Some Sliding Mode Control Design Approaches

also called Variable Structure Control

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Three approaches

Recall the condition for a sliding mode on the surface $\sigma(\mathbf{x}) = 0$:

$$\sigma\dot{\sigma} < 0.$$

Several common approaches for controller design include:

1. switching control based on signs of $\sigma x_1, \ldots \sigma x_n$
2. equivalent control: divides the problem into two parts
3. reaching model control: designs reaching mode dynamics so that $\sigma \to 0$.

Given process model $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)$, choose acceptable switching function $\sigma(\mathbf{x})$.
Then, *read on*...

Design based on signs of switching function and state

1. Choose state feedback form $u = \mathbf{K}\mathbf{x}$, where elements of $\mathbf{K} = [k_1 \dots k_n]$ may be switching functions.
2. The condition for a sliding mode will be:

$$\begin{aligned} \sigma\dot{\sigma}(\mathbf{x}) &= \sigma\left(\nabla_{\mathbf{x}}\sigma\right)\dot{\mathbf{x}} \\ &= \sigma\left(\nabla_{\mathbf{x}}\sigma\right)\mathbf{f}(\mathbf{x},u) \\ &= \sigma x_1\left[\phi_1(\mathbf{x}) + k_1\right] + \cdots + \sigma x_n\left[\phi_n(\mathbf{x}) + k_n\right] \qquad (1) \end{aligned}$$

3. Find upper bounds $\gamma_i > |\phi_i(\mathbf{x})|$, $i = 1, \dots, n$.
4. Choose k_i so that each term in (1) is negative. A typical form:

$$k_i = \begin{cases} -\gamma_i & \text{, for } \sigma x_i > 0 \\ \gamma & \text{, for } \sigma x_i < 0 \end{cases} \qquad (2)$$

Equivalent control approach

1. Write control input u in two parts:

$$u = u_{eq} + u_{sw}$$

   where

   - u_{eq}: input that satisfies $\dot{\sigma} = 0$ on surface $\sigma = 0$ in the absence of disturbances and uncertainties, and
   - u_{sw}: input that ensures sliding existence condition $\sigma\dot{\sigma} < 0$ in the presence of disturbances and uncertainties

2. Solve for equivalent control u_{eq}, using the relationship:

$$\dot{\sigma} = (\nabla_{\mathbf{x}}\sigma)\,\mathbf{f}(\mathbf{x}, u_{eq}) = 0 \tag{3}$$

3. Choose switching control to ensure $\sigma\dot{\sigma} < 0$. A typical choice is based on sign of σ:

$$u_{sw} = \begin{cases} -K & \text{, for } \sigma > 0 \\ K & \text{, for } \sigma < 0 \end{cases} \tag{4}$$

Reaching mode control approach

Many variations are possible. Here are two:

- Choose Lyapunov function $V(\mathbf{x}) = \sigma^2$, and design u so that $\dot{V} = \sigma\dot{\sigma} < 0$ (negative definite). Then $\sigma = 0$ is an asymptotically stable surface, i.e. state converges to surface.
- Design u so that

  $$\dot{\sigma} = -\alpha\sigma.$$

  Then

  $$\sigma(t) = \sigma(0)e^{-\alpha t}, \text{for } t \geq 0.$$

Note: If convergence to $\sigma = 0$ is not in finite time, then total invariance may be lost, because true sliding mode does not occur.