Nash Equilibrium in Shared Electric Vehicle Charging Stations: Optimizing Charging Time and Resource Allocation

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***Abstract*—Navigation systems play a pivotal role in optimizing the allocation of public charging stations for electric vehicles (EVs), aiming to reduce overall charging time by leveraging comprehensive data analysis. However, current systems typically focus solely on travel time and treat the allocation challenge as a routing problem. This study expands the scope by incorporating the queuing time at charging stations as a critical component of the total charging time, alongside the travel time on the road. Recognizing that both roads and charging stations are susceptible to congestion, we introduce a joint-resource congestion game to model the dynamic interactions between EVs and these shared resources.**

***Index Terms*—electric vehicle, public charging station, congestion game, resource allocation**

I. Introduction

As electric vehicles (EVs) steadily increase in popularity, shared charging stations have emerged as vital elements of urban infrastructure. Nevertheless, their swift integration presents considerable challenges, such as competition among EV users, prolonged waiting times, and suboptimal energy usage. These difficulties stem from the decentralized and self-serving decision-making behavior exhibited by EV owners, who are compelled to evaluate aspects such as charging expenses and waiting durations when choosing charging stations. Addressing these challenges is critical to enhance both the effectiveness and equity of collectively used charging resources.

This paper uses a game-theoretic framework to analyze and optimize the interaction among EV users in shared charging stations. We model the problem as a normal-form congestion game, where each EV user is a player whose goal is to minimize individual cost, and we investigate how strategic decisions affect the overall system performance.

The payoff of each user includes direct costs, waiting-time costs, and the effect of the strategies of other users. The concept of Nash equilibrium is applied to identify stable outcomes where no user can improve their payoff unilaterally, thus reflecting the intrinsic dynamics of decentralized decision-making. This is important to fill the critical literature gap in user interactions of shared charging facilities, as opposed to most works that either treat EV users in isolation or focus on the integration of renewable energies. Incorporating the competitive dynamics related to shared resources into the analysis, this work opens a way to gain insight on efficient resource allocation and how pricing mechanisms or incentives might be implemented to achieve off-peak charging.

II. Related Works

A. Shortest Path and Heuristic-Based Methods

Early studies on EV navigation typically employed shortest path algorithms, like Dijkstra or A*, to help EVs find the nearest charging stations. These approaches were efficient but did not account for factors such as queuing at charging stations or dynamic pricing.

Heuristic-based methods, such as rule-based navigation, have also been employed to find charging stations while minimizing specific costs (e.g., travel distance or time). However, these methods often lacked a holistic view of the system, especially in competitive multi-agent environments.

B. Queuing Theory

Queuing theory has been widely applied to predict waiting times at charging stations. For example, some studies modeled each charging station as an M/M/1 queue to estimate the expected waiting time. These models helped EVs decide which charging station to choose to minimize queuing delays.

However, traditional queuing models often assume a stationary arrival process, which might not be realistic in dynamic traffic conditions where EVs change their route based on real-time information.

C. Congestion Game Model

The study introduces a **joint-resource congestion game** model, which considers both road segments and charging stations as congestible resources. This approach extends the traditional congestion game framework that typically focuses on a single type of resource, such as either road networks or charging stations. By modeling both roads and stations as shared resources, Zhang et al. (2019) provide a more realistic representation of the real-world competition faced by multiple EVs seeking charging opportunities. The joint-resource congestion game is a non-cooperative game where a finite number of EVs compete for limited resources, and a Nash equilibrium is proven to exist within this setting. At this equilibrium, no individual EV can improve its outcome by unilaterally changing its strategy, given the strategies chosen by other vehicles.

D. Q-Learning for Nash Equilibrium

To solve the allocation problem and reach Nash equilibrium, Zhang et al. (2019) employ a **Q-learning algorithm**. This reinforcement learning approach is particularly suitable for dynamic and unstable real-time traffic environments, as it does not require a pre-defined model of the environment. The Q-learning agent iteratively learns an optimal allocation strategy by interacting with its environment, which consists of roads and charging stations, adjusting its actions based on the congestion status observed. One of the major advantages of Q-learning is its ability to converge to an optimal solution even under stochastic and uncertain conditions, making it highly effective in complex urban environments with fluctuating charging demand and traffic congestion.

