Mathematical Formulas and Concepts Used in the Code

1 Introduction

Occupancy grid mapping represents the environment probabilistically by dividing space into cells marked as *free*, *occupied*, or *unknown*. This is summary shows the formulas used in the code. [1, 2, 3].

2 Robot and Sensor Model

The robot pose in the world frame is

$$\mathbf{p}_r^W = (x_r, y_r), \quad \theta_r$$

where (x_r, y_r) is position and θ_r is yaw. A sensor i mounted on the robot has fixed offset and orientation

$$\mathbf{p}_{S_i}^R = (x_s, y_s), \quad \theta_{S_i}^R.$$

2.1 Coordinate Transform

Sensor world-frame position:

$$\mathbf{p}_{S_i}^W = \mathbf{p}_r^W + R(\theta_r)\mathbf{p}_{S_i}^R, \quad R(\theta_r) = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix}.$$

Orientation: $\phi_{S_i}^W = \theta_r + \theta_{S_i}^R$.

2.2 Time-of-Flight to Range

An ultrasonic measurement t_i yields distance

$$d_i = \tfrac{c \cdot t_i}{2}, \qquad c \approx 343 \text{ m/s}.$$

3 World-to-Grid Mapping

The grid is a square of side `size_m` with resolution res. World point (x, y) maps to grid indices

$$j = \left\lfloor \tfrac{x + \texttt{origin\_x}}{\text{res}} \right\rfloor, \quad i = \left\lfloor \tfrac{\texttt{origin\_y} - y}{\text{res}} \right\rfloor.$$

4 Inverse Sensor Model

Occupancy probability of a cell m given z_t:

$$p(\text{occ} \mid z_t) = \begin{cases} p_{\text{free}}, & r < d_i - \delta, \\ p_{\text{occ}}, & |r - d_i| \leq \delta, \\ p_{\text{unk}}, & r > d_i + \delta, \end{cases}$$

with $r = \|\mathbf{m} - \mathbf{p}_{S_i}^W\|$. Log-odds update:

$$L_t(m) = L_{t-1}(m) + \ell(m; z_t), \quad \ell = \log \frac{p}{1-p}.$$

5 Discretization of the Ultrasonic Cone

Ultrasonic sensors emit a cone with half-angle $\beta = \frac{\text{FOV}}{2}$. Following [1], we approximate with N_{RAYS} beams:

$$a_k = -\beta + \tfrac{2\beta}{N_{\text{RAYS}}-1} k, \quad \phi_k = \phi_{S_i}^W + a_k.$$

Each ray is marched in steps Δr, marking:

- free cells (L_{FREE}) for $r < d_i - \delta$,
- occupied cells (L_{OCC}) at $r \approx d_i$,
- no update beyond $d_i + \delta$.

6 Implementation Summary

- Grid stored as 1D array of log-odds with size $W \times H$.
- Input CSV: $(t, x_r, y_r, \theta_r, d_1, \ldots, d_n)$.
- Each reading processed with Digital Differential Analyzer (DDA) ray-tracing.
- Traversed cells $\rightarrow$ free, endpoint $\rightarrow$ occupied.
- Exported as PGM image: black = occupied, white = free, gray = unknown.

References

[1] H. Moravec and A. Elfes, "High resolution maps from wide angle sonar," *ICRA*, 1985.

[2] S. Thrun, "Learning Occupancy Grids With Forward Sensor Models," School of Computer Science, Carnegie Mellon University, Pittsburgh, PA, 1998.

[3] R. Siegwart, I. Nourbakhsh, D. Scaramuzza, *Introduction to Autonomous Mobile Robots*, MIT Press, 2011.