CAP 5415 Computer Vision

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Edge Detection

Lecture-3

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- Gradient operators
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- Gradient operators
  - Prewit
  - Sobel
- Laplacian of Gaussian (Marr-Hildreth)

Contents

- Gradient operators
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  - Sobel
- Laplacian of Gaussian (Marr-Hildreth)
- Gradient of Gaussian (Canny)

Edge detection

Source: D. Lowe

Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image

Source: D. Lowe

Edge detection

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  - Intuitively, most semantic and shape information from the image can be encoded in the edges

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Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels

Source: D. Lowe

Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Source: D. Lowe

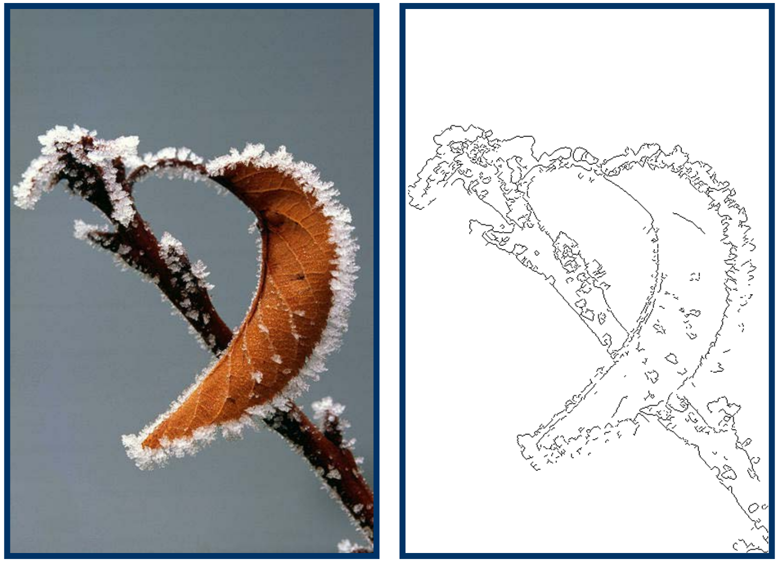
Example

Example

Example

Example

Example



An Application

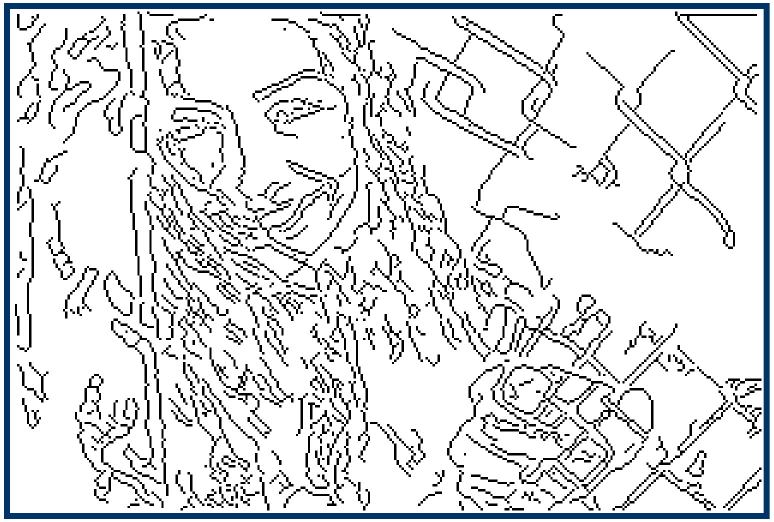
- What is an object?
- How can we find it?

An Application

- What is an object?
- How can we find it?

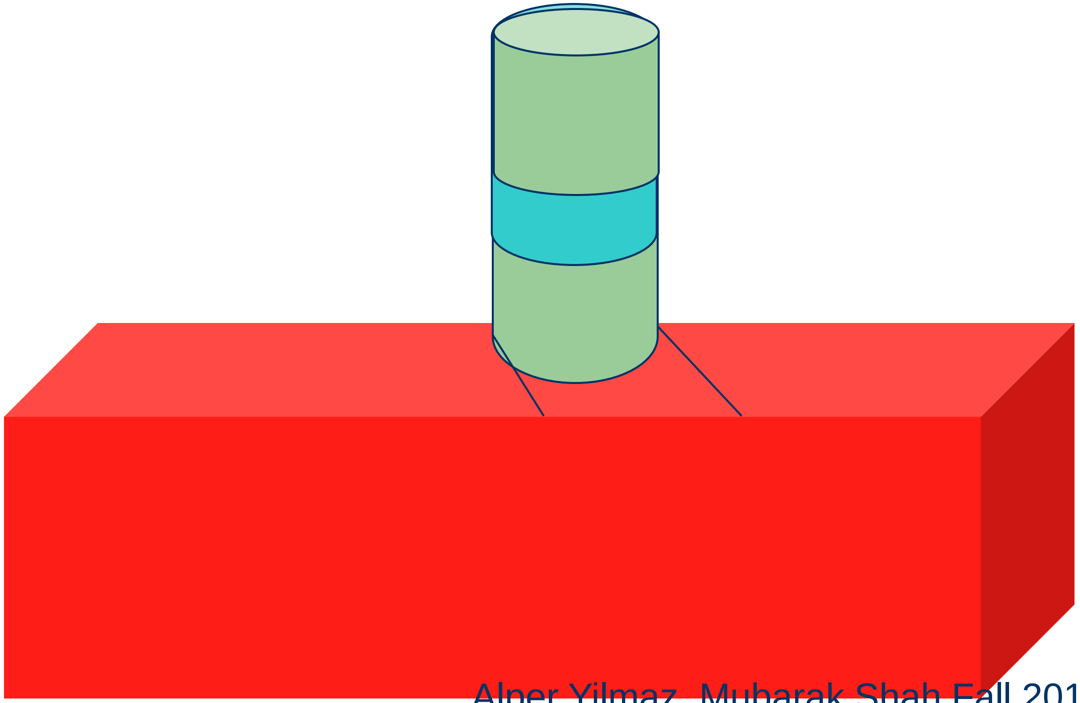
An Application

- What is an object?
- How can we find it?



Edge Detection in Images

- At edges intensity or color changes



Origin of Edges

Source: Steve Seitz

Origin of Edges

- Edges are caused by a variety of factors

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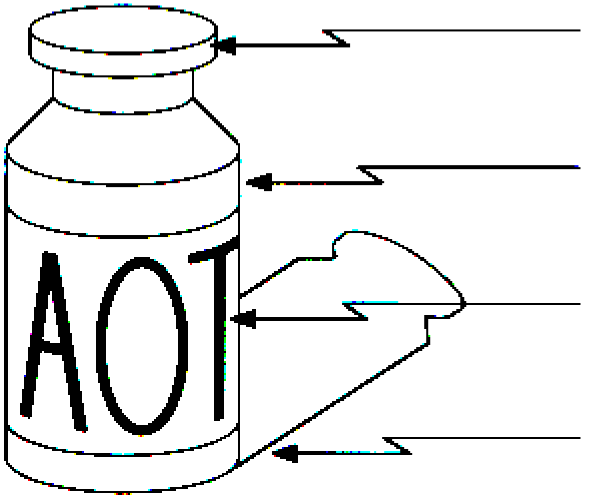
Origin of Edges



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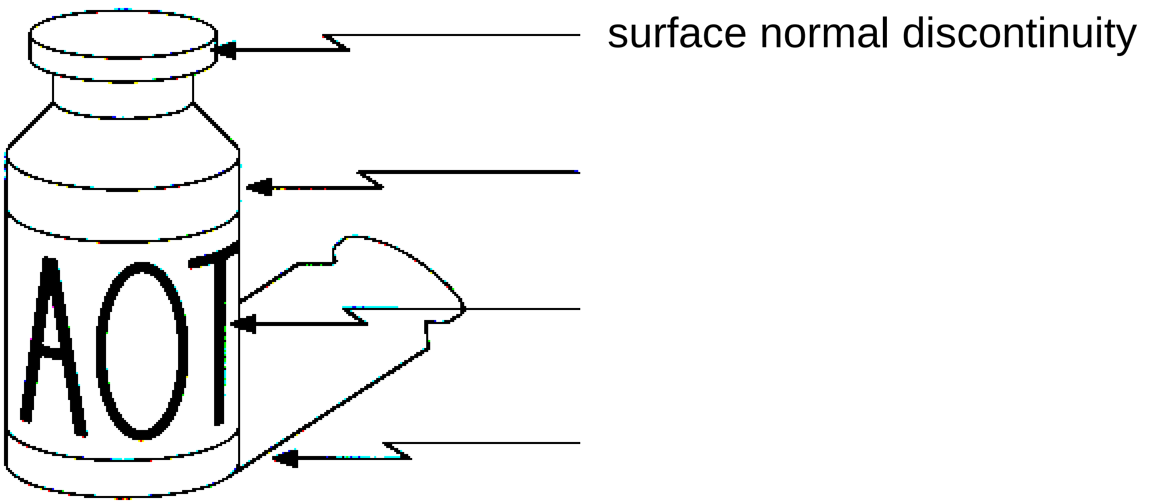
Origin of Edges



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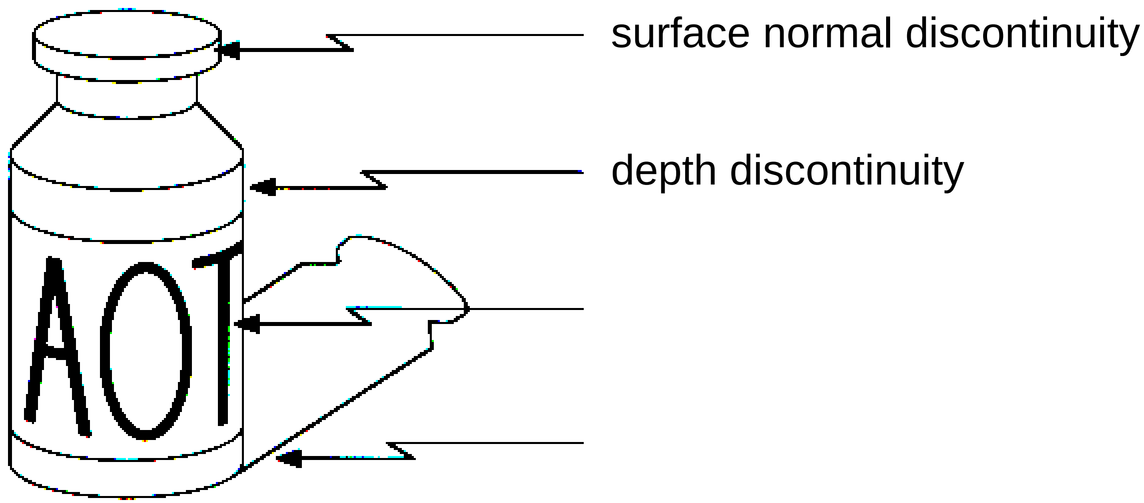
Origin of Edges



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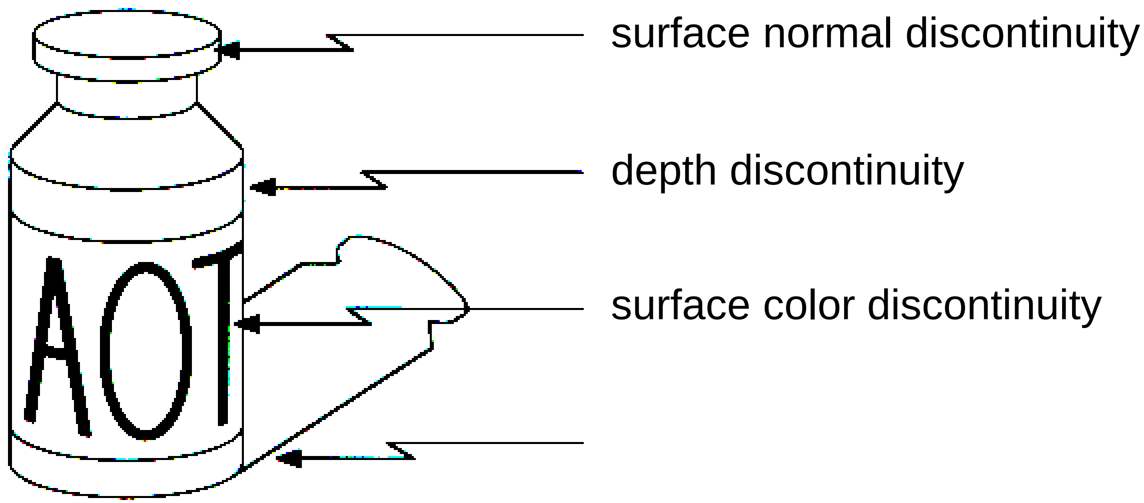
Origin of Edges



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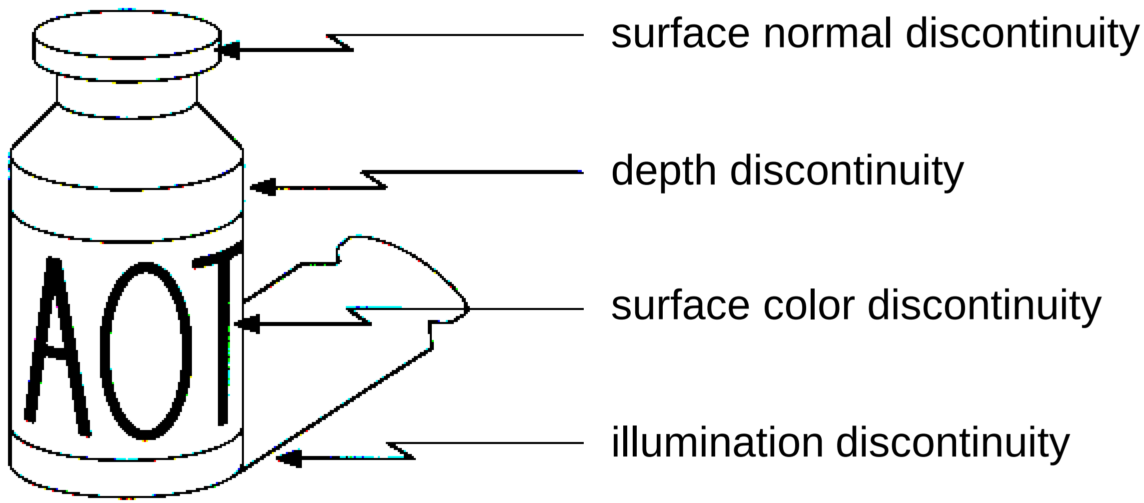
Origin of Edges

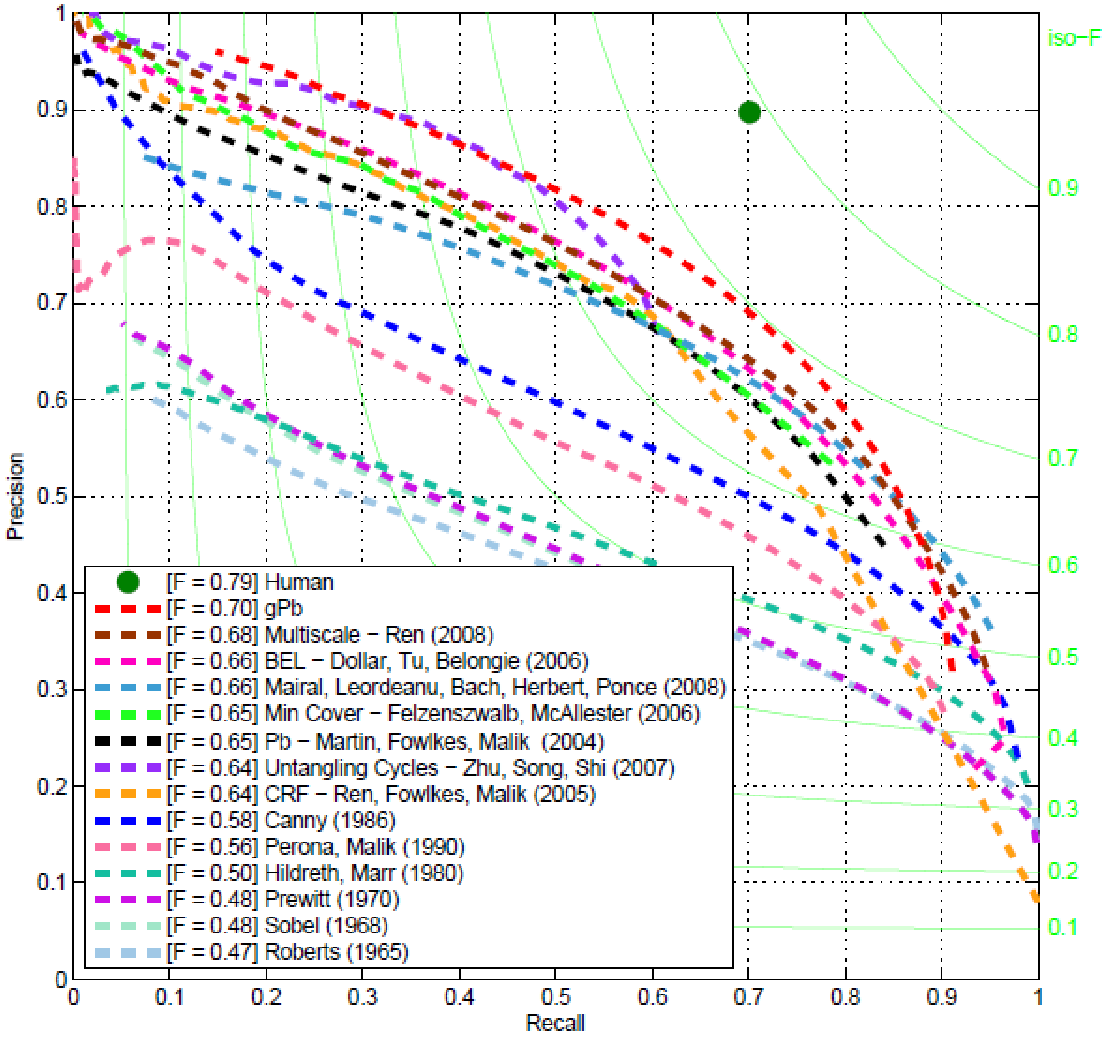


- Edges are caused by a variety of factors

Source: Steve Seitz

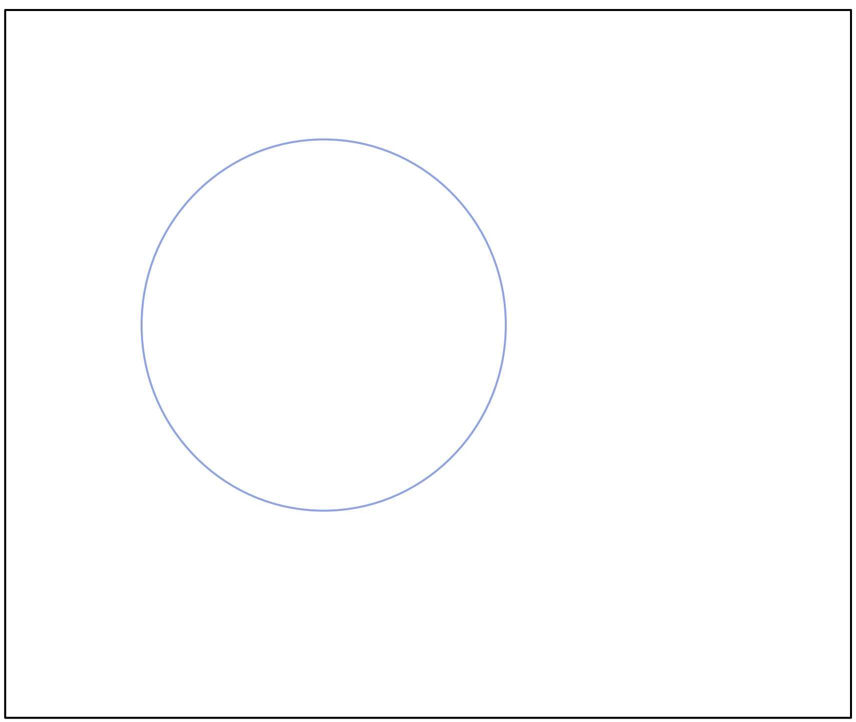
45 years of boundary detection

45 years of boundary detection

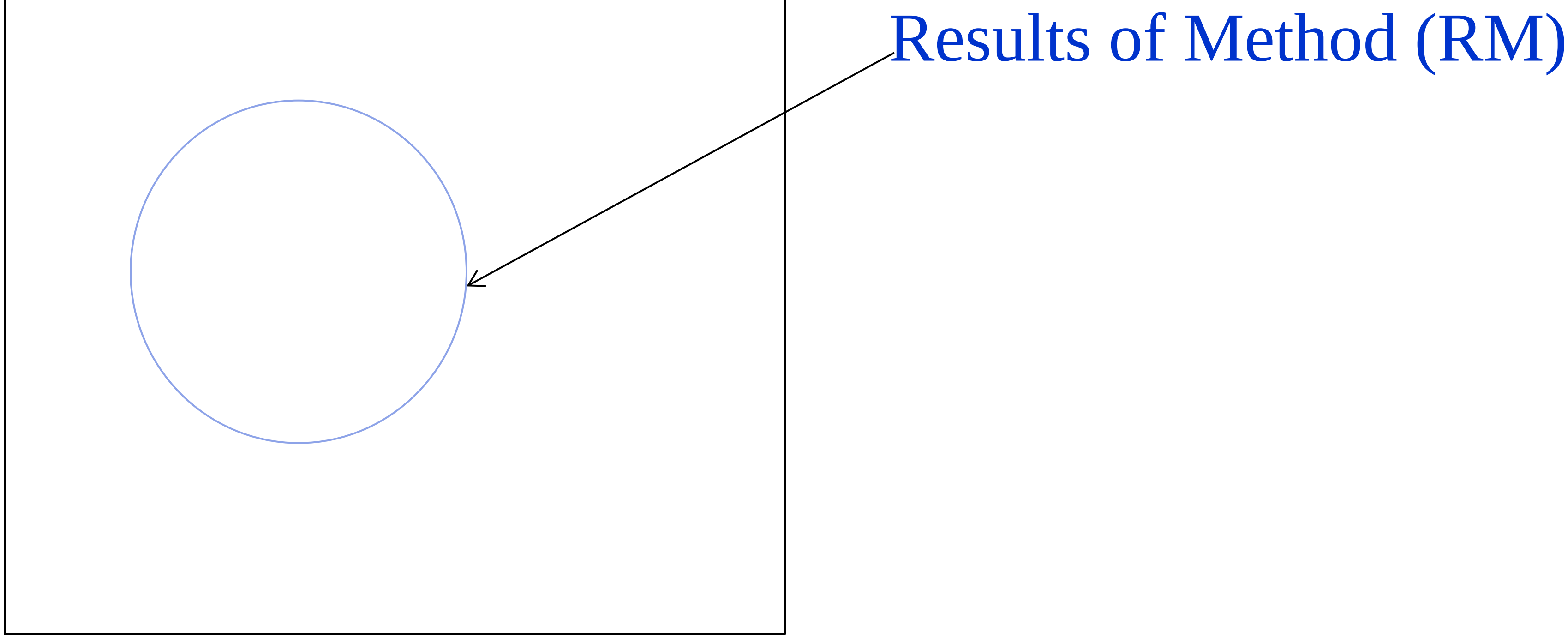


Source: Arbelaez, Maire, Fowlkes, and Malik. TPAMI 2011 (pdf)

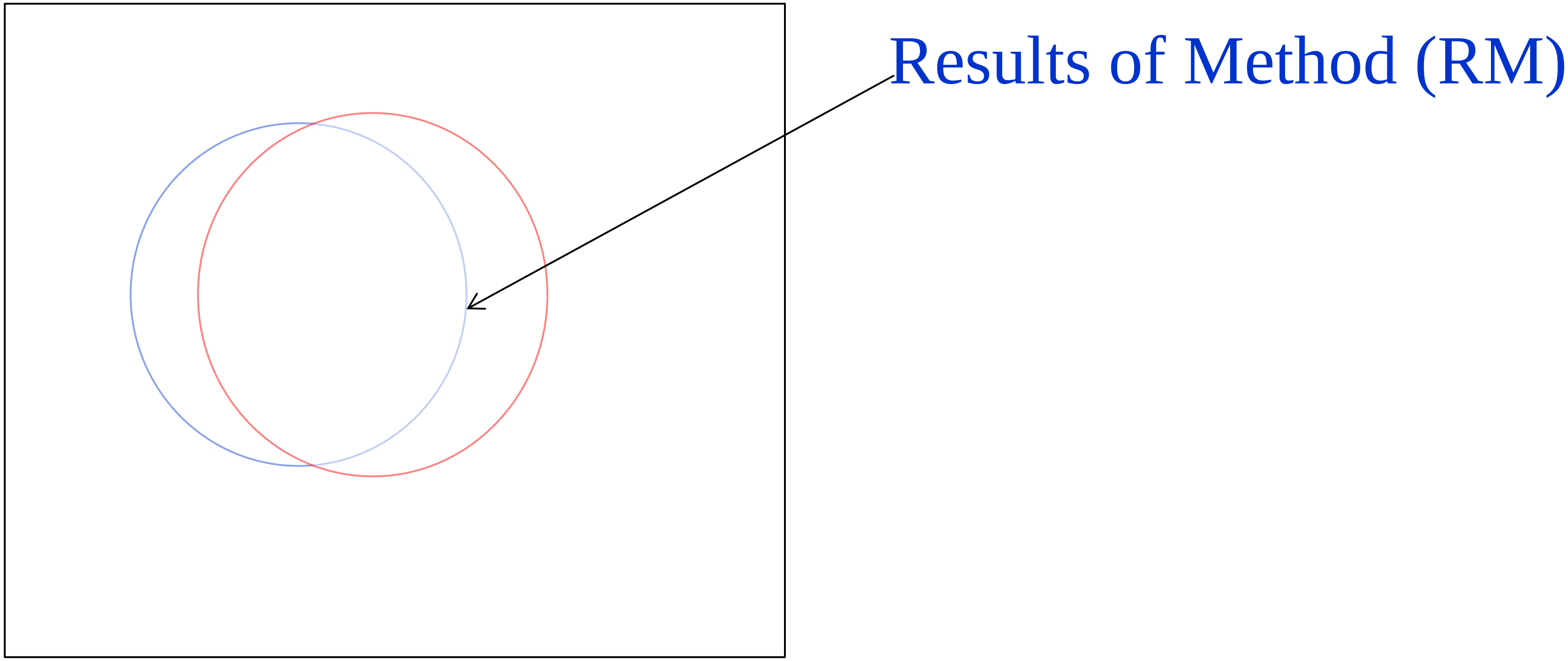
Evaluation Metrics



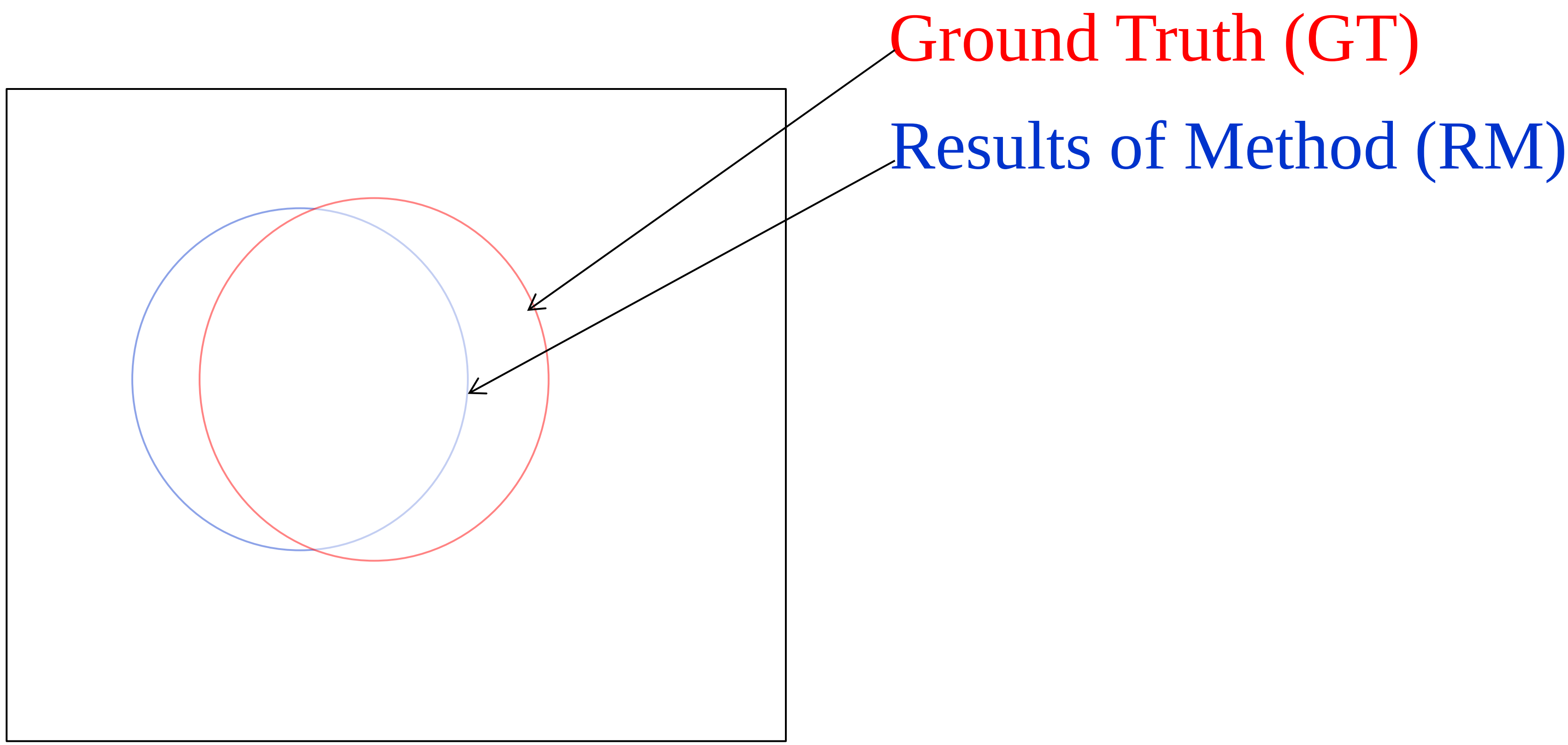
Evaluation Metrics



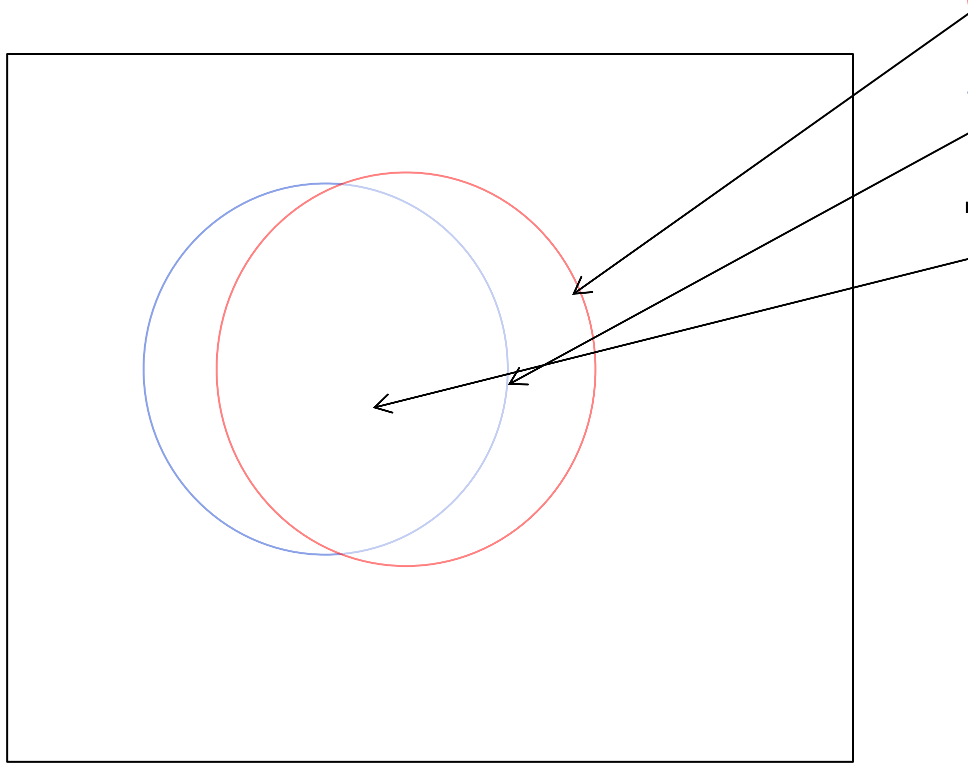
Evaluation Metrics



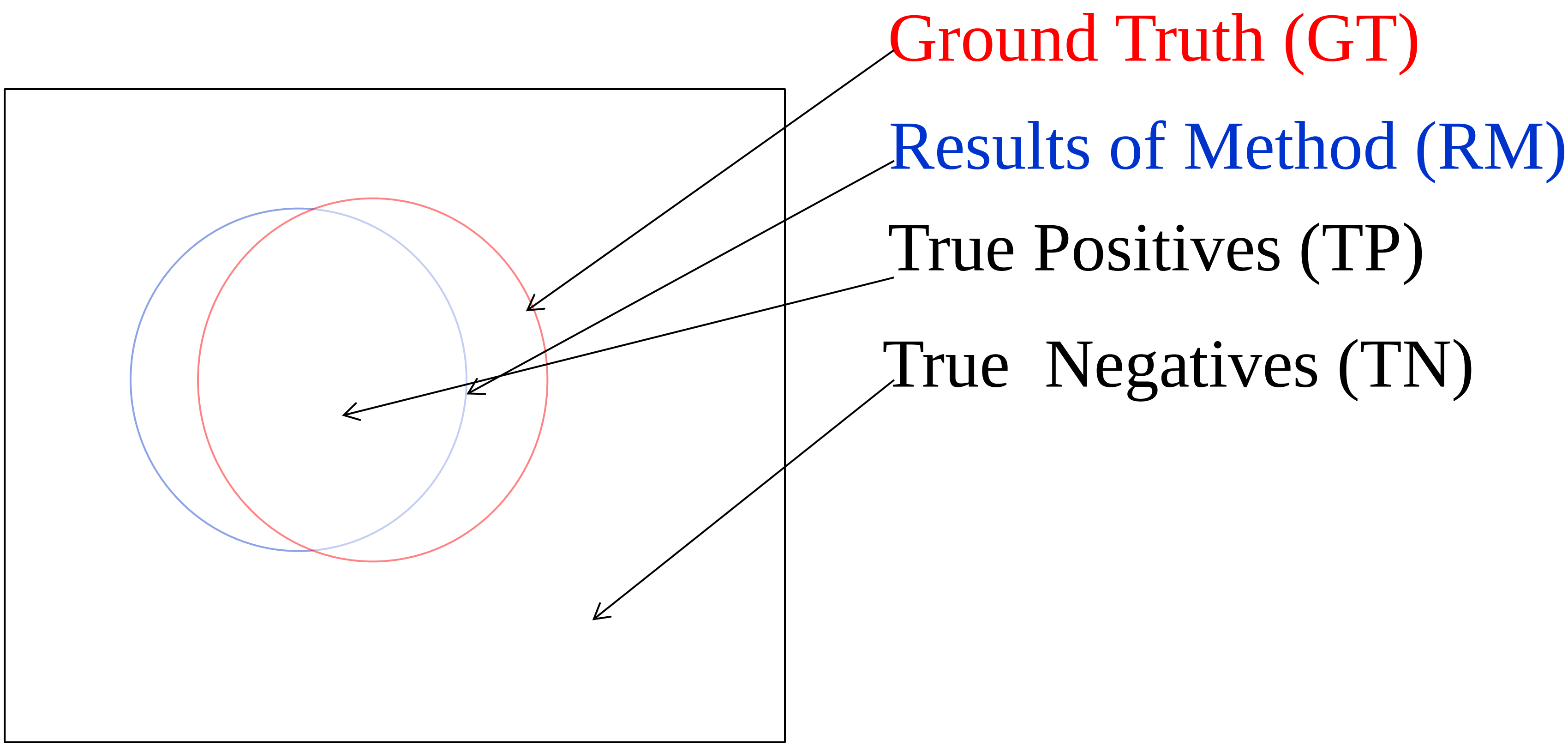
Evaluation Metrics



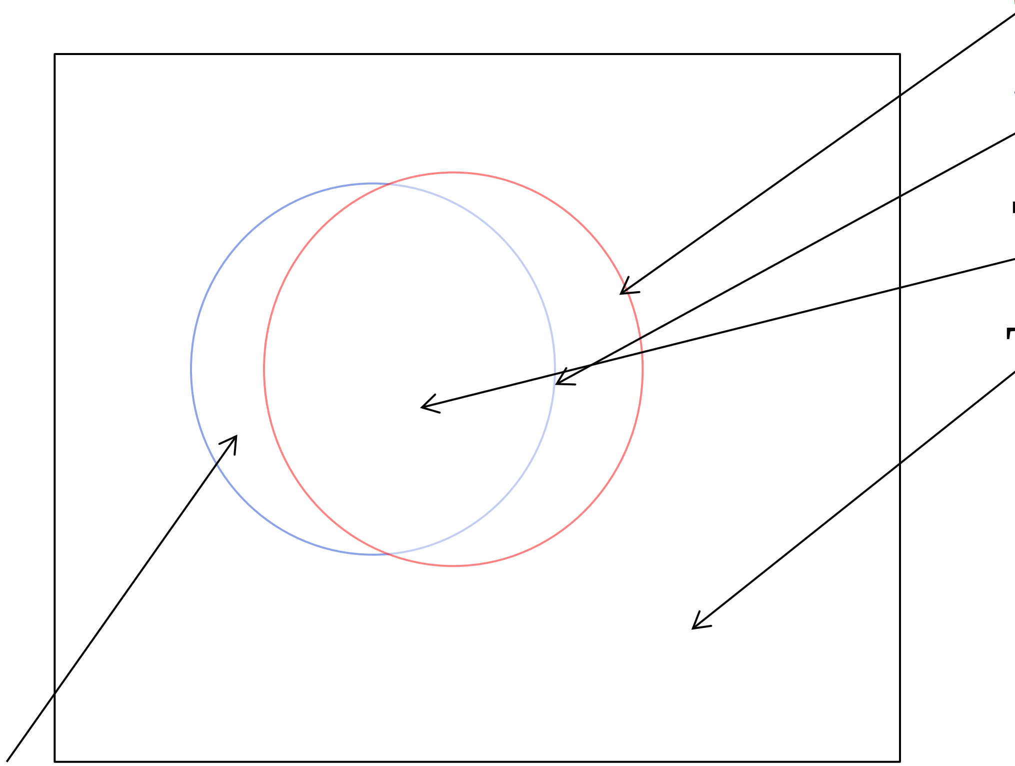
Evaluation Metrics



Evaluation Metrics



Evaluation Metrics



Ground Truth (GT)