Zhang et al. (2019) also highlight that Q-learning outperforms heuristic-based algorithms, such as genetic algorithms (GA), in terms of convergence speed. This demonstrates the practical efficiency of Q-learning for resource allocation in scenarios involving multiple EVs, making it a preferable choice in real-world implementations where swift decision-making is critical.

III. Methodology

A. Model Overview

The proposed system for EV navigation to charging stations is modeled as a multi-agent environment where multiple EVs, acting as rational agents, aim to minimize their overall costs by selecting the most suitable charging station. Each EV is treated as a player in the game, and the interactions between EVs are modeled using game-theoretic principles. The system adopts a complete information approach, meaning that all EVs have access to real-time data about charging stations, including their availability, pricing, and peak times.

The model incorporates key elements such as travel time, queuing time, charging cost, and charging speed to determine the optimal charging station for each EV. The decision-making process for each EV considers both individual objectives and the collective impact on the charging infrastructure.

B. Assumptions

To streamline the problem and focus on the core aspects of EV navigation, the following assumptions are made:

- **Complete Information**: All EVs have real-time access to information regarding charging station parameters, including charging fees, total number of charging spots, number of available spots, and peak times.
- **Rational Agents**: Each EV behaves rationally and aims to minimize its total cost, which includes travel time, queuing time, and charging costs.
- **Fixed Number of Charging Stations**: The number and locations of charging stations are fixed and known to all EVs.
- **Charging Speed Variability**: Charging stations may offer different charging speeds, affecting the overall time required for charging. This information is available to all EVs.
- **Traffic Conditions**: Real-time traffic conditions are considered when calculating travel time to charging stations.
- **No Vehicle-to-Grid Interaction**: The model does not account for vehicle-to-grid interactions, focusing solely on the charging aspect.

C. Charging Station Parameters

The model utilizes several parameters to evaluate and select charging stations:

- **Charging Fees**: The cost per unit of electricity at each charging station. Fees may vary based on the time of day, with higher prices during peak hours.
- **Total and Available Spots**: The total number of charging spots at each station and the number currently available. This helps EVs estimate potential queuing times.
- **Peak Time Data**: Historical and real-time data on peak usage times for each charging station. This information assists EVs in avoiding stations that are likely to be congested.
- **Charging Speeds**: Different charging stations may offer varying charging speeds (e.g., Level 2, DC fast charging). The model accounts for the impact of charging speed on the total time required for charging.
- **Travel Time**: The time required to reach a charging station from the current location of the EV, influenced by real-time traffic conditions.

Each of these parameters is utilized by the EVs to assess the total cost of choosing a particular charging station. The objective is to develop a decision-making process that allows EVs to minimize their total cost while balancing the load across multiple charging stations to prevent congestion.

D. Queuing Theory Integration

To accurately model the queuing time at charging stations, Queuing Theory is employed, specifically the **M/M/c** model. This approach allows for the calculation of average waiting times based on the arrival rate of EVs and the service rate of charging stations.

1) Queuing System Parameters:

- **Arrival Rate** (λ): The average rate at which EVs arrive at a charging station (e.g., vehicles per hour).
- **Service Rate** (μ): The service capacity of each charging point, i.e., the number of vehicles a charging point can serve per hour.
- **Number of Servers** (c): The number of charging points available at the charging station.

- **System Capacity**: Whether the system allows an infinite queue or has a finite capacity.

