EXO1

I/ * $L_1 = \{a^n b^m c^{n+m} \quad n \geq 1, m \geq 0\}$ 1,25

$\hookrightarrow G_1 : (S \to aSc / aAc \; ; \; A \to bAc / \varepsilon)$

* $L_2 = \{a^n cc b^p \; / \; n > 0, p \geq 0\}$

$\hookrightarrow G_2 \; (S \to aS / accA \; ; \; A \to bA / \varepsilon)$ 1,25

$\equiv \; S \to AccB \; ; \; A \to aA / a \; , \; B \to bB / \varepsilon$

II) * $G_1 = a\overset{1}{S}bB / \overset{2}{\varepsilon} \quad B \overset{3}{\to} b$

$S \xrightarrow{1 \; n} a^n S(bB)^n \xrightarrow{3} a^n (b\cancel{B})^n \xrightarrow{3 \; n} a^n \underbrace{(bb) \ldots (bb)}_{n \text{ fois}}$

$L(G) \Rightarrow \{a^n (bb)^n \quad n \geq 0\}$ 1,25

* $G_2 = S \overset{1}{\to} AccB \; ; \; A \overset{2}{\to} aAb / \overset{3}{\varepsilon} \quad B \to b\overset{4}{B}a / \overset{5}{ba}$

$A \xrightarrow{2 \; n} a^n A b^n \xrightarrow{3} a^n b^n \quad n \geq 1$

$B \xrightarrow{4 \; m-1} b^m B a^m \xrightarrow{5} b^m a^m \quad m \geq 1$

$L(G) = \{a^n b^n cc b^m a^m \quad n \geq 0, m \geq 1\}$ 1,25

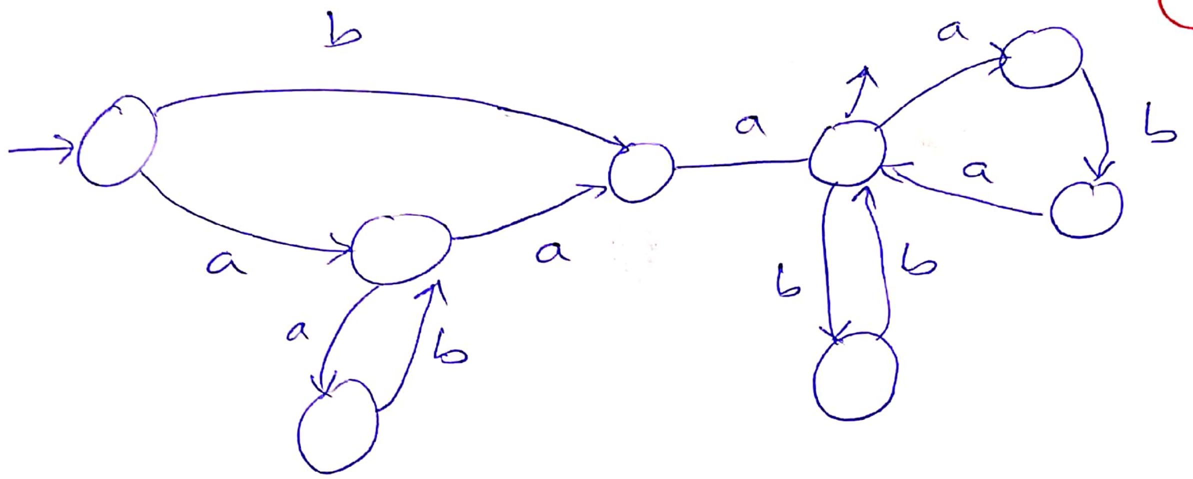
5pts

Ex2

5,5

Ex $(b + a(ab)^*a)\,a\,(aba + bb)^*$

1,25



II) a)

$$\begin{cases} S_1 = aS_2 + bS_3 + cS_4 \\ S_2 = cS_2 + bS_1 \\ S_3 = cS_3 + aS_4 \\ S_4 = aS_2 + cS_1 + \varepsilon \end{cases}$$

0,75

b) On a l'automate déterministe.