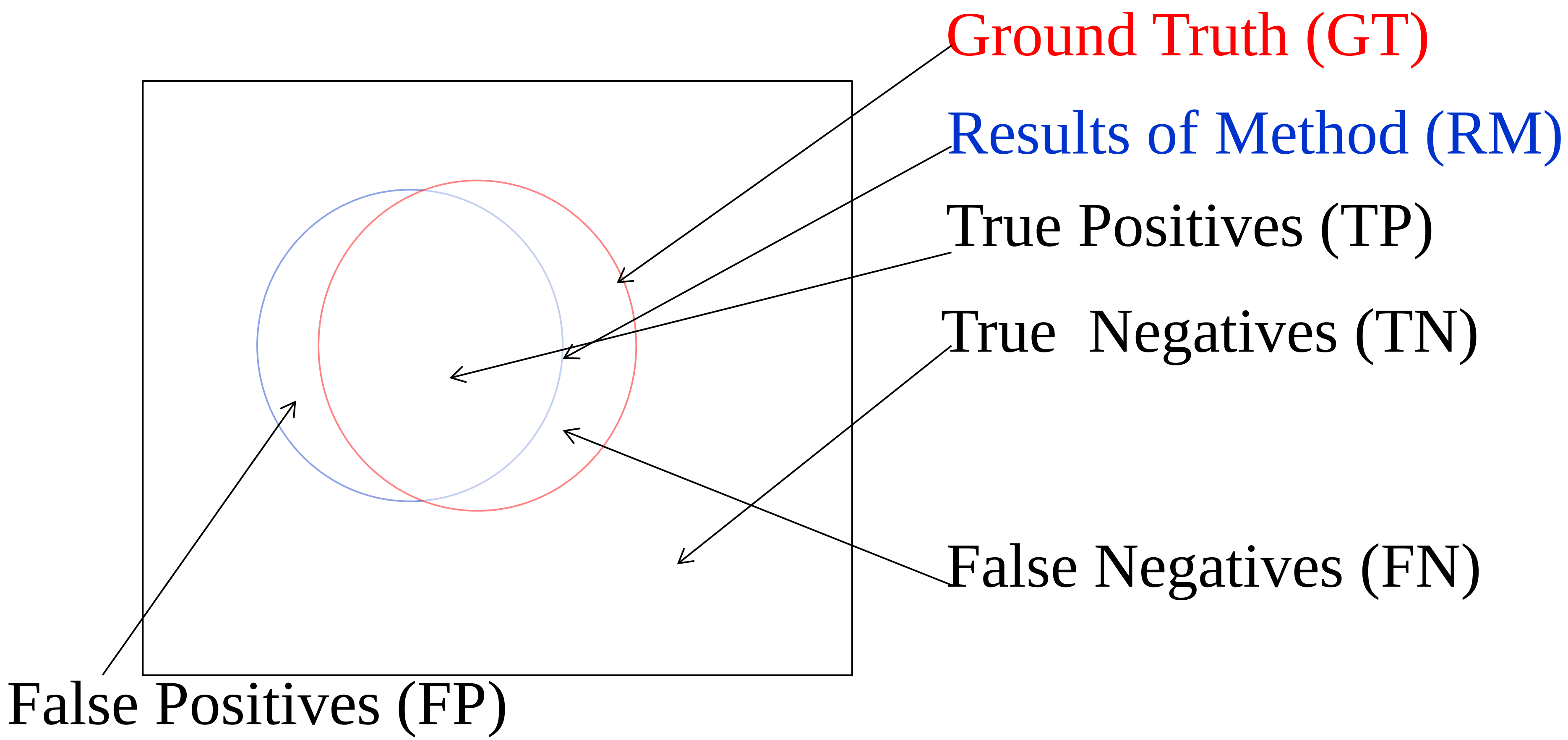
Results of Method (RM)

True Positives (TP)

True Negatives (TN)

False Positives (FP)

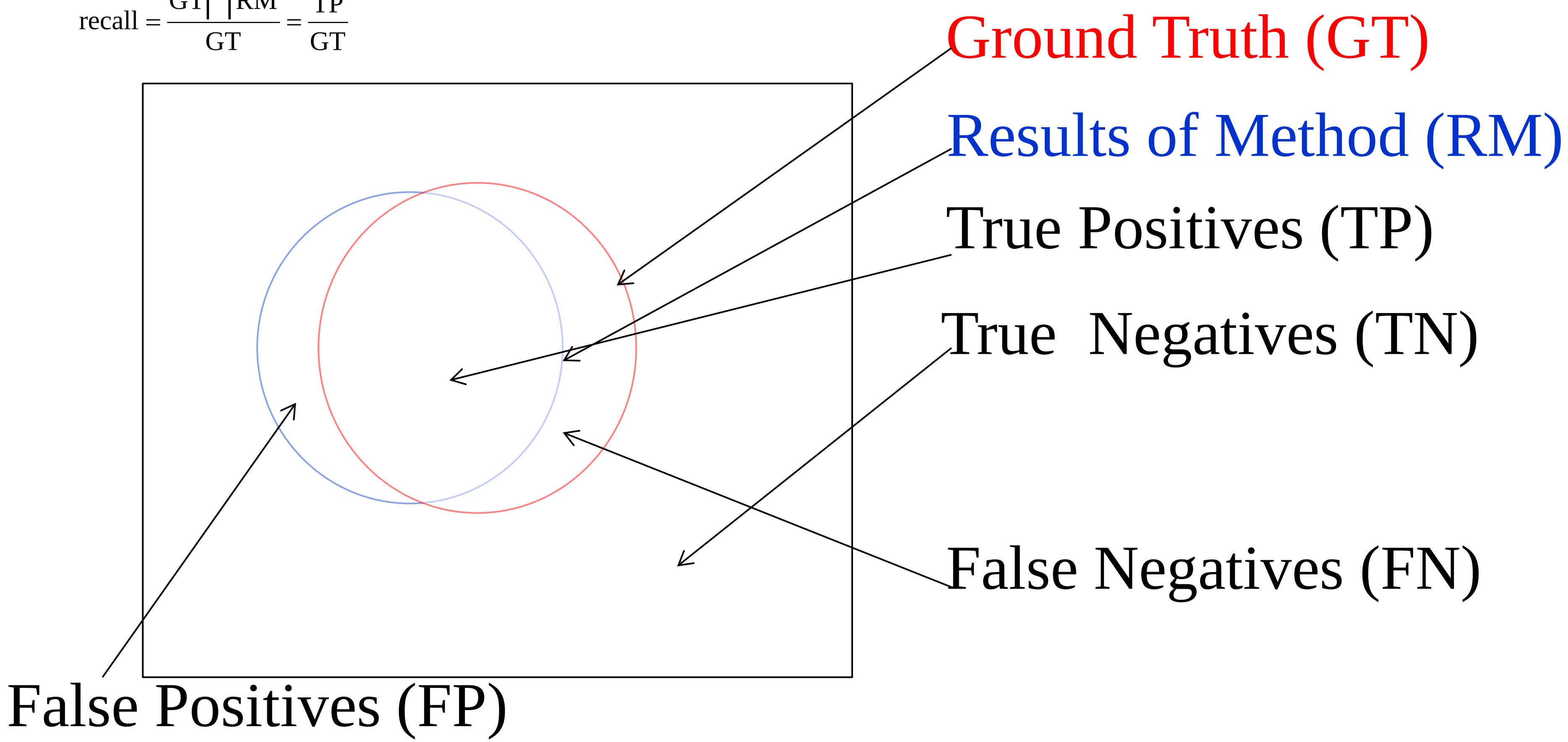
Evaluation Metrics

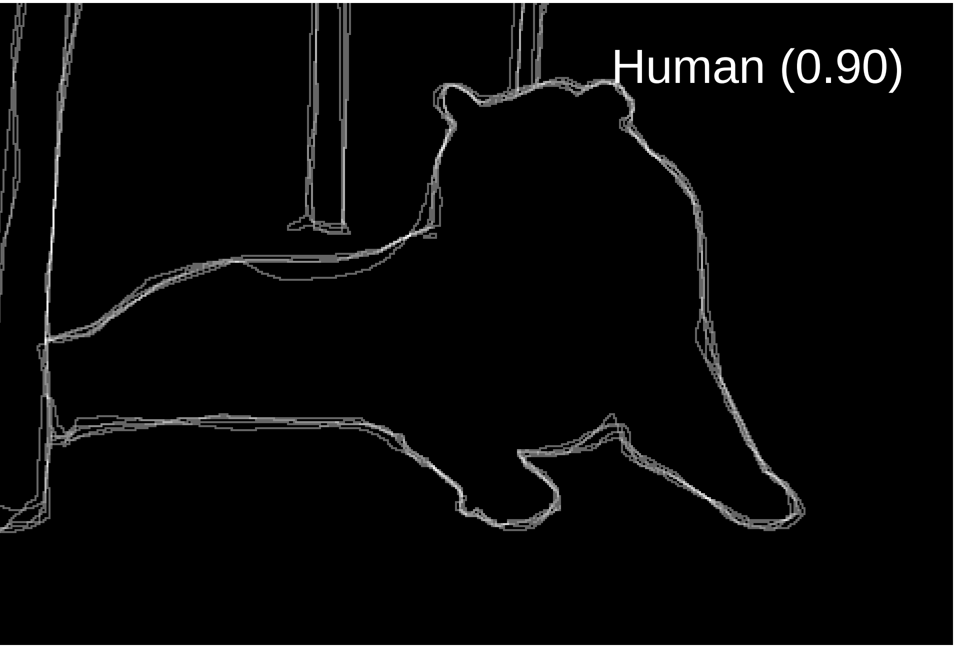


Evaluation Metrics

$$\text{precision} = \frac{GT \bigcap RM}{RM} = \frac{TP}{RM}$$

$$\text{recall} = \frac{GT \bigcap RM}{GT} = \frac{TP}{GT}$$

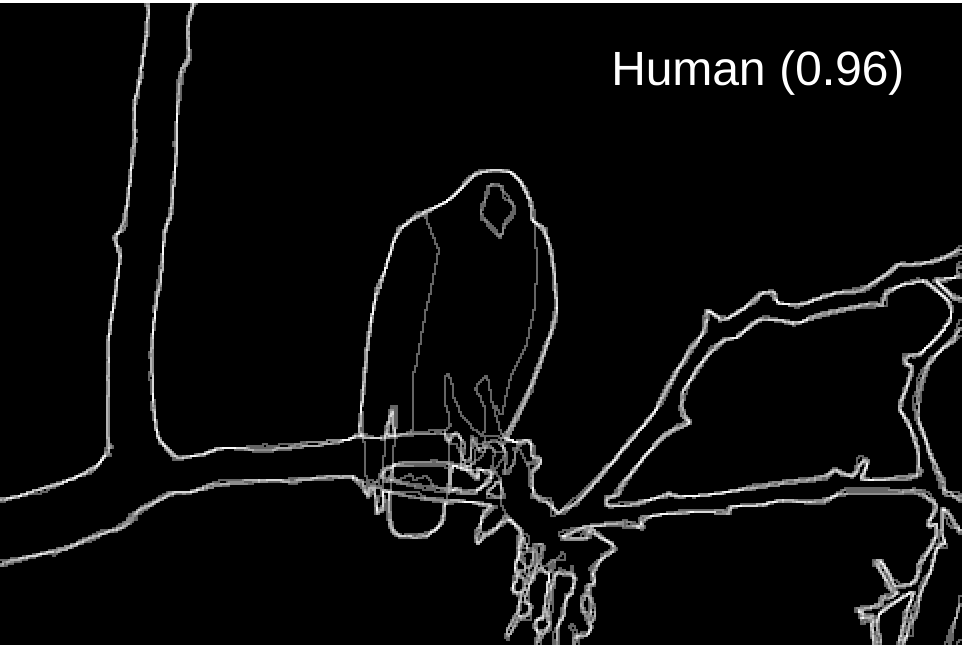




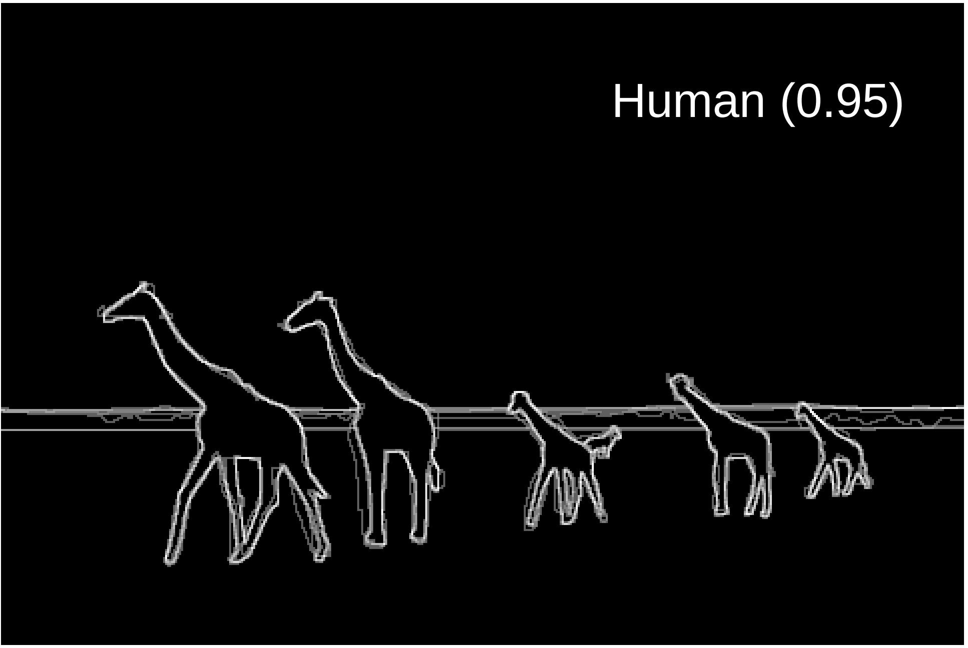
Slide Credit: James Hays

For more:
http://www.eecs.berkeley.edu/Research/Projects/CS/vision/bsds/bench/html/108082-color.html

Results



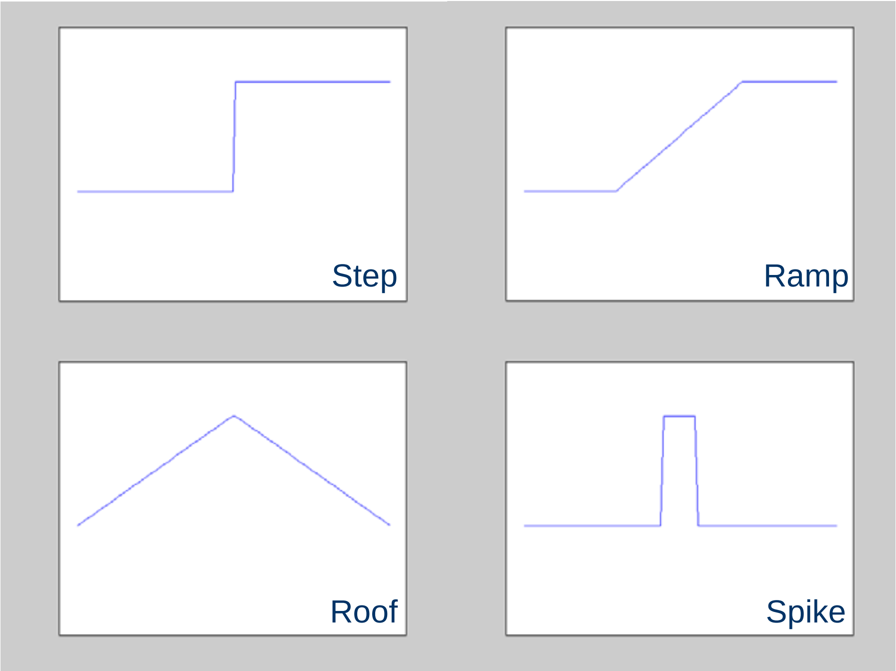
Slide Credit: James Hays



Slide Credit: James Hays

What is an Edge?

- Discontinuity of intensities in the image
- Edge models
  - Step
  - Roof
  - Ramp
  - Spike



Closeup of edges

Source: D. Hoiem

Closeup of edges

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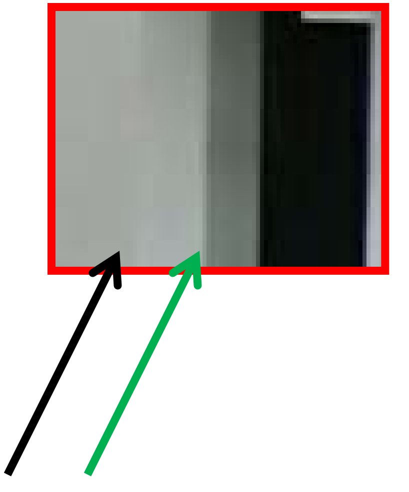
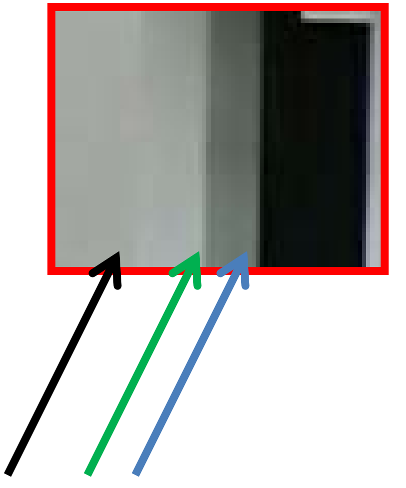
Closeup of edges



Source: D. Hoiem

Closeup of edges



Source: D. Hoiem

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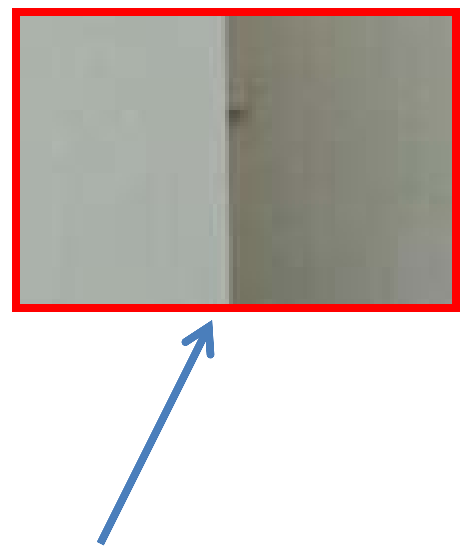


Source: D. Hoiem

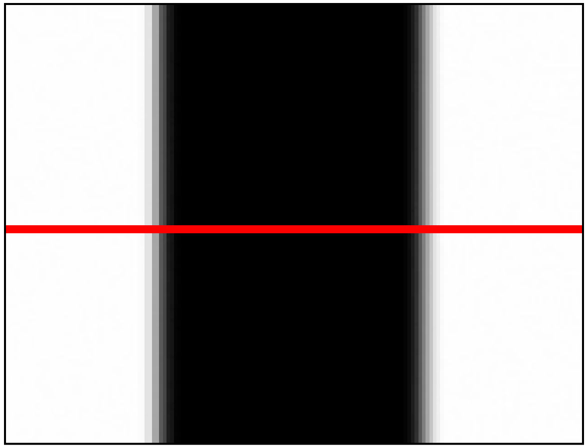
Closeup of edges

Source: D. Hoiem

Characterizing edges

- An edge is a place of rapid change in the image intensity function

image



Slide Credit: James Hays

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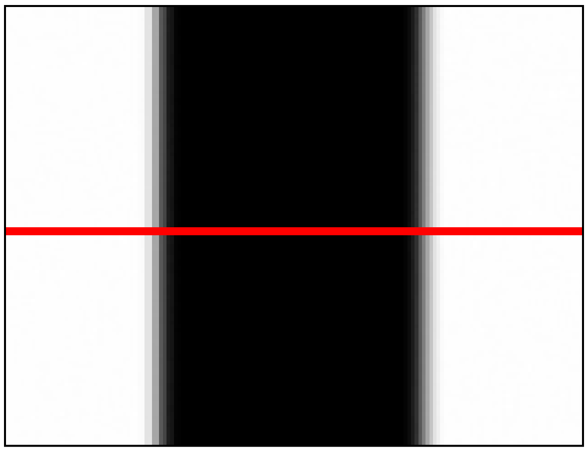
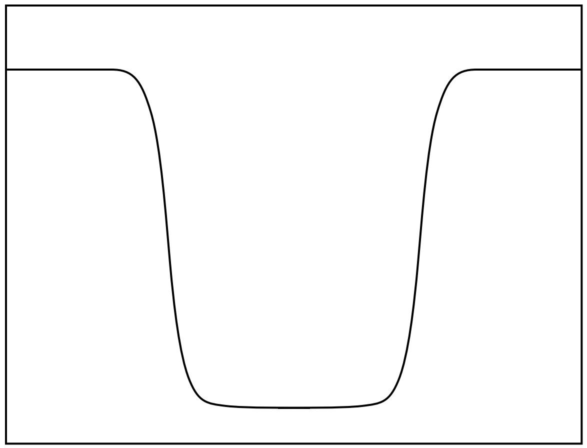


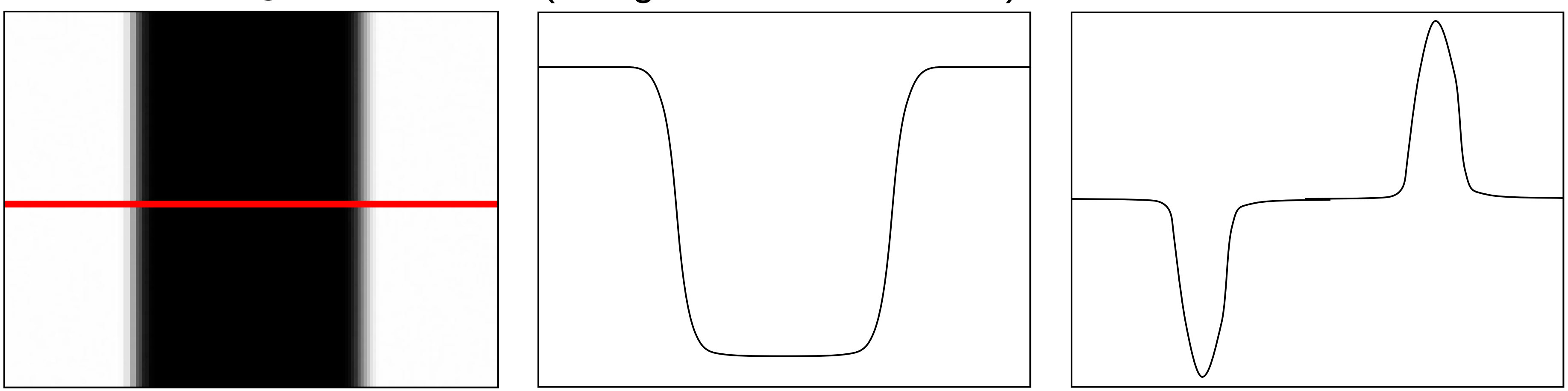


Slide Credit: James Hays

Characterizing edges

- An edge is a place of rapid change in the image intensity function

image | intensity function (along horizontal scanline) | first derivative



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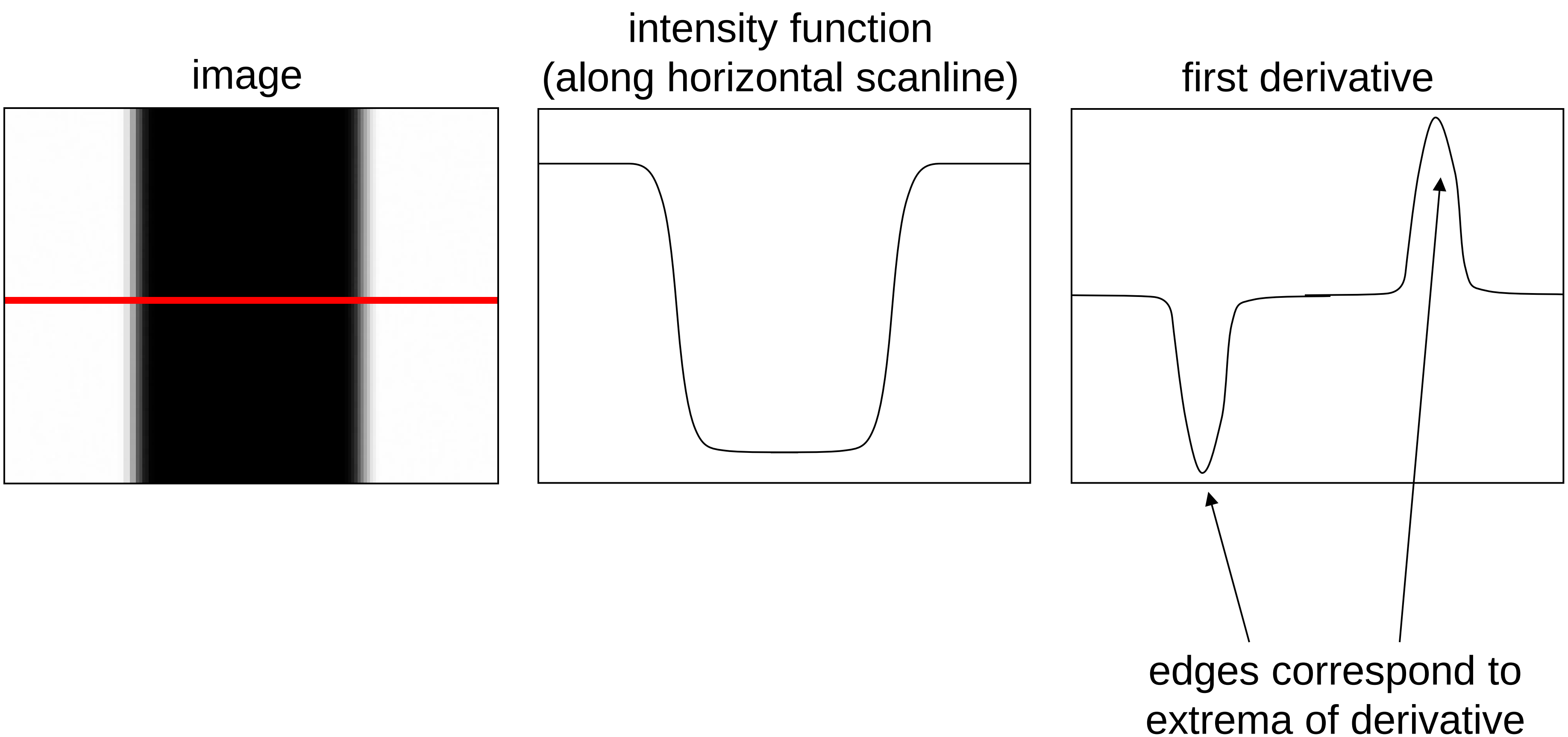
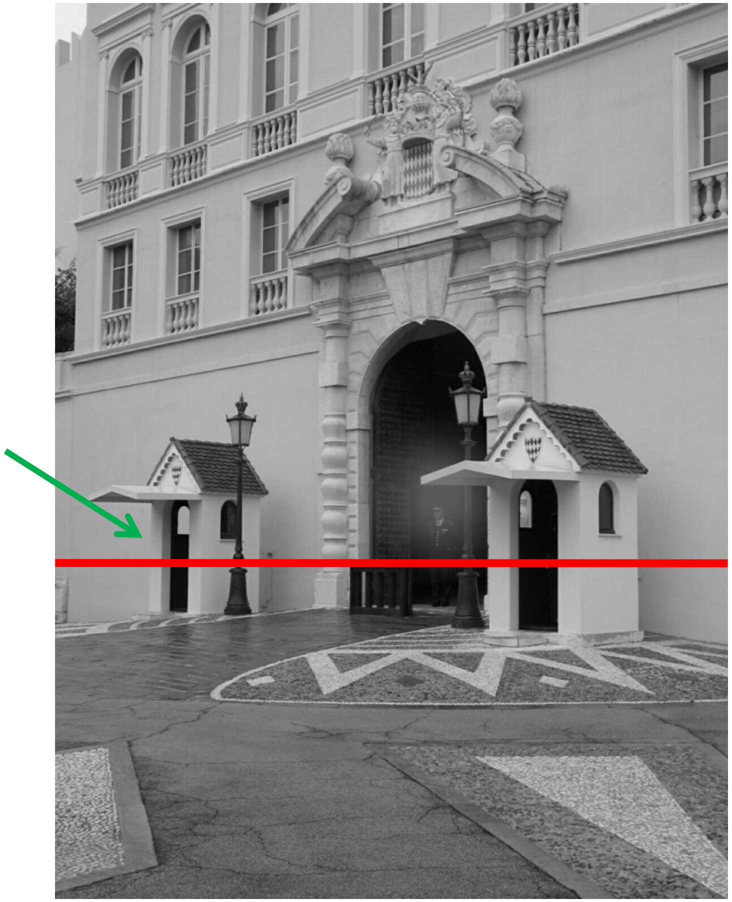
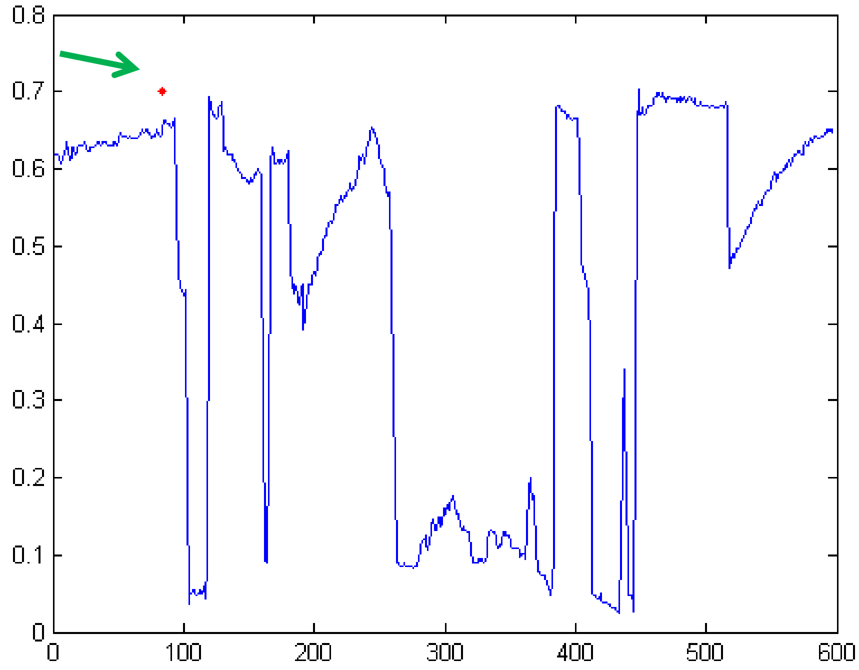


