

Algebra 1 Series 1

Exercise 1.

Let P, Q, R be three propositions

Write a truth table for the following propositions

$$(1) P \vee \bar{P}, (2) P \wedge \bar{P}, (3) [(P \Rightarrow Q) \Rightarrow R], (4) [P \Rightarrow (Q \Rightarrow R)].$$

What do you conclude?

Exercise 2.

Are the following propositions true or false:

$$(1) \exists n \in \mathbb{N}: [n + 1 < 0]; (2) \forall x \in \mathbb{R}, \forall y \in \mathbb{R}: [(x + y - xy = 1) \Leftrightarrow (x = 1 \vee y = 1)]$$

Exercise 3.

Specify the negation of the following propositions and then indicate whether they are true or false:

$$(1) \forall x \in \mathbb{R}: [(x^2 + |x| + 1 \geq 0) \wedge (|x| < 3)], (2) \exists n \in \mathbb{N}^*, \forall x \in \mathbb{R}_+: \frac{x^{2n}}{1+x} > 1$$

Exercise 4.

Use the case by case proof to prove the validity of the following cases:

$$(1) \forall n \in \mathbb{N}: [2 \text{ divide } n^3 - n], (2) \forall x \in \mathbb{R}: [|x - 1| \leq x^2 - x + 1]$$

Exercise 5.

Let be the following propositions:

$$(P) \forall x \in \mathbb{R}^*: \left[\frac{6x+1}{x^2} > 0 \Rightarrow x \leq 0 \right]$$

1. Give the opposite contrast of the proposition (P).
2. Is the proposition (P) true or false? Explain your answer.
3. Give the negation of the proposition (P).

Exercise 6.

Use the proof by counterexample to prove that the following proposition is wrong

$$(1) \forall x \in \mathbb{R} : \left[x + \frac{1}{x} \geq 2 \right], (2) \forall (x, y) \in \mathbb{R}^2: \sqrt{x+y} = \sqrt{x} + \sqrt{y}$$

Exercise 7.

- 1- Prove using backward reasoning that $\sqrt[3]{7}$ is a solid number ($\sqrt[3]{7} \notin \mathbb{Q}$).
- 2- Generalize for a deaf number $\sqrt[n]{a}$, i.e. $\sqrt[n]{a} \notin \mathbb{Q}$.

Exercise 8.

Let the two predicates

$$(1) \forall n \in \mathbb{N}, (P_n) : \left[1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \right]; (2) (Q_n) [2 \text{ divide } (3^n + 1)]$$

- 1- Prove using induction proof the validity of the proposition (P_n) .
- 2- Prove the validity of the implication $(\forall n \in \mathbb{N}: Q_n \Rightarrow Q_{n+1})$.
- 3- Prove that the proposition (Q_n) is false.

Exercise 9.

The students Ahmed, Ali, and Salem obtained the following marks: 06, 10, and 16 in the algebra exam. Considering that the following four propositions are true:

- (Ahmad's score is 06) \Rightarrow (Ali's score is 10)
- (Ali's score 6) \Rightarrow (Salem's score 10)
- (Ahmed's score is not 10) \Rightarrow (Ali's score is 16)
- (Salem's score 16) \Rightarrow (Ali's score 06).

1- Fill in the following table, considering the validity of the case from its falsehood:

Suggested possibility	(P_1)	(P_2)	(P_3)	(P_4)
Ahmed's score is 6, Ali's score is 10, and Salem's score is 16				
Ahmed's score is 6, Ali's score is 16, and Salem's score is 10				
Ahmed's score is 10, Ali's score is 6, and Salem's score is 16				
Ahmed's score is 10, Ali's score is 16, and Salem's score is 6				
Ahmed's score is 16, Ali's score is 10, and Salem's score is 6				
Ahmed's score is 16, Ali's score is 6, and Salem's score is 10				

2- If the four propositions are correct, what is each student's score?