Describing Function Analysis Example

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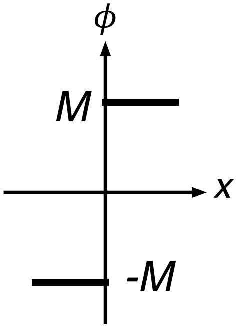
Relay nonlinearity



Can be modeled as

$$\phi = M\,\mathrm{sgn}(x)$$

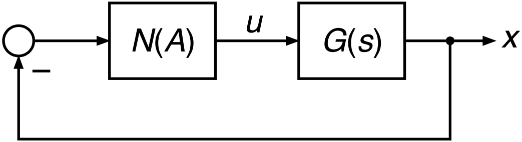
where M is the magnitude.

Describing function is

$$N(A) = \frac{4M}{\pi A}$$

where A is the amplitude of the input sinusoid.

The control system



Linear plant:

$$G(s) = \frac{1}{(s+1)^3}$$

Nonlinear feedback control:

$$u = -M\,\mathrm{sgn}(x)$$

Let $M = \pi/4$. Then $N(A) = 1/A$.

Nyquist-like analysis

From the closed-loop characteristic equation, comes the relationship:

$$G(\omega) = \frac{-1}{N(A)}.$$

For this example, the result is:

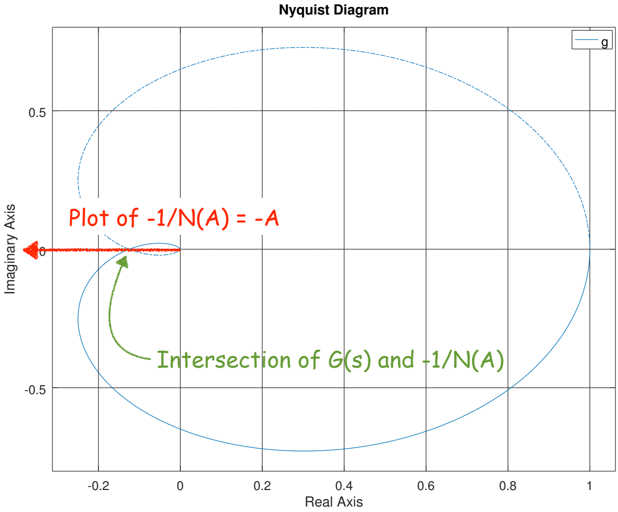
$$G(\omega) = -A.$$

Plot these on the same diagram:

- Nyquist plot of $G(\omega)$
- $-1/N(A) = -A$, for $A \in [0, \infty)$.

Prediction of limit cycle

Study intersection of $G(\omega)$ and $-1/N(A)$, i.e. negative real axis.



Intersection details

1. On negative real axis, the intersection ≈ -0.125, so $A \approx 0.125$.
2. On $G(\omega)$ curve, the frequency $\omega \approx 1.7$ rad/s.
   Hint: It is easier to study the Bode diagram... find frequency where the gain equals 0.125 or -18 dB.

Summary: The intersection that satisfies

$$G(\omega) = \frac{-1}{N(A)} = -A$$

predicts a limit cycle characterized by:

- amplitude $A \approx 0.125$
- period $T = 2\pi/\omega \approx 3.7$ s.

Simulation result

x: output, u: input