Project 4: Feedback Linearizing Control

1)

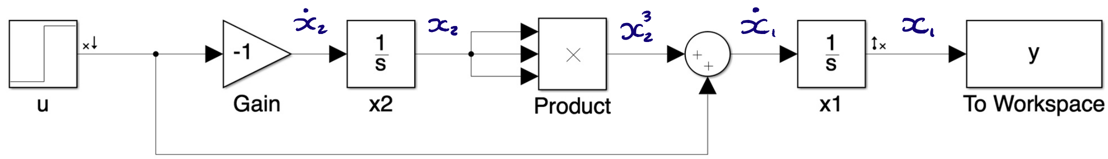

Figure 1: Model of Nonlinear System

$$\dot{x} = f(\underline{x}) + g(x)u = \begin{bmatrix} x_2^3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$y = h(x) = x_1$$

$$u = a(x) + b(x) v$$

$$f(\underline{x}) = \begin{bmatrix} x_2^3 \\ 0 \end{bmatrix} \qquad g(x)u = \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$

$$h(\underline{x}) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T \underline{x}$$

$$\dot{y} = L_f h(\underline{x}) + L_g h(x) u \to 0 \qquad (n = 1)$$

$$= \dot{x}_1 + 0 \qquad (r = 1)$$

$$= x_2^3 + u$$

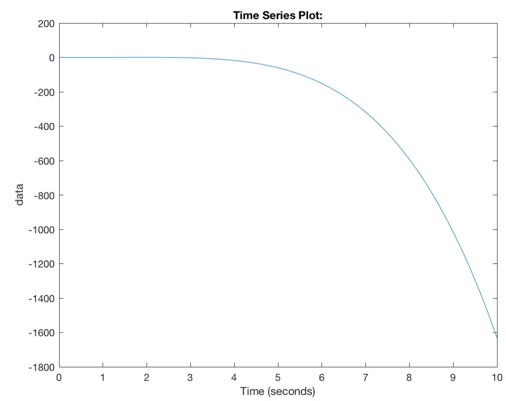

Figure 2: Unit-Step Response of Nonlinear System showing unstable output.

$$u = \frac{1}{L_g L_f h(x)} \left(-L_f h(x) + V \right)$$

$$= \frac{1}{\cancel{L_g x_1} \to 1} \left(-x_2^3 + V \right) \to V = x_2^3 + u$$

$$V = -K\underline{z}$$

$$\underline{z} = T(x) = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2^3 + u \end{bmatrix} \qquad \dot{z}_1 = x_2^3 + u$$

$$\dot{y} = \dot{x}_1 = x_2^3 + u$$

$$u = -x_2^3 + -y + v$$

$$\dot{y} = -y + u$$

$$Ys = -Y + V$$

$$Y(s+1) = U$$

$$\frac{Y}{V} = \frac{1}{s+1} \leftarrow \text{pole/eigenvalue as specified}$$

Find open loop transfer function.

$$\frac{G(s)}{1 + G(s)} = \frac{1}{s+1} \qquad G(s) = \frac{1 + G(s)}{s+1}$$

$$G(s) = \frac{1}{s}$$

Internal Dynamics and Simulation

For analyzing the zero dynamics we must choose a variable that is independent of y. Since x_2 is not dependent on y we analyze the internal dynamics of the system by studying $\dot{x}_2$ both analytically and by simulating in Simulink.

$$\dot{x}_2 = -u$$

Since we designed $u = -x_2^3 + v - y$

$\dot{x}_2 = x_2^3 - v - y$ which yields a nonlinear, unbounded and unstable internal dynamics. This is confirmed by Simulink as well.

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An error occurred while running the simulation and the simulation was terminated
Caused by:
 • Derivative of state '1' in block 'nonlinear_model/Compensated Nonlinear /Nonlinear
   System/dx2->x2 ' at time 2.3136077677688252 is not finite. The simulation will be
   stopped. There may be a singularity in the solution.  If not, try reducing the step
   size (either by reducing the fixed step size or by tightening the error tolerances)
Component: Simulink | Category: Block error
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Figure 3: Error from Simulink when integrating x2.

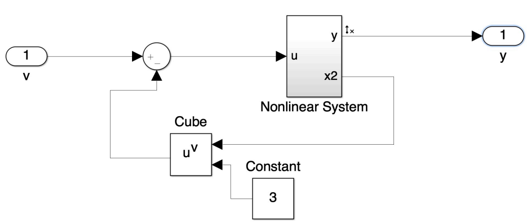


Figure 4: Feedback Linearized System. Note that the nonlinear system in Figure 1 has been obfuscated into a subsystem block.

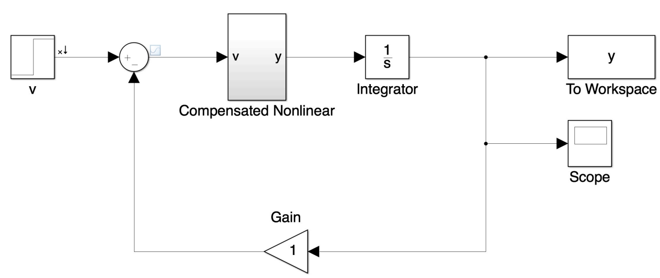


Figure 5: Complete System

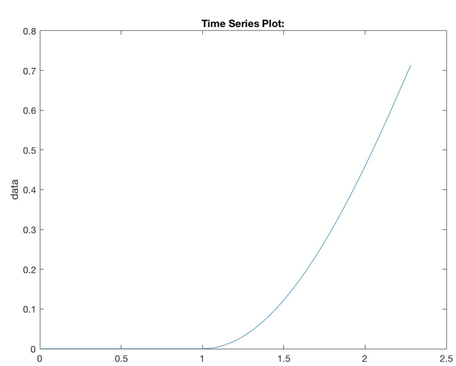


Figure 6: Unstable behavior noted before singularity occurs.

2.) $\dot{x}_1 = \sin(x_2)$ $\qquad n=4,\ r=2$

$$\dot{x}_2 = x_1^4 \cos(x_2) + u$$

$$y = x_1$$

$$\dot{y} = \dot{x}_1 = \sin(x_2)$$

$$\ddot{y} = \cos(x_2)\,\dot{x}_2 = \cos(x_2)\left(x_1^4 \cos(x_2) + u\right)$$

$$= \cos^2(x_2)\,x_1^4 + \cos(x_2)\,u$$

$$u = \frac{-\cos^2(x_2)\,x_1^4}{\cos(x_2)} = -\cos(x_2)\,x_1^4 + v$$

Substituting,

$$\ddot{y} = \cos^2(x_2)\,x_1^4 + \cos(x_2)\left(-\cos(x_2)\,x_1^4\right)$$

$$= \cancel{\cos^2(x_2)\,x_1^4} - \cancel{\cos^2(x_2)\,x_1^4} + v$$

Tracking

$\ddot{y} = v$ $\qquad$ (Valid as long as $\cos(x_2) \neq 0 \rightarrow x_2 \neq \frac{\pi}{2} n$)

$e = y(t) - y_d(t)$ $\qquad$ (where $y_d(t) \equiv x_{d_1}(t)$)

$v = \ddot{y}_d - k_1 e - k_2 \dot{e}$

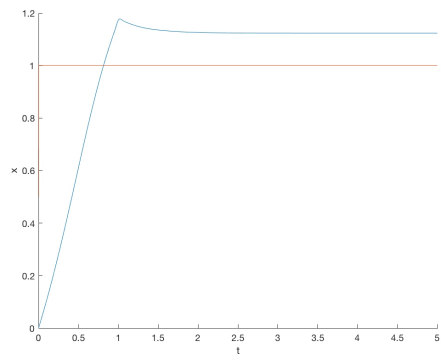
Internal Dynamics

$$\dot{x}_2 = \cancel{x_1^4 \cos(x_2)} + \left(\cancel{-\cos(x_2)\,x_1^4} + (\ddot{y}_d - k_1 e - k_2 \dot{e}\right)$$

$$\dot{x}_2 = \ddot{y}_d - k_1 e - k_2 \dot{e}$$

Since we can assume $\ddot{y}_d$, e, and $\dot{e}$ are bounded, the ID are stable.