2) Key Metrics in the M/M/c Model:

a) System Utilization (ρ): System utilization represents the load on each server and is calculated as:

$$\rho = \frac{\lambda}{c\mu} \tag{1}$$

For the system to be stable, $\rho < 1$ must hold.

b) Probability of Zero Customers in the System (P_0): P_0 is the probability that there are no EVs in the system and is given by:

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^c}{c!} \cdot \frac{1}{1-\rho}\right]^{-1} \tag{2}$$

c) Average Number of Vehicles in Queue (L_q): L_q denotes the average number of vehicles waiting in the queue:

$$L_q = \frac{\frac{(\lambda/\mu)^c}{c!} \cdot \rho}{(1-\rho)^2} \cdot P_0 \tag{3}$$

d) Average Waiting Time in Queue (W_q): W_q represents the average time a vehicle spends waiting in the queue:

$$W_q = \frac{L_q}{\lambda} \tag{4}$$

3) Calculation Steps: To compute the average waiting time, follow these steps:

1) **Calculate System Utilization** (ρ):
$$\rho = \frac{\lambda}{c\mu} \tag{5}$$
Ensure that $\rho < 1$ for system stability.
2) **Calculate** P_0: Use Equation 2 to determine the probability of zero vehicles in the system.
3) **Calculate the Average Number of Vehicles in Queue** (L_q): Substitute P_0 into Equation 3 to find the average number of vehicles waiting.
4) **Calculate the Average Waiting Time** (W_q): Use Equation 4 and the arrival rate λ to determine the average waiting time.

E. Driver Types and Weighting Factors

1) Introduction to Driver Types: In the context of EV navigation and charging station selection, drivers exhibit different preferences and priorities based on their commuting patterns and charging needs. To accurately model these heterogeneous preferences, we categorize drivers into distinct types, each characterized by specific weighting factors (α and β) that influence their decision-making process.

2) Driver Type Categories: We define two primary driver types:

- **Commuter Drivers**
- **Casual Drivers**

Each driver type is associated with different priorities regarding travel time and charging costs, as reflected in their respective α and β values.

3) Weighting Factors: The weighting factors determine the relative importance of travel time (T_{travel}) and charging cost (C_{charge}) in the utility function of each driver type. Specifically:

- **Commuter Drivers**: Prioritize minimizing travel time over charging costs.
- **Casual Drivers**: Prioritize minimizing charging costs over travel time.

TABLE I
DRIVER TYPES AND THEIR CORRESPONDING WEIGHTING FACTORS

Driver Type	α	β
Commuter	0.8	0.2
Casual	0.2	0.8

F. Game-Theoretic Framework

The interactions between multiple EVs selecting charging stations are modeled using game-theoretic principles. This framework allows for the analysis of strategic decision-making among EVs aiming to minimize their individual costs while considering the actions of other EVs.

1) Game Formulation: The problem of EV navigation to charging stations is formulated as a non-cooperative congestion game involving multiple players (EVs). Each EV is a rational agent that aims to minimize its total cost, which includes travel time, queuing time, and charging costs. The strategy for each EV is to choose the optimal charging station from a set of available options. The payoff for each EV is defined as the negative of its total cost, meaning that each EV aims to minimize its own cost while competing with other EVs for limited charging resources.

2) Nash Equilibrium: The concept of Nash equilibrium is employed to analyze the outcome of the congestion game. A Nash equilibrium occurs when no EV can unilaterally change its strategy to achieve a lower cost, given the strategies of all other EVs. In the context of EV charging, a Nash equilibrium represents a stable state where each EV has chosen a charging station such that any deviation would result in a higher total cost.

3) Congestion Game Perspective: The problem is viewed as a congestion game, where the cost for each EV depends on the number of other EVs choosing the same charging station. Congestion games are a type of game where the utility (or cost) of each player is affected by the number of players selecting the same resource. In this case, the charging stations are the shared resources, and the queuing time at each station increases as more EVs choose the same station.

From the perspective of an EV, the goal is to select a charging station that minimizes its total cost while avoiding congestion. Each EV must anticipate the actions of other EVs and choose a strategy that balances its own objectives with the potential impact of congestion. Congestion games are known to have at least one pure strategy Nash equilibrium, providing a stable solution for the system. By modeling the charging station selection problem as a congestion game, the interactions between EVs can be better understood, and efficient solutions can be derived to balance the load across charging stations.

G. Formalizing the Congestion Game

To apply Congestion Games to the EV charging station selection problem, the game must be formally defined by specifying the players, strategies, and payoff functions.

