Sliding Mode Control Design Example: The Equivalent Control Approach

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1 The Plant

The plant is modeled by a nonlinear state variable model:

$$\dot{x}_1 = x_2 \tag{1}$$
$$\dot{x}_2 = 10 \sin x_1 - x_2 + u \tag{2}$$

2 Sliding Mode Controller Design

2.1 Switching Surface

Choose a linear switching surface equation, such as:

$$\sigma(x) = 4x_1 + x_2. \tag{3}$$

If the system state is on the surface defined by $\sigma(x) = 0$, then the two state variables are related by:

$$x_2 = -4x_1. \tag{4}$$

Substituting (4) into the model (1) yields the "sliding mode," a reduced order dynamic described by:

$$\dot{x}_1 = -4x_1. \tag{5}$$

The sliding mode response takes on the form e^{-4t}, which is stable and has a time constant of 1/4.

2.2 Controller Design

The equivalent control approach divides the the control u into two parts:

$$u = u_{eq} + u_{sw}.$$

Here, u_{eq} is called the equivalent control, and u_{sw} is a switching component. The equivalent control causes the state of the ideal system to have dynamics consistent with $\sigma(x) = 0$. In practice, the actual system has uncertain or changing parameters and disturbances. Therefore, the switching component u_{sw} is used to force the state back onto the surface $\sigma(x) = 0$.

2.2.1 Equivalent control design

If the state x is on the surface $\sigma(x) = 0$, then the equivalent control u_{eq} will cause the state to evolve on the surface, under the assumption of perfect modeling and no disturbances. The condition of staying on the surface can be expressed as:

$$\dot{\sigma}(x) = 0. \tag{6}$$

From the surface equation (3) and nonlinear process model (1)-(2), the equivalent control condition (6) can be examined:

$$\begin{aligned} \dot{\sigma}(x) &= 0 \\ 4\dot{x}_1 + \dot{x}_2 &= 0 \\ 4x_2 + 10\sin x_1 - x_2 + u_{eq} &= 0 \end{aligned}$$

Solving for the equivalent control yields:

$$u_{eq} = -10\sin x_1 - 3x_2. \tag{7}$$

2.2.2 The switching control design

The switching control is designed to force the state back onto the surface $\sigma(s) = 0$. In other words, the switching control is exerted whenever $\sigma \neq 0$. One approach for the switching control is apply a control effort opposite to the sign of σ. For example, if σ is positive, then the control should try to reduce σ.

$$u_{sw} = -K\,\mathrm{sgn}(\sigma) \tag{8}$$

The magnitude of K would be related to the uncertainty or disturbance in the system. A large uncertainty or large disturbance would lead to a large K.

3 Simulation Results

Explanation of the simulation model The SIMULINK diagram for the control system is shown in Fig. 1(a). Details of the plant block are in Fig. 1(b). The plant diagram differs slightly from the model (1)-(2) in two ways:

1. The magnitude of the simulated nonlinearity is 15, whereas the design assumption is that the magnitude is 10. The simulation will examine the response to this modeling error.

2. The simulated state x is now the *error state*. In other words, the error state is the difference between the process state and a desired reference signal (a step function). The reference signal is a step change of 90°, or $\pi/2$ radians.

In Fig. 1(a), the state x is used to construct the switching surface function sigma. State feedback is also used to build the equivalent control (7). Note that the equivalent control is based on the modeling assumption for the nonlinearity. The switching control (8) with value $K = 20$ is added to the equivalent control within the block labeled VSC. The output of that block is the controller output or plant input, labeled u.

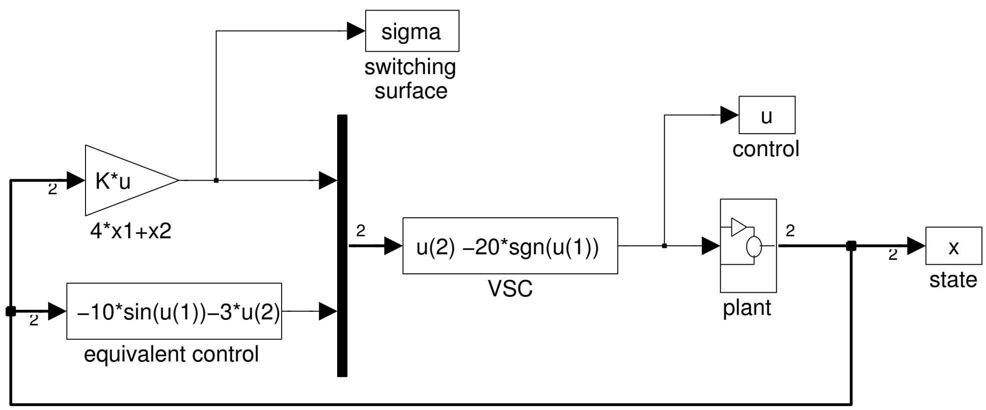
The state response The simulated error state response is plotted in Fig. 2. Output error is the solid plot. The initial error is $-\pi/2$. The derivative of output error is the dashed plot. The first-order sliding mode response may be barely perceptible in the output error (solid plot), but is very evident in the derivative of output error. The sliding mode response starts begins at approximate time $t = 0.2$.

