Project #5: Variable Structure & Backstepping Control

ELEC 7560/7566 – Summer 2017

Due: Wednesday, August 2

Consider the nonlinear system (Van der Pol oscillator with input u)

$$\ddot{y} + (y^2 - 1)\dot{y} + y = u$$

Using state variables $x_1 = y$, $x_2 = \dot{y}$, write the state variable model of the system. Then, answer one of the following two problems (choose one design approach).

1 Sliding Mode Control

(50 pts)

1. Pick a switching surface equation $\sigma(x) = 0$ that corresponds to a sliding mode with unit time constant (equal to one).
2. For a sliding mode to exist, trajectories near $\sigma(x) = 0$ must satisfy the condition $\sigma\dot{\sigma} < 0$. Try to find a control u that satisfies the sliding mode condition.
3. Study the behavior of the closed loop system by computer simulation. Create a phase portrait, showing the response from several initial conditions. Note: Numerical integration methods that employ variable step size often struggle with sliding mode systems, because the step size reduces to find accurate results around the zero crossings of the function $\sigma(x) = 0$. Try using a fixed step size routine.

2 Backstepping Control Approach

(50 pts) Two cycles of the backstepping approach are to be performed.

1. In the first cycle, treat x_2 as the input to the x_1 dynamic, and design a pseudo-control (or "virtual control") that yields a stable linear dynamic with unit time constant.
2. In the second cycle, design the control that cancels nonlinearities, and yields a stable dynamic with unit time constant.
3. Study the behavior of the closed loop system by computer simulation. Create a phase portrait, showing the response from several initial conditions.