

Project #1: Classical Phase Plane Analysis

ELEC 7560/7566 – Summer 2017

Due: Wednesday, August 2

Consider the second-order nonlinear model $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, where $\mathbf{x} = [x_1 \ x_2]^T$ and $\mathbf{f}(\mathbf{x})$ is the vector field:

$$\begin{aligned} f_1(\mathbf{x}) &= x_2^2 - 1 \\ f_2(\mathbf{x}) &= x_1^2 - 1. \end{aligned} \tag{1}$$

Nullclines (4 points) Describe and sketch the four nullclines of the phase portrait. Hint: There are two nullclines satisfying $(\dot{x}_1 = 0, \dot{x}_2 \neq 0)$, and two nullclines satisfying $(\dot{x}_1 \neq 0, \dot{x}_2 = 0)$.

Equilibrium points (4 points) Describe and sketch the four equilibrium points (equilibrium states). Hint: Study the intersections of nullclines.

Linearization (8 points) Find the linear approximate model around each equilibrium point.

Linear analysis (8 points) Study each of the four linear models (study the eigenvalues), and describe the associated phase portraits.

Limit cycle theorems Apply Poincaré-Hopf's theorem (8 points), Bendixson's theorem (8 points), and the Poincaré-Bendixson theorem (8 points) to the analysis, and try to answer these questions:

1. Could a limit cycle (closed trajectory in the phase plane) exist in the neighborhood of the first equilibrium point? the second equilibrium point? the third equilibrium point? the fourth equilibrium point?
2. Could a limit cycle enclose a combination of two equilibrium points?
3. Could a limit cycle enclose a combination of three equilibrium points?

4. Could a limit cycle enclose all four equilibrium points?

Computer-based graphical analysis (12 points) Study the nonlinear vector fields and/or phase portrait using `pplane` or other computer tool. Compare computer-based analysis to the earlier findings. Specifically:

1. Discuss the extent to which linearization and linear analysis is a good approximation of the nonlinear model (1).
2. Discuss the extent to which the various limit cycle analysis theorems agree with the computer-based graphical results.

Write a report to summarize the findings. Total points: 60.