Project #2: Analysis and Controller Design Using Lyapunov's Direct Method

ELEC 7560/7566 – Summer 2017

Due: Wednesday, August 2

Consider the nonlinear plant model

$$\ddot{y} + 2y^3\dot{y} + y = u.$$

1. **Nonlinear model** – Define state variables as $x_1 = y$, $x_2 = \dot{y}$, and derive a state variable model of the nonlinear system, taking the form:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u). \tag{1}$$

2. **Equilibrium point** – Show that the origin $(x_1 = 0, x_2 = 0)$ is an equilibrium state for (1) when $u = 0$.

3. **Linear model** – Find a linear approximate model $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u$ for dynamics near the equilibrium.

4. **Linear analysis** – Analyze stability of the equilibrium using the linear model. In other words, analyze the eigenvalues of matrix $\mathbf{A}$. Assume input $u = 0$ for this analysis.

5. **Phase portraits** – Generate phase portraits (around the equilibrium) for both the nonlinear state variable model and the linear state variable model. (Let input $u = 0$.) The tool pplane.jar can be downloaded from the Internet to produce phase portraits. Compare the nonlinear and linear phase portrait diagrams. What does linear analysis reveal about stability of the equilibrium in the nonlinear model?

6. **Linear control design** – Consider a linear state feedback control $u = -\mathbf{K}\mathbf{x}$. That is, $\mathbf{K}$ is a 1×2 matrix, so that $u = -k_1x_1 - k_2x_2$. Design the gain matrix $\mathbf{K}$ so that the closed-loop system matrix $\mathbf{A} - \mathbf{b}\mathbf{K}$ has eigenvalues at $-1 \pm j1$. The MATLAB command place can be used to compute the gains k_1 and k_2.

7. **Linear control of the linear model** – Write state equations for the closed loop system constructed from the linear state variable model (Prob. 3) and the linear control $u = -Kx$ (Prob. 6). Create a phase portrait around the equilibrium.

8. **Linear control of the nonlinear model** – Write state equations for the closed loop system constructed from the nonlinear model (Prob. 1) and linear control $u = -\mathbf{K}\mathbf{x}$ (Prob. 6). Create a phase portrait around the equilibrium. Compare to the portrait created for Problem 7, both near the equilibrium and "far" from the equilibrium.

9. **Nonlinear control design** – Using Lyapunov's Direct Method of stability analysis, design an asymptotically stabilizing nonlinear feedback control $u = g(x_1, x_2)$ for the nonlinear state variable model.

10. **Nonlinear control of the nonlinear model** – Write state equations for the closed loop system constructed from the nonlinear model (Prob. 1) and nonlinear control $u = g(\mathbf{x})$ (Prob. 9). Create a phase portrait around the origin. Compare to the portraits created for Problem 7 and Problem 8.

Please submit a handwritten report when finished, or submit the report with the other projects at the end of the semester. Each problem above is worth 10 points.