Slide Credit: James Hays

Intensity profile





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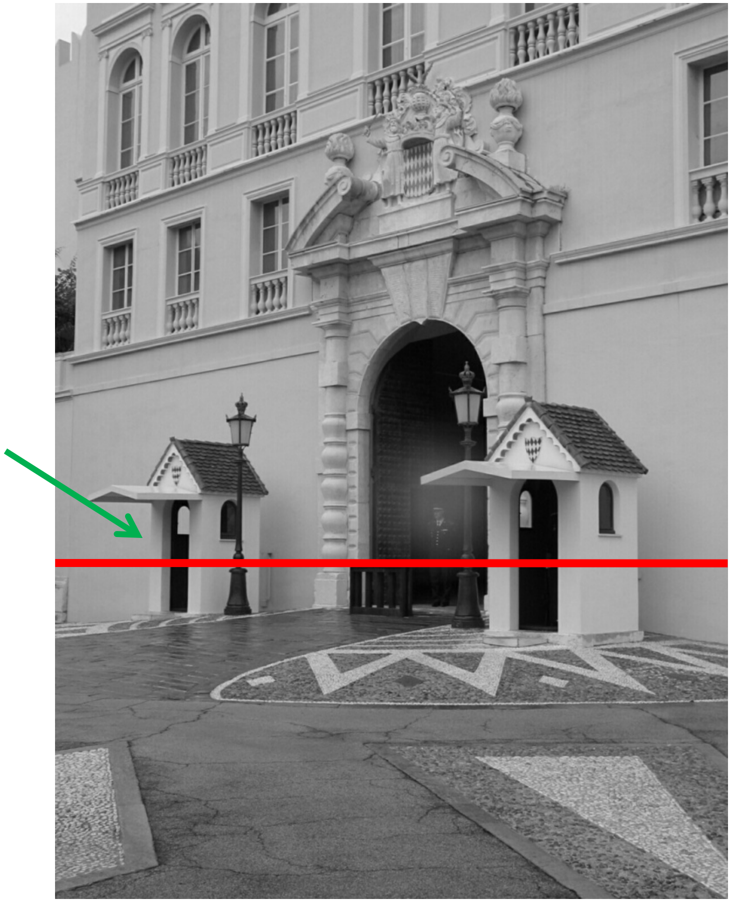
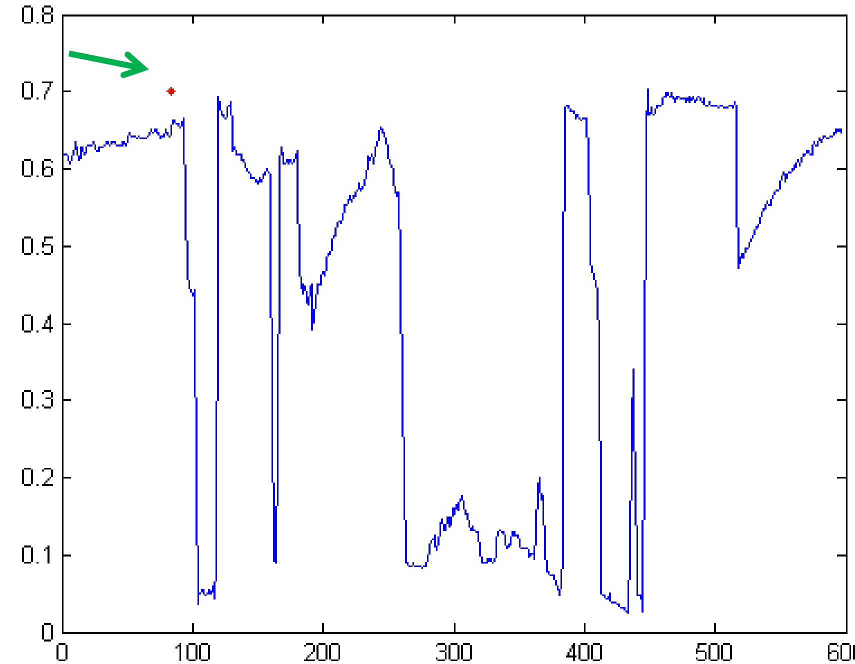
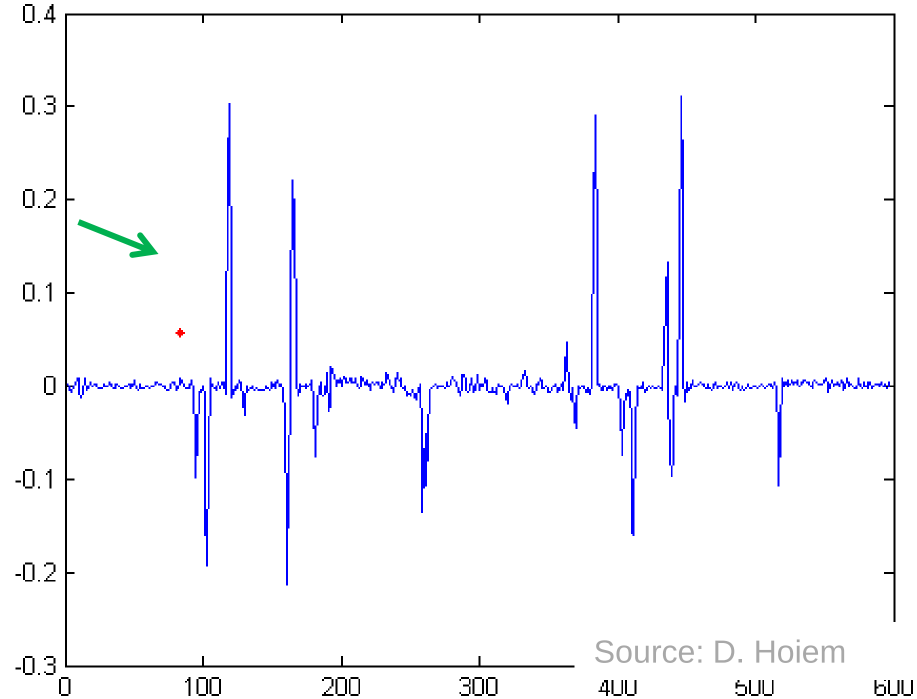
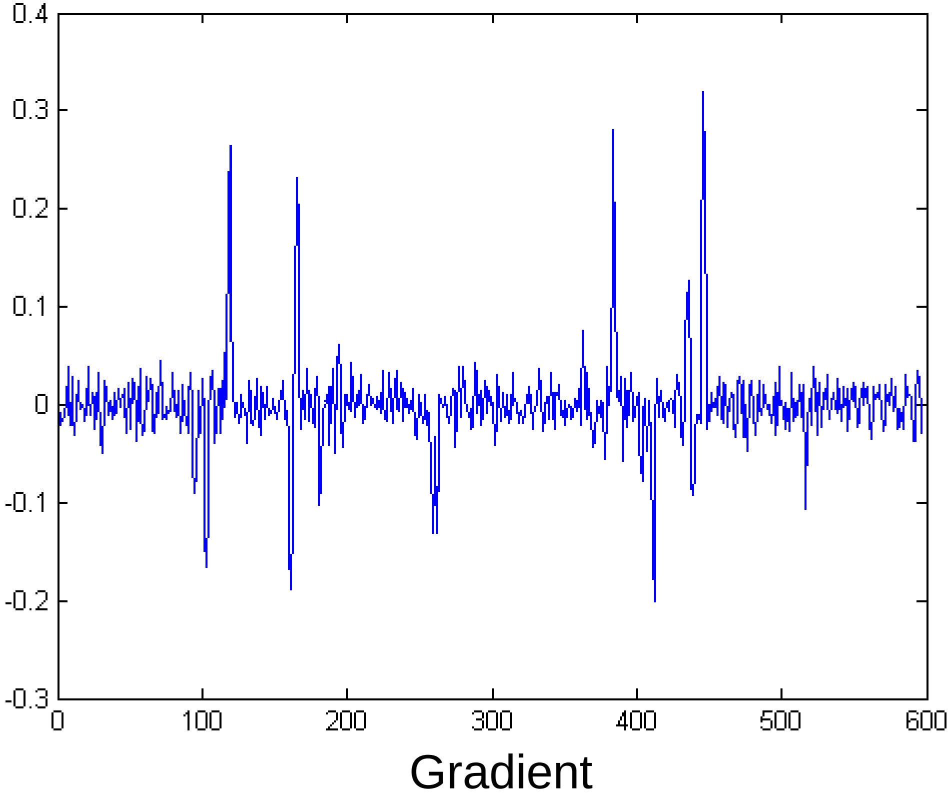






Source: D. Hoiem

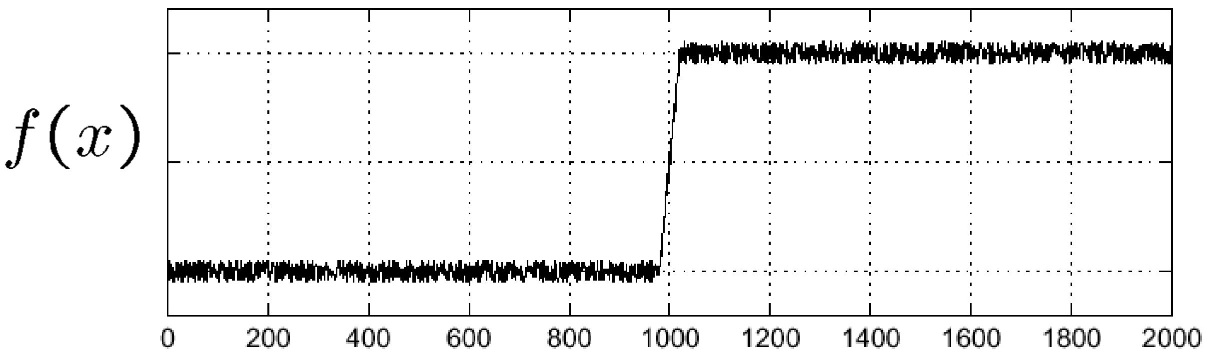
With a little Gaussian noise



Source: D. Hoiem

Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



Source: S. Seitz

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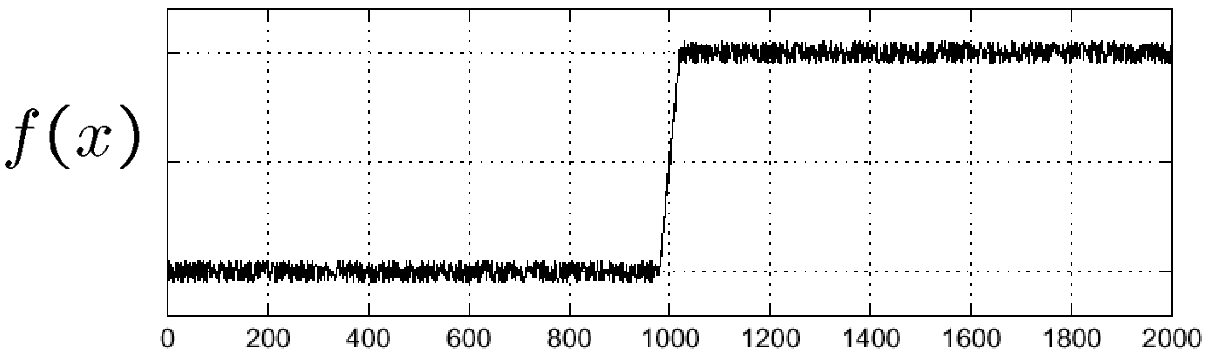
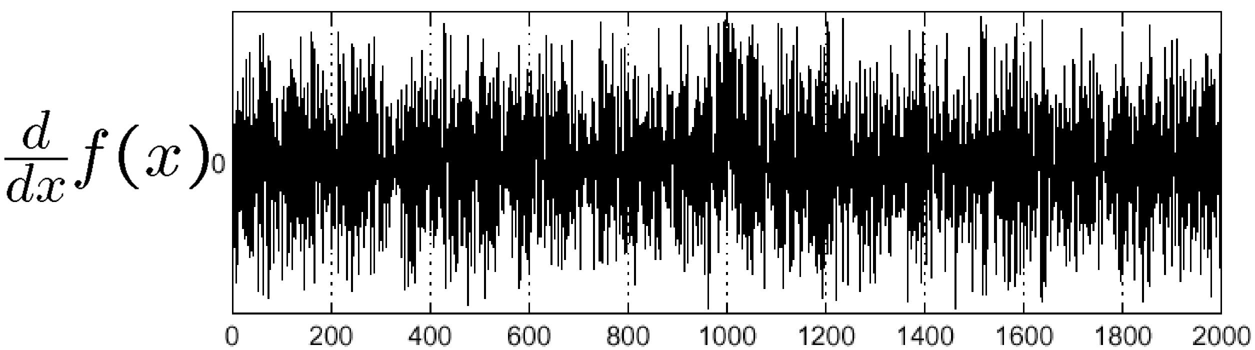




Where is the edge?

Source: S. Seitz

Effects of noise

Source: D. Forsyth

Effects of noise

- Difference filters respond strongly to noise

Source: D. Forsyth

Effects of noise

- Difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors

Source: D. Forsyth

Effects of noise

- Difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response

Source: D. Forsyth

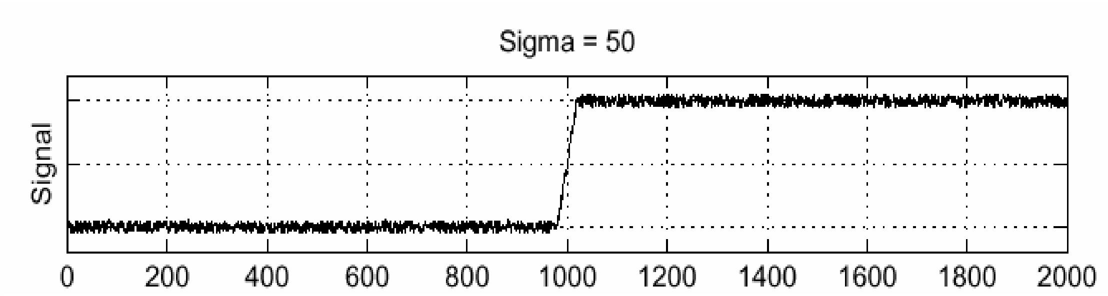
Effects of noise

- Difference filters respond strongly to noise
  – Image noise results in pixels that look very different from their neighbors
  – Generally, the larger the noise the stronger the response
- What can we do about it?

Source: D. Forsyth

Solution: smooth first

f

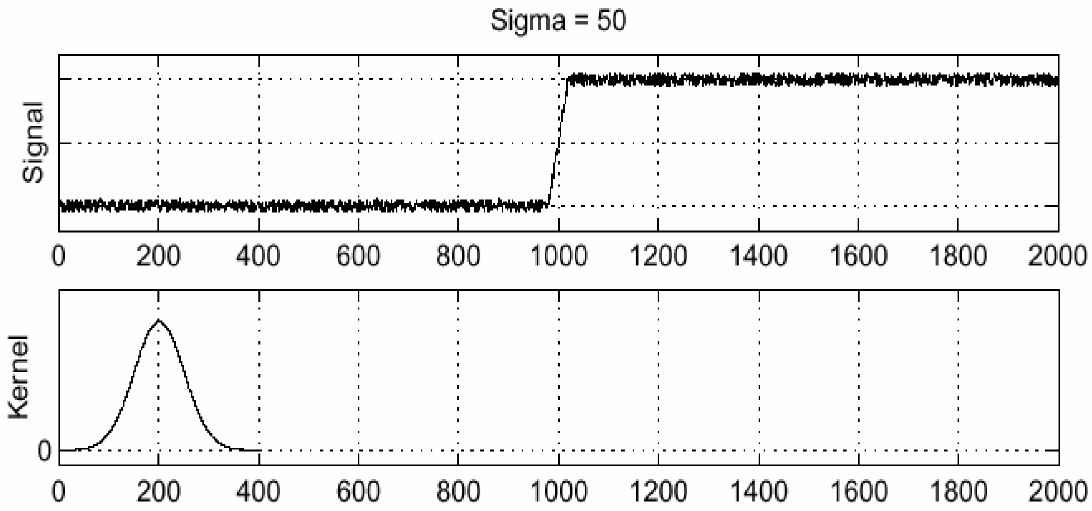


Source: S. Seitz

Solution: smooth first

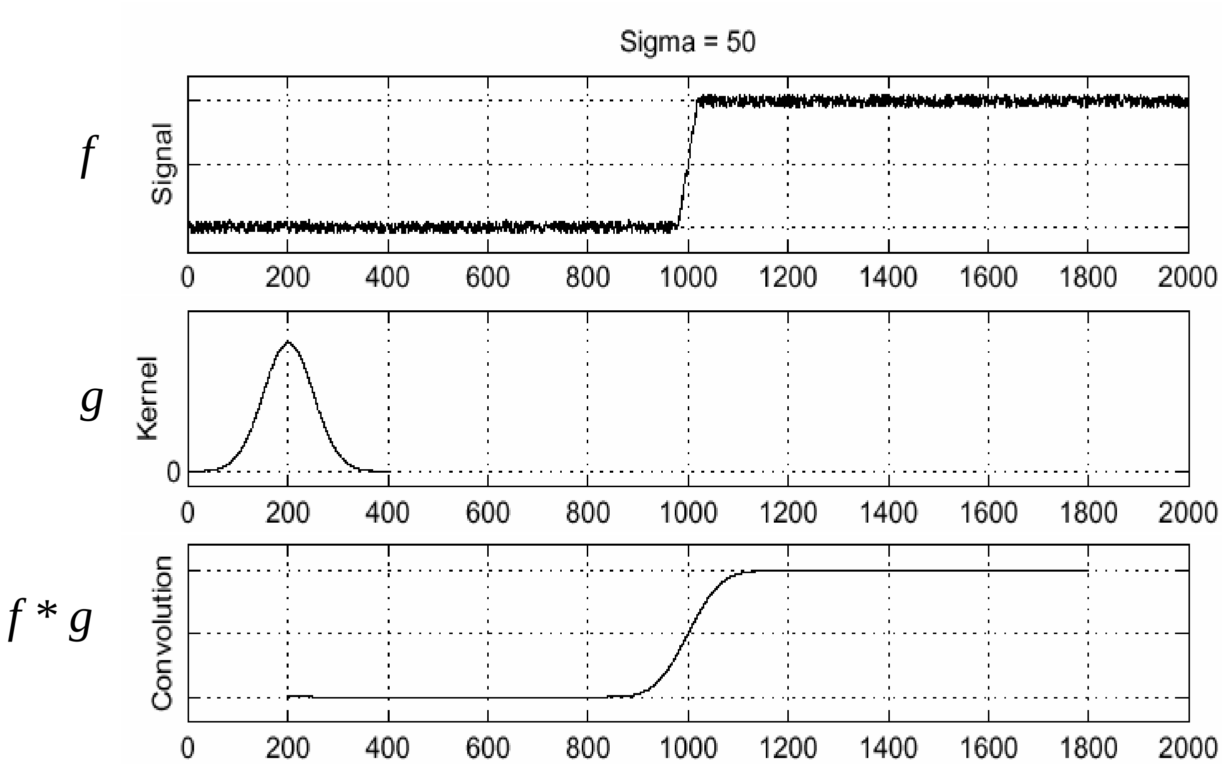
f

g



Source: S. Seitz

Solution: smooth first



Source: S. Seitz

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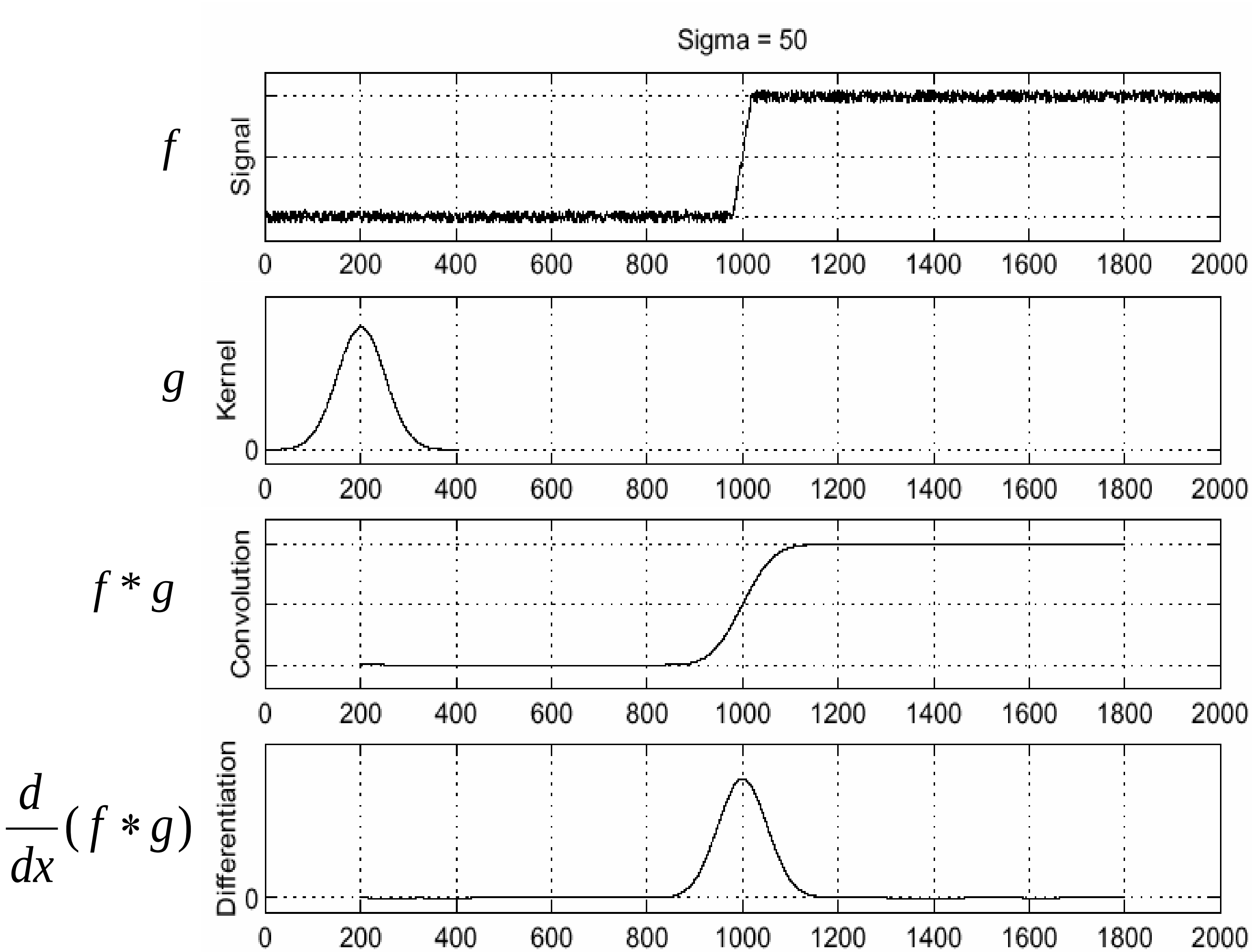


- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

Derivative theorem of convolution

Source: S. Seitz

Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative:

Source: S. Seitz

Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative:
$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

- This saves us one operation:

Source: S. Seitz

Derivative theorem of convolution

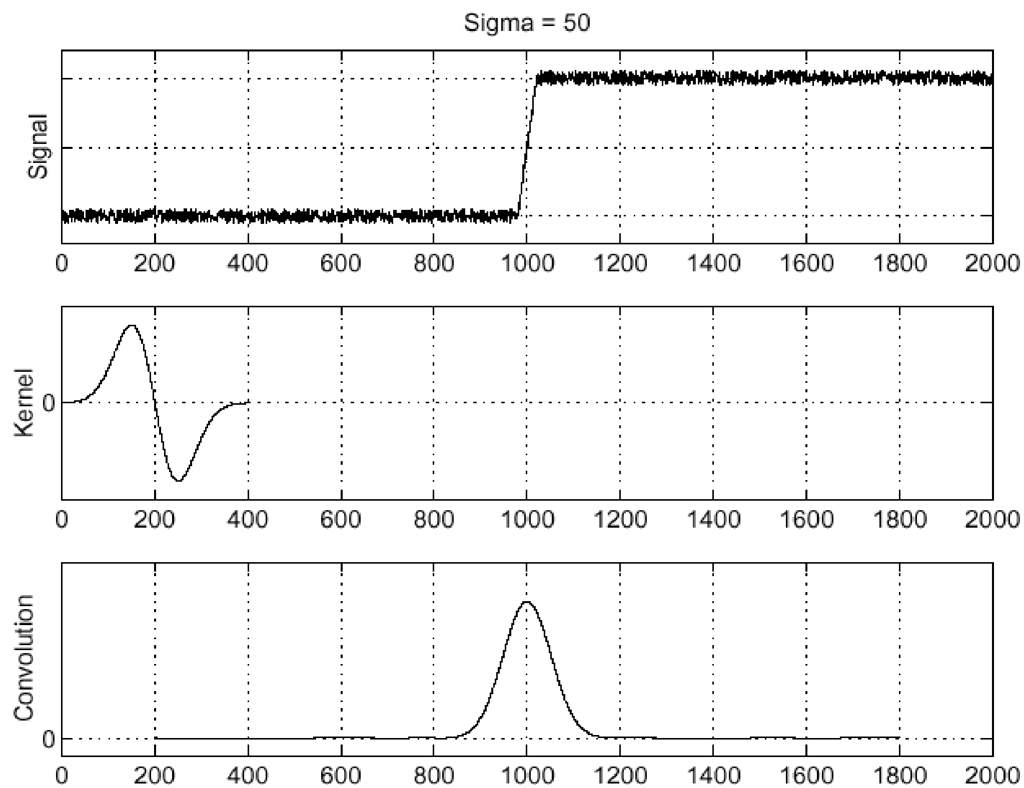
- Differentiation is convolution, and convolution is associative:
$$\frac{d}{dx}(f * g) = f * \frac{d}{dx}g$$

- This saves us one operation:

f

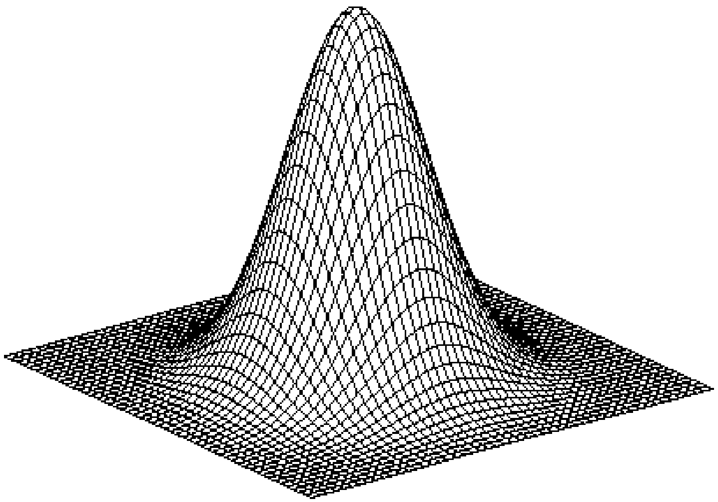
$\frac{d}{dx}g$

$f * \frac{d}{dx}g$

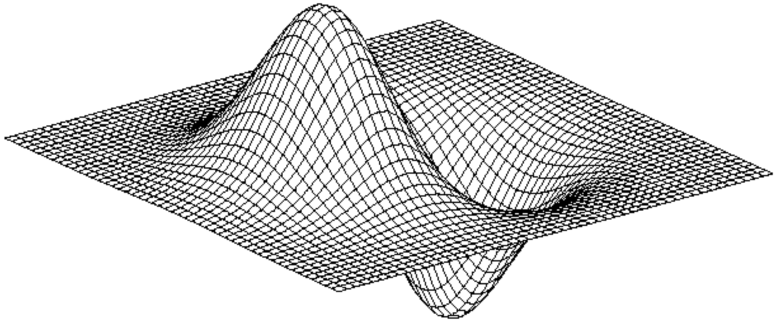


Source: S. Seitz

Derivative of Gaussian filter



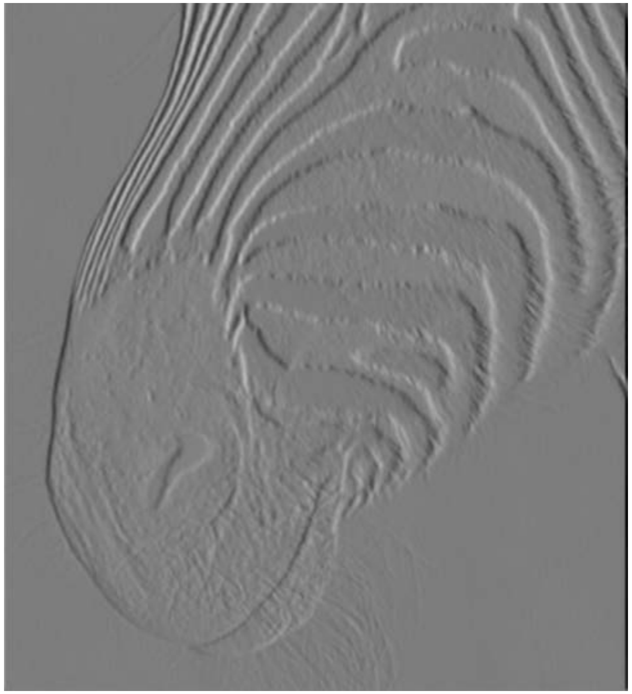
* [1 -1] =



Tradeoff between smoothing and localization

Source: D. Forsyth

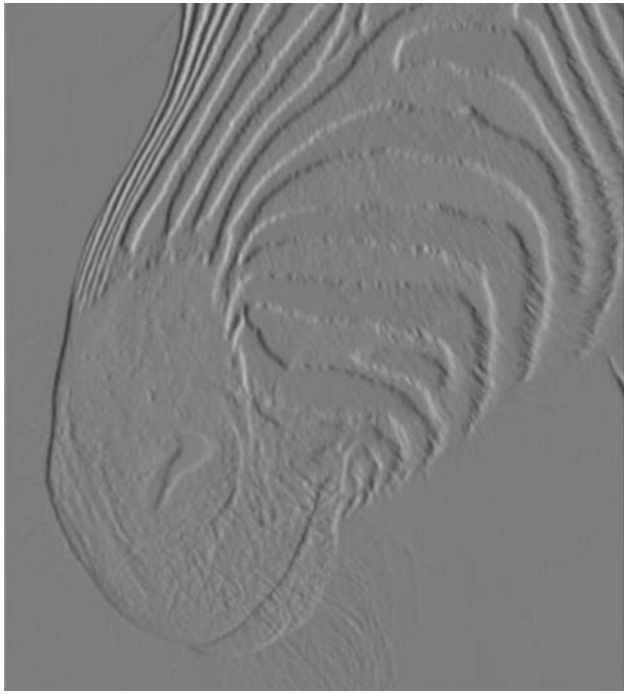
Tradeoff between smoothing and localization



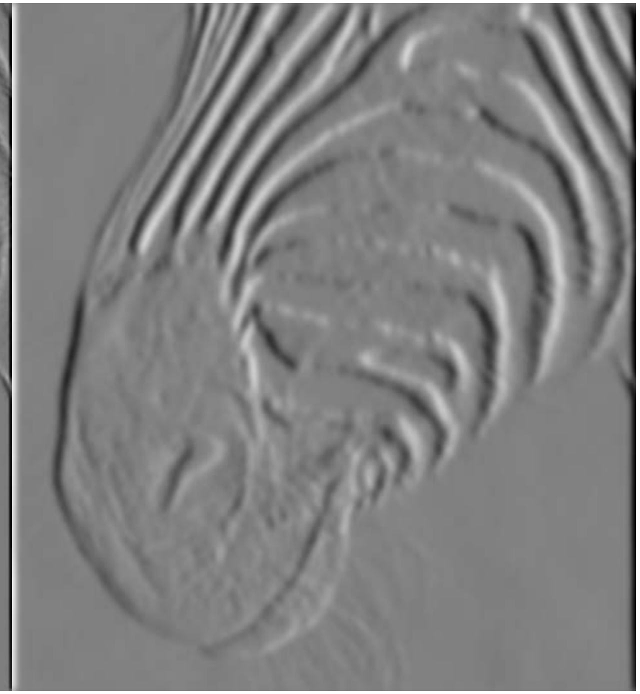
1 pixel

Source: D. Forsyth

Tradeoff between smoothing and localization



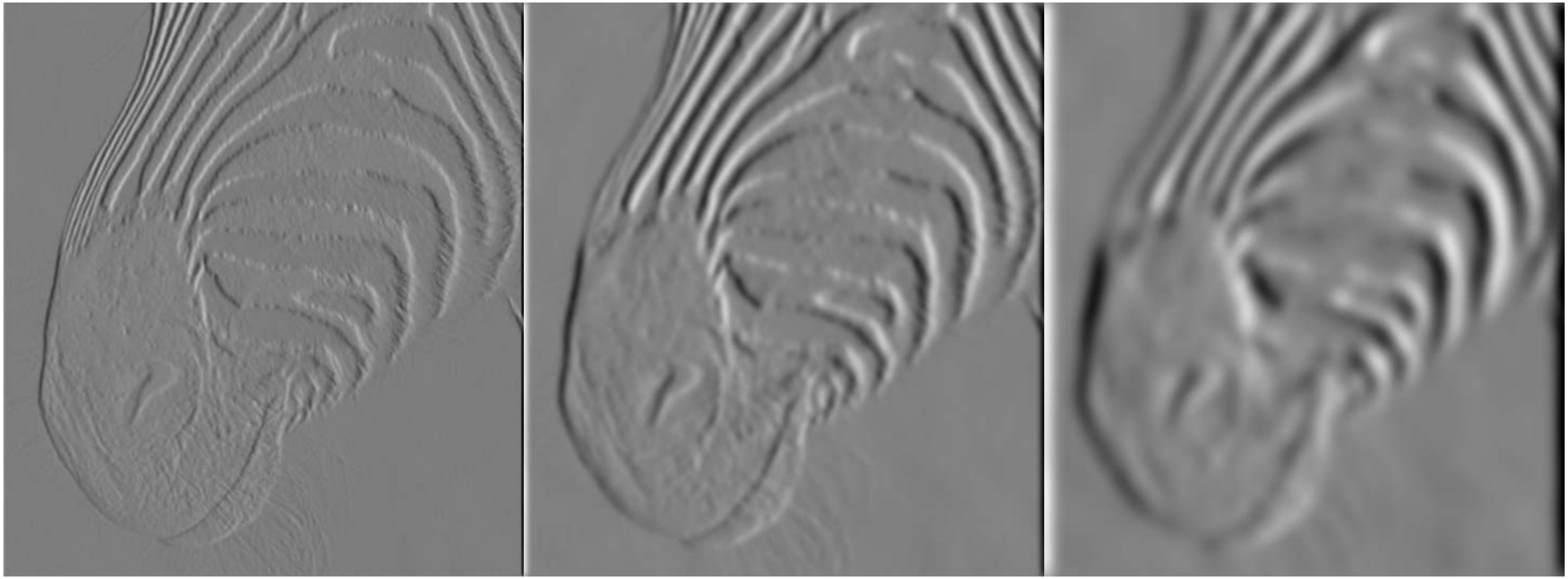
1 pixel



3 pixels

Source: D. Forsyth

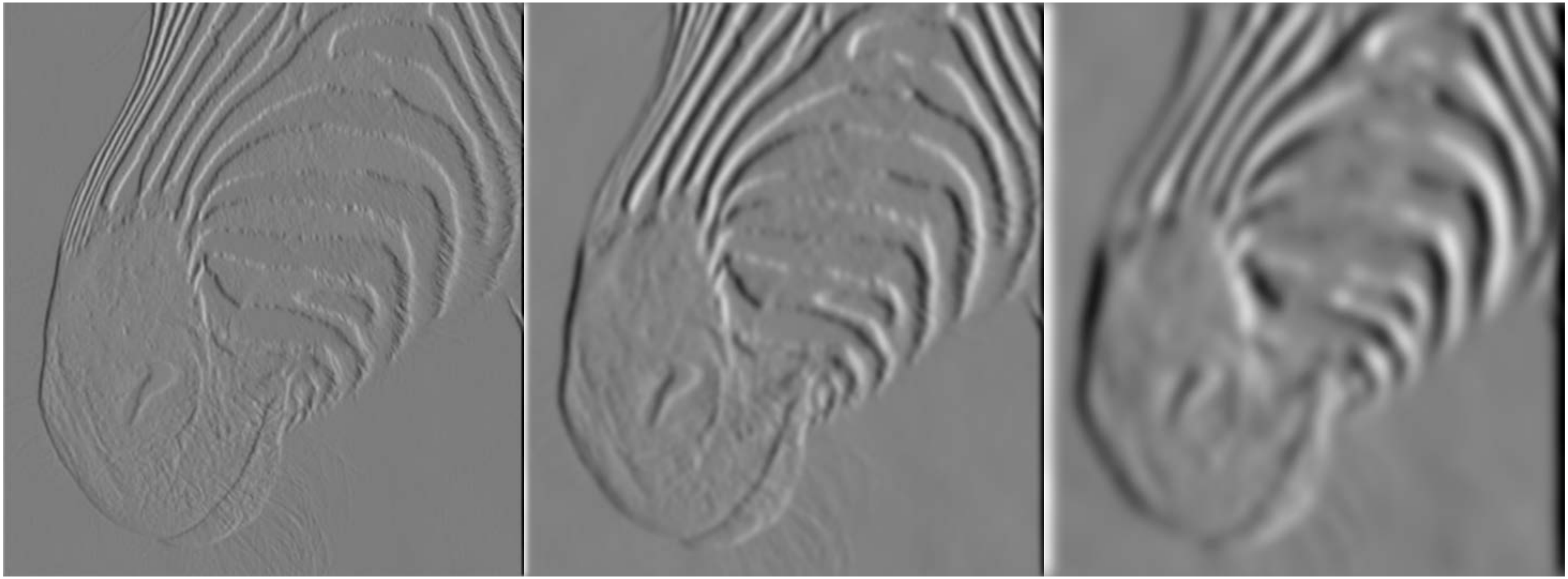
Tradeoff between smoothing and localization



1 pixel 3 pixels 7 pixels

Source: D. Forsyth

Tradeoff between smoothing and localization



1 pixel 3 pixels 7 pixels

- Smoothed derivative removes noise, but blurs edge. Also finds edges at different “scales”.