The state response in a phase portrait is shown in Fig. 3. Reaching and sliding modes are evident. Chatter of the state trajectory around the surface $\sigma(x) = 0$ might be detected upon very close inspection.

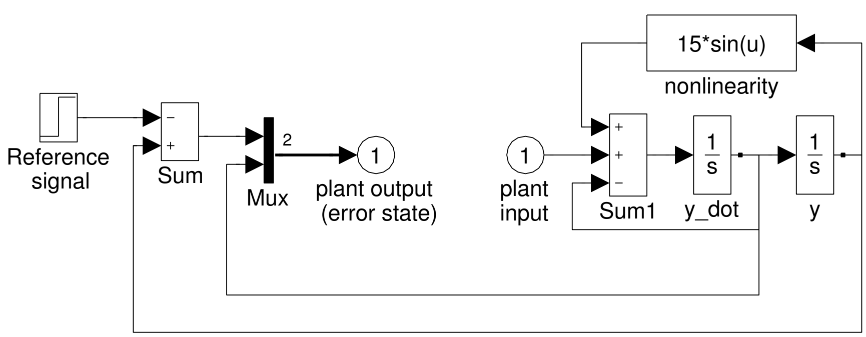
The switching surface Behavior of the sliding mode switching equation (3) is plotted in Fig. 4. The time periods for reaching and sliding are evident. The chatter around $\sigma = 0$ might be perceptible, as the plot is a bit thicker beyond time $t = 0.2$.

The plant input The plant input or controller output u is plotted in Fig. 5. Here, the control chatter is evident. The chatter is not significant at the output variable (solid curve in Fig. 2), due to the lowpass nature of the plant. The sliding mode period begins after time $t = 0.2$. The smooth equivalent control u_{eq} and the switching control u_{sw} are evident.

VSC of a nonlinear system using the equivalent control approach.



(a)



(b)

Figure 1: (a) Variable structure control using equivalent control approach. (b) Details of the plant.

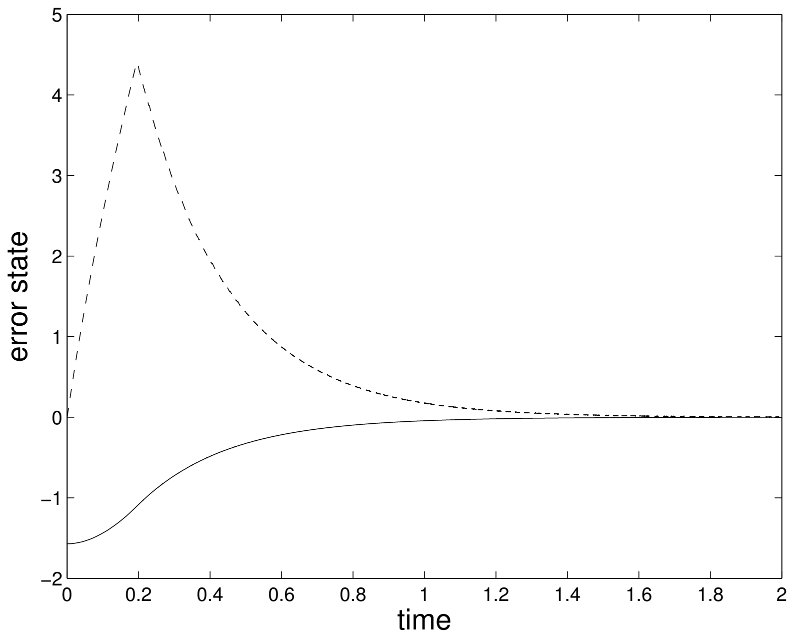


Figure 2: Error state responses: (solid) output error, (dashed) derivative of output error. Sliding mode response starts at approximately $t = 0.2$.