1) Players (N): Each EV seeking to charge its battery is considered a player in the game.

$$N = \{EV_1, EV_2, \dots, EV_N\} \tag{6}$$

2) Strategy Space (S): The set of available charging stations constitutes the strategy space.

$$S = \{s_1, s_2, \dots, s_M\} \tag{7}$$

3) Payoff Function (U_i): The payoff for each EV is the negative of its total cost, encompassing travel time, queuing time, and charging costs.

$$U_i = [\alpha (T_{\text{travel}}(s_i) + T_{\text{queue}}(s_i)) + \beta C_{\text{charge}}(s_i)] \tag{8}$$

Where:

- α and β are weighting factors specific to the driver type of EV i.
- $T_{\text{travel}}(s_i)$: Travel time to charging station s_i, determined using real-time traffic data and pathfinding algorithms such as A* or Dijkstra's algorithm.
- $T_{\text{queue}}(s_i)$: Queuing time at charging station s_i, modeled using the **M/M/c** queuing theory approach.
- $C_{\text{charge}}(s_i)$: Charging cost at charging station s_i.

H. Calculation of Charging Time and Cost

1) Travel Time (T_{travel}): Travel time is determined using real-time traffic data, which accounts for current road conditions, traffic congestion, and the distance to the station. Algorithms such as A* or Dijkstra's algorithm are utilized to find the shortest path to the charging station, incorporating traffic data to estimate the travel duration, similar to navigation applications like Google Maps.

2) Queuing Time (T_{queue}): Queuing time is modeled using the **M/M/c** queuing theory approach, as detailed in Section III-A.

3) Charging Cost (C_{charge}): The charging cost is calculated as:

$$C_j = r_j \times E_i \tag{9}$$

Where:

- **Rate per kWh** (r_j): The cost of electricity per kilowatt-hour at charging station j. This rate can vary based on factors such as time of day, dynamic pricing, and location.
- **Energy Required** (E_i): The amount of energy needed by EV i to reach its desired level of charge, depending on the current battery level, desired state of charge, and vehicle efficiency.

I. Decision-Making Process

Each EV evaluates the total cost associated with each available charging station by combining travel time, queuing time, and charging cost. The EV then selects the charging station that minimizes its total cost while considering the strategies of other EVs to avoid congestion and optimize overall system efficiency.

J. Conclusion of Methodology

By integrating queuing theory and game-theoretic principles, the proposed methodology provides a robust framework for modeling EV navigation to charging stations. This approach enables the analysis of strategic interactions among EVs and facilitates the development of efficient charging station selection strategies that minimize individual and collective costs.

IV. Results

TABLE II
PAYOFF MATRIX (P1: COMMUTER, P2: CASUAL)

P1 \ P2	**Station 1**	**Station 2**
Station 1	5.58, 12.35	3.40, 5.79
Station 2	5.86, 10.98	8.99, 5.98

In this table, Player 1 (P1: Commuter) and Player 2 (P2: Casual) have different objectives: - Player 1 (Commuter): Prioritizes minimizing travel and waiting time. - Player 2 (Casual): Focuses on minimizing the cost of charging, as price is a dominant factor for them.

- At **(Station 1, Station 2)**, the Nash equilibrium is observed. Player 1 chooses Station 1 because it is near and minimizes their travel time, even though the charging cost is higher. Player 2, on the other hand, opts for Station 2 to benefit from its lower charging cost, despite the longer travel time.
- If both players choose **Station 1**, Player 2 faces a high cost of 12.35 because of the expensive charging rate, even though Player 1 finds Station 1 reasonable for minimizing time.
- Conversely, at **(Station 2, Station 2)**, both players incur moderate payoffs, with Player 1 facing a slightly higher cost due to the travel time penalty.

The Nash equilibrium at **(Station 1, Station 2)** reflects a logical division of strategies based on their priorities: time versus cost.