Source: D. Forsyth

Detecting Discontinuities

- Image derivatives

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon) - f(x)}{\varepsilon}\right) \Rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}) - f(x)}{\partial x}$$

- Convolve image with derivative filters

Detecting Discontinuities

- Image derivatives

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0} \left(\frac{f(x+\varepsilon) - f(x)}{\varepsilon} \right) \Rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}) - f(x)}{\partial x}$$

- Convolve image with derivative filters

Backward difference [-1 1]

Detecting Discontinuities

- Image derivatives

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0} \left(\frac{f(x+\varepsilon) - f(x)}{\varepsilon} \right) \Longrightarrow \frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}) - f(x)}{\partial x}$$

- Convolve image with derivative filters

Backward difference	[-1 1]
Forward difference	[1 -1]

Detecting Discontinuities

- Image derivatives

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon) - f(x)}{\varepsilon} \right) \quad \Rightarrow \quad \frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}) - f(x)}{\partial x}$$

- Convolve image with derivative filters

Backward difference	[-1 1]
Forward difference	[1 -1]
Central difference	[-1 0 1]

Derivative in Two-Dimensions

- Definition
- Approximation
- Convolution kernels

Derivative in Two-Dimensions

- Definition

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon,y) - f(x,y)}{\varepsilon}\right)$$

- Approximation

- Convolution kernels

Derivative in Two-Dimensions

- Definition

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}\right) \qquad \frac{\partial f(x,y)}{\partial y} = \lim_{\varepsilon \to 0}\left(\frac{f(x, y+\varepsilon) - f(x,y)}{\varepsilon}\right)$$

- Approximation

- Convolution kernels

Derivative in Two-Dimensions

- Definition

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon,y) - f(x,y)}{\varepsilon}\right) \qquad \frac{\partial f(x,y)}{\partial y} = \lim_{\varepsilon \to 0}\left(\frac{f(x,y+\varepsilon) - f(x,y)}{\varepsilon}\right)$$

- Approximation

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x_{n+1},y_m) - f(x_n,y_m)}{1}$$

- Convolution kernels

Derivative in Two-Dimensions

- Definition

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}\right) \qquad \frac{\partial f(x,y)}{\partial y} = \lim_{\varepsilon \to 0}\left(\frac{f(x, y+\varepsilon) - f(x,y)}{\varepsilon}\right)$$

- Approximation

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x_{n+1}, y_m) - f(x_n, y_m)}{1} \qquad \frac{\partial f(x,y)}{\partial y} \approx \frac{f(x_n, y_{m+1}) - f(x_n, y_m)}{1}$$

- Convolution kernels

Derivative in Two-Dimensions

- Definition

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$$f_x=\begin{bmatrix}1 & -1\end{bmatrix}$$

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- Convolution kernels

$$f_x = \begin{bmatrix} 1 & -1 \end{bmatrix} \qquad f_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Image Derivatives

Image I

$$I_x = I * \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$I_y = I * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

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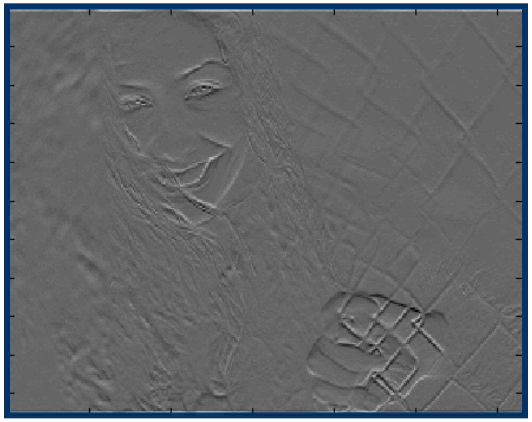

$$I_y = I * \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Derivatives and Noise

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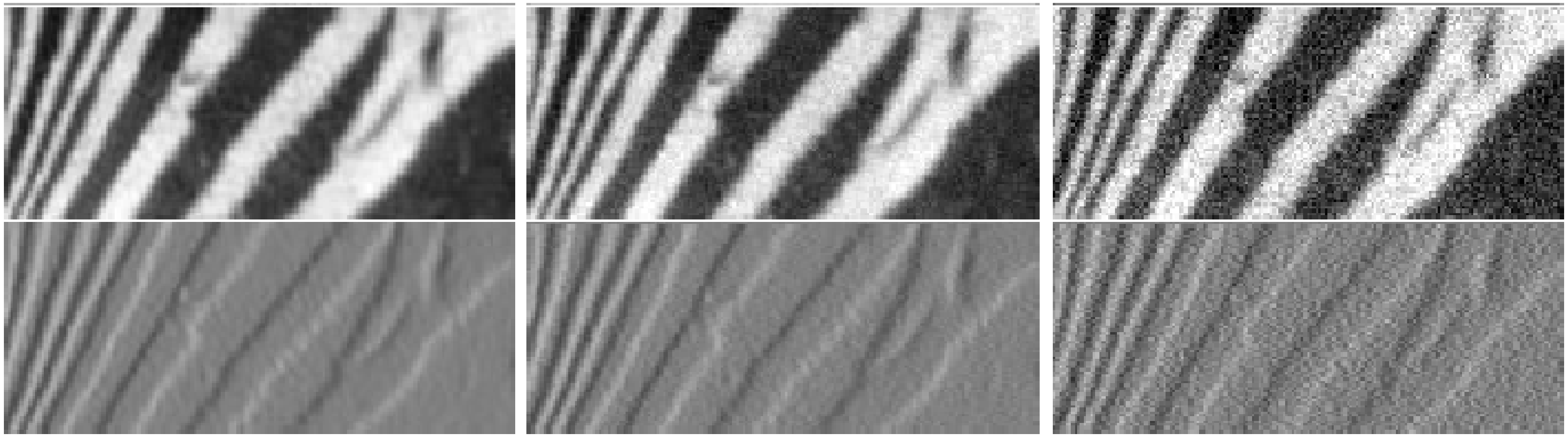
- What is to be done?
  - Neighboring pixels look alike
  - Pixel along an edge look alike
  - Image smoothing should help
    - Force pixels different from their neighbors (possibly noise) to look like neighbors

Derivatives and Noise



Increasing noise →

Zero mean additive gaussian noise

Image Smoothing

Image Smoothing

- Expect pixels to “**be like**” their neighbors
  – Relatively few reflectance changes

Image Smoothing

- Expect pixels to “**be like**” their neighbors
  - Relatively few reflectance changes
- Generally expect noise to be independent from pixel to pixel
  - Smoothing suppresses noise

Gaussian Smoothing

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$$g(x,y) = e^{\frac{-(x^2+y^2)}{2o^2}}$$

Gaussian Smoothing

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- Scale of Gaussian σ
  - As σ increases, more pixels are involved in average
  - As σ increases, image is more blurred
  - As σ increases, noise is more effectively suppressed

Edge Detectors

Edge Detectors

- Gradient operators
  - Prewit
  - Sobel

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- Laplacian of Gaussian (Marr-Hildreth)

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- Gradient operators
  - Prewit
  - Sobel
- Laplacian of Gaussian (Marr-Hildreth)
- Gradient of Gaussian (Canny)

Prewitt and Sobel Edge Detector

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- Compute derivatives
  - In x and y directions

Prewitt and Sobel Edge Detector

- Compute derivatives
  - In x and y directions
- Find gradient magnitude

Prewitt and Sobel Edge Detector

- Compute derivatives
  - In x and y directions
- Find gradient magnitude
- Threshold gradient magnitude

Prewitt Edge Detector

Prewitt Edge Detector

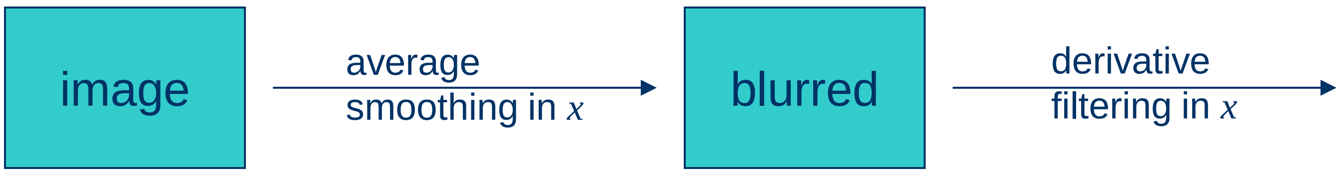
image

Prewitt Edge Detector

Prewitt Edge Detector

image $\xrightarrow{\text{average smoothing in } x}$ blurred

Prewitt Edge Detector



Prewitt Edge Detector

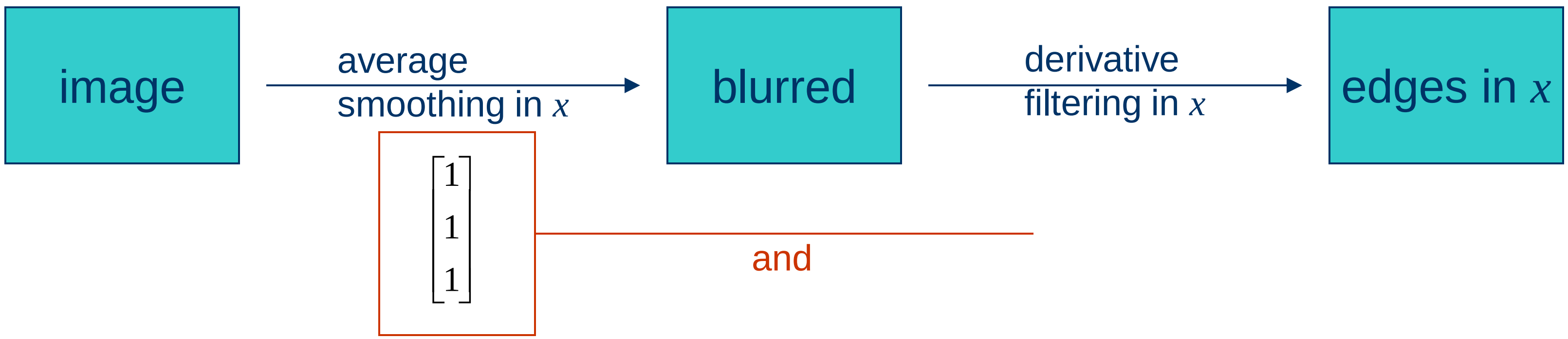
image $\xrightarrow[\text{smoothing in } x]{\text{average}}$ blurred $\xrightarrow[\text{filtering in } x]{\text{derivative}}$ edges in x

Prewitt Edge Detector

$$\text{image} \xrightarrow[\text{smoothing in } x]{\text{average}} \text{blurred} \xrightarrow[\text{filtering in } x]{\text{derivative}} \text{edges in } x$$

$$\boxed{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$$

Prewitt Edge Detector

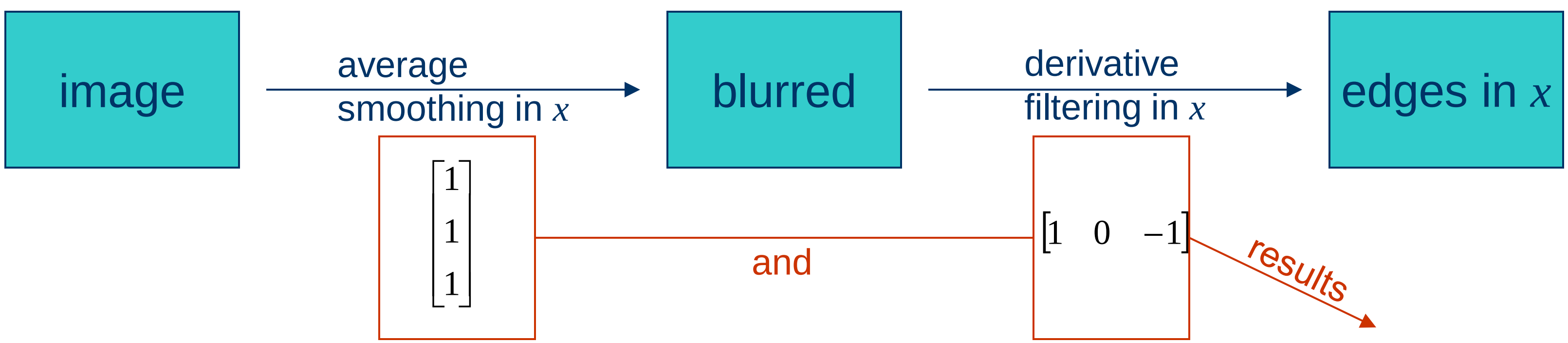


Prewitt Edge Detector

image $\xrightarrow[\text{smoothing in } x]{\text{average}}$ blurred $\xrightarrow[\text{filtering in } x]{\text{derivative}}$ edges in x

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

Prewitt Edge Detector



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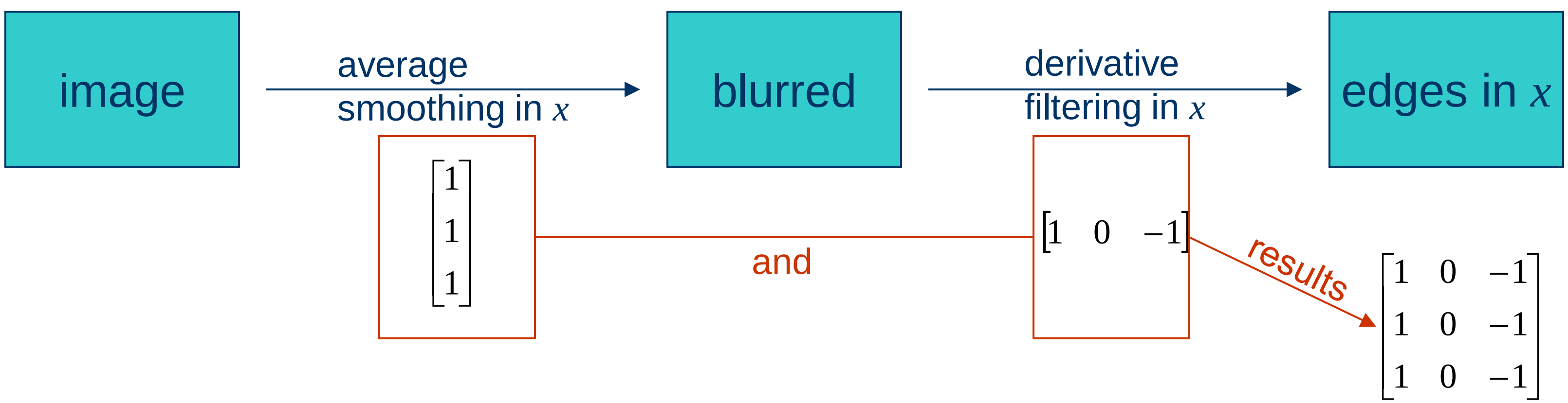
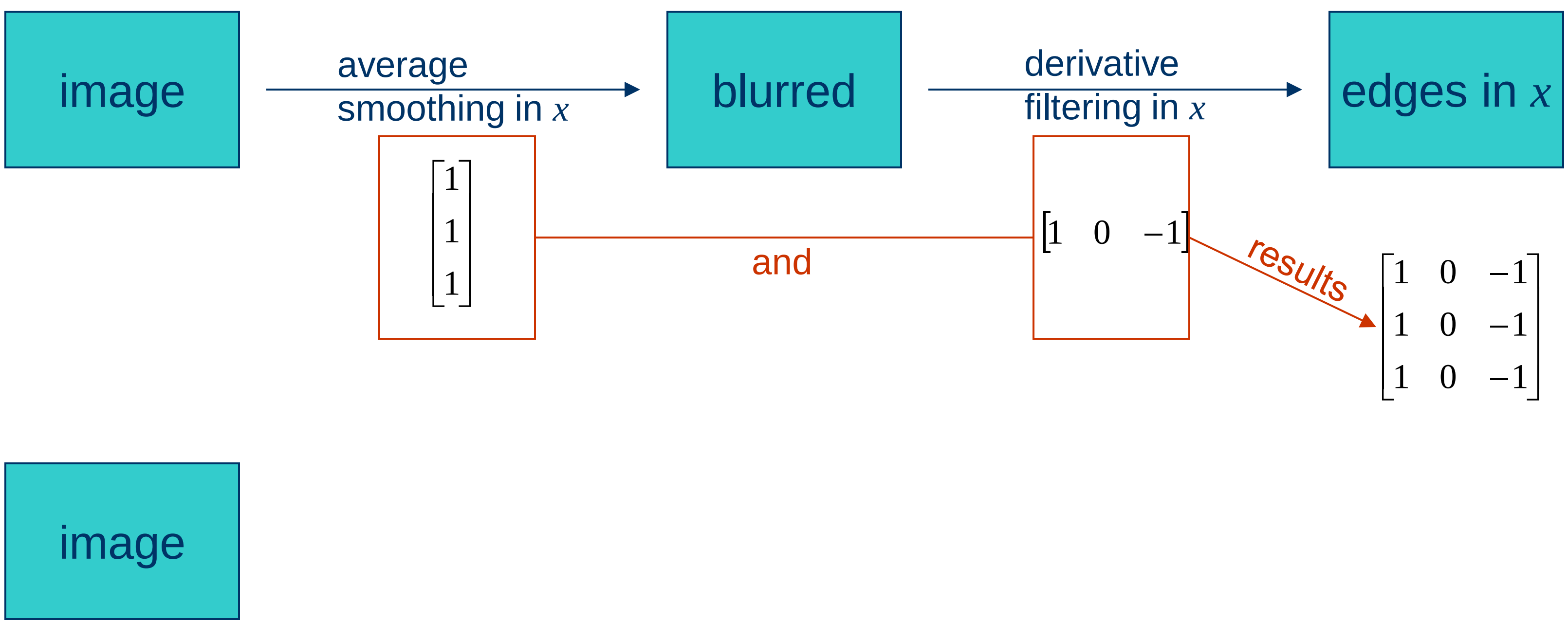
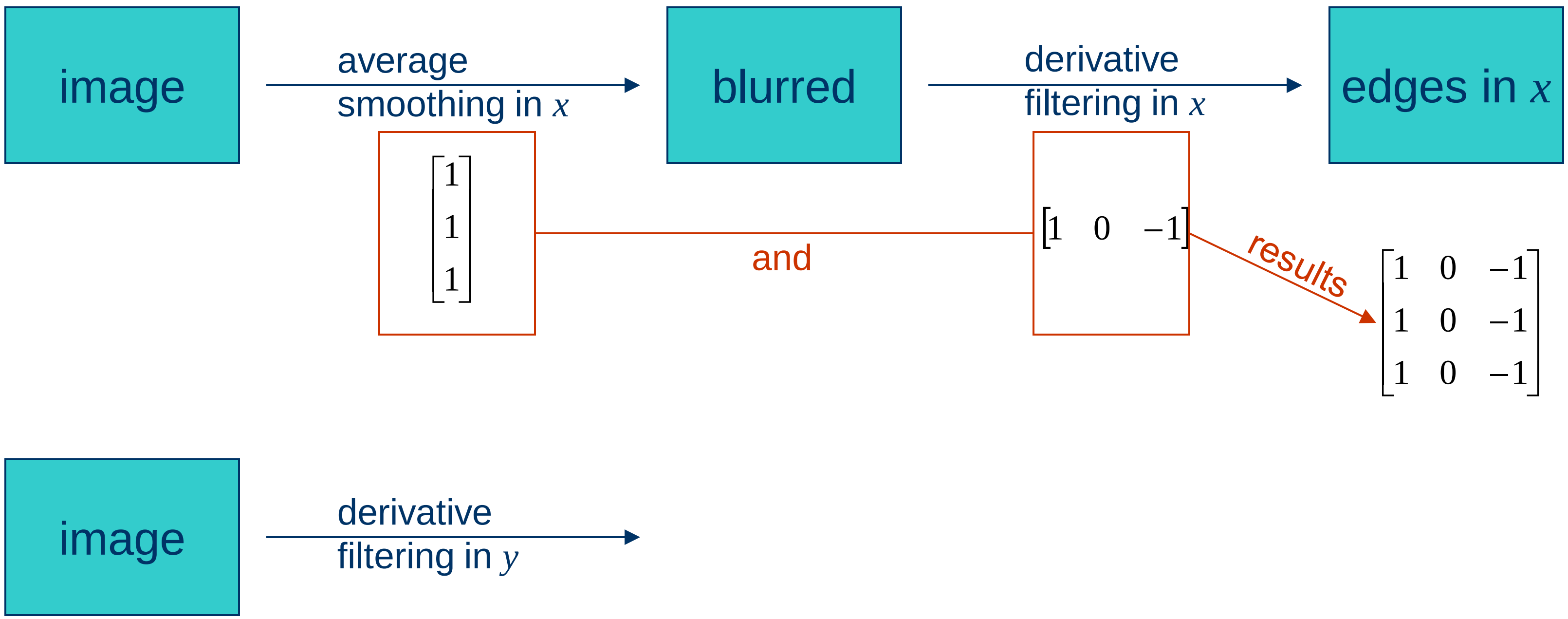


Prewitt Edge Detector



Prewitt Edge Detector



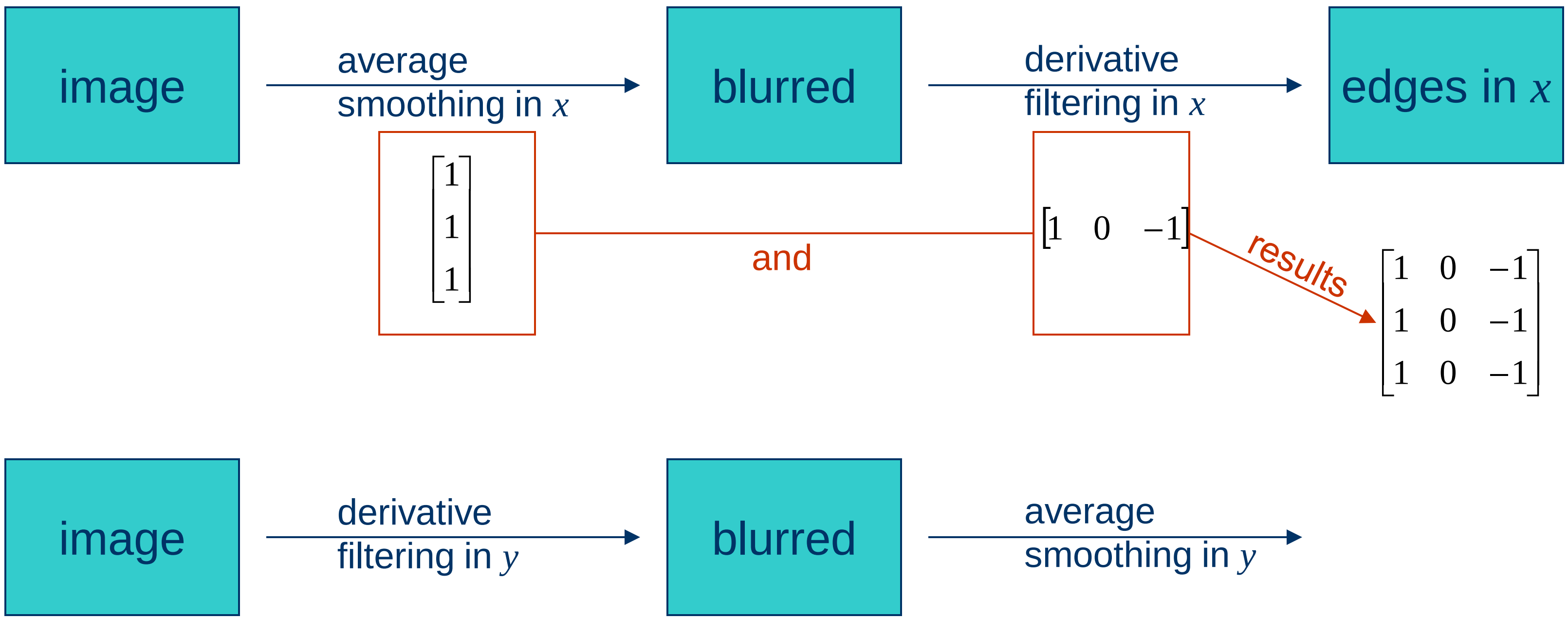
Prewitt Edge Detector

image $\xrightarrow[\text{smoothing in } x]{\text{average}}$ blurred $\xrightarrow[\text{filtering in } x]{\text{derivative}}$ edges in x

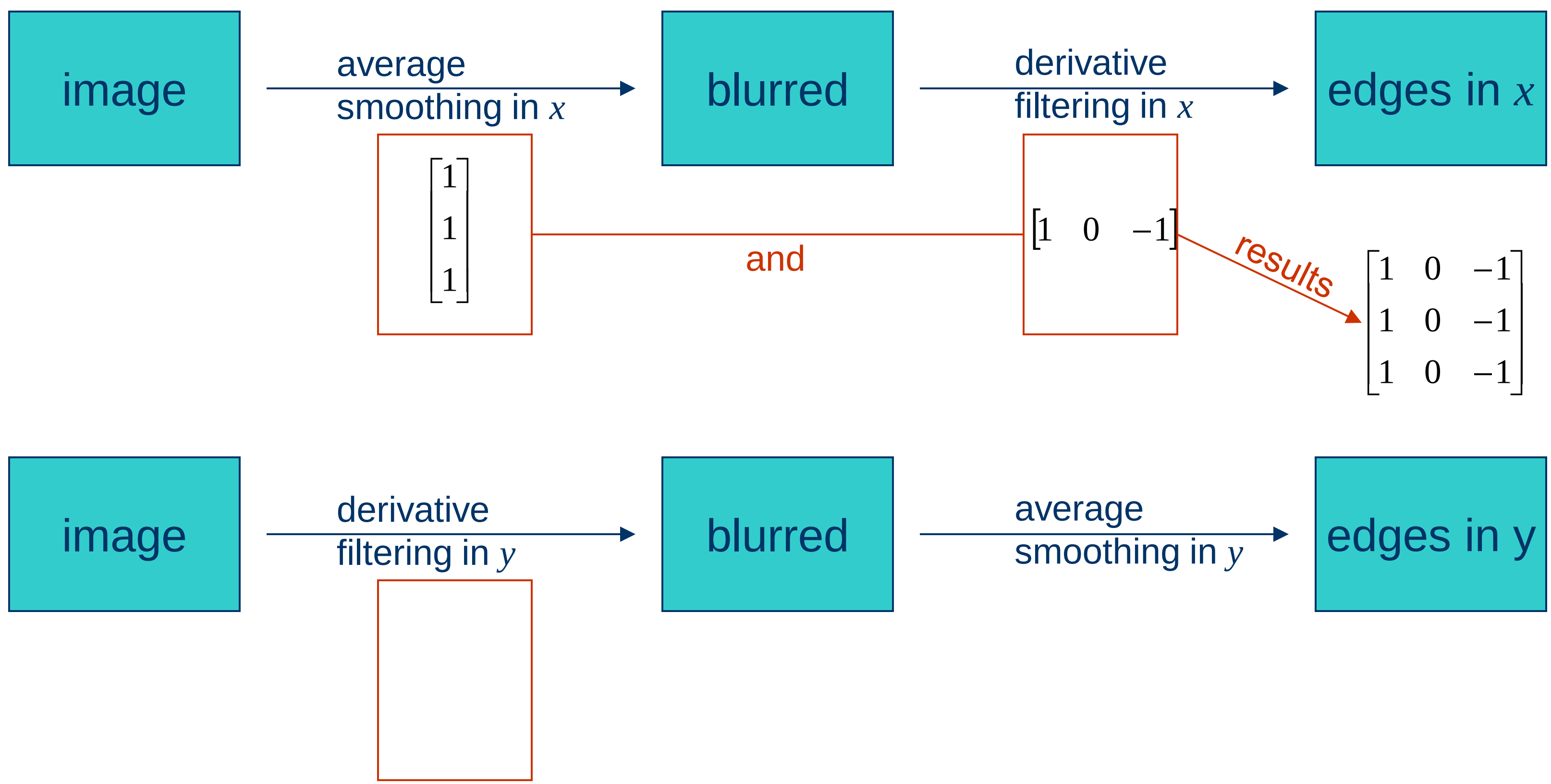
$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$ results $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$

image $\xrightarrow[\text{filtering in } y]{\text{derivative}}$ blurred

Prewitt Edge Detector



Prewitt Edge Detector



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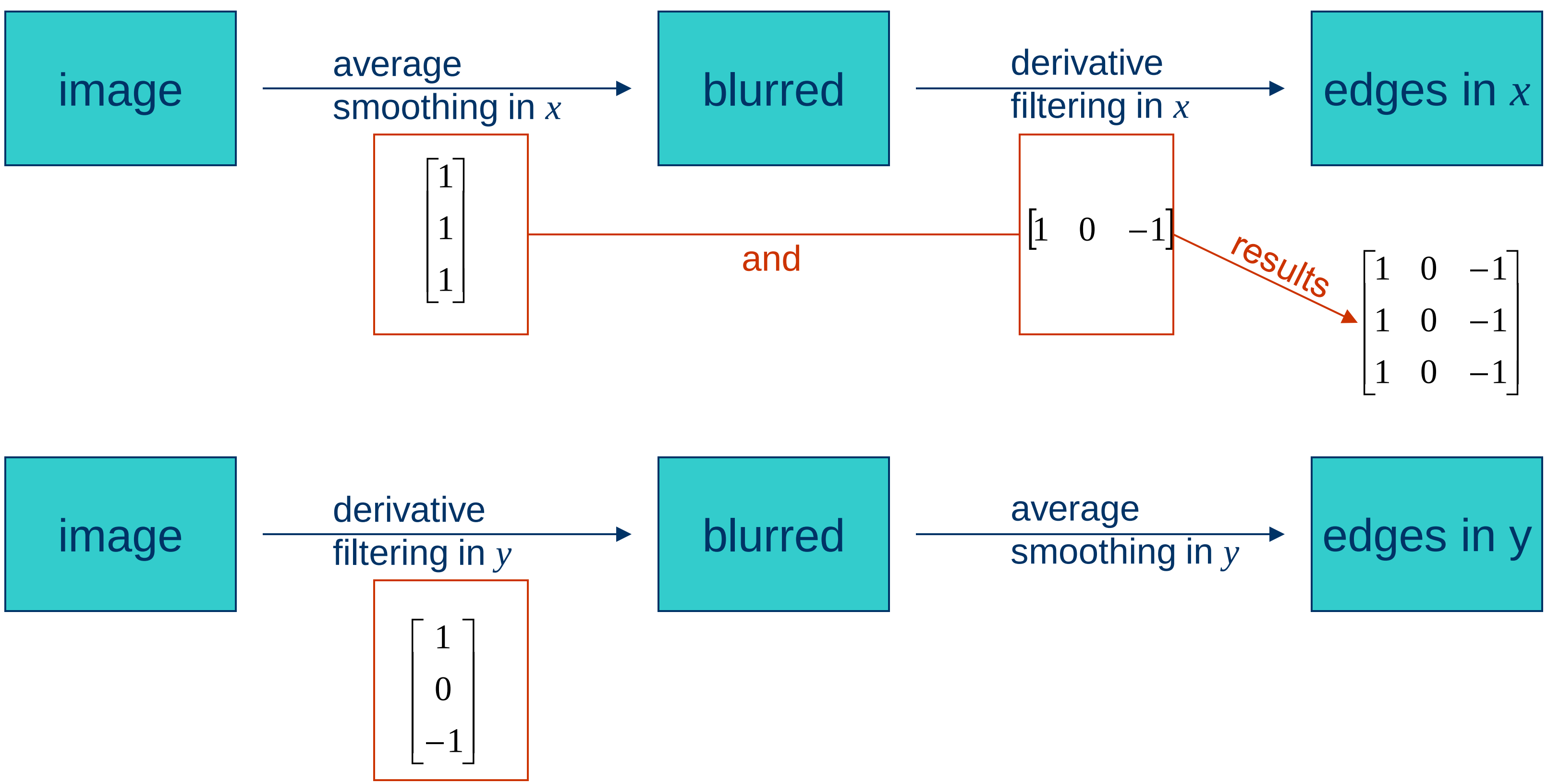
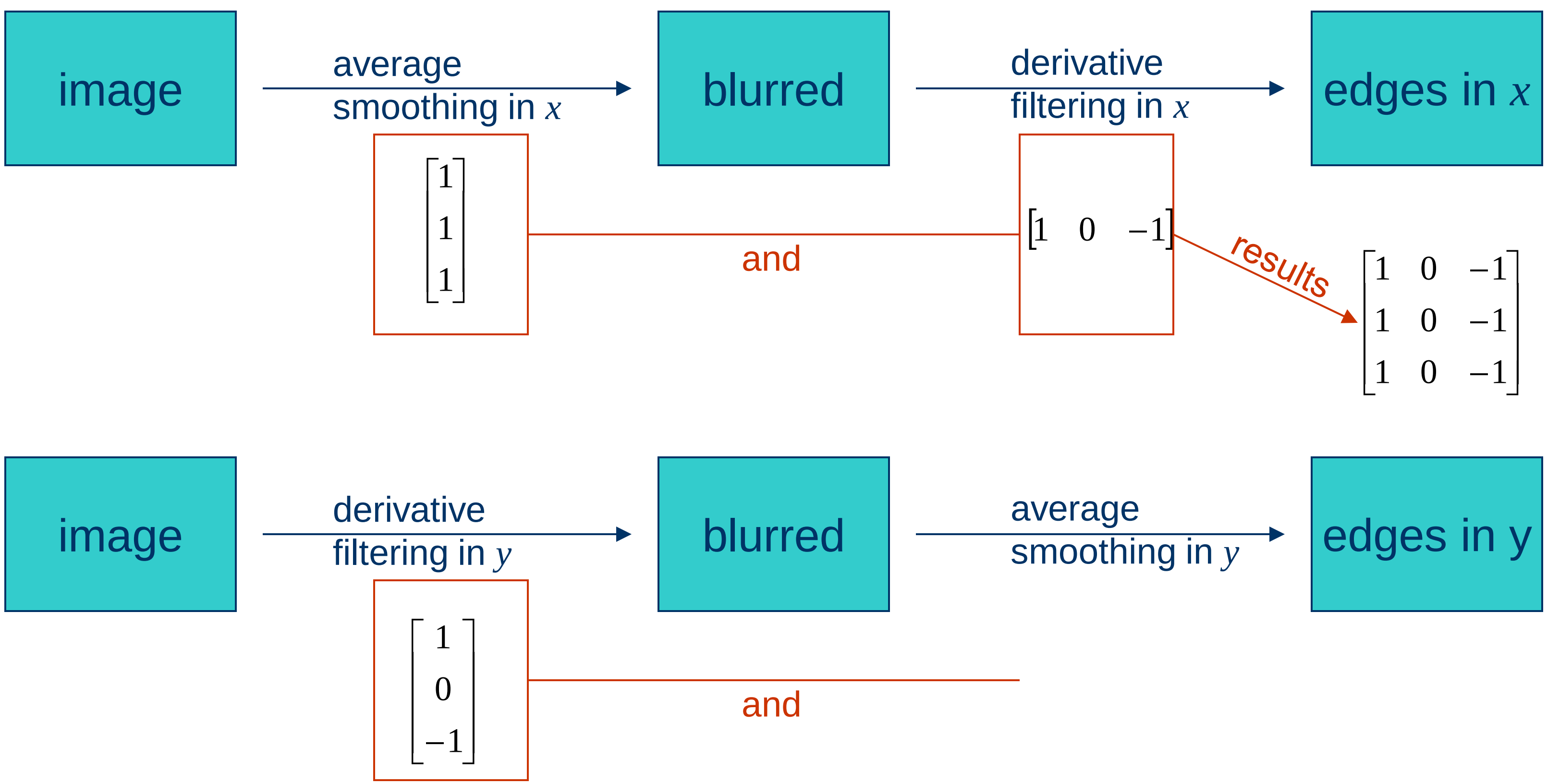