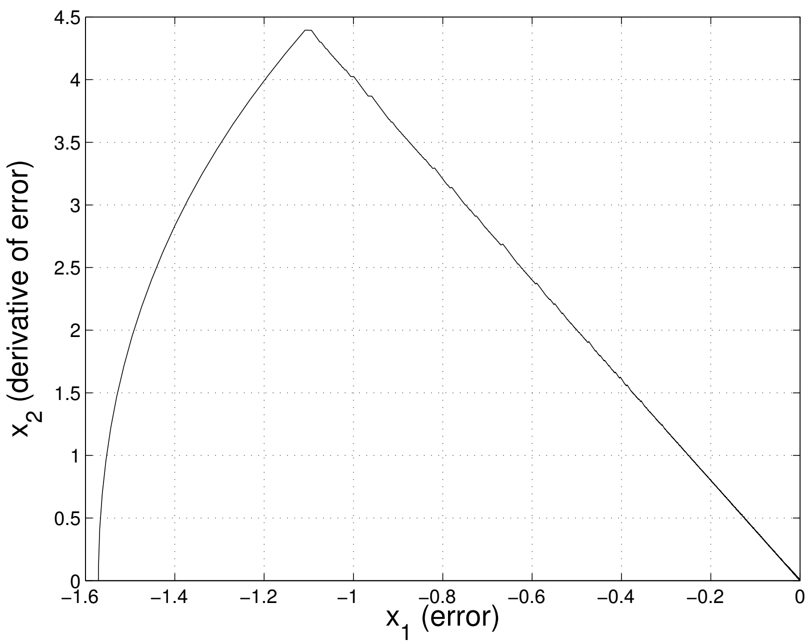


Figure 3: Variable structure system phase portrait. The initial state is at $(-1.57, 0)$, and the state moves toward the origin at the right. The initial response (curved part of the plot) is called the "reaching mode." The switching surface $\sigma(x) = 0$ is the straight line with slope -4.

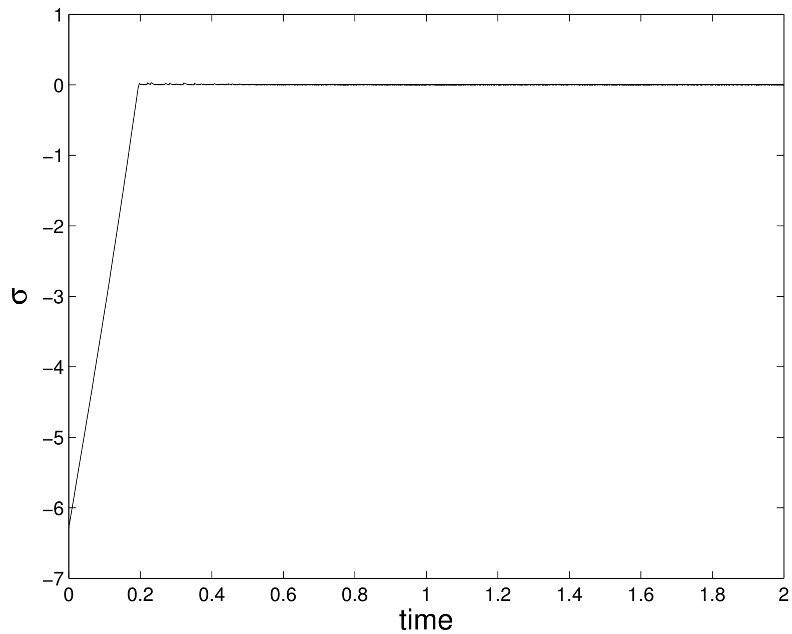


Figure 4: Switching "surface" function $\sigma(x)$ vs. time t.

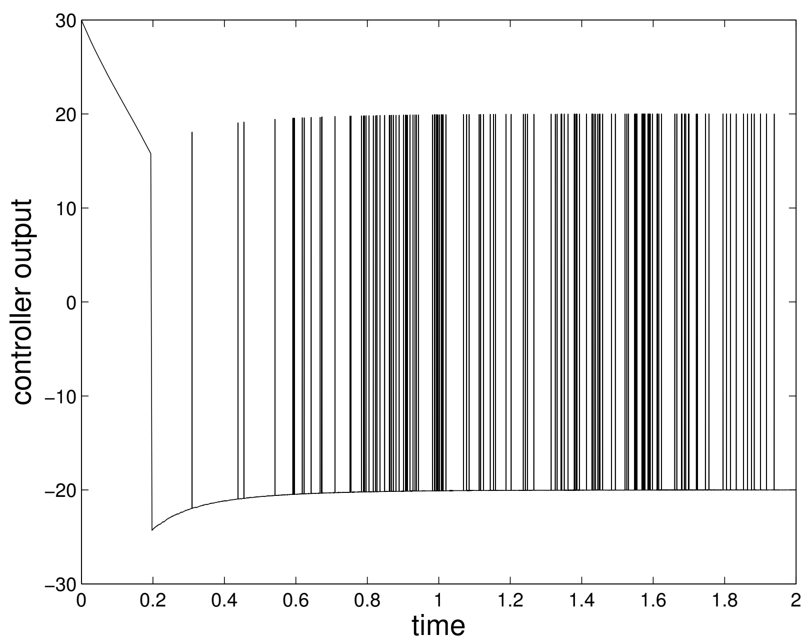


Figure 5: Plant input (controller output) u. The sliding mode begins after time $t = 0.2$. The equiivalent control u_{eq} and switching control u_{sw} are clearly visible.