Project 5: Backstepping Control

$$\frac{d^2x}{dt^2} - \mu(1-x^2)\frac{dx}{dt} + x = 0$$

$$\ddot{y} + (y^2-1)\dot{y} + y = u$$

$$\ddot{y} - (1-y^2)\dot{y} + y = u, \quad \mu = 1$$

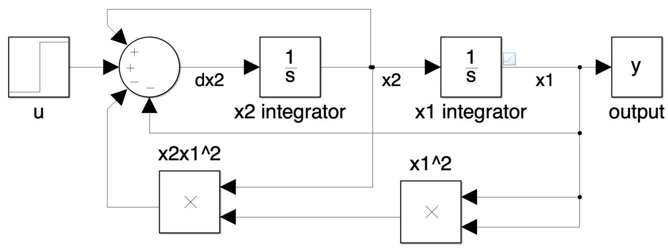
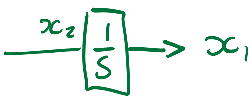


Figure 1: Van Der Pol Oscillator Block Diagram in strict feedback form

$$\boxed{\begin{aligned} x_1 &= y \\ \dot{x}_1 &= \dot{y} \\ x_2 &= \dot{y} \\ \dot{x}_2 &= \ddot{y} \end{aligned}}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -x_1 - x_2 x_1^2 + x_2 + u$$

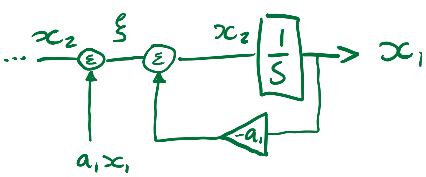


- Treat x_2 as a pseudo-input,

$$x_2 = -a_1 x_1$$

$$V_1(x_1) = \frac{1}{2}x_1^2$$

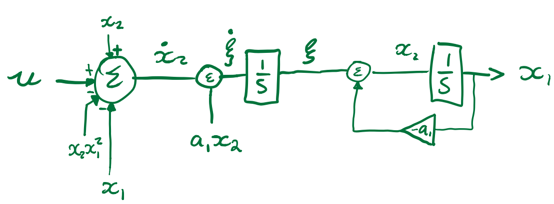
$$\dot{V}_1 = x_1\dot{x}_1 = x_1 x_2 = -a_1 x_1^2 < 0 \quad (\text{neg def if } a_1 > 0)$$



$$\xi = x_2 + a_1 x_1$$

$$\dot{\xi} = \dot{x}_2 + a_1\dot{x}_1 = a_1 x_2 + (-x_1 - x_2 x_1^2 + x_2 + u)$$

- Propagate $a_1 x_1$ backwards



$$V_2(x_1, \xi) = V_1(x_1) + \frac{1}{2}\xi^2 \leftarrow \text{pos def.}$$

$$\dot{V}_2(x_1, \xi) = \dot{V}_1(x_1) + \xi\dot{\xi}, \leftarrow \text{pseudo-input}$$