	a	b	c
1	2	3	4
2	—	1	2
3	4	—	3
4	2	—	1

0,25

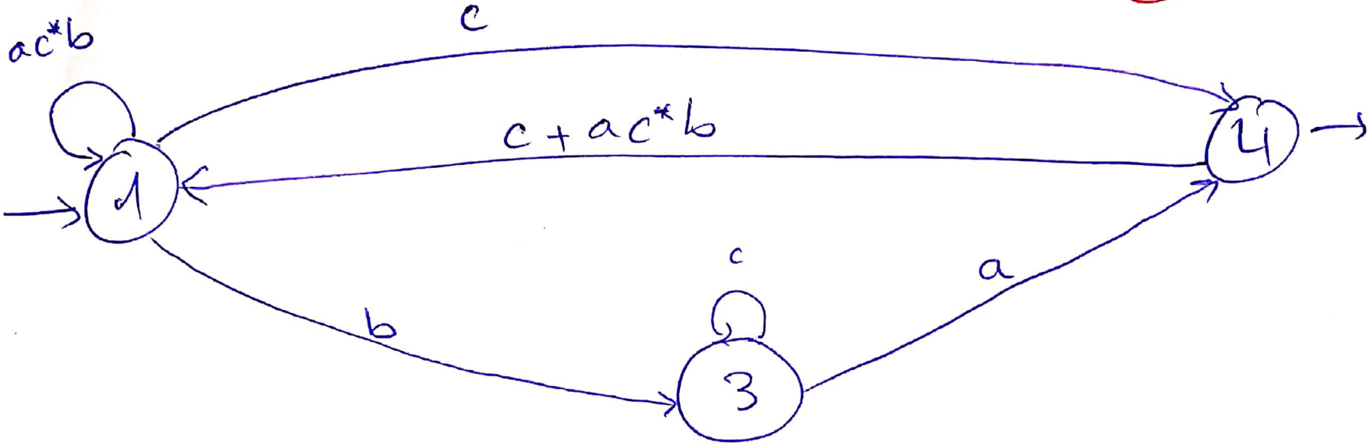
$\{1, 2, 3\}\ \{4\} \to Q_F$ 0,25

Alors ensemble indécomposable

Donc 0,5 l'automate est minimal

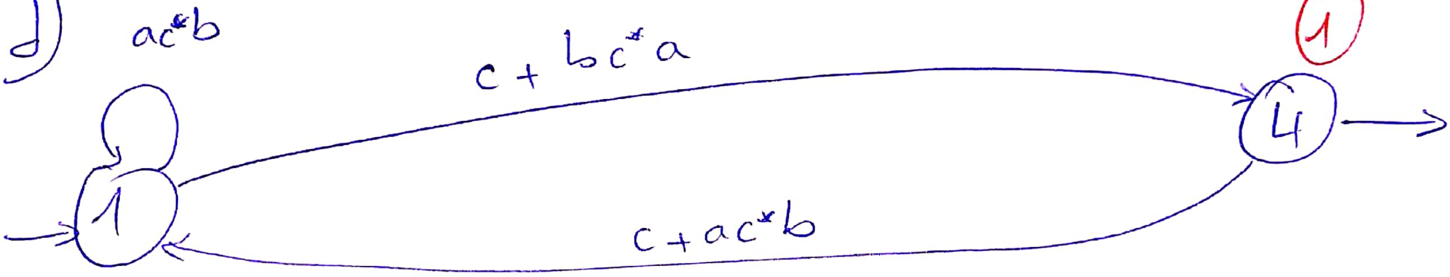
Après l'élimination de l'état 2

1



d)

1



e) $\left(ac^*b + (c + bc^*a)(c + ac^*b)\right)^* \cdot (c + bc^*a)$ 0,5

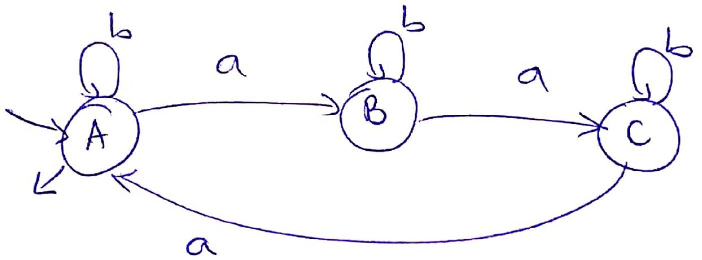
Ex3:

7

L_1: $b^*ab^*ab^*$

A_1)

1

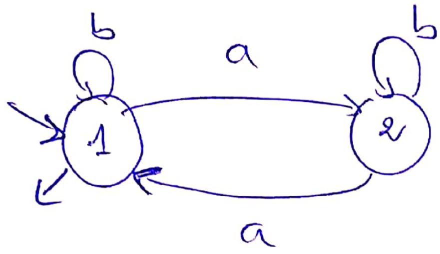


Déterministe et Complet.

