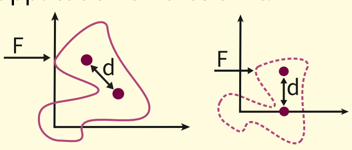
6. SYSTEM OF PARTICLES AND ROTATIONAL MOTION

RIGID BODY

A body with perfectly definite and unchanging shape due to the application of force on it.



Rotational Equilibrium

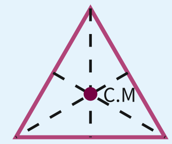
$$\sum \vec{\tau}_{ext} = 0$$

Translational Equilibrium

$$\sum \vec{F}_{ext} = 0$$

CENTRE OF MASS

The point where whole mass of system is supposed to be concentrated.



1. Position of centre of mass depends upon shaped, size and distribution of mass of the body.
2. The centre of mass of an object will have only translation motion.
3. Centre of mass & center of gravity coincide for a small body.
4. Centre of mass of rigid bodies is independent of the state i.e rest or motion of the body.

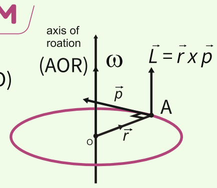
Position of Centