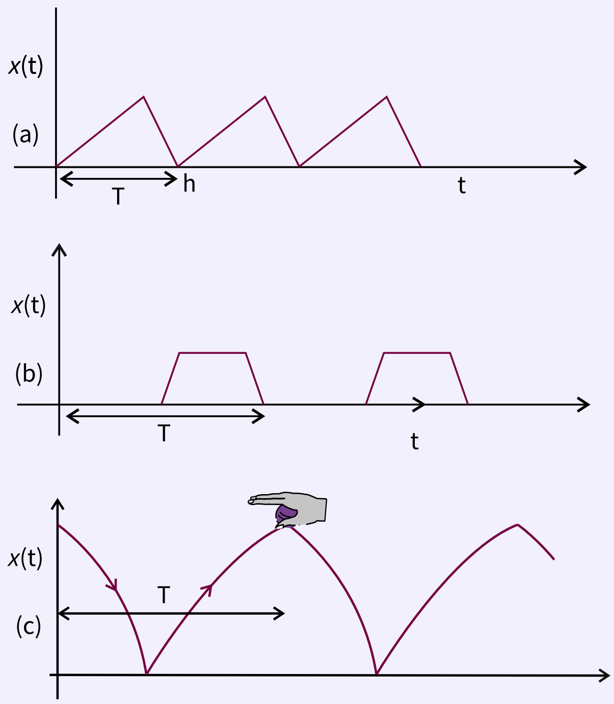
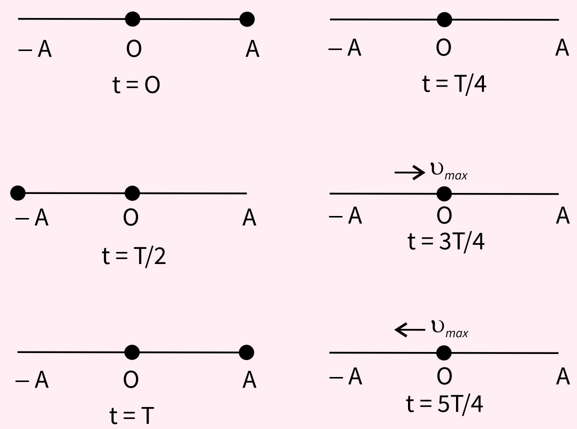
13. OSCILLATIONS

Examples of periodic motion. The period T is shown in each case.





The location of the particle in SHM at the discrete values t = 0, T/4, T/2, 3T/4, T, 5T/4.

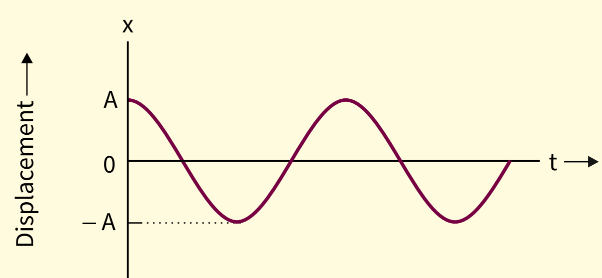
Characteristics of Linear S.H.M

Differential Equation of S.H.M $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Displacement $x = A \sin(\omega t + \phi)$

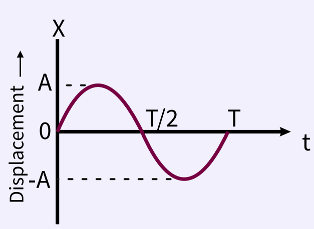
Velocity $V = \frac{dx}{dt} = \omega A \cos(\omega t + \phi)$

Acceleration $a = \frac{d^2x}{dt^2} = -\omega^2 A \sin(\omega t + \phi) = -\omega^2 X$

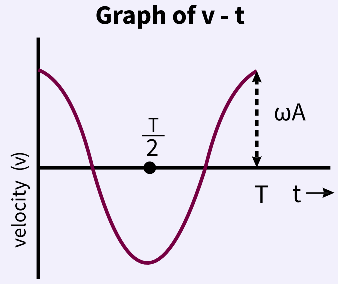


Displacement as a continuous function of time for simple harmonic motion.

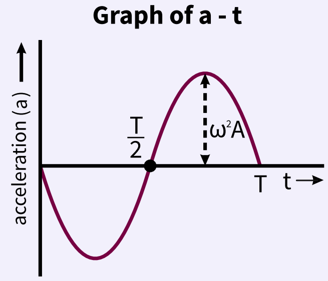
Graph of X - t



Graph of v - t



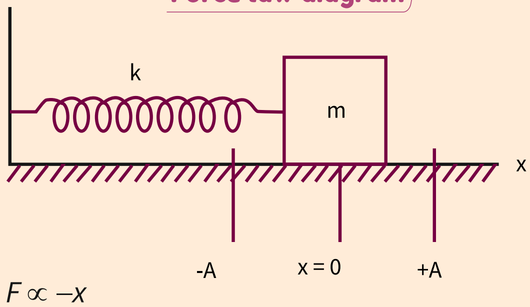
Graph of a - t



x(t) : Displacement x as a function of time t
A : amplitude
ω : angular frequency
ωt + ϕ : Phase (time - dependent)
ϕ : Phase constant

The meaning of standard symbols in

Force law diagram



$F \propto -x$

$F = -\kappa x$

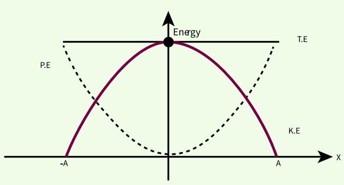
Time Period Calculation

(1) Force → $\vec{F} = -m\omega^2\vec{x}$ or $\vec{F} = -k\vec{x}$;

$\left(\omega = \sqrt{\frac{k}{m}}\right)$ Time period $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

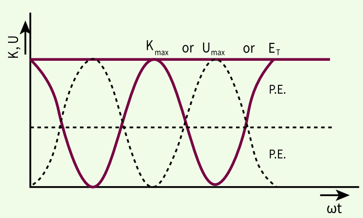
K - spring Constant

Energy of Linear S.H.M



→ P.E → $U = \frac{1}{2}Kx^2$

→ K.E → $KE = \frac{1}{2}K(A^2 - x^2)$

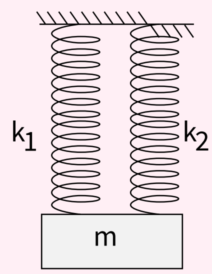


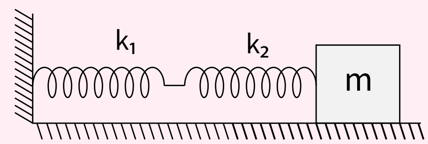
→ P.E → $U = \frac{1}{2}KA^2\sin^2(\omega t + \phi)$

→ K.E → $KE = \frac{1}{2}KA^2\cos^2(\omega t + \phi)$

Spring Block System

Time Period → $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$





(i) $K_{eq} = K_1 + K_2$

$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$

(ii) $K_{eq} = \frac{K_1K_2}{K_1 + K_2}$;

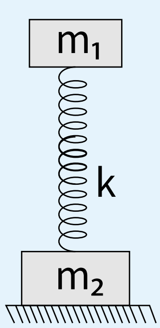
$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{K_1K_2}}$

Two Blocks Spring System

Reduced Mass: $\mu = \frac{m_1m_2}{m_1 + m_2}$

$T = 2\pi\sqrt{\frac{m_1m_2}{K(m_1 + m_2)}} = 2\pi\sqrt{\frac{\mu}{k}}$



Simple Pendulum

$\alpha = -\frac{mgL}{I}\theta$

Time period $= 2\pi\sqrt{\frac{\ell}{g}}$