Prewitt Edge Detector



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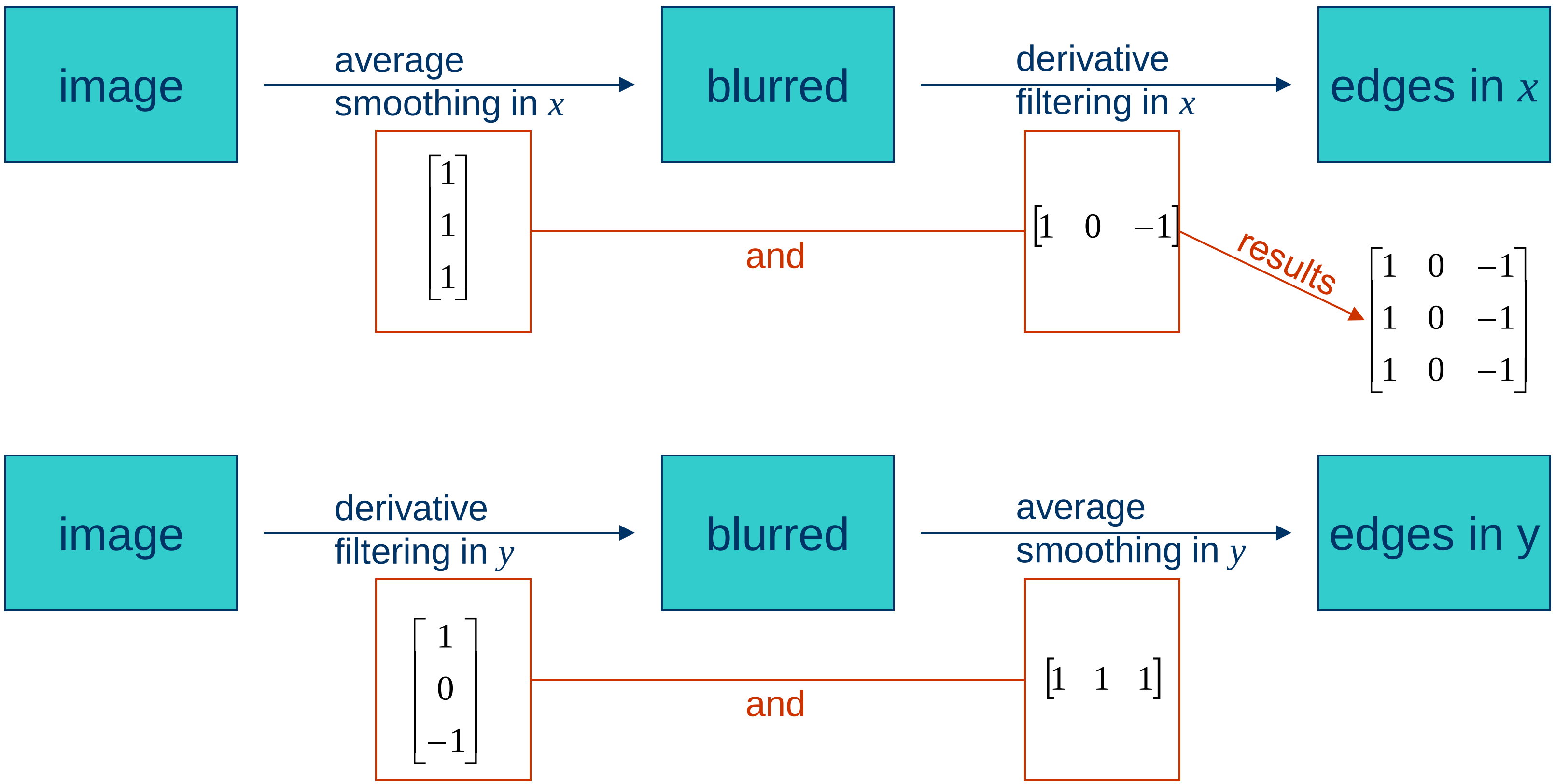

Prewitt Edge Detector

image $\xrightarrow{\text{average smoothing in } x}$ blurred $\xrightarrow{\text{derivative filtering in } x}$ edges in x

$$\begin{bmatrix}1\\1\\1\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}1 & 0 & -1\end{bmatrix} \quad \xrightarrow{\text{results}} \quad \begin{bmatrix}1 & 0 & -1\\1 & 0 & -1\\1 & 0 & -1\end{bmatrix}$$

image $\xrightarrow{\text{derivative filtering in } y}$ blurred $\xrightarrow{\text{average smoothing in } y}$ edges in y

$$\begin{bmatrix}1\\0\\-1\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}1 & 1 & 1\end{bmatrix} \quad \xrightarrow{\text{results}}$$

Prewitt Edge Detector

$$\text{image} \xrightarrow{\text{average smoothing in } x} \text{blurred} \xrightarrow{\text{derivative filtering in } x} \text{edges in } x$$

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$$\begin{bmatrix}1\\0\\-1\end{bmatrix} \text{ and } \begin{bmatrix}1 & 1 & 1\end{bmatrix} \xrightarrow{\text{results}} \begin{bmatrix}1 & 1 & 1\\0 & 0 & 0\\-1 & -1 & -1\end{bmatrix}$$

Sobel Edge Detector

Sobel Edge Detector

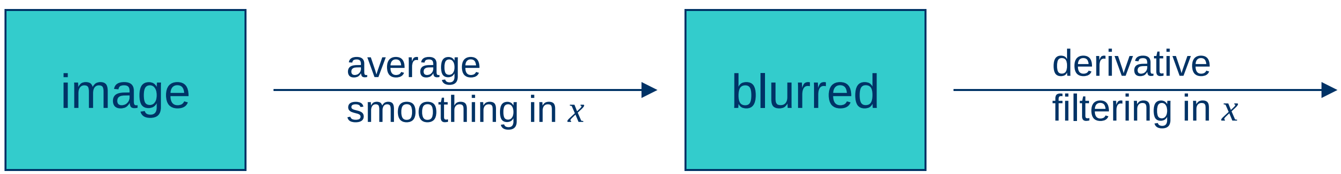
image

Sobel Edge Detector

image $\xrightarrow[\text{smoothing in } x]{\text{average}}$

Sobel Edge Detector

Sobel Edge Detector



Sobel Edge Detector

image $\xrightarrow[\text{smoothing in } x]{\text{average}}$ blurred $\xrightarrow[\text{filtering in } x]{\text{derivative}}$ edges in x

Sobel Edge Detector

image $\xrightarrow[\text{smoothing in } x]{\text{average}}$ blurred $\xrightarrow[\text{filtering in } x]{\text{derivative}}$ edges in x

$$\boxed{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}$$

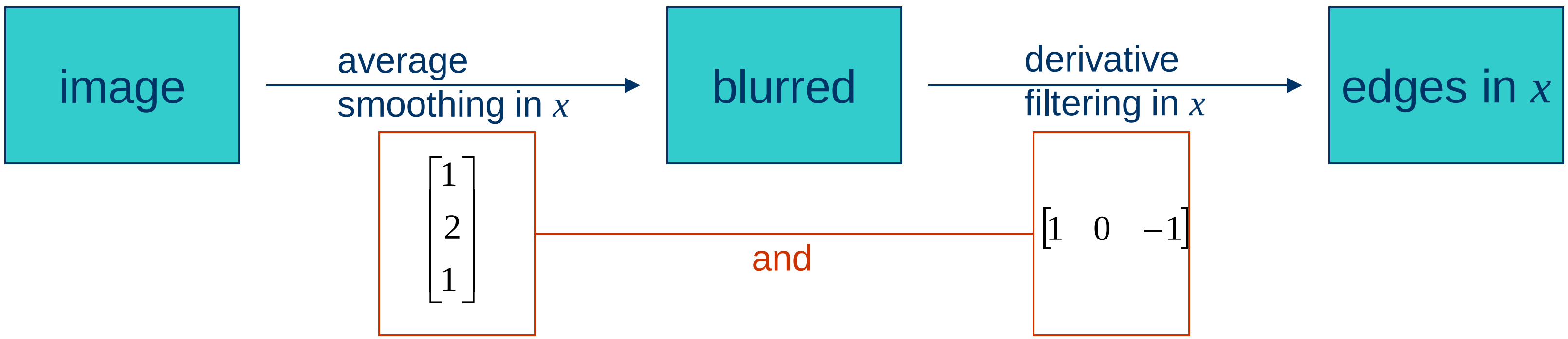
Sobel Edge Detector

image $\xrightarrow{\text{average smoothing in } x}$ blurred $\xrightarrow{\text{derivative filtering in } x}$ edges in x

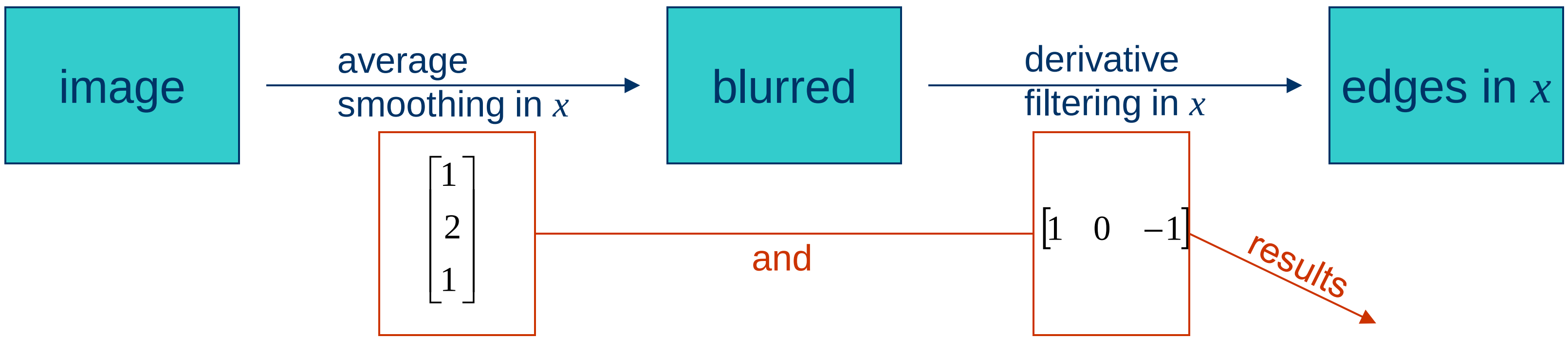
$$\boxed{\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}}$$

and

Sobel Edge Detector



Sobel Edge Detector



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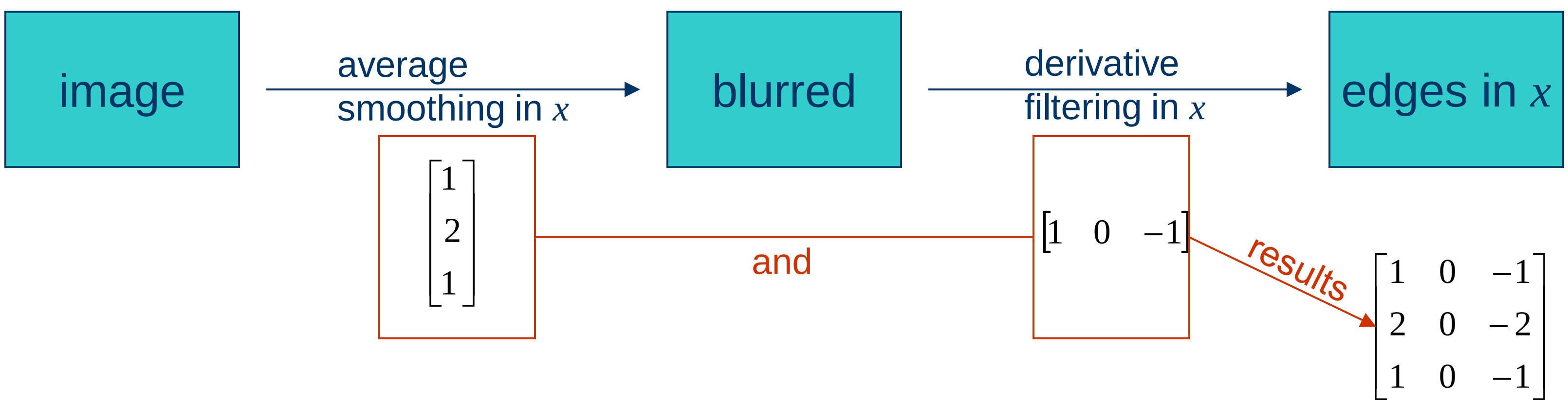

Sobel Edge Detector

image $\xrightarrow{\text{average smoothing in } x}$ blurred $\xrightarrow{\text{derivative filtering in } x}$ edges in x

$$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \quad \xrightarrow{\text{results}} \quad \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

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image $\xrightarrow{\text{derivative filtering in } y}$

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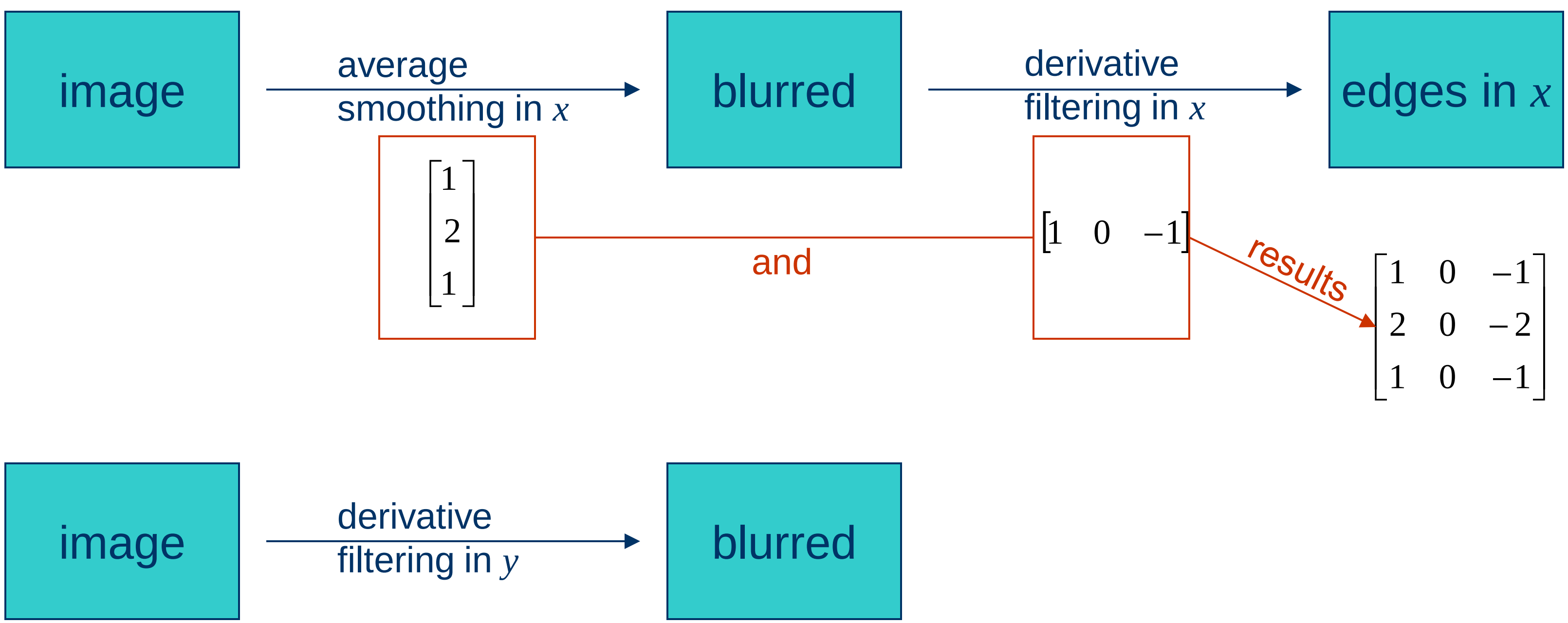

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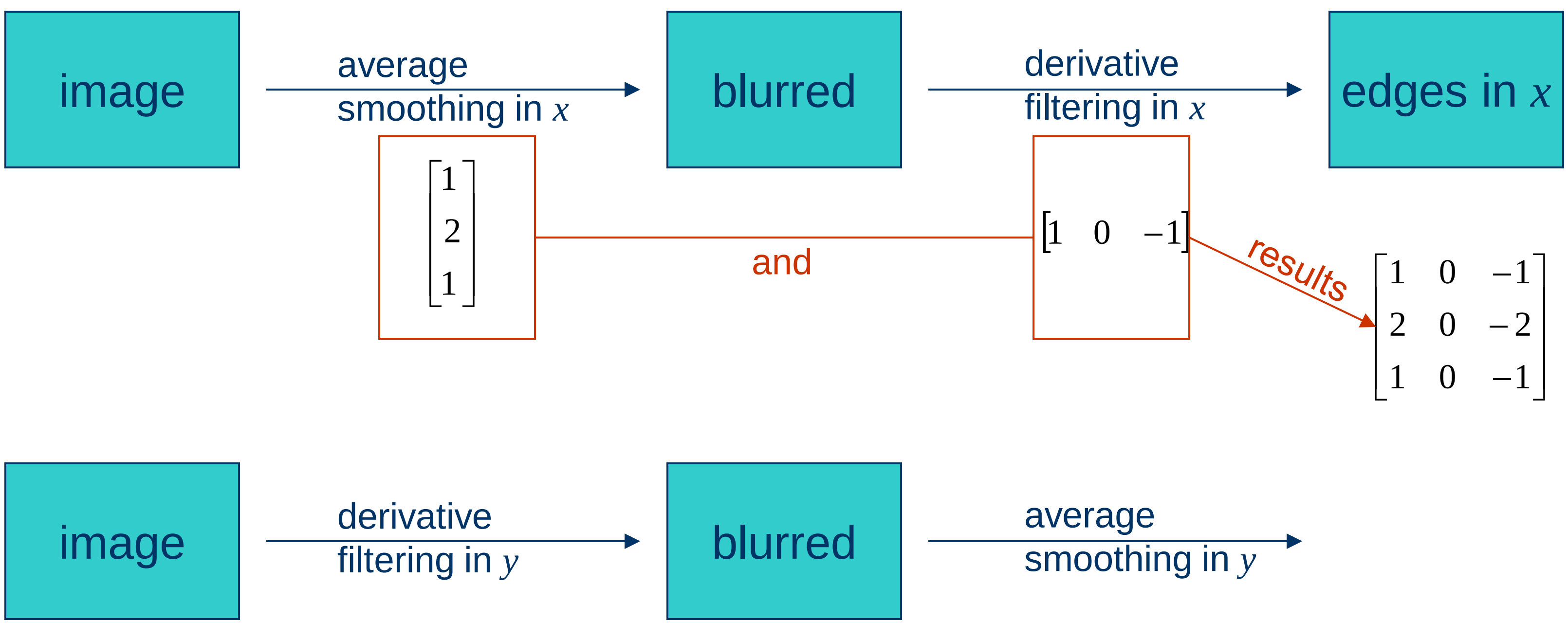

Sobel Edge Detector

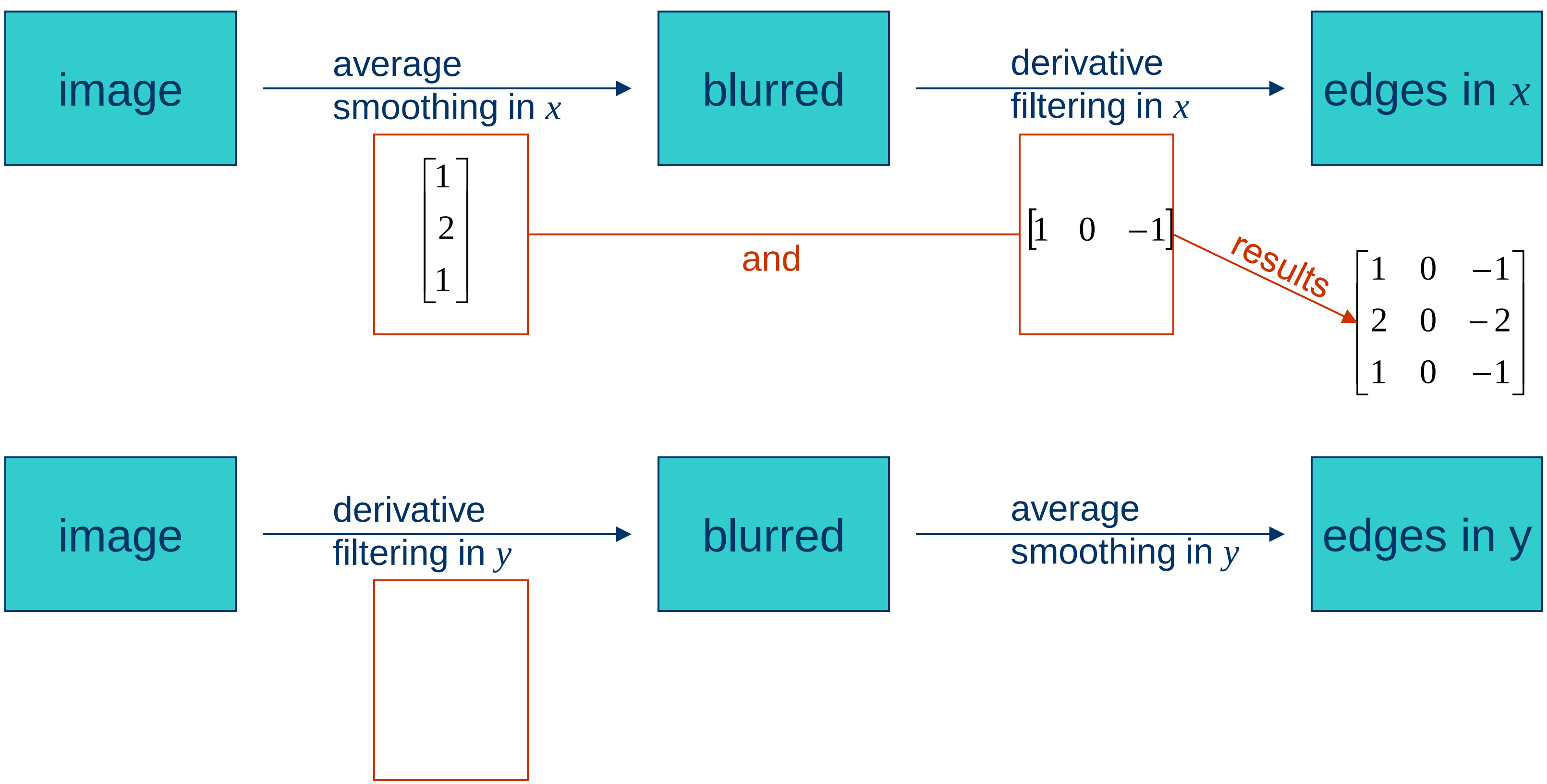

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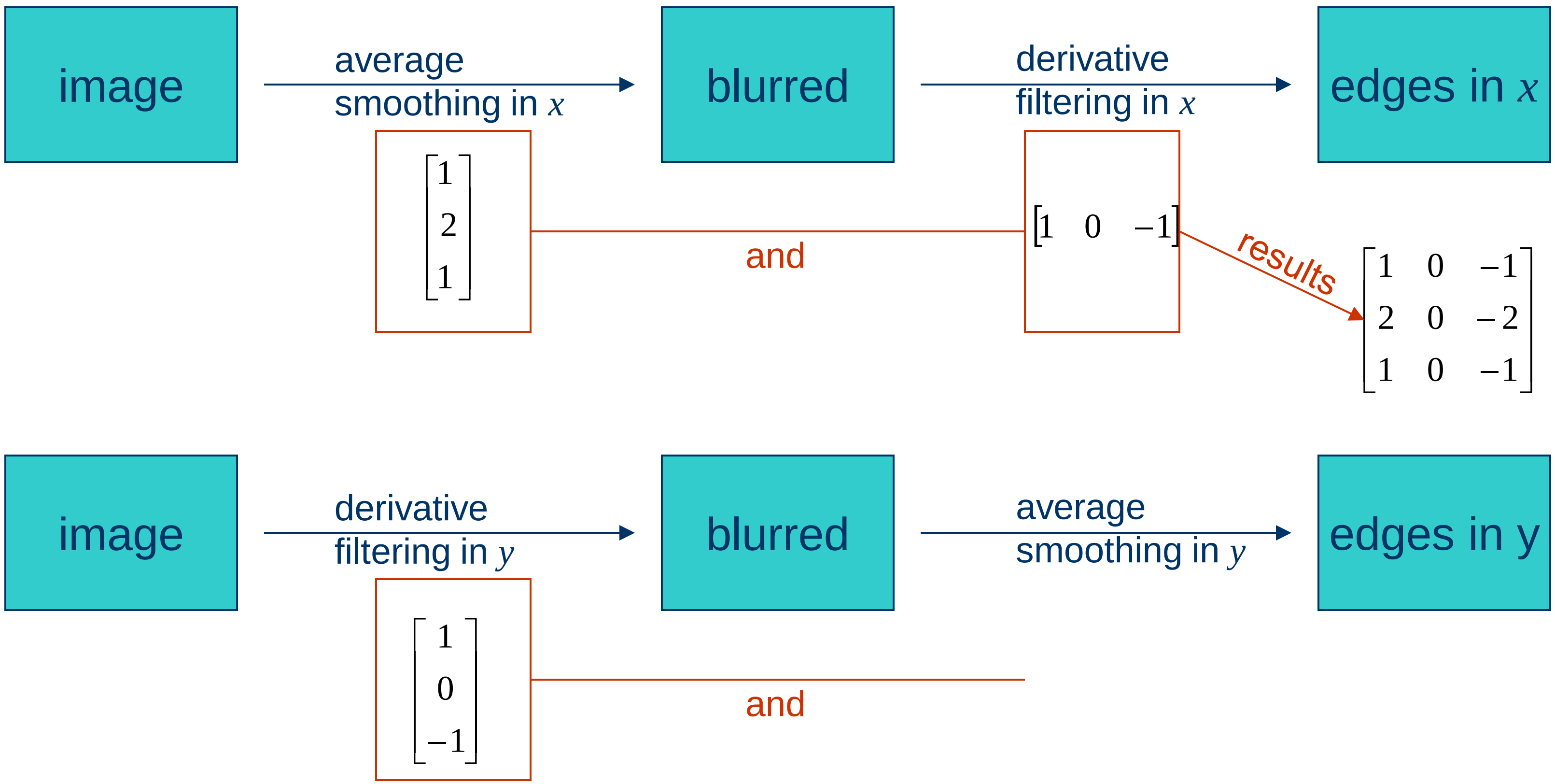
image $\xrightarrow{\text{derivative filtering in } y}$ blurred $\xrightarrow{\text{average smoothing in } y}$ edges in y

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Sobel Edge Detector



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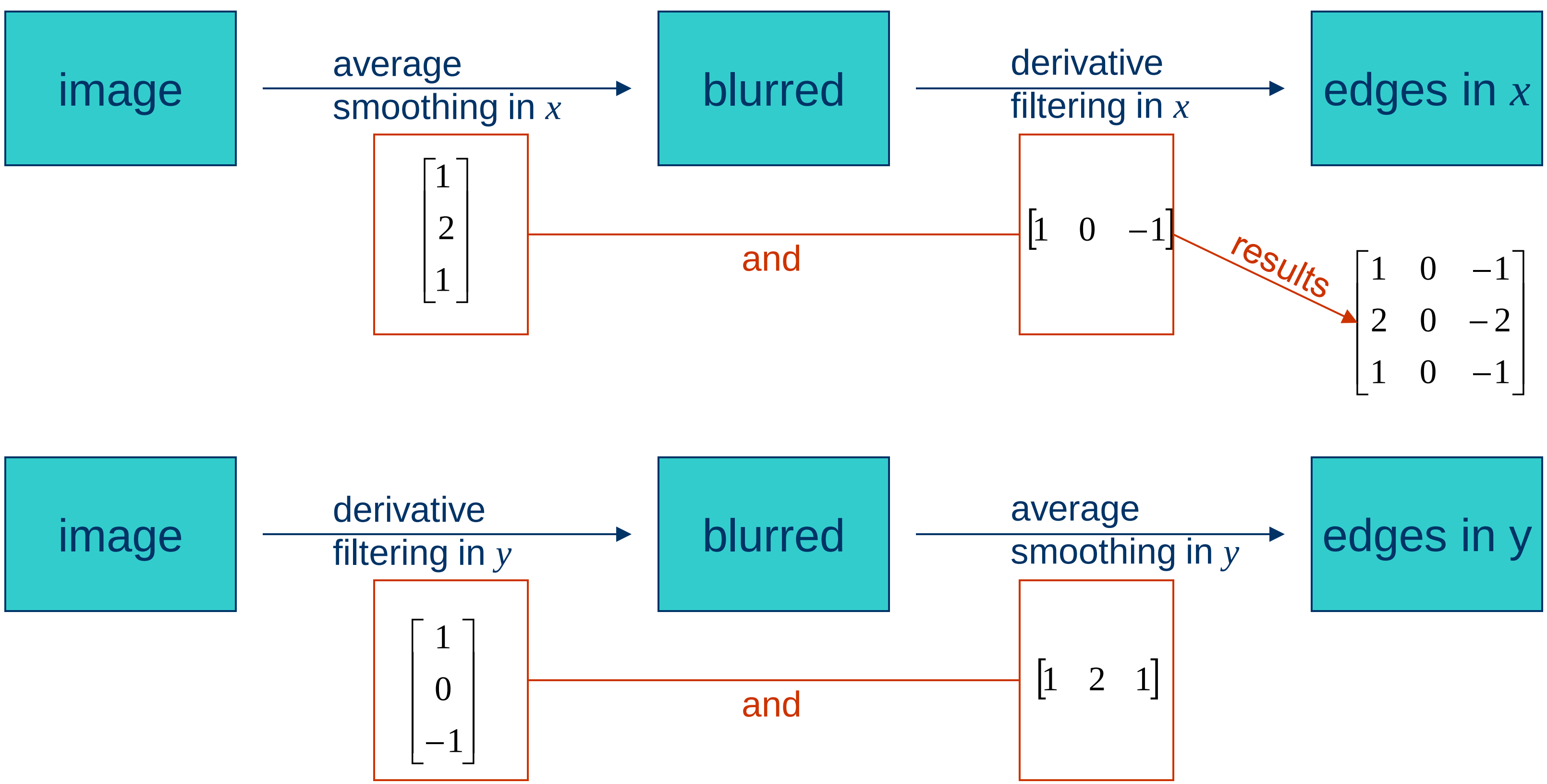

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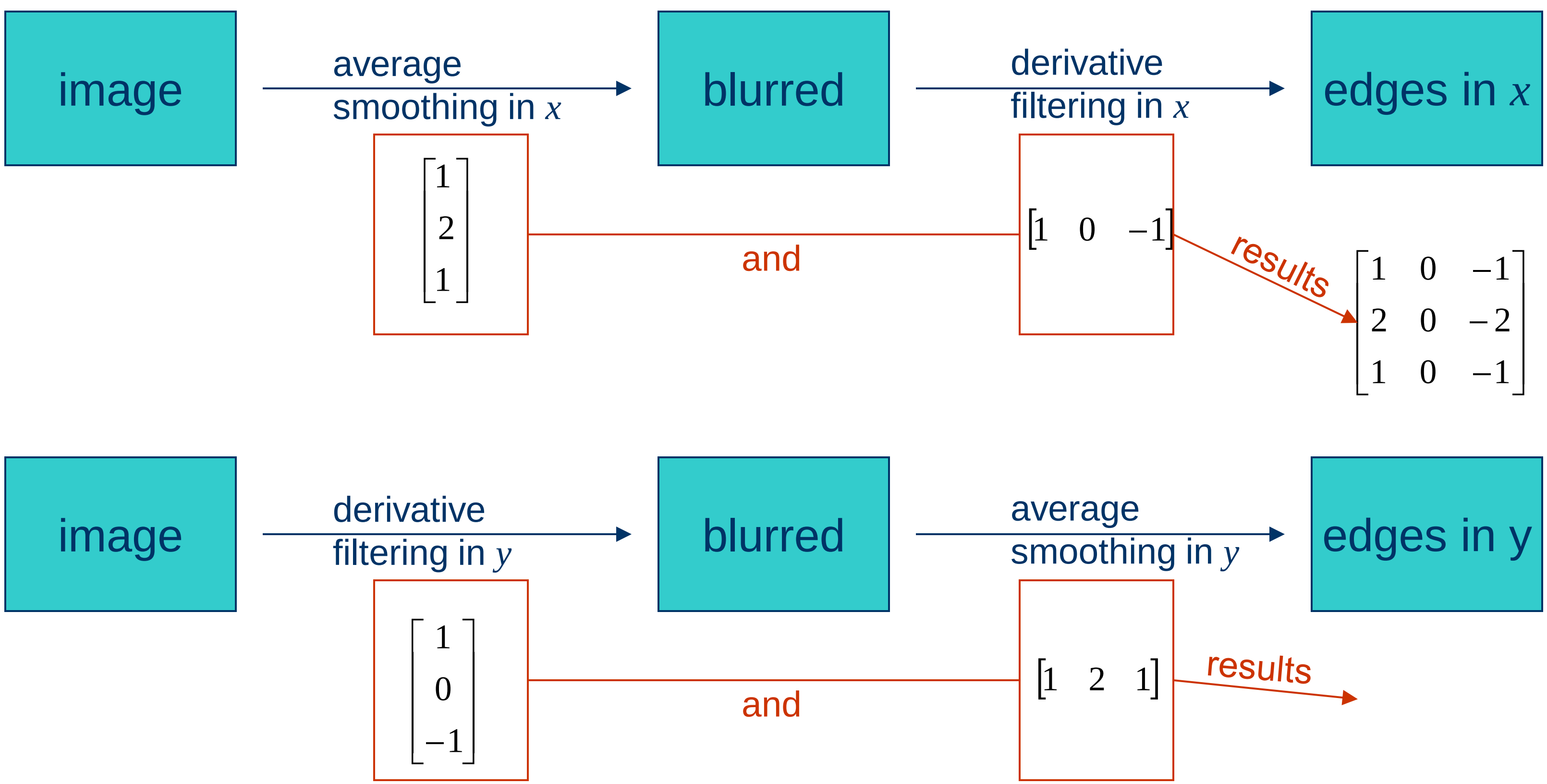

