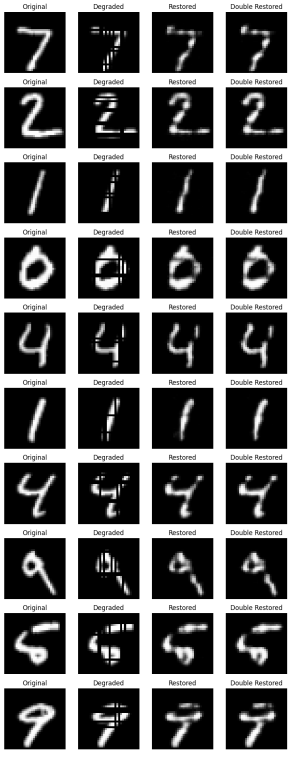
(a) Blur



(b) Gaussian Noise



(c) Salt&Pepper



(d) LinesRows

Figure 3: Restoration for IGN.