$$= -a_1 x_1^2 + x_2(-x_1 - x_2 x_1^2 + x_2 + u)$$

$$= -a_1 x_1^2 - x_1 x_2 - x_2^2 x_1^2 + x_2^2 + x_2 u$$

$$< 0$$

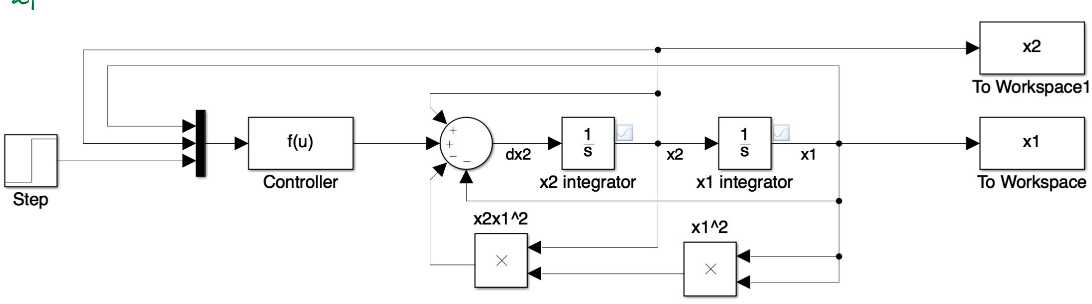
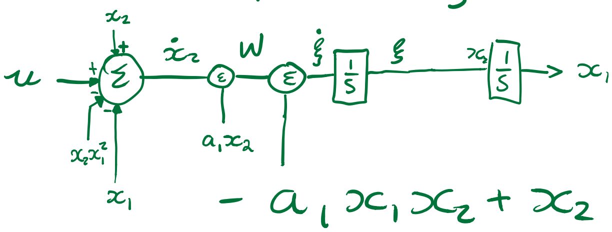


Figure 2: Full Control System

Stabilize ξ



$$-a_1 x_1 x_2 + x_2 + u = 0$$

$$u = a_1 x_2 (x_1 + 1)$$

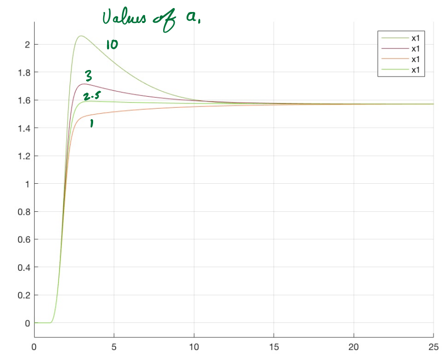


Figure 3: Stable van der Pol output with various values of a1

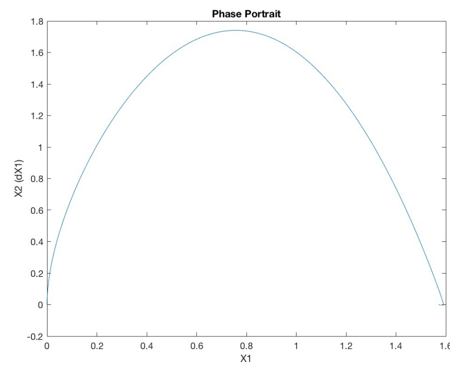


Figure 4: Stable Phase portrait a1 = 2.5

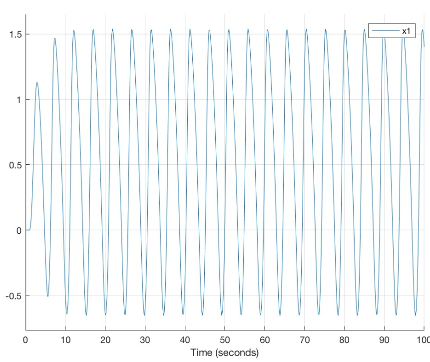


Figure 5: Output without controller

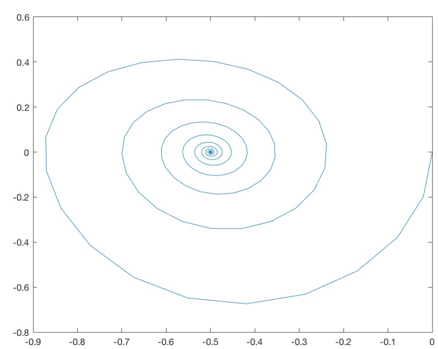


Figure 6: Output with unstable behavior

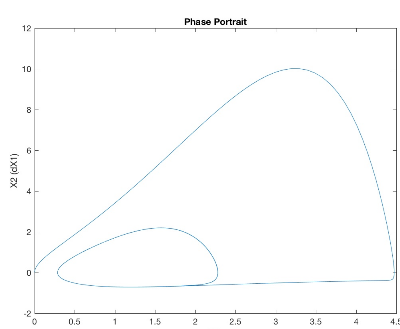


Figure 7: Suspect this was chaotic behavior.