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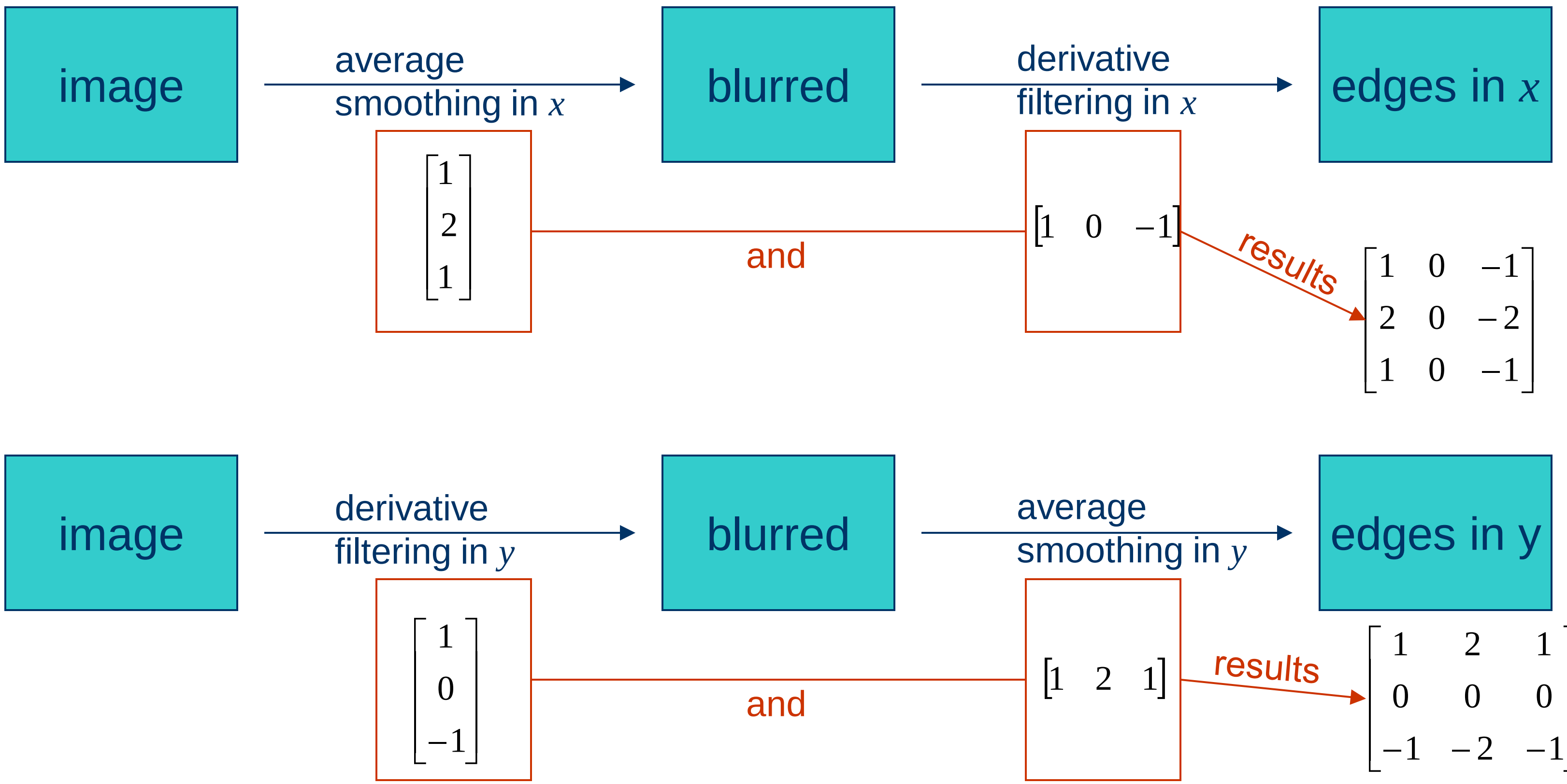

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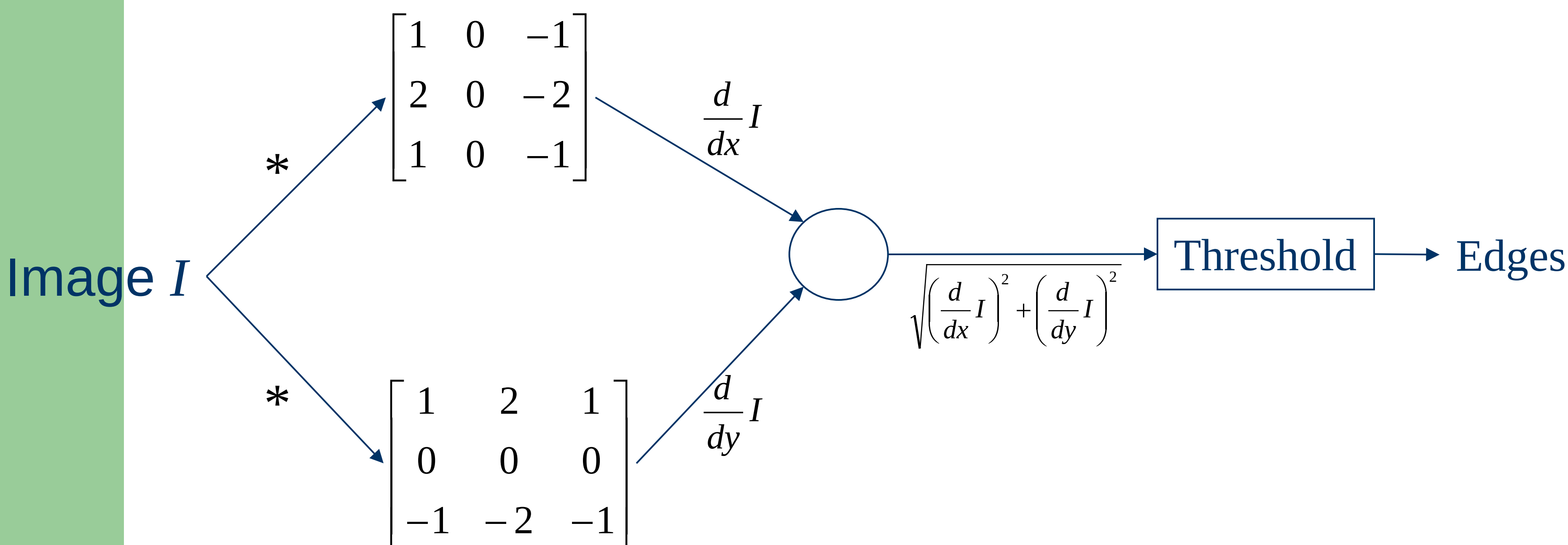
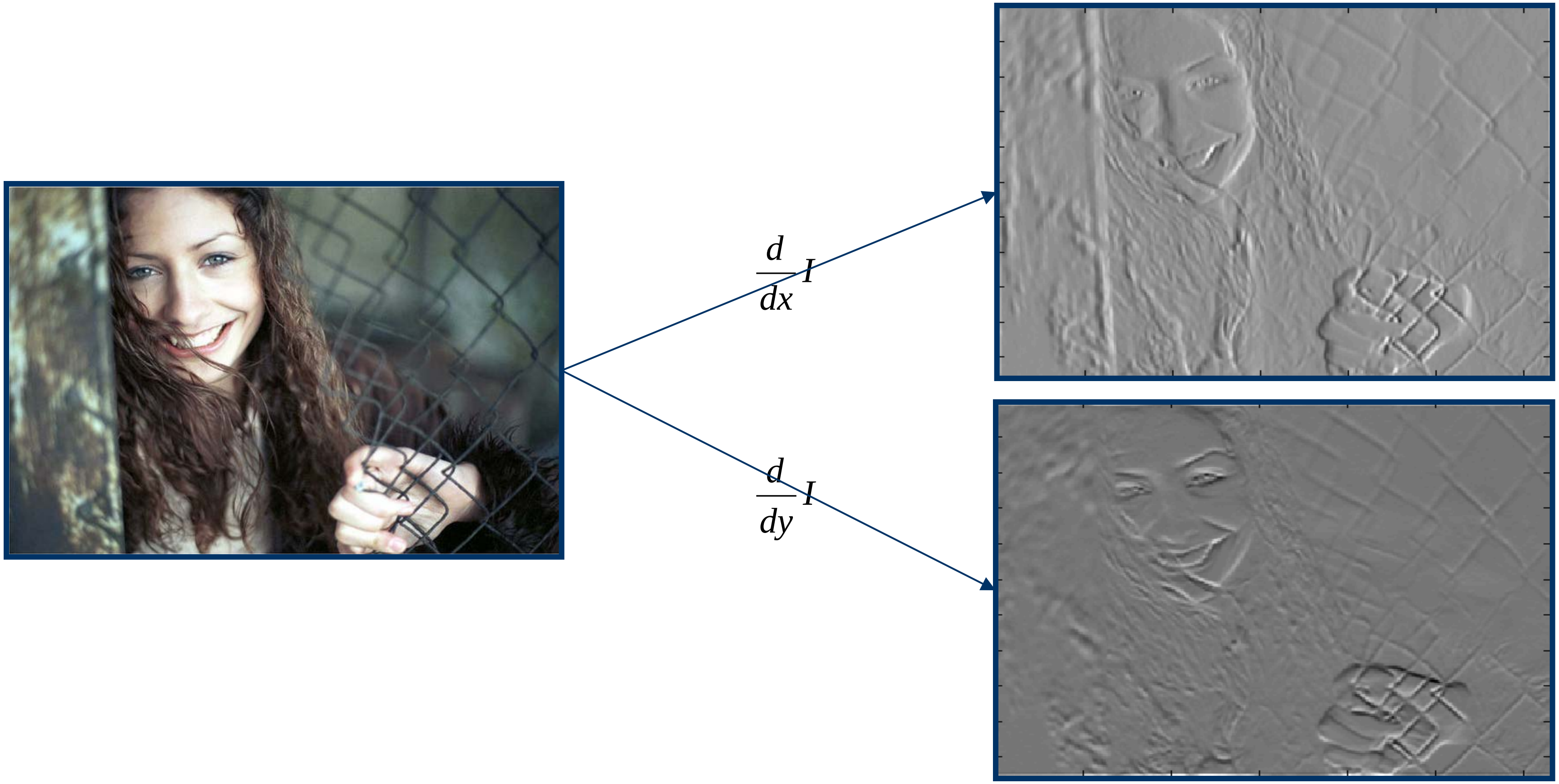


Sobel Edge Detector



Sobel Edge Detector

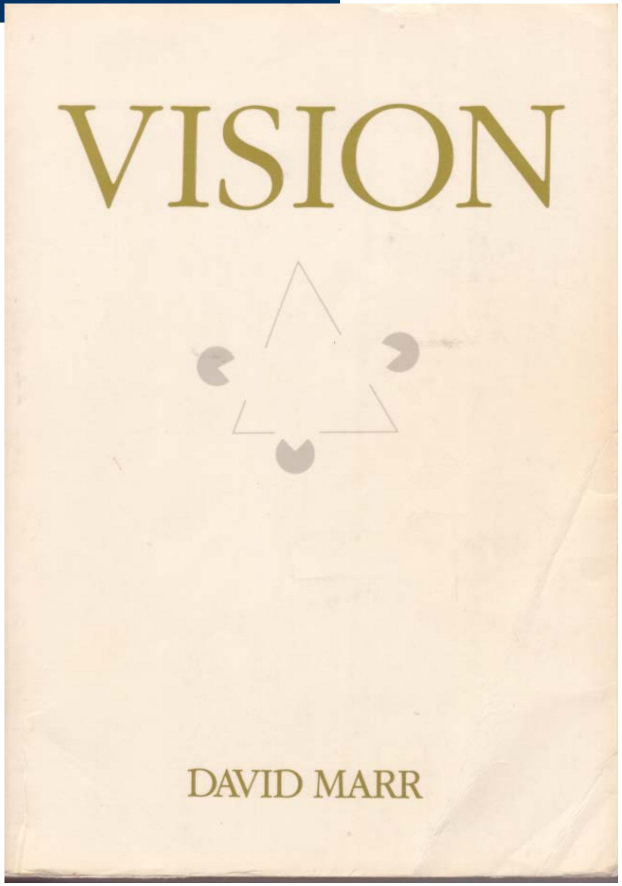
$$\Delta = \sqrt{\left(\frac{d}{dx} I\right)^2 + \left(\frac{d}{dy} I\right)^2}$$

$$\Delta \geq Threshold = 100$$

Marr Hildreth Edge Detector

David Marr

David Marr with Poggio and Francis Crick



Ellen Hildtreh

Hubel and Weisel, Noble Prize, 1981

For their discoveries concerning information processing in the visual system

Proceedings of the Royal Society of London. Series B. Biological Sciences

rspb.royalsocietypublishing.org

Published 29 February 1980 doi: 10.1098/rspb.1980.0020

Proc. R. Soc. Lond. B **29 February 1980** vol. 207 no. 1167 **187-217**

Theory of Edge Detection

D. Marr and E. Hildreth

Abstract

A theory of edge detection is presented. The analysis proceeds in two parts. (1) Intensity changes, which occur in a natural image over a wide range of scales, are detected separately at different scales. An appropriate filter for this purpose at a given scale is found to be the second derivative of a Gaussian, and it is shown that, provided some simple conditions are satisfied, these primary filters need not be orientation-dependent. Thus, intensity changes at a given scale are best detected by finding the zero values of $\nabla ^{2}$G(x, y)* I(x, y) for image I, where G(x, y) is a two-dimensional Gaussian distribution and $\nabla ^{2}$ is the Laplacian. The intensity changes thus discovered in each of the channels are then represented by oriented primitives called zero-crossing segments, and evidence is given that this representation is complete. (2) Intensity changes in images arise from surface discontinuities or from reflectance or illumination boundaries, and these all have the property that they are spatially localized. Because of this, the zero-crossing segments from the different channels are not independent, and rules are deduced for combining them into a description of the image. This description is called the raw primal sketch. The theory explains several basic

Marr Hildreth Edge Detector

Marr Hildreth Edge Detector

- Smooth image by Gaussian filter → S

Marr Hildreth Edge Detector

- Smooth image by Gaussian filter ➔ S
- Apply Laplacian to S

Marr Hildreth Edge Detector

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  - Used in mechanics, electromagnetics, wave theory, quantum mechanics and Laplace equation

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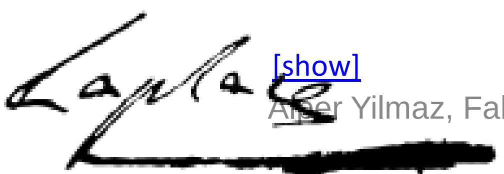
  - Scan along each row, record an edge point at the location of zero-crossing.

Marr Hildreth Edge Detector

- Smooth image by Gaussian filter ➔ S
- Apply Laplacian to S
  - Used in mechanics, electromagnetics, wave theory, quantum mechanics and Laplace equation
- Find zero crossings
  - Scan along each row, record an edge point at the location of zero-crossing.
  - Repeat above step along each column

1842.

Born	23 March 1749 Beaumont-en-Auge, Normandy, France
Died	5 March 1827 (aged 77) Paris, France
Nationality	French
Fields	Astronomer and Mathematician
Institutions	École Militaire (1769–1776)
Alma mater	University of Caen
Academic advisors	Jean d'Alembert Christophe Gadbled Pierre Le Canu
Doctoral students	Siméon Denis Poisson
Known for	[show]
Signature	

Pierre-Simon Laplace (1749–1827).
Posthumous portrait by Madame Feytaud,

He was not a modest person and thought he is the best mathmatician in France at the time

Source: Wikipedia

Marr Hildreth Edge Detector

- Gaussian smoothing

- Find Laplacian

Marr Hildreth Edge Detector

- Gaussian smoothing

$$\overbrace{S}^{\text{smoothed image}} = \overbrace{g}^{\text{Gaussian filter}} * \overbrace{I}^{\text{image}}$$

- Find Laplacian

Marr Hildreth Edge Detector

- Gaussian smoothing

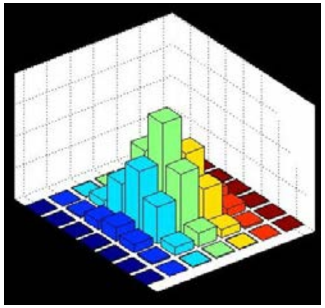
$$\overbrace{S}^{\text{smoothed image}} = \overbrace{g}^{\text{Gaussian filter}} * \overbrace{I}^{\text{image}} \qquad g = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Find Laplacian

Marr Hildreth Edge Detector

- Gaussian smoothing

$$\overbrace{S}^{\text{smoothed image}} = \overbrace{g}^{\text{Gaussian filter}} * \overbrace{I}^{\text{image}} \qquad g = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



- Find Laplacian

Marr Hildreth Edge Detector

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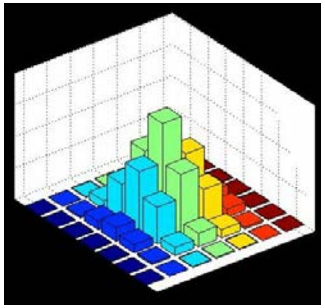
$$\overbrace{S}^{\text{smoothed image}} = \overbrace{g}^{\text{Gaussian filter}} * \overbrace{I}^{\text{image}} \qquad g = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



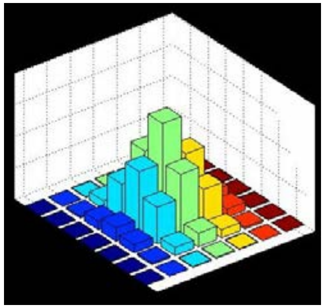
- Find Laplacian

$$\Delta^2 S = \overbrace{\frac{\partial^2}{\partial x^2} S}^{\text{second order derivative in } x} + \overbrace{\frac{\partial^2}{\partial y^2} S}^{\text{second order derivative in } y}$$

Marr Hildreth Edge Detector

- Gaussian smoothing

$$\overbrace{S}^{\text{smoothed image}} = \overbrace{g}^{\text{Gaussian filter}} * \overbrace{I}^{\text{image}} \qquad g = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2+y^2}{2\sigma^2}}$$



- Find Laplacian

$$\Delta^2 S = \overbrace{\frac{\partial^2}{\partial x^2} S}^{\text{second order derivative in } x} + \overbrace{\frac{\partial^2}{\partial y^2} S}^{\text{second order derivative in } y}$$

- ∇ is used for gradient (first derivative)
- Δ^2 is used for Laplacian (Second derivative)

Marr Hildreth Edge Detector

- Deriving the Laplacian of Gaussian (LoG)

Marr Hildreth Edge Detector

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$$\Delta^2 S = \Delta^2 (g * I) = (\Delta^2 g) * I$$

Marr Hildreth Edge Detector

- Deriving the Laplacian of Gaussian (LoG)

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Marr Hildreth Edge Detector

- Deriving the Laplacian of Gaussian (LoG)

$$\Delta^2 S = \Delta^2 (g * I) = (\Delta^2 g) * I$$

$$g = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$g_x = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} \left(-\frac{2x}{2\sigma^2} \right)$$

Marr Hildreth Edge Detector

- Deriving the Laplacian of Gaussian (LoG)

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$$\Delta^2 g = -\frac{1}{\sqrt{2\pi}\sigma^3} \left(2 - \frac{x^2+y^2}{\sigma^2} \right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Marr Hildreth Edge Detector

- Deriving the Laplacian of Gaussian (LoG)

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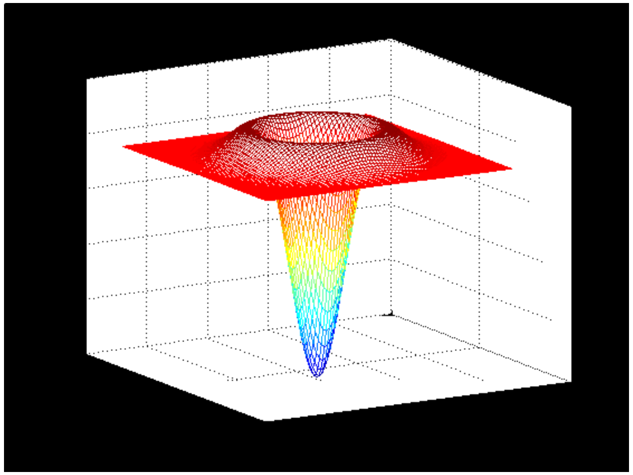

Gaussian

Gaussian

$$g(x) = e^{\frac{-x^2}{2o^2}}$$

Gaussian

$$g(x) = e^{\frac{-x^2}{2o^2}}$$

Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

Standard deviation

Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

Standard deviation

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$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

Standard deviation

x

Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

Standard deviation

x

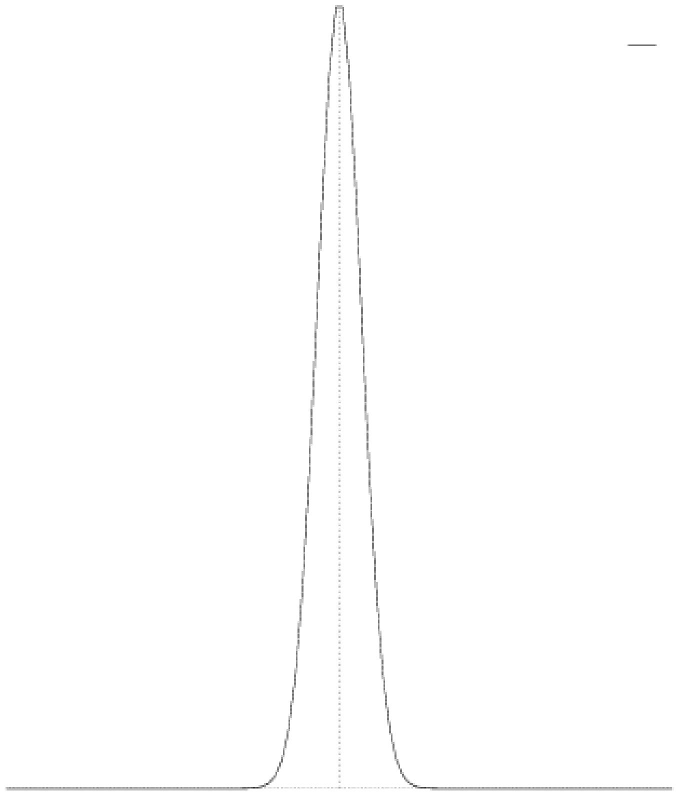
g(x)

Gaussian

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

Standard deviation

x	-3	-2	-1	0	1	2	3
g(x)	.011	.13	.6	1	.6	.13	.011



2-D Gaussian

2-D Gaussian

$$g(x, y) = e^{\frac{-(x^2 + y^2)}{2\sigma^2}}$$

2-D Gaussian

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$$\sigma = 2$$

2-D Gaussian

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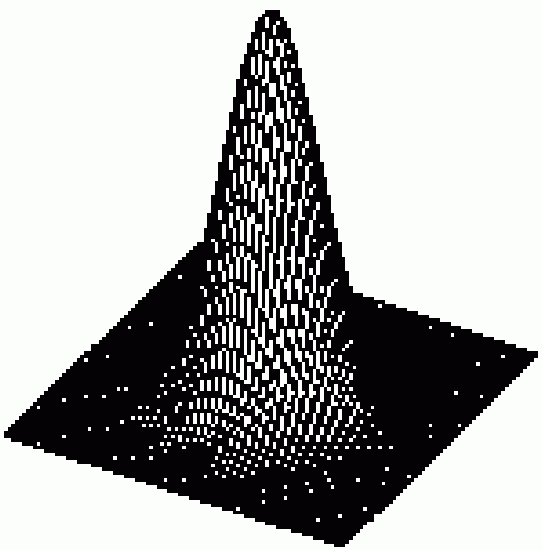
0	0	0	0	1	2	2	2	1	0	0	0	0
0	0	1	3	6	9	11	9	6	3	1	0	0
0	1	4	11	20	30	34	30	20	11	4	1	0
0	3	11	26	50	73	82	73	50	26	11	3	0
1	6	20	50	93	136	154	136	93	50	20	6	1
2	9	30	73	136	198	225	198	136	73	30	9	2
2	11	34	82	154	225	255	225	154	82	34	11	2
2	9	30	73	136	198	225	198	136	73	30	9	2
1	6	20	50	93	136	154	136	93	50	20	6	1
0	3	11	26	50	73	82	73	50	26	11	3	0
0	1	4	11	20	30	34	30	20	11	4	1	0
0	0	1	3	6	9	11	9	6	3	1	0	0
0	0	0	0	1	2	2	2	1	0	0	0	0

$$\sigma = 2$$

2-D Gaussian



LoG Filter

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3}\left(2 - \frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

LoG Filter

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3}\left(2-\frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

			Y				
0.0008	0.0066	0.0215	0.031	0.0215	0.0066	0.0008	
0.0066	0.0438	0.0982	0.108	0.0982	0.0438	0.0066	
0.0215	0.0982	0	-0.242	0	0.0982	0.0215	
0.031	0.108	-0.242	-0.7979	-0.242	0.108	0.031	X
0.0215	0.0982	0	-0.242	0	0.0982	0.0215	
0.0066	0.0438	0.0982	0.108	0.0982	0.0438	0.0066	
0.0008	0.0066	0.0215	0.031	0.0215	0.0066	0.0008	

Finding Zero Crossings

Finding Zero Crossings

- Four cases of zero-crossings :

Finding Zero Crossings

- Four cases of zero-crossings :
  - {+,-}

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  - {-,+}

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  - {+,-}
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  - {-,+}
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  - compute slope of zero-crossing

Finding Zero Crossings

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  - {+,-}
  - {+,0,-}
  - {-,+}
  - {-,0,+}
- Slope of zero-crossing {a, -b} is |a+b|.
- To mark an edge
  - compute slope of zero-crossing
  - Apply a threshold to slope

On the Separability of Gaussian

- Two-dimensional Gaussian can be separated into 2 one-dimensional Gaussians

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n^2 multiplications

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$$g(x) = e^{-\left(\frac{x^2}{2\sigma^2}\right)}$$

$$g_1 = g(x) = \left[.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011\right]$$

On the Separability of Gaussian

- Two-dimensional Gaussian can be separated into 2 one-dimensional Gaussians

$$h(x,y) = I(x,y) * g(x,y)$$

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$$h(x,y) = \left(I(x,y) * g_1(x)\right) * g_2(y)$$

$2n$ multiplications

$$g(x) = e^{-\left(\frac{x^2}{2\sigma^2}\right)}$$

$$g_1 = g(x) = \begin{bmatrix} .011 & .13 & .6 & 1 & .6 & .13 & .011 \end{bmatrix}$$

$$g_2 = g(y) = \begin{bmatrix} .011 \\ .13 \\ .6 \\ 1 \\ .6 \\ .13 \\ .011 \end{bmatrix}$$

On the Separability of LoG

- Similar to separability of Gaussian filter
  - Two-dimensional LoG can be separated into 4 one-dimensional Convolutions

On the Separability of LoG

On the Separability of LoG

$$\Delta^2 S = \Delta^2 (g * I) = (\Delta^2 g) * I = I * (\Delta^2 g)$$

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Requires n^2 multiplications

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$$\Delta^2 S = \Delta^2 (g * I) = (\Delta^2 g) * I = I * (\Delta^2 g)$$

Requires n^2 multiplications

$$\Delta^2 S = (I * g_{xx}(x)) * g(y) + (I * g_{yy}(y)) * g(x)$$

On the Separability of LoG

$$\Delta^2 S = \Delta^2 (g * I) = \left(\Delta^2 g\right) * I = I * \left(\Delta^2 g\right)$$

Requires n^2 multiplications

$$\Delta^2 S = \left(I * g_{xx}(x)\right) * g(y) + \left(I * g_{yy}(y)\right) * g(x)$$

Requires $4n$ multiplications

Seperability

Gaussian Filtering

Seperability

Gaussian Filtering

Image

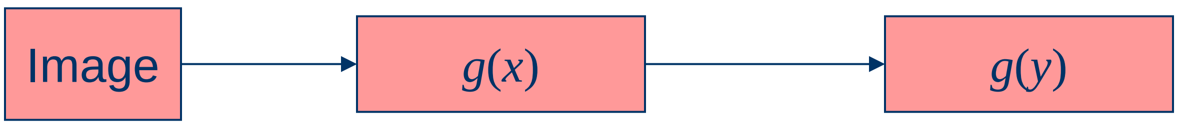
Seperability

Gaussian Filtering

Image $\longrightarrow$ $g(x)$

Seperability

Gaussian Filtering



Seperability

Gaussian Filtering

$$\boxed{\text{Image}} \longrightarrow \boxed{g(x)} \longrightarrow \boxed{g(y)} \longrightarrow I * g$$

Seperability

Gaussian Filtering

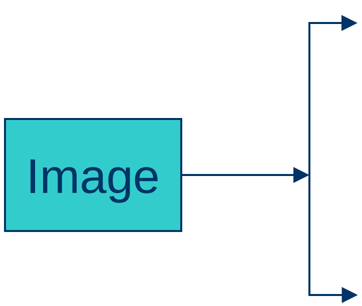
$$\text{Image} \longrightarrow \boxed{g(x)} \longrightarrow \boxed{g(y)} \longrightarrow I * g$$

Laplacian of Gaussian Filtering

Seperability

Gaussian Filtering

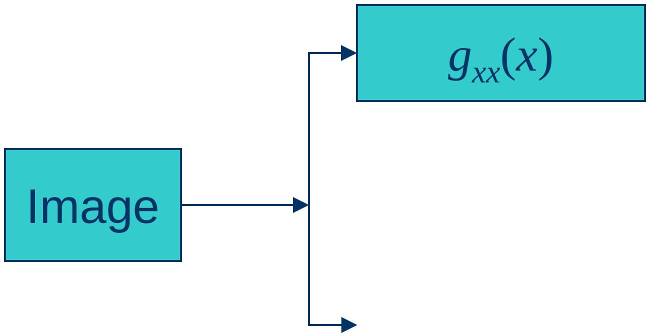
Laplacian of Gaussian Filtering



Seperability

Gaussian Filtering

Laplacian of Gaussian Filtering



Seperability

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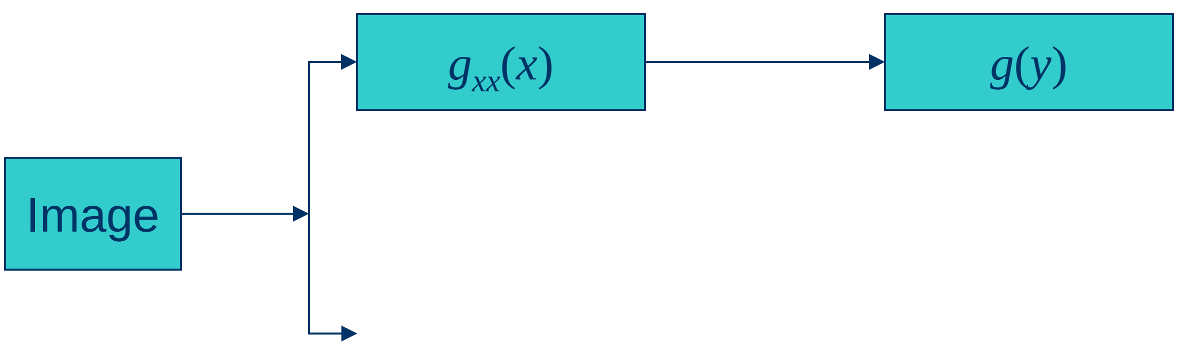

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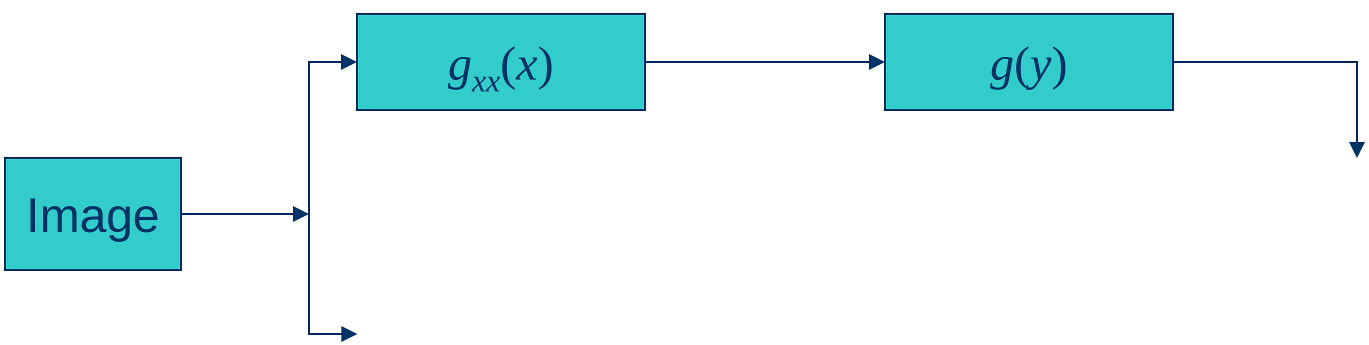
Laplacian of Gaussian Filtering



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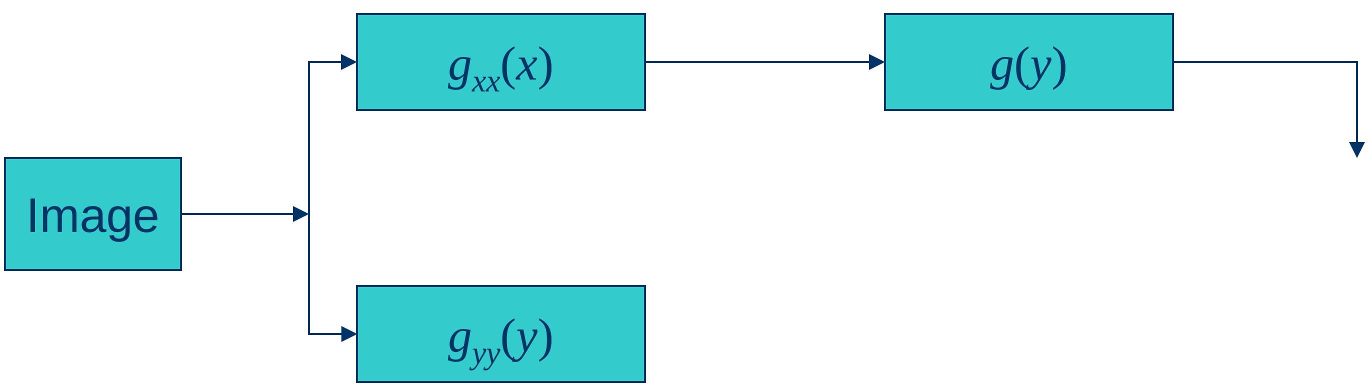
Gaussian Filtering

Laplacian of Gaussian Filtering



Seperability

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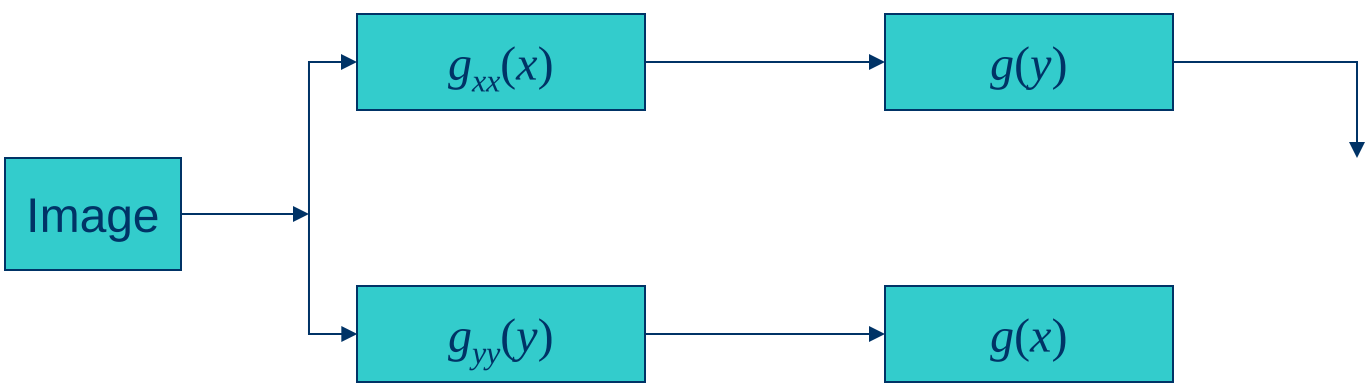
Laplacian of Gaussian Filtering