Using ode45 to simulate in Matlab:



3) Input State Linearization

$$\dot{x}_1 = x_1 - x_1 u_1 = x_1(1-u_1)$$
$$\dot{x}_2 = (1-\ln(x_3))x_2 - x_2 u_1 = (1-z_2)x_2 - x_2 u_1$$
$$\dot{x}_3 = -x_1 x_3 - x_3 u_1 + u_2 = -x_3(u_1 + x_1) + u_2$$

$$z_1 = \ln\left(\frac{x_1}{x_2}\right) = \ln(x_1) - \ln(x_2)$$
$$z_2 = \ln(x_3)$$
$$z_3 = \ln(x_1)$$

$$\dot{z}_2 = \frac{\dot{x}_3}{x_3} = \frac{u_2}{x_3} - x_1 - u_1$$

$$\dot{z}_3 = \frac{\dot{x}_1}{x_1} = \boxed{1-u_1}$$

$$\dot{z}_1 = \frac{\dot{x}_1}{x_1} - \frac{\dot{x}_2}{x_2}$$

$$= -u_1 - \frac{u_1 \cancel{x_2} + \cancel{x_2}(1-z_2)}{\cancel{x_2}}$$

$$= 1 - u_1 - \left[\frac{(1-z_2)\cancel{x_2} - \cancel{x_2}u_1}{\cancel{x_2}}\right]$$

$$\cancel{1} - \cancel{u_1} \cancel{-1} + z_2 + \cancel{u_1}$$

$$\boxed{= z_2}$$

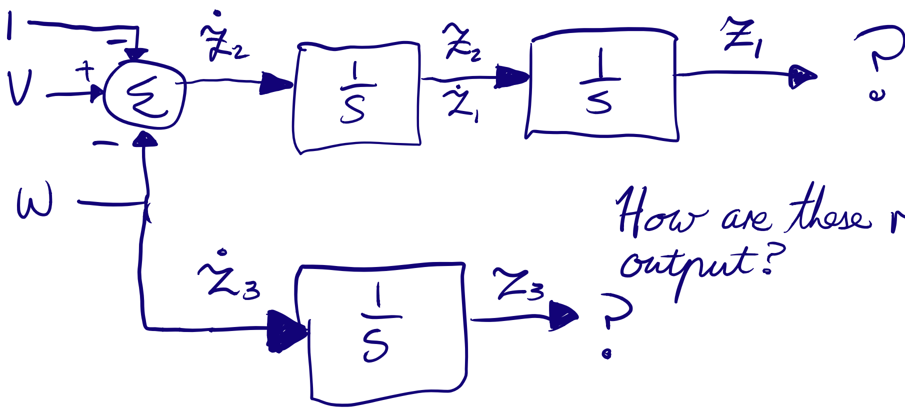
$$\dot{z}_2 = \frac{u_2}{x_3} - x_1 - u_1, \quad u_2 = x_3 x_1 + v$$

$$u_1 = 1 + w$$

$$= \frac{\cancel{x_3} x_1}{\cancel{x_3}} \cancel{- x_1} - u_1$$

$$= -u_1 + v = -(1+w) + v$$

$$\dot{z}_1 = z_2$$
$$\dot{z}_2 = v - w - 1$$
$$\dot{z}_3 = 1 - u_1 = 1 - (1 + w) = w$$
$$u_2 = x_3 x_1 + v$$
$$v = \dots$$
$$w = \dots$$



How are these related to my output?