L_2

A_2)

1



L_3

A_3)

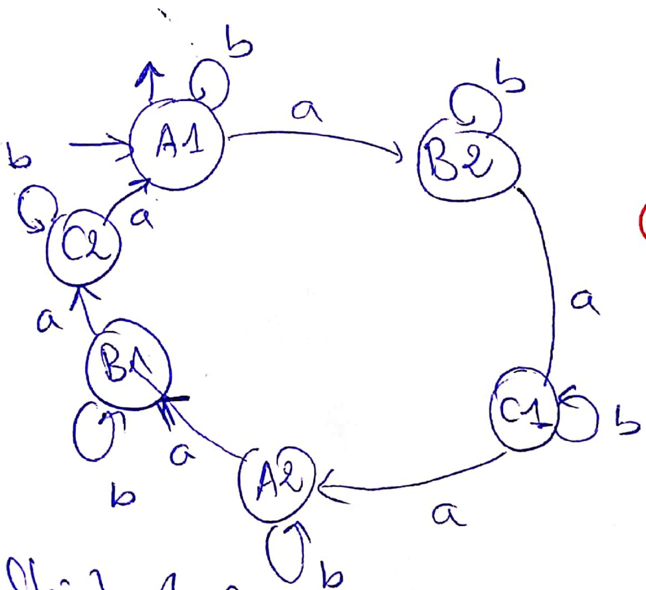
$A_3 = A_1 \times A_2$

	a	b
A1	B2	A1
A2	B1	A2
B1	C2	B1
B2	C1	B2
C1	A2	C1
C2	A1	C2

Etat - entrée: A1

Etats - Sortie: A1

2

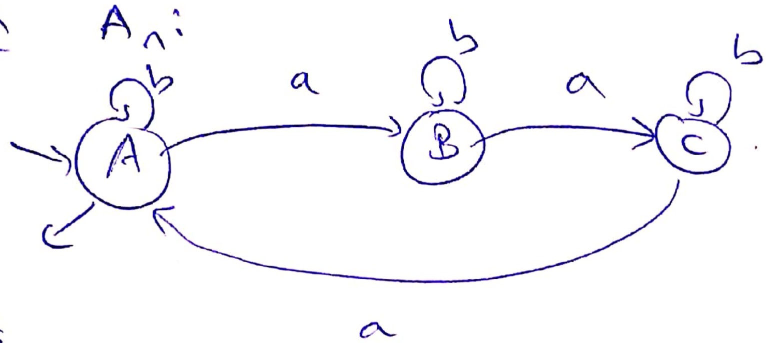


multiple de 3 et multiple de 2 en même temps ⇝ Donc multiple de 6 = 3×2

4) $L_4 = L_2 \cap \overline{L_1}$

$A_4 = A_2 \times A'_1$

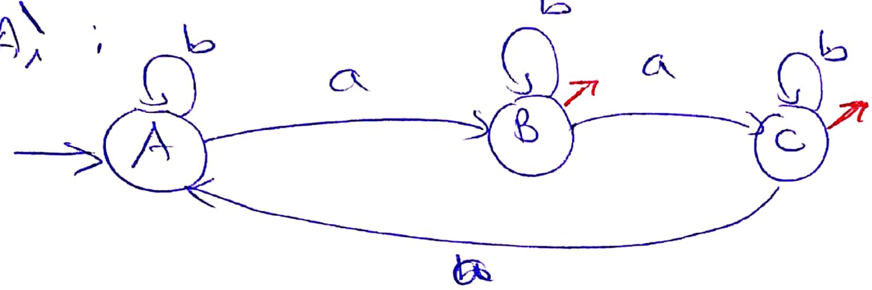
<u>On a</u> A_1:



Déterministe et Complet.