Seperability

Gaussian Filtering

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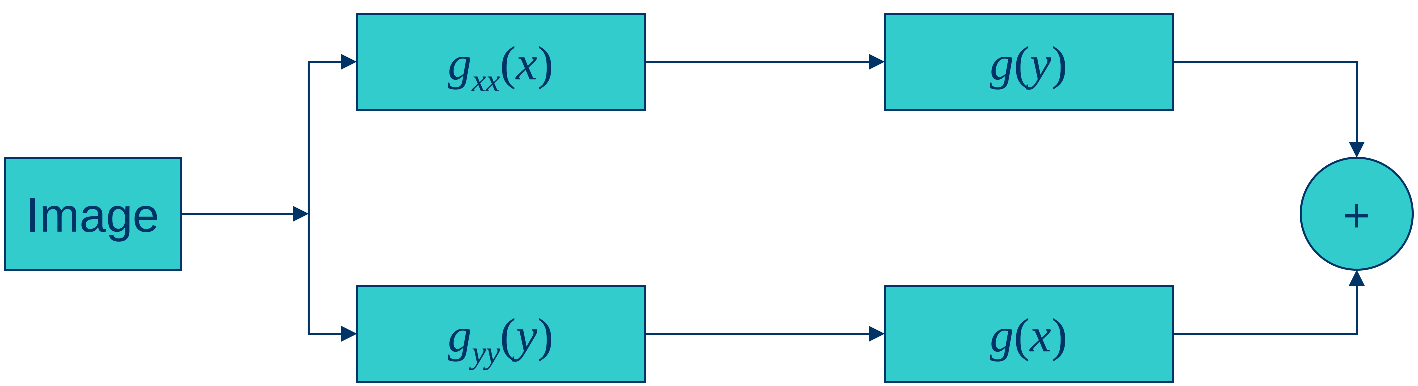

Seperability

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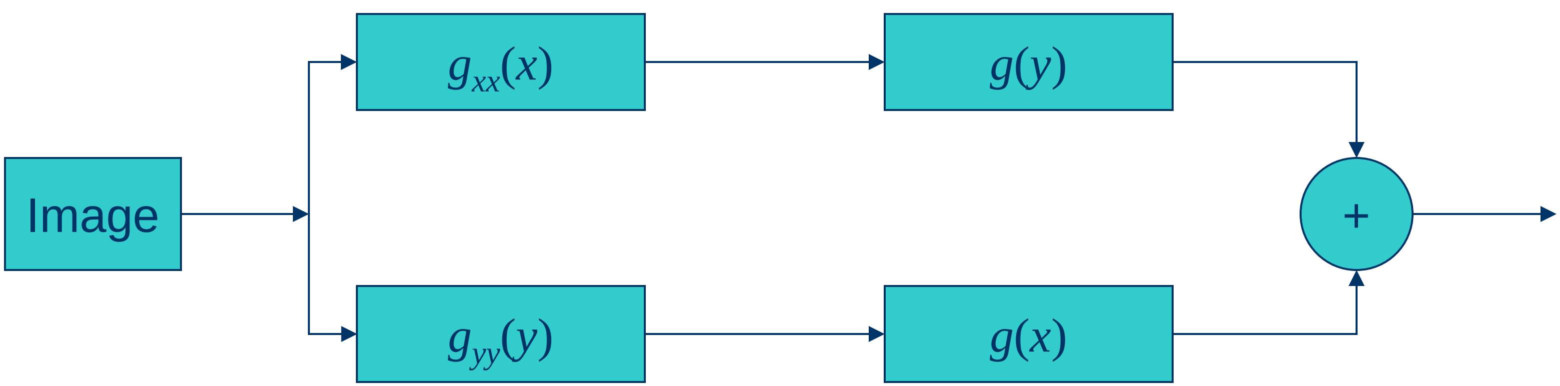

Seperability

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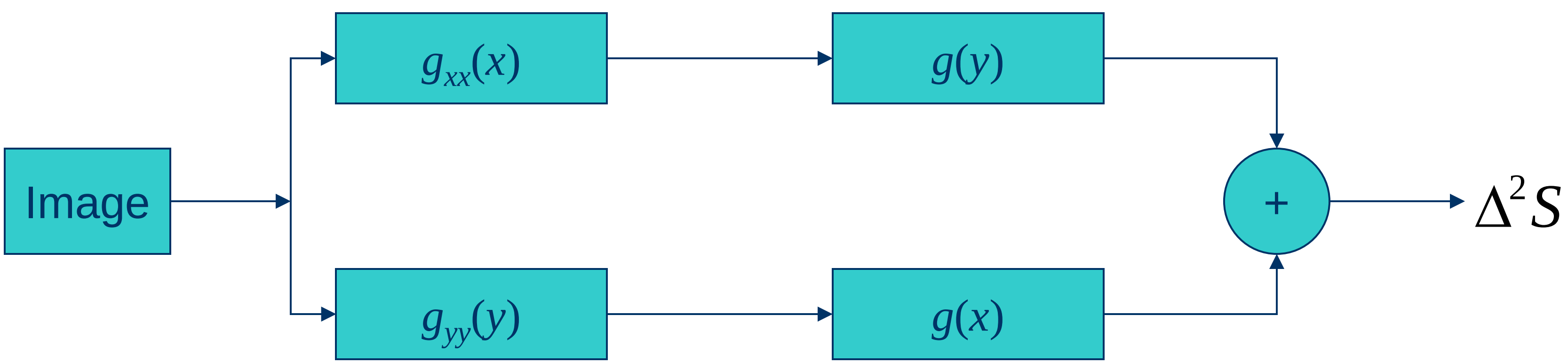


Example

I

Example

$$I$$

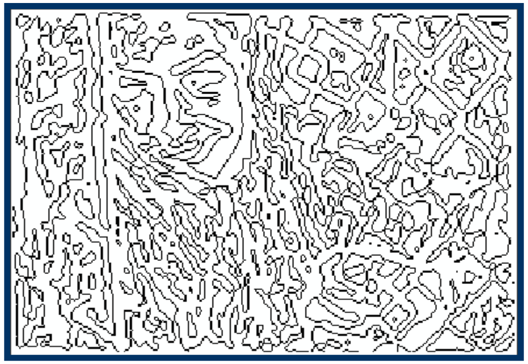
$$I * \left(\Delta^2 g\right)$$

Example

I

$I * \left(\Delta^2 g\right)$

Zero crossings of $\Delta^2 S$



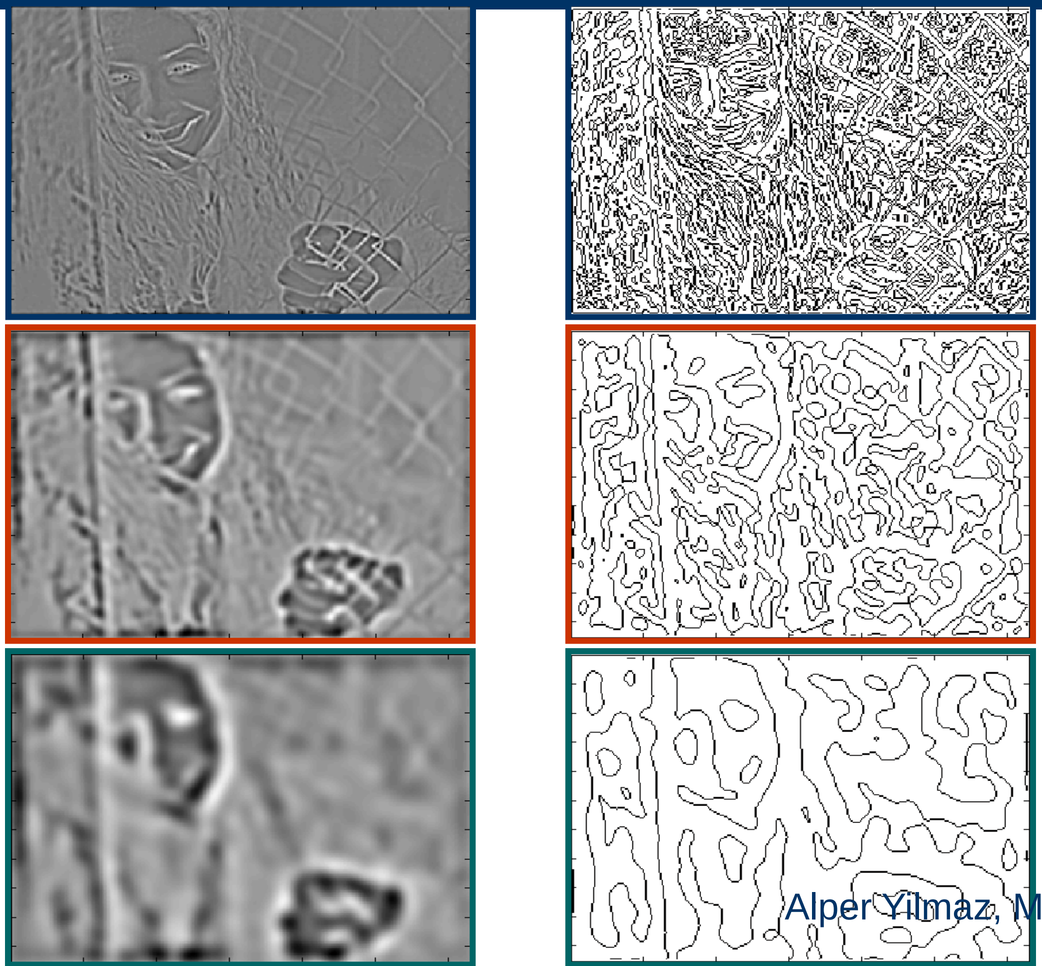
Example

$$\Delta^2 G_\sigma = -\frac{1}{\sqrt{2\pi}\sigma^3}\left(2-\frac{x^2+y^2}{\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$\sigma = 1$

$\sigma = 3$

$\sigma = 6$



LoG Algorithm

$$\Delta^2 g(x, y)$$

$$g(x),\ g_{xx}(x),\ g(y),\ g_{yy}(y)$$

LoG Algorithm

- Apply LoG to the Image: Either
  - Use 2D filter $\Delta^2 g(x, y)$
  - Or Use 4 1D filters $g(x),\ g_{xx}(x),\ g(y),\ g_{yy}(y)$

LoG Algorithm

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  - Or Use 4 1D filters $g(x),\ g_{xx}(x),\ g(y),\ g_{yy}(y)$
- Find zero-crossings from each row
- Find slope of zero-crossings
- Apply threshold to slope and mark edges

Quality of an Edge

Quality of an Edge

- Robust to noise

Quality of an Edge

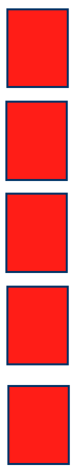
- Robust to noise
- Localization

Quality of an Edge

- Robust to noise
- Localization
- Too many or too less responses

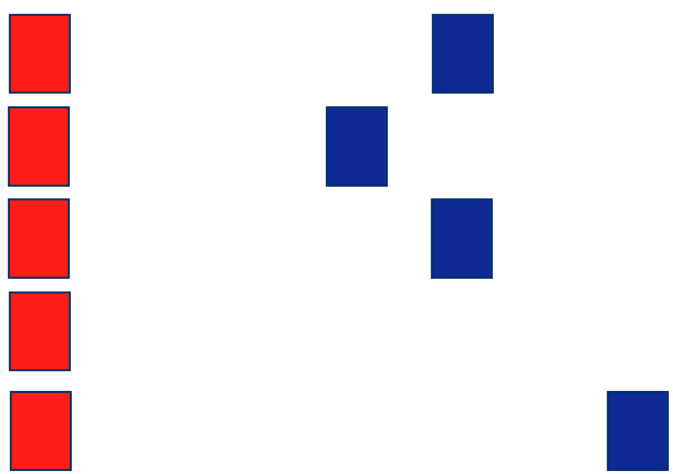
Quality of an Edge

Quality of an Edge



True
edge

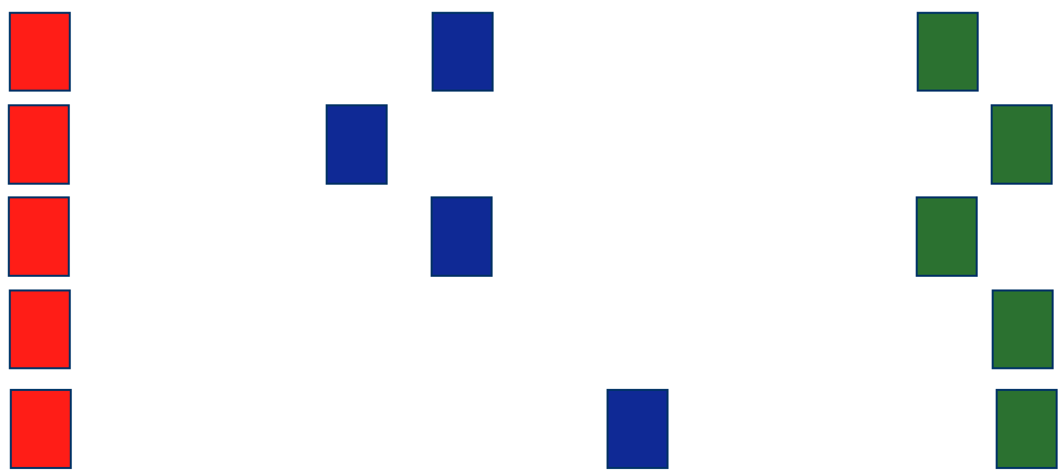
Quality of an Edge



True edge

Poor robustness to noise

Quality of an Edge

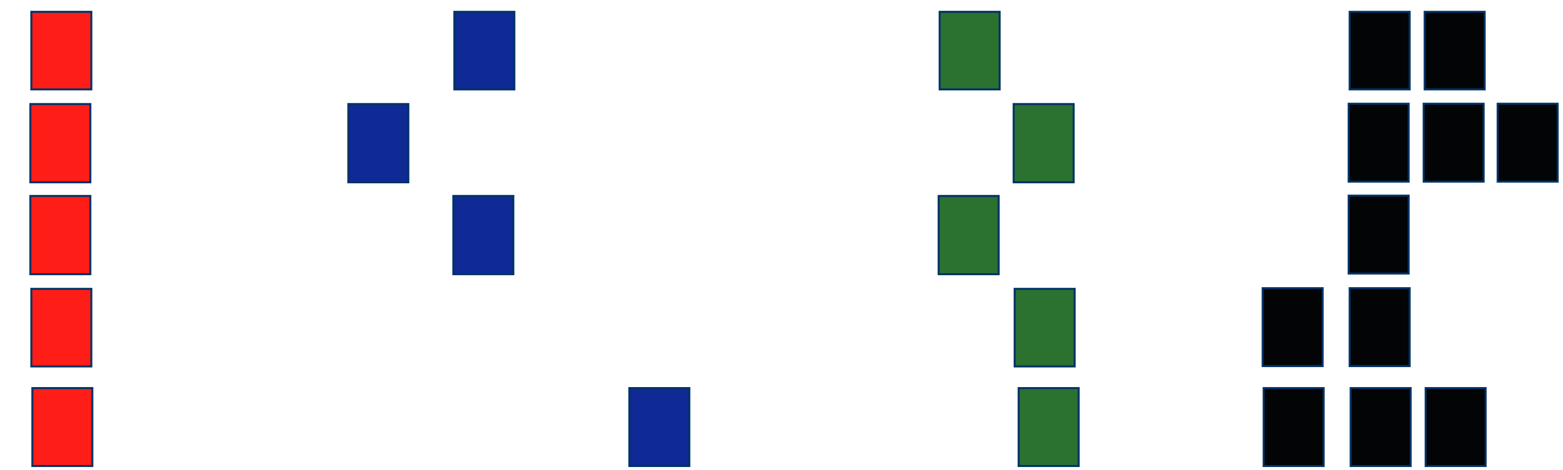


True edge

Poor robustness to noise

Poor localization

Quality of an Edge



True edge

Poor robustness to noise

Poor localization

Too many responses

Canny Edge Detector

John Canny

Technical Report 720

Finding Edges and Lines in Images

John Francis Canny

MIT Artificial Intelligence Laboratory

Canny Edge Detector

Canny Edge Detector

- Criterion 1: Good Detection: The optimal detector must minimize the probability of false positives as well as false negatives.

Canny Edge Detector

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Canny Edge Detector

- Criterion 1: Good Detection: The optimal detector must minimize the probability of false positives as well as false negatives.
- Criterion 2: Good Localization: The edges detected must be as close as possible to the true edges.
- Single Response Constraint: The detector must return one point only for each edge point.

Canny Edge Detector Steps

Canny Edge Detector Steps

1. Smooth image with Gaussian filter

Canny Edge Detector Steps

1. Smooth image with Gaussian filter
2. Compute derivative of filtered image

Canny Edge Detector Steps

1. Smooth image with Gaussian filter
2. Compute derivative of filtered image
3. Find magnitude and orientation of gradient

Canny Edge Detector Steps

1. Smooth image with Gaussian filter
2. Compute derivative of filtered image
3. Find magnitude and orientation of gradient
4. Apply “Non-maximum Suppression”

Canny Edge Detector Steps

1. Smooth image with Gaussian filter
2. Compute derivative of filtered image
3. Find magnitude and orientation of gradient
4. Apply “Non-maximum Suppression”
5. Apply “Hysteresis Threshold”

Canny Edge Detector
First Two Steps

- Smoothing
- Derivative

Canny Edge Detector First Two Steps

- Smoothing

$$S = I * g(x, y) = g(x, y) * I$$

- Derivative

Canny Edge Detector
First Two Steps

- Smoothing

$$S = I * g(x,y) = g(x,y) * I \qquad g(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Derivative

Canny Edge Detector
First Two Steps

- Smoothing

$$S = I * g(x,y) = g(x,y) * I$$

$$g(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Derivative

$$\nabla S = \nabla(g * I) = (\nabla g) * I$$

Canny Edge Detector
First Two Steps

- Smoothing

$$S = I * g(x,y) = g(x,y) * I \qquad g(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Derivative

$$\nabla S = \nabla\left(g * I\right) = \left(\nabla g\right) * I$$

$$\nabla S = \begin{bmatrix} g_x \\ g_y \end{bmatrix} * I = \begin{bmatrix} g_x * I \\ g_y * I \end{bmatrix}$$

Canny Edge Detector First Two Steps

- Smoothing

$$S = I * g(x,y) = g(x,y) * I$$

$$g(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

- Derivative

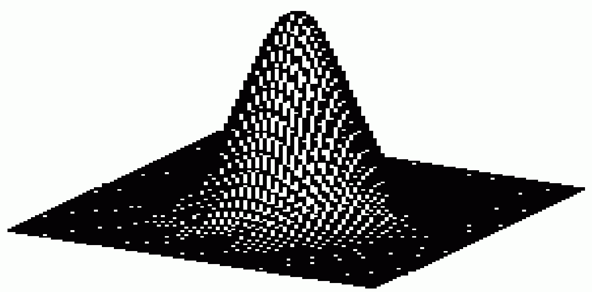
$$\nabla S = \nabla(g * I) = (\nabla g) * I$$

$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \end{bmatrix} = \begin{bmatrix} g_x \\ g_y \end{bmatrix}$$

$$\nabla S = \begin{bmatrix} g_x \\ g_y \end{bmatrix} * I = \begin{bmatrix} g_x * I \\ g_y * I \end{bmatrix}$$

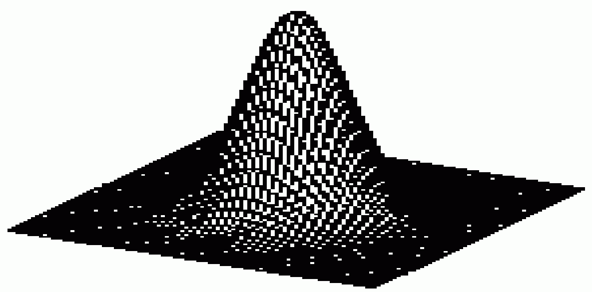
Canny Edge Detector
Derivative of Gaussian

Canny Edge Detector
Derivative of Gaussian



$g(x, y)$

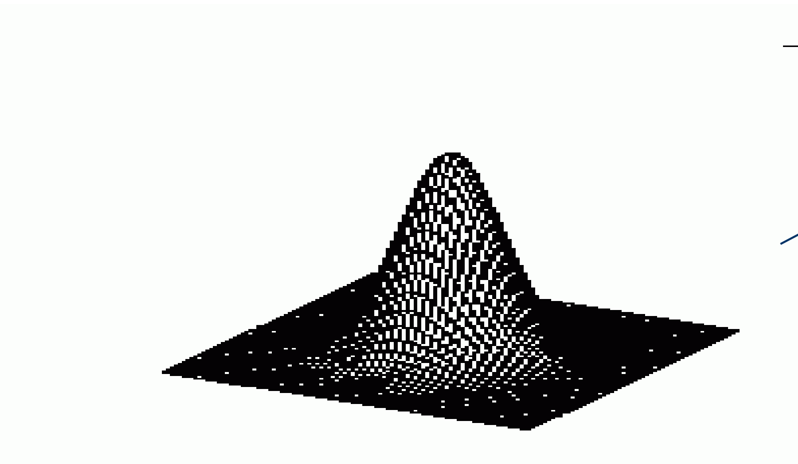
Canny Edge Detector
Derivative of Gaussian



$g(x, y)$

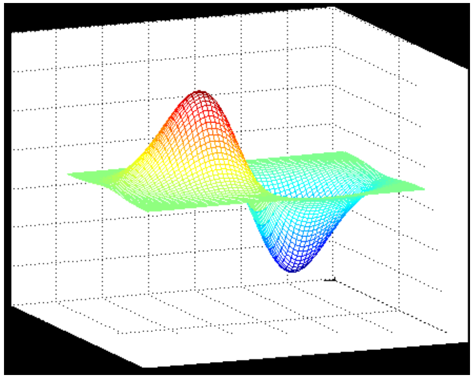
$g_x(x, y)$

Canny Edge Detector
Derivative of Gaussian

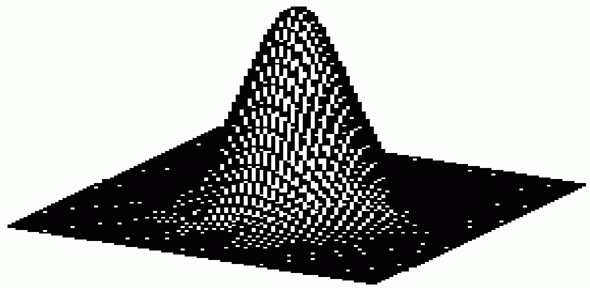


$g(x, y)$

$g_x(x, y)$

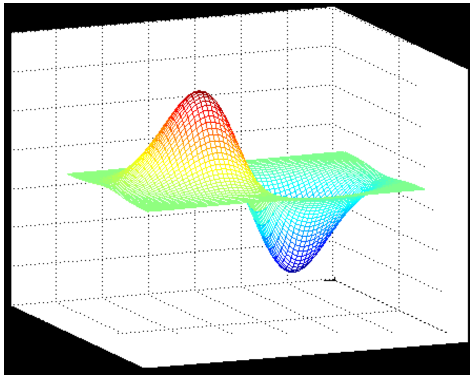


Canny Edge Detector
Derivative of Gaussian



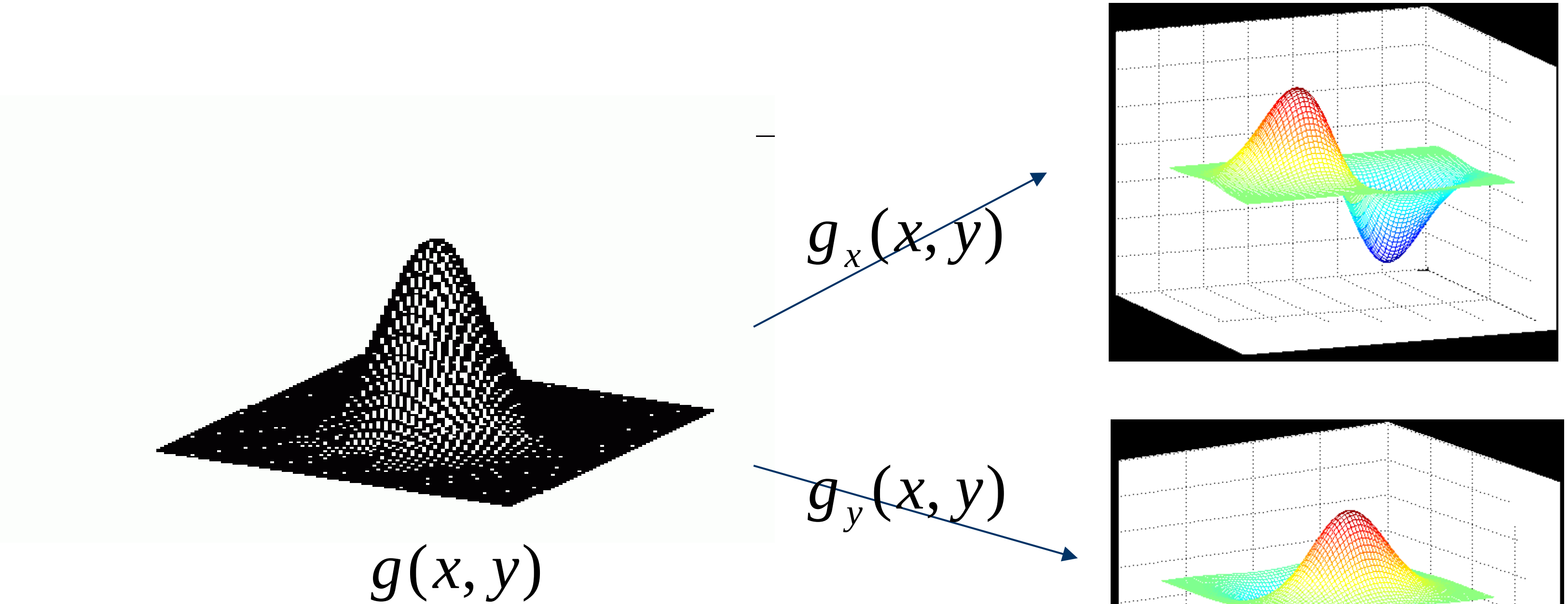
$g(x,y)$

$g_x(x,y)$



$g_y(x,y)$

Canny Edge Detector
Derivative of Gaussian



Canny Edge Detector First Two Steps

Canny Edge Detector
First Two Steps

I

Canny Edge Detector
First Two Steps

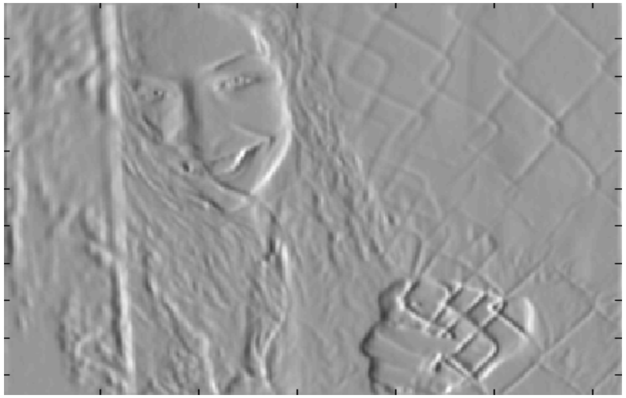
I

S_x

Canny Edge Detector
First Two Steps

I

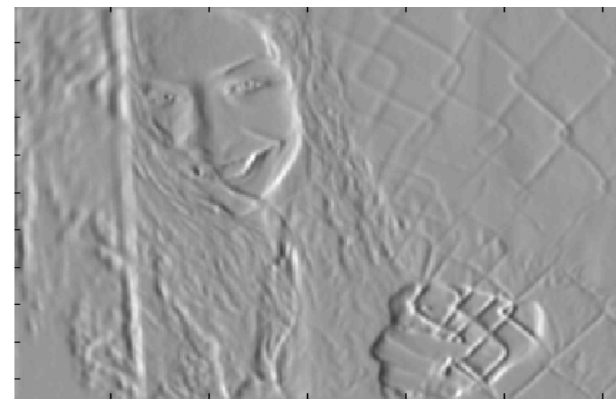
S_x



Canny Edge Detector
First Two Steps

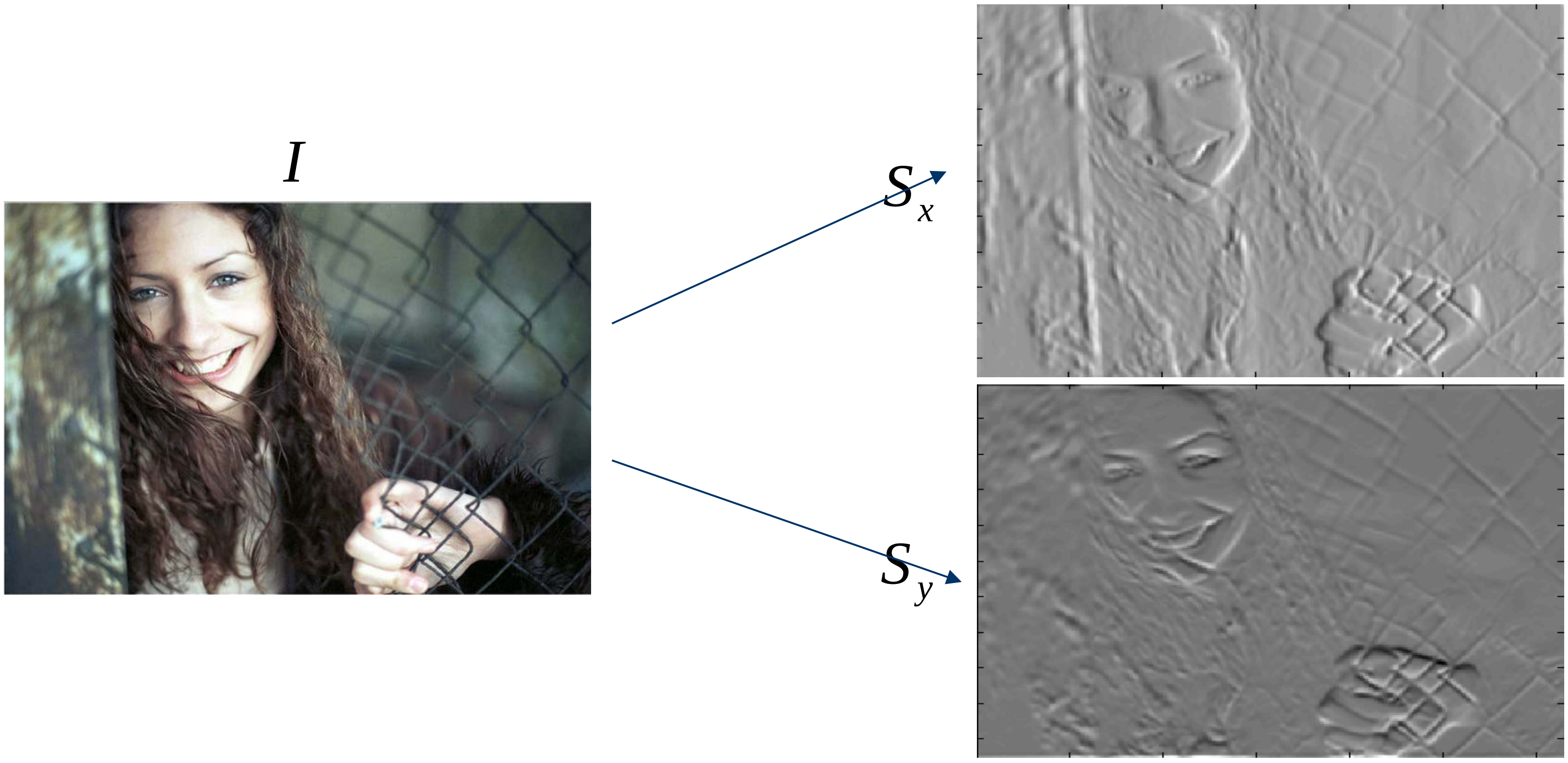
I

S_x



S_y

Canny Edge Detector
First Two Steps



Alper Yilmaz, Mubarak Shah Fall 2012, UCF

Canny Edge Detector
Third Step

- Gradient magnitude and gradient direction

Canny Edge Detector
Third Step

- Gradient magnitude and gradient direction

(S_x, S_y) Gradient Vector

$$\text{magnitude} = \sqrt{(S_x^2 + S_y^2)}$$

$$\text{direction} = \theta = \tan^{-1} \frac{S_y}{S_x}$$

Canny Edge Detector
Third Step

- Gradient magnitude and gradient direction

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$\text{magnitude} = \sqrt{(S_x^2 + S_y^2)}$

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image

Canny Edge Detector
Third Step

- Gradient magnitude and gradient direction

(S_x, S_y) Gradient Vector

magnitude $= \sqrt{(S_x^2 + S_y^2)}$

direction $= \theta = \tan^{-1} \frac{S_y}{S_x}$

image

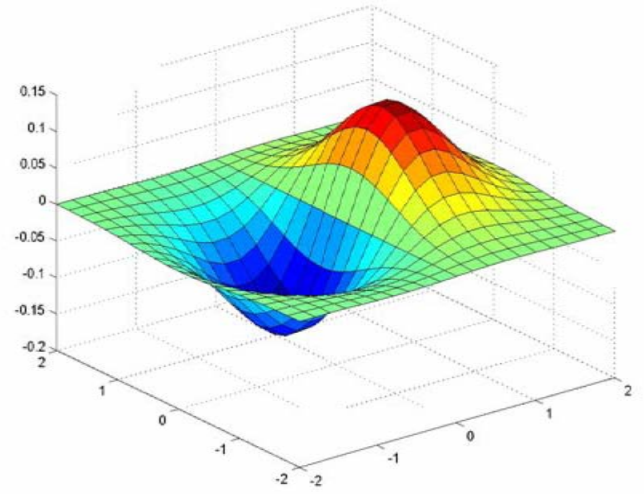
gradient magnitude

Example

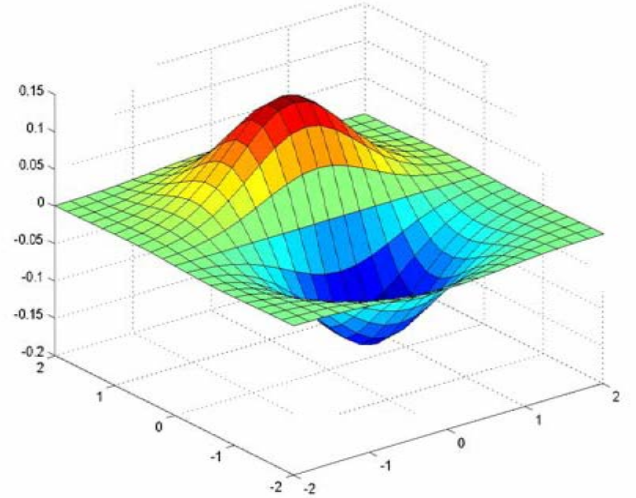
original image (Lena)