TABLE III
PAYOFF MATRIX (P1: COMMUTER, P2: COMMUTER)

P1 \ P2	**Station 1**	**Station 2**
Station 1	3.66, 3.66	3.61, 6.18
Station 2	6.15, 3.57	6.29, 6.29

In this scenario, both players are commuters who prioritize travel and waiting time equally. The analysis reveals the following:

- At **(Station 1, Station 1)**, a Nash equilibrium occurs because both players minimize their time-based costs. Each player's payoff is symmetric (3.66, 3.66), and neither can improve their cost by unilaterally switching to Station 2.
- If one player moves to **Station 2** while the other stays at Station 1, the player at Station 2 faces a higher travel time and waiting cost, leading to an increased payoff (e.g., 6.15 for P1).
- At **(Station 2, Station 2)**, both players incur slightly higher costs (6.29) compared to Station 1 due to the increased travel time associated with Station 2. However, because the waiting time is balanced, it becomes a suboptimal choice compared to the Nash equilibrium at (Station 1, Station 1).

The Nash equilibrium at **(Station 1, Station 1)** is intuitive because both players seek to minimize their overall time cost, and Station 1 offers the closest proximity.

TABLE IV
PAYOFF MATRIX (P1: CASUAL, P2: CASUAL)

P1 \ P2	**Station 1**	**Station 2**
Station 1	12.12, 12.12	10.81, 5.39
Station 2	4.06, 8.14	6.05, 6.05

In this table, both players are casual users who prioritize minimizing the charging cost over waiting and travel time. The following observations emerge:

- At **(Station 2, Station 2)**, the Nash equilibrium occurs because both players choose Station 2, which has the lowest charging cost (0.17 per kWh). Their symmetric payoffs (6.05, 6.05) reflect this choice.
- At **(Station 1, Station 1)**, both players incur high costs (12.12, 12.12) because Station 1 has an expensive charging rate. This is clearly suboptimal for cost-sensitive casual players.
- If one player switches to **Station 2** while the other remains at Station 1, the player at Station 2 benefits from a much lower cost. For example: - At **(Station 1, Station 2)**, P2 achieves a cost of 5.39, much better than at Station 1. - At **(Station 2, Station 1)**, P1 incurs a low cost of 4.06, while P2 incurs a higher cost at Station 1.

The Nash equilibrium at **(Station 2, Station 2)** reflects the logical outcome where both casual players select the cheapest charging station, despite the travel time being higher.

V. CONCLUSION

A. Summary of Research

This thesis developed and analyzed a navigation system for Electric Vehicles (EVs) to charging stations using game theory. The primary objective was to design a model that enables EV users to select optimal charging stations by minimizing their total costs, which encompass travel time, queuing time, and charging expenses. To achieve this, the study integrated queuing theory and game-theoretic principles, accounting for heterogeneous driver types characterized by distinct weighting factors (α and β).

B. Key Findings

The research yielded several significant findings:

- **Game-Theoretic Model**: A non-cooperative congestion game was successfully formulated, where each EV acts as a rational agent aiming to minimize its individual cost. The model effectively captures the strategic interactions among multiple EVs competing for limited charging resources.
- **Driver Type Differentiation**: By categorizing drivers into **Commuter** and **Casual** types with respective weighting factors ($\alpha = 0.8, \beta = 0.2$ for Commuters and $\alpha = 0.2, \beta = 0.8$ for Casuals), the model accommodates diverse user preferences, leading to more personalized and realistic charging station selections.
- **Queuing Theory Integration**: The incorporation of the **M/M/c** queuing model provided accurate estimations of average waiting times at charging stations. This integration allows EV users to make informed decisions based on both current and predicted congestion levels.
- **Nash Equilibrium Analysis**: The study demonstrated that the congestion game reaches a Nash equilibrium, ensuring a stable state where no EV can unilaterally reduce its total cost by changing its charging station choice. This equilibrium facilitates balanced load distribution across multiple charging stations, mitigating excessive congestion.

C. Implications of the Study

The findings of this research have significant implications for both EV users and charging station operators:

- **Enhanced User Experience**: By providing EV users with cost-effective and time-efficient charging options, the navigation system enhances overall user satisfaction and promotes the adoption of electric vehicles.
- **Efficient Infrastructure Utilization**: Charging station operators can leverage the insights from the game-theoretic model to optimize the placement and capacity of charging stations, ensuring equitable distribution of resources and reducing bottlenecks during peak times.
- **Policy Formulation**: Policymakers can utilize the study's outcomes to develop strategies that encourage balanced EV charging behaviors, such as incentivizing off-peak charging or implementing dynamic pricing mechanisms.

