Sliding Mode Dynamics

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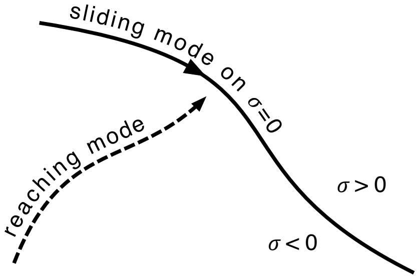
Sliding and Reaching Modes

Consider a scalar-valued function $\sigma(\mathbf{x})$, where $\mathbf{x} \in \mathrm{R}^n$ and $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)$.

switching surface - The surface defined by $\sigma = 0$.

sliding mode - dynamics that satisfy $\sigma = 0$, i.e. dynamics "on" the switching surface

reaching mode - dynamics that are not the sliding mode, i.e. dynamics "off" the switching surface



Describing the sliding mode

The sliding mode is most easily described for the case of single input u. Furthermore,

1. the process state equation is in controllable form:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\vdots \\
\dot{x}_n &= -a_0 x_1 - \cdots - a_{n-1} x_n + g(\mathbf{x}) u + \delta(t)
\end{aligned}
\tag{1}
$$

   where $g(\mathbf{x})$ is a scalar-valued nonlinearity, and $\delta(t)$ is a disturbance.

2. The switching surface is linear in the state :

$$\sigma(\mathbf{x}) = c_0 x_1 + \cdots + c_{n-2} x_{n-1} + x_n = 0. \tag{2}$$

From (2), the sliding mode satisfies a constraint:

$$x_n = -c_0 x_1 - \cdots - c_{n-1} x_{n-1}. \tag{3}$$

Substituting (3) into process model (1) yields the model of the sliding mode:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ &= -c_0 x_1 - \cdots - c_{n-2} x_{n-1}. \end{aligned} \tag{4}$$

Characteristics of the sliding mode

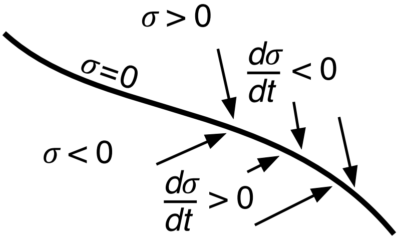
reduced order - The original process is order n, but the sliding mode (4) is order $n-1$.

characteristic equation - The sliding mode (4) has characteristic equation:

$$s^{n-1} + c_{n-2}s^{n-2} + \cdots + c_1 s + c_0 = 0$$

total invariance - The sliding mode (4) is completely independent of the process parameters $a_0, \ldots a_{n-1}$, nonlinearity $g(\mathbf{x})$ and disturbance $\delta(t)$.

Conditions for existence of the sliding mode



If $\sigma > 0$, then $\dot{\sigma} < 0$.
If $\sigma < 0$, then $\dot{\sigma} > 0$.
In summary,

$$\sigma\dot{\sigma} < 0 \tag{5}$$

General design concepts for sliding mode control

Given the process dynamic $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)$:

1. Choose a surface equation $\sigma(\mathbf{x})$ such that dynamics on $\sigma = 0$ are asymptotically stable.
2. Design input u to satisfy sliding mode existence condition $\sigma\dot{\sigma} < 0$.
   - Hint: Consider the Lypunov function $V(\mathbf{x}) = \sigma^2(\mathbf{x})$, and design u so that $\dot{V}(\mathbf{x})$ is negative definite.
   - Note:
$$\dot{\sigma}(\mathbf{x}) = (\nabla_{\mathbf{x}}\sigma)\,\dot{\mathbf{x}} = (\nabla_{\mathbf{x}}\sigma)\,\mathbf{f}(\mathbf{x}, u)$$

Some references

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