Slide Credit: James hays

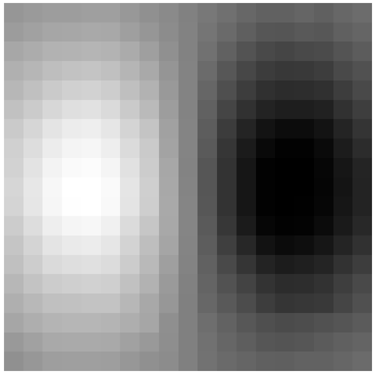
Derivative of Gaussian filter

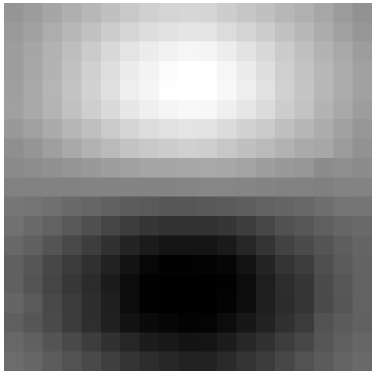


x-direction



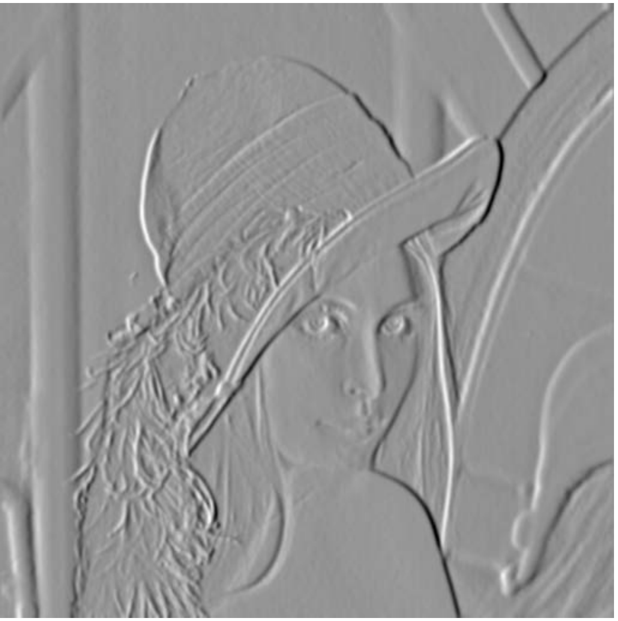
y-direction





Slide Credit: James hays

Compute Gradients (DoG)



X-Derivative of Gaussian

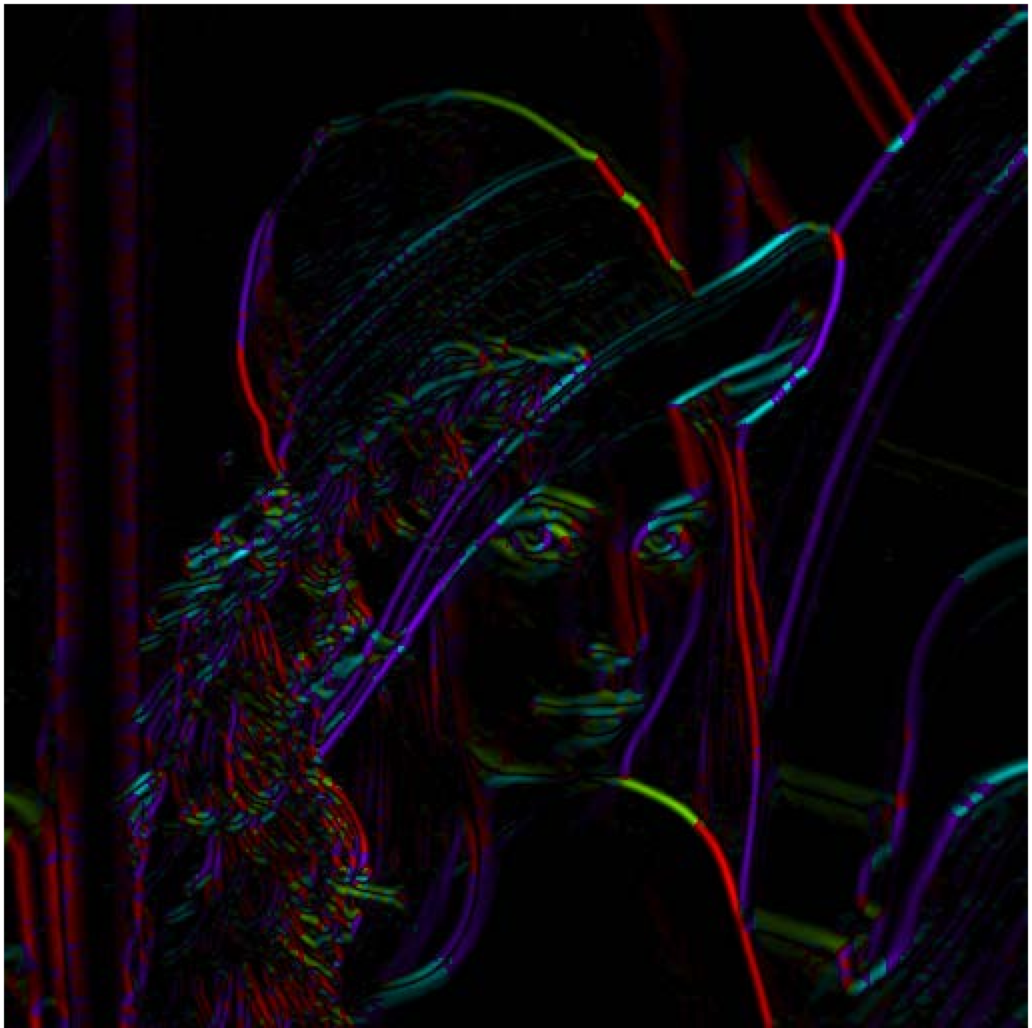
Y-Derivative of Gaussian

Gradient Magnitude

Slide Credit: James hays

Get Orientation at Each Pixel

- Threshold at minimum level
- Get orientation



theta = atan2(gy, gx)

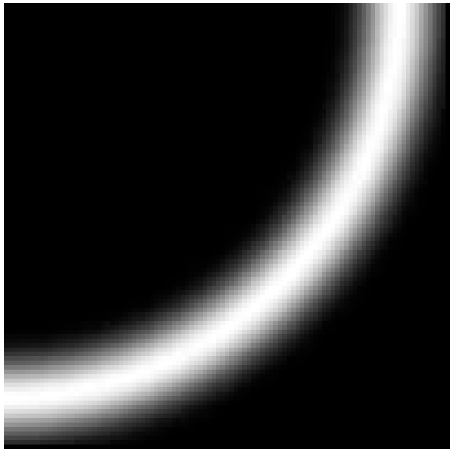
Slide Credit: James hays

Canny Edge Detector
Fourth Step

- Non maximum suppression

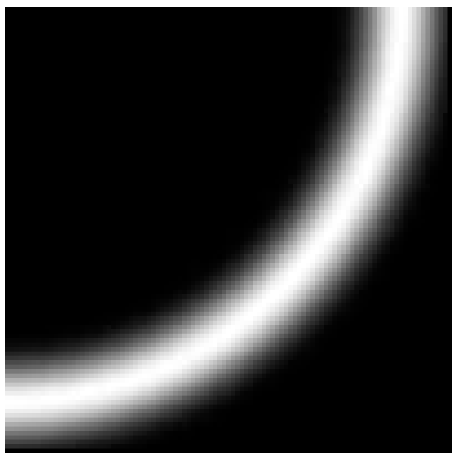
Canny Edge Detector
Fourth Step

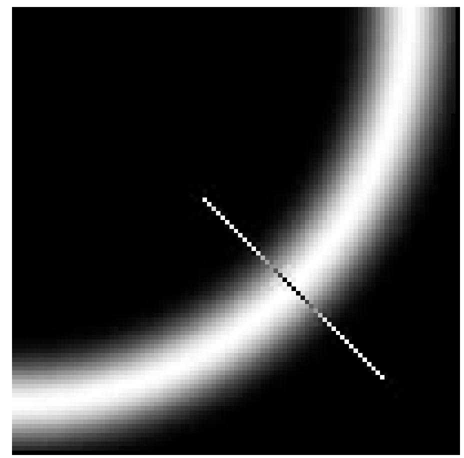
- Non maximum suppression



Canny Edge Detector
Fourth Step

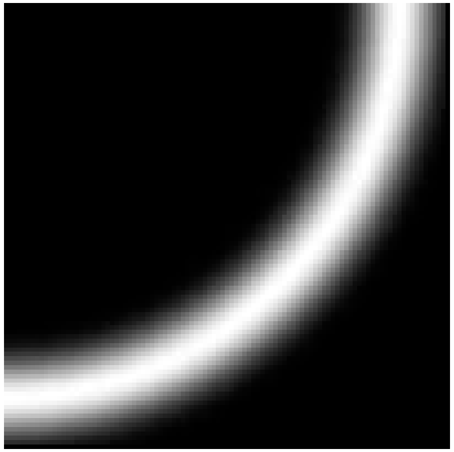
- Non maximum suppression

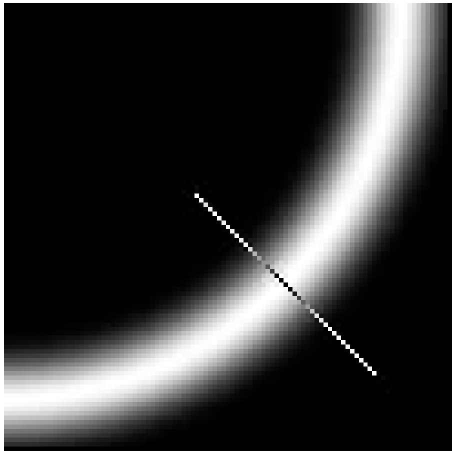




Canny Edge Detector
Fourth Step

- Non maximum suppression





We wish to mark points along the curve where the **magnitude is largest**. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues: at which point is the maximum, and where is the next one?

Canny Edge Detector
Non-Maximum Suppression

- Suppress the pixels in $|\nabla S|$ which are not local maximum

Canny Edge Detector
Non-Maximum Suppression

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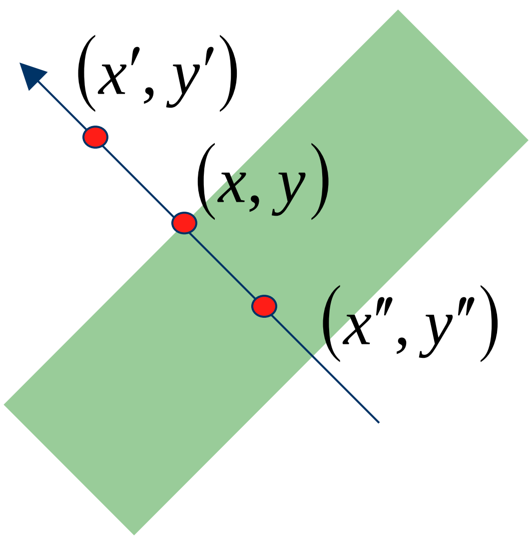

Canny Edge Detector
Non-Maximum Suppression

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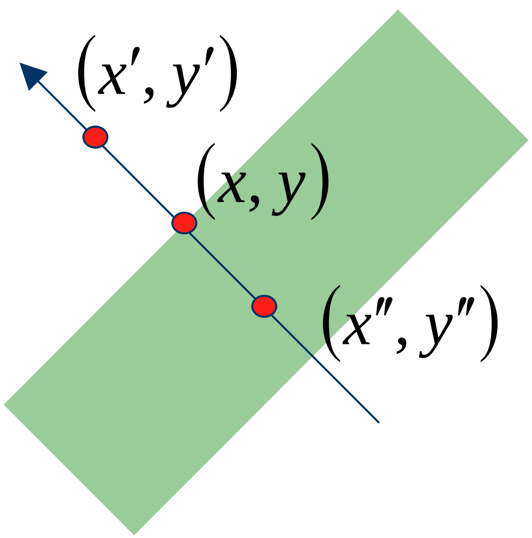

$$M(x,y)=\begin{cases} |\nabla S|(x,y) & \text{if } |\nabla S|(x,y) > |\Delta S|(x',y') \\ & \& \, |\Delta S|(x,y) > |\Delta S|(x'',y'') \\ 0 & \text{otherwise} \end{cases}$$

Canny Edge Detector
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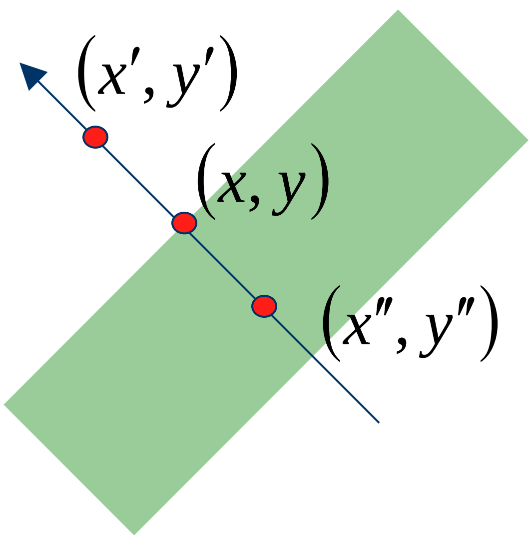

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x’ and **x**” are the neighbors of **x** along normal direction to an edge

Canny Edge Detector
Non-Maximum Suppression

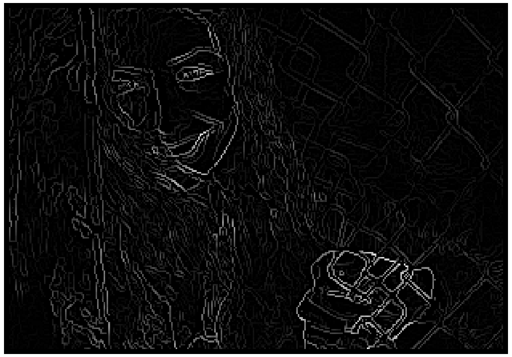
Canny Edge Detector
Non-Maximum Suppression

$$|\Delta S| = \sqrt{S_x^2 + S_y^2}$$

Canny Edge Detector
Non-Maximum Suppression

$$|\Delta S| = \sqrt{S_x^2 + S_y^2}$$

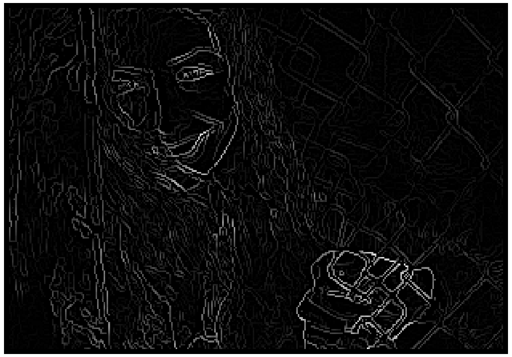
$$M$$



Canny Edge Detector
Non-Maximum Suppression

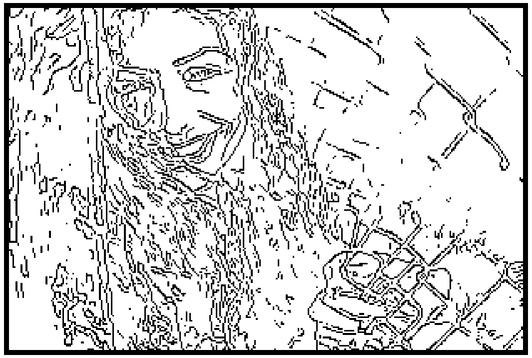
$$|\Delta S| = \sqrt{S_x^2 + S_y^2}$$

M



For visualization

$M \geq Threshold = 25$



Canny Edge Detector
Hysteresis Thresholding

Canny Edge Detector
Hysteresis Thresholding

- If the gradient at a pixel is

Canny Edge Detector
Hysteresis Thresholding

- If the gradient at a pixel is
  - above "**High**", declare it as an '**edge pixel**'

Canny Edge Detector Hysteresis Thresholding

- If the gradient at a pixel is
  - above "**High**", declare it as an '**edge pixel**'
  - below "**Low**", declare it as a "**non-edge-pixel**"

Canny Edge Detector
Hysteresis Thresholding

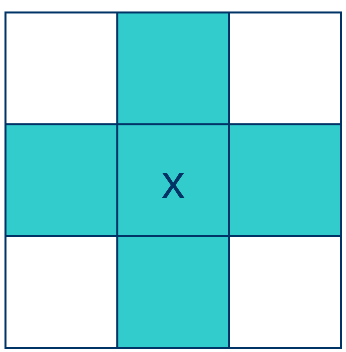
- If the gradient at a pixel is
  - above “**High**”, declare it as an ‘**edge pixel**’
  - below “**Low**”, declare it as a “**non-edge-pixel**”
  - **between** “low” and “high”
    - Consider its neighbors iteratively then declare it an “edge pixel” if it is **connected** to an ‘edge pixel’ **directly** or via pixels **between** “low” and “high”.

Canny Edge Detector
Hysteresis Thresholding

- Connectedness

Canny Edge Detector
Hysteresis Thresholding

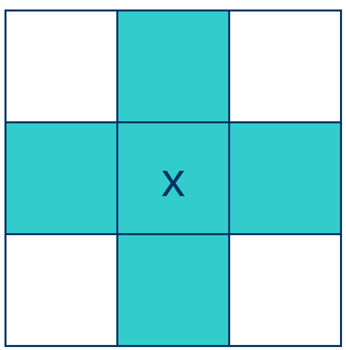
- Connectedness



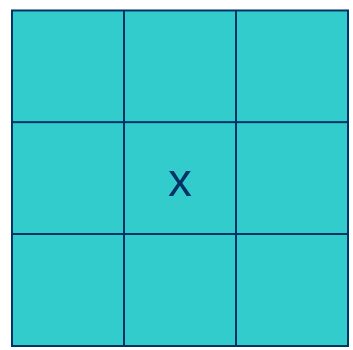
4 connected

Canny Edge Detector
Hysteresis Thresholding

- Connectedness



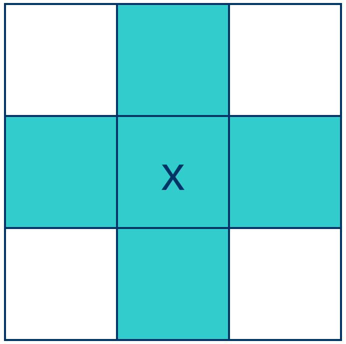
4 connected



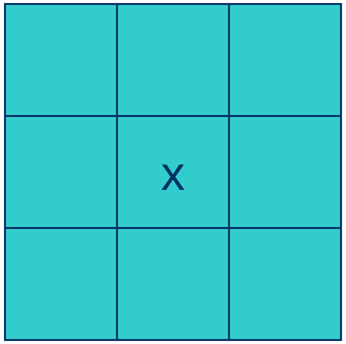
8 connected

Canny Edge Detector
Hysteresis Thresholding

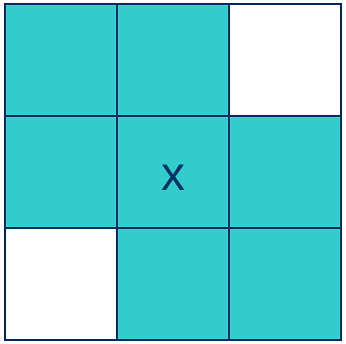
- Connectedness



4 connected



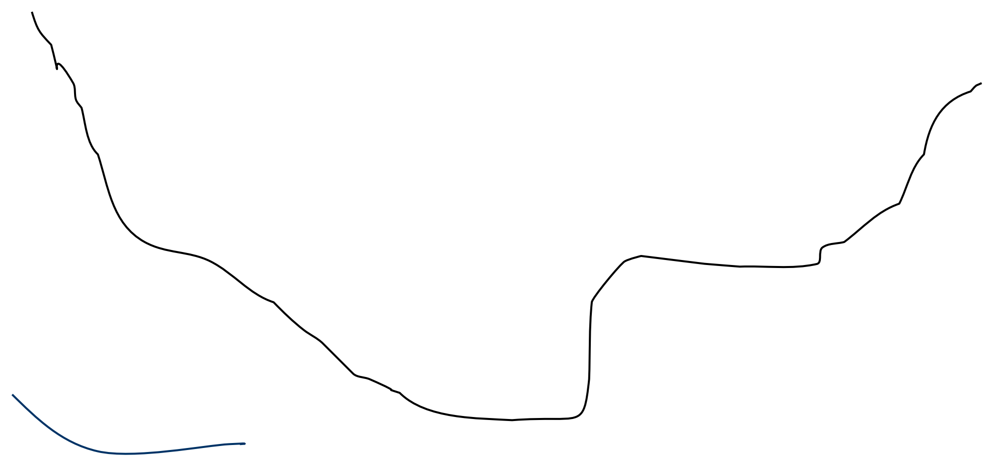
8 connected



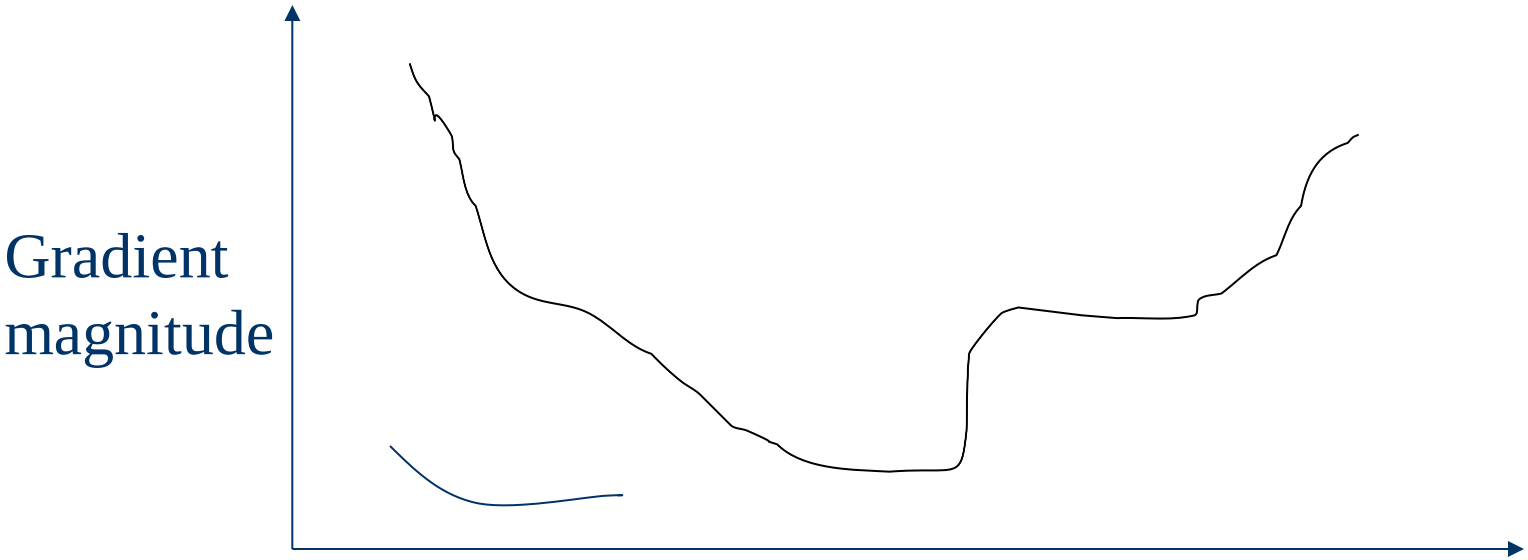
6 connected

Canny Edge Detector
Hysteresis Thresholding

Canny Edge Detector
Hysteresis Thresholding

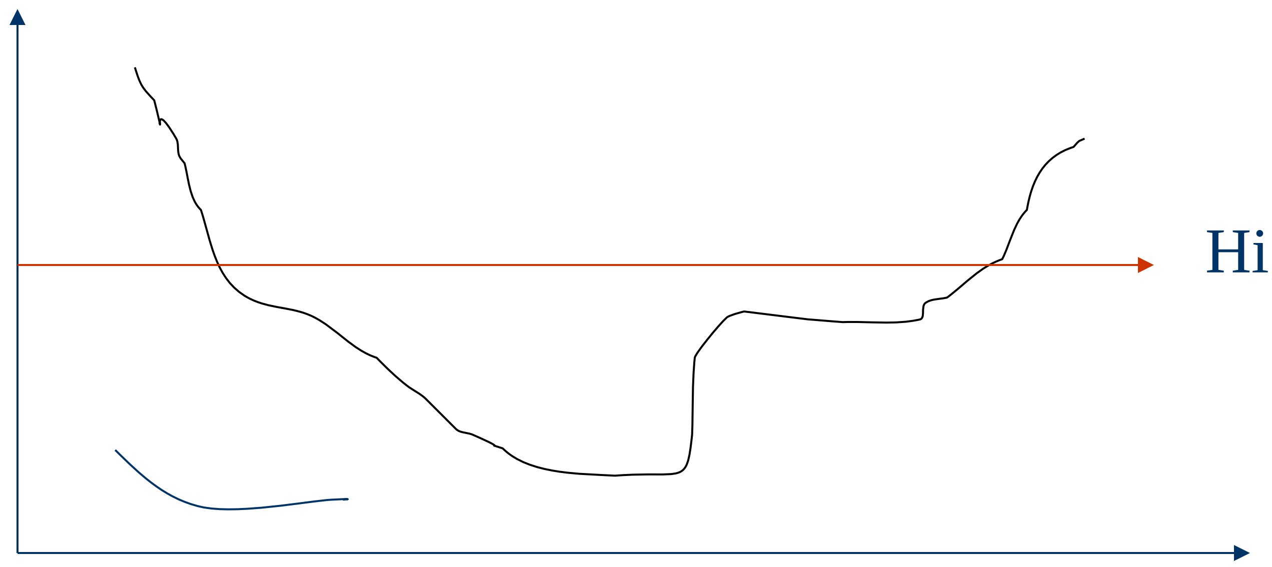


Canny Edge Detector
Hysteresis Thresholding



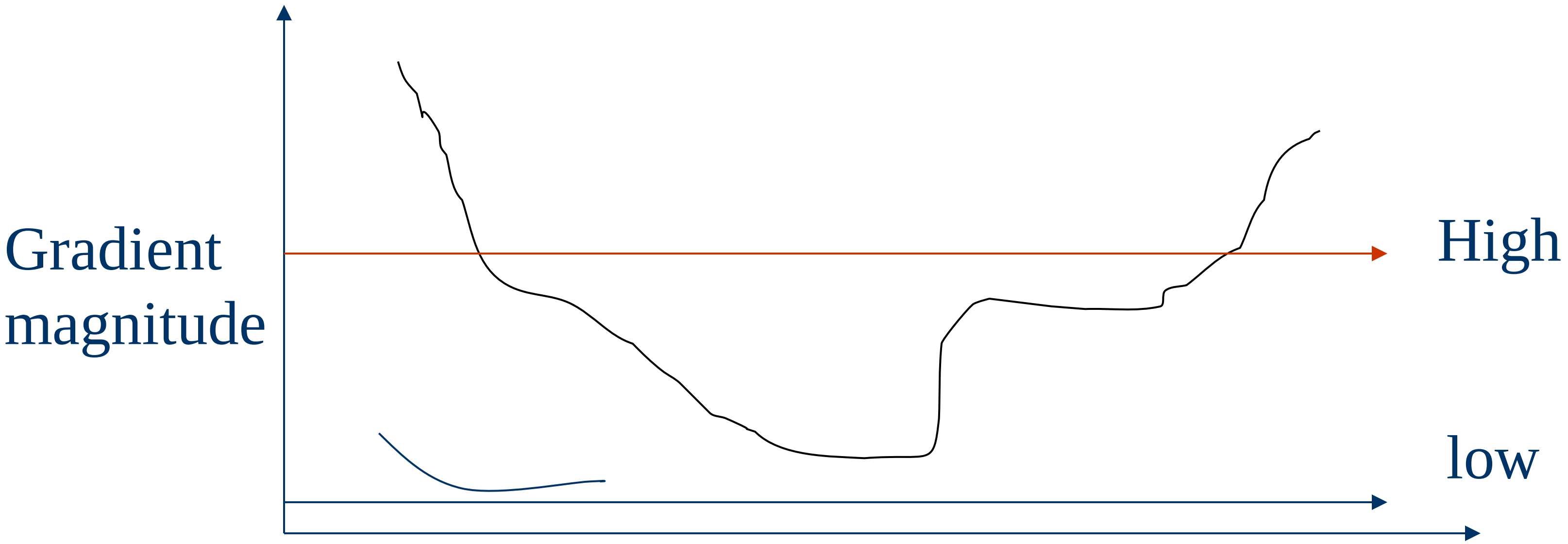
Canny Edge Detector
Hysteresis Thresholding

Gradient magnitude



High

Canny Edge Detector
Hysteresis Thresholding



Canny Edge Detector
Hysteresis Thresholding

Canny Edge Detector
Hysteresis Thresholding

- Scan the image from left to right, top-bottom.

Canny Edge Detector
Hysteresis Thresholding

- Scan the image from left to right, top-bottom.
  - The gradient magnitude at a pixel is above a high threshold declare that as an edge point

Canny Edge Detector
Hysteresis Thresholding

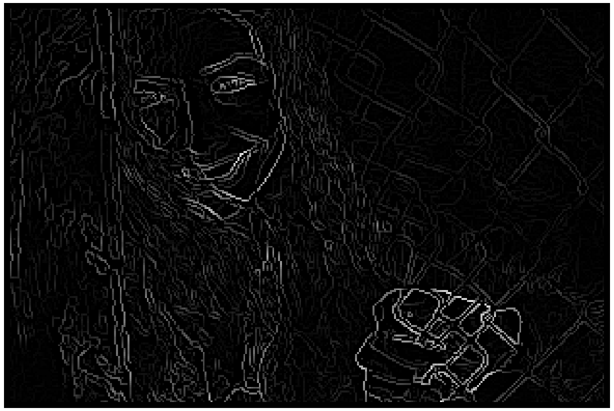
- Scan the image from left to right, top-bottom.
  - The gradient magnitude at a pixel is above a high threshold declare that as an edge point
  - Then recursively consider the *neighbors* of this pixel.

Canny Edge Detector
Hysteresis Thresholding

- Scan the image from left to right, top-bottom.
  - The gradient magnitude at a pixel is above a high threshold declare that as an edge point
  - Then recursively consider the *neighbors* of this pixel.
    - If the gradient magnitude is above the low threshold declare that as an edge pixel.

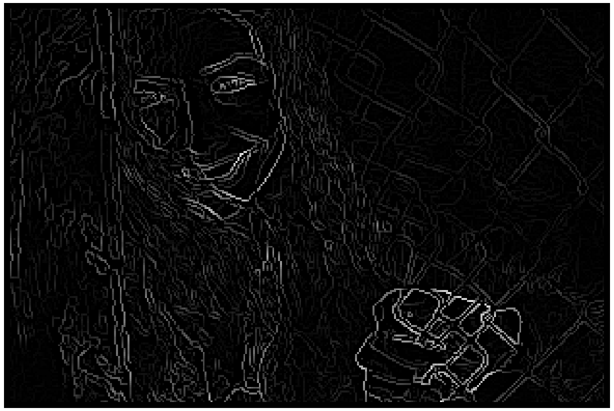
Canny Edge Detector
Hysteresis Thresholding

Canny Edge Detector
Hysteresis Thresholding

M 

Canny Edge Detector
Hysteresis Thresholding

M

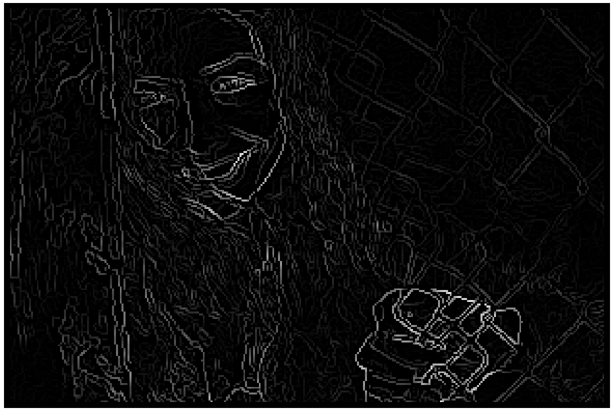


regular

$M \geq 25$

Canny Edge Detector
Hysteresis Thresholding

M



regular

$M \geq 25$

Hysteresis

$High = 35$

$Low = 15$

Before Non-max Suppression

Slide Credit: James hays

After non-max suppression

Slide Credit: James hays

Hysteresis thresholding

- Threshold at low/high levels to get weak/strong edge pixels
- Do connected components, starting from strong edge pixels

Slide Credit: James hays

Final Canny Edges


Slide Credit: James hays

Effect of σ (Gaussian kernel spread/size)

original

Canny with $\sigma = 1$

Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Source: S. Seitz

Matlab functions

Matlab functions

- Edge: Find Edges in intensity image

Matlab functions

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➢ returns a binary image with 1's where the function finds edges in Image and 0's elsewhere.

Matlab functions

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➢ EDGE supports six different edge-finding methods

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Prewitt

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  Sobel
  Prewitt
  Canny
  Roberts

Matlab functions

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Sobel

Prewitt

Canny

Roberts

Laplacian of Gaussian

Matlab functions

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Sobel

Prewitt

Canny

Roberts

Laplacian of Gaussian

zero-cross

Matlab functions

Matlab functions

Examples

Matlab functions

Examples

BW1 = edge(Image,'log‘,Thresh,SIGMA)

Matlab functions

Examples

BW1 = edge(Image,'log‘,Thresh,SIGMA)

- Specifies the Laplacian of Gaussian method

Matlab functions

Examples

BW1 = edge(Image,'log‘,Thresh,SIGMA)

- Specifies the Laplacian of Gaussian method
- Ignore all the edges weaker than threshold.

Matlab functions

Examples

BW1 = edge(Image,'log‘,Thresh,SIGMA)

- Specifies the Laplacian of Gaussian method
- Ignore all the edges weaker than threshold.
- Sigma specifies the standard deviation of LoG filter.

Matlab functions

Examples

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BW1 = edge(Image,‘canny‘,Thresh,SIGMA)

Matlab functions

Examples

BW1 = edge(Image,'log‘,Thresh,SIGMA)

- Specifies the Laplacian of Gaussian method
- Ignore all the edges weaker than threshold.
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BW1 = edge(Image,‘canny‘,Thresh,SIGMA)

- Specifies the Canny method

Matlab functions

Examples

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BW1 = edge(Image,‘canny‘,Thresh,SIGMA)

- Specifies the Canny method
- Thresh is two element vector for low and high threshold value.

Matlab functions

Examples

BW1 = edge(Image,'log‘,Thresh,SIGMA)

- Specifies the Laplacian of Gaussian method
- Ignore all the edges weaker than threshold.
- Sigma specifies the standard deviation of LoG filter.

BW1 = edge(Image,‘canny‘,Thresh,SIGMA)

- Specifies the Canny method
- Thresh is two element vector for low and high threshold value.
- Sigma specifies the standard deviation of Gaussian filter.

Matlab functions

Matlab functions

- imgradient: Gradient magnitude and direction of an image

Matlab functions

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  [Gmag,Gdir] = imgradient(Image,Method)

Matlab functions

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Methods includes

Matlab functions

- imgradient: Gradient magnitude and direction of an image

[Gmag,Gdir] = imgradient(Image,Method)

Methods includes

- Prewitt

Matlab functions

- imgradient: Gradient magnitude and direction of an image

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Methods includes

- Prewitt
- Robert

Matlab functions

- imgradient: Gradient magnitude and direction of an image

  [Gmag,Gdir] = imgradient(Image,Method)

Methods includes

- Prewitt
- Robert
- Central difference

Suggested Reading

Suggested Reading

- Chapter 2, Mubarak Shah, “Fundamentals of Computer Vision”

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- Chapter 2, Mubarak Shah, “Fundamentals of Computer Vision”
- Richard Szeliski, "Computer Vision: Algorithms and Applications".
  - 4.2