<u>Donc</u>

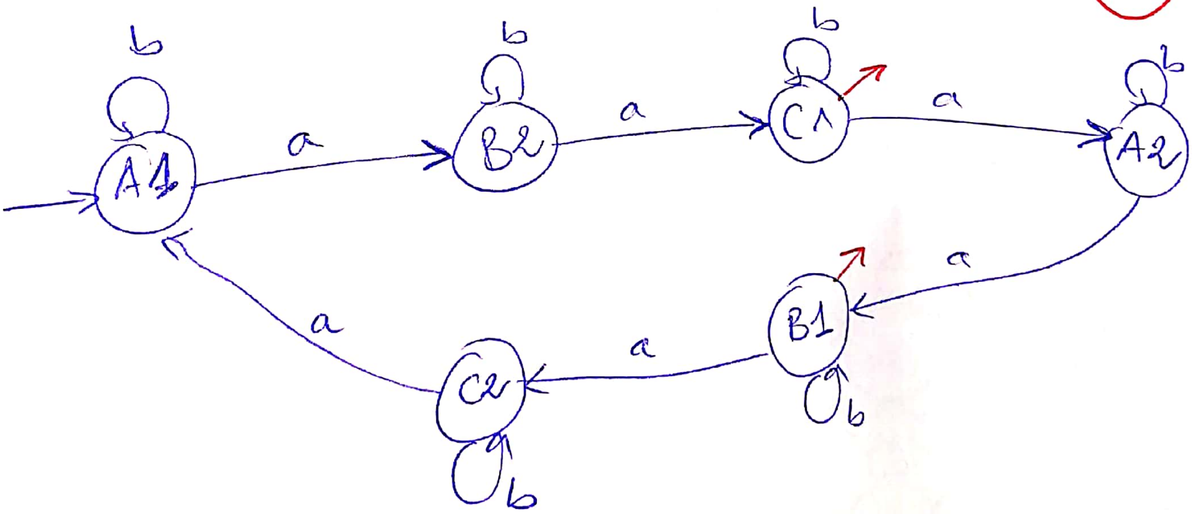
A'_1 :



<u>A_4:</u> le m̂ que A_3 mais: Etats sortie.

$\{B1, C1\}$

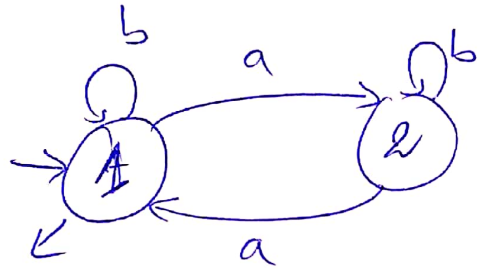
1



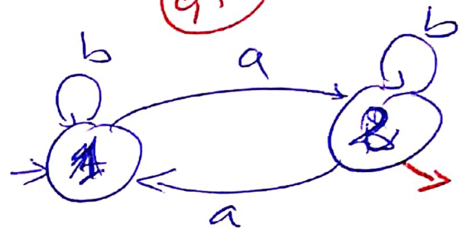
1 On a A_1' (0,5)

2 On a A_2 est un automate déterministe et complet Donc

A_2^*

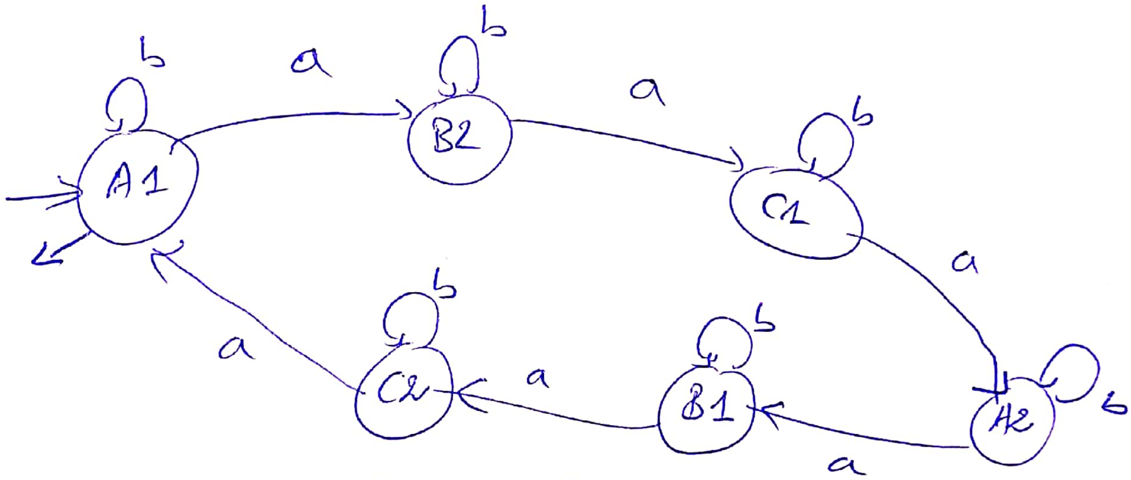


$\rightsquigarrow A_2'$ (0,5)