D. Limitations

While the study provides a robust framework for EV navigation and charging station selection, it is subject to certain limitations:

- **Time Constraints for Large-Scale Experiments**: Due to limited time, the study was unable to conduct more extensive experiments to fully validate the proposed model. Larger-scale experiments are essential to assess the model's scalability and effectiveness in diverse real-world scenarios.
- **Static Pricing Assumption**: The model assumes fixed charging fees, whereas, in reality, dynamic pricing is prevalent. Incorporating dynamic pricing could further enhance the model's realism.
- **Limited Driver Types**: Only two driver types were considered. Expanding the categorization to include additional profiles (e.g., **Leisure**, **Business**) could provide a more nuanced understanding of user preferences.
- **Simplified Traffic Conditions**: The study utilized real-time traffic data for travel time estimation but did not account for future traffic predictions or variability over different timescales.
- **Ignoring Vehicle-to-Grid Interactions**: The model does not consider interactions between EVs and the power grid, such as demand response or energy storage capabilities, which could influence charging behaviors.

E. Recommendations for Future Research

Building upon the foundations laid by this thesis, future research can explore several avenues to enhance and extend the current model:

- **Dynamic Pricing Models**: Integrate dynamic pricing strategies into the game-theoretic framework to reflect real-world pricing fluctuations and their impact on EV charging decisions.
- **Expanded Driver Profiles**: Incorporate a broader range of driver types with varying priorities and constraints to capture a more comprehensive spectrum of user behaviors.
- **Advanced Traffic Modeling**: Employ predictive analytics and machine learning techniques to forecast traffic conditions, enabling more accurate travel time estimations and proactive charging station selection.
- **Vehicle-to-Grid (V2G) Interactions**: Explore the role of V2G technologies in influencing EV charging patterns, potentially leading to synergistic benefits for both users and the power grid.
- **Real-World Implementation and Testing**: Deploy the proposed navigation system in a real-world setting to validate the model's efficacy and gather empirical data for further refinement.
- **Conducting Larger-Scale Experiments**: Allocate more time and resources to perform extensive experiments that can comprehensively validate the model across various scenarios and user behaviors.

F. Conclusion

In conclusion, this thesis successfully developed a game-theoretic model for EV navigation to charging stations, integrating queuing theory and differentiated driver types to optimize charging station selection. The model not only provides a theoretical framework for understanding EV charging behaviors but also offers practical insights for enhancing user experiences and infrastructure management. Despite its limitations, the study lays a solid groundwork for future advancements in smart transportation systems and sustainable energy management.

Although time constraints prevented the execution of more extensive experiments to fully validate the proposed model, future work will continue to refine and enhance the model. Conducting larger-scale experiments will be essential to assess the model's scalability, robustness, and applicability in diverse real-world scenarios. This ongoing development will ensure the navigation system's effectiveness and reliability, ultimately providing EV users with optimized charging experiences and contributing to the broader adoption of electric vehicles.

VI. Project Description

Mohammed Alnajim did the introduction + participated in methodology + dataset extraction + simulation + result Yi-Lei Li did related works methodology

References

[1] Zhang, Li, Ke Gong, and Maozeng Xu. "Congestion control in charging stations allocation with Q-learning." *Sustainability* 11.14 (2019): 3900.

[2] Donald Gross, John F. Shortle, James M. Thompson, Carl M. Harris, *Fundamentals of Queueing Theory*, 5th Edition, Wiley, 2018.

[3] Liu, Mingxi, et al. "Decentralized charging control of electric vehicles in residential distribution networks." IEEE Transactions on Control Systems Technology 27.1 (2017): 266-281.

[4] Zhang, Ke, et al. "Optimal charging schemes for electric vehicles in smart grid: A contract theoretic approach." IEEE Transactions on Intelligent Transportation Systems 19.9 (2018): 3046-3058.

[5] Laha, Aurobinda, et al. "Game theory based charging solution for networked electric vehicles: A location-aware approach." IEEE Transactions on Vehicular Technology 68.7 (2019): 6352-6364.