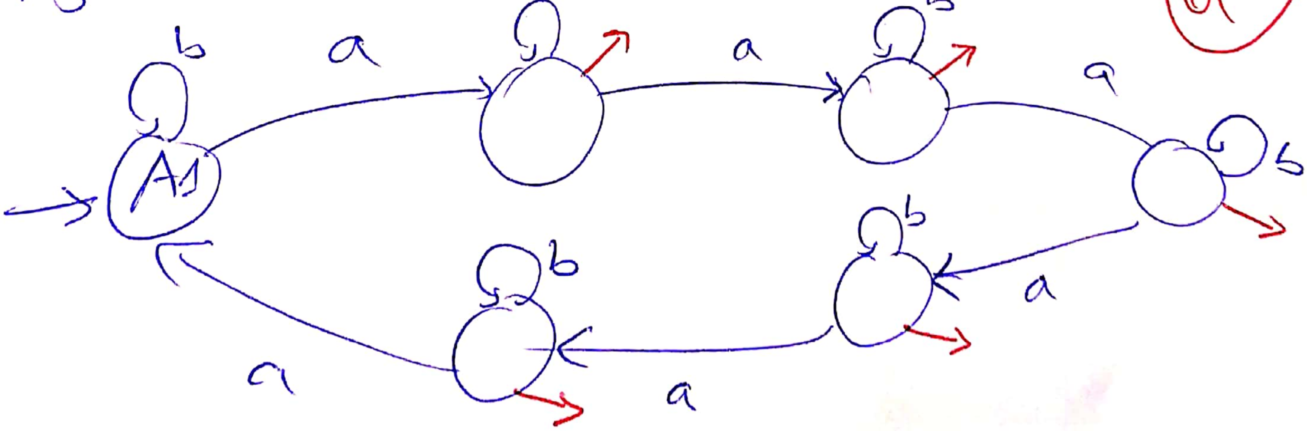
3 :

A_3



A_3 : Deterministe et Complet.

A_3' (0,5)



Exo4 : ~~eq~~, b (0,5) + (0,5)   6

$L = w \in \{a,b,c\}^* \; / \; |w|_a = |w|_b + |w|_c$

On a le nbre de (a) = nbr(b) + nbr(c) si ~~[illegible]~~ (B)

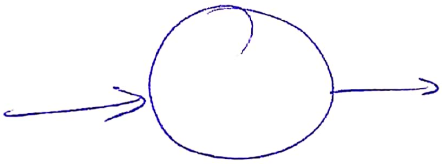
Donc lorsque on lu (a) A, ~~[illegible]~~ empile (A) ou depile (B)

lu (b) ou lu (c), on depile (A)

" ou empile (B)

(1) (1,5)

a.Z/AZ	b.Z/BZ	c.Z/BZ
a.A/AA	b.B/BB	c.B/BB
a.B/ε	b.A/ε	c.A/ε



L'automate est deterministe. (0,5)

1/ $S \to aSb / aSc / bSa / cSa / \varepsilon$

G n'est pas propre:

Rendre G propre:

① Ajout sommet S' ; $S' \to S$ 0,25

② Eliminer $S \to \varepsilon$ 0:

$\hookrightarrow$ $S' \to S$

$S \to aSb / aSc / bSa / cSa$ 0,5

$S \to ab / ac / ba / ca$

③,④ n'existe pas de symbole inaccessible ni improductive. 0,25

Donc G est propre.

transformation

$S \to aSb \Rightarrow \begin{cases} S \to AX \\ A \to a \\ X \to SB \\ B \to b \end{cases}$

$S \to aSc \Rightarrow \begin{cases} S \to AY \\ Y \to SC \\ C \to a \end{cases}$

$S \to bSa \Rightarrow \begin{cases} S \to BZ \\ Z \to SA \\ A \to a \end{cases}$

$S \to cSa \Rightarrow \{ S \to CZ$ 1

$S \to ab \Rightarrow (S \to AB)$

$S \to ac \Rightarrow (S \to AC)$

$S \to ba \Rightarrow (S \to BA)$

$S \to ca \Rightarrow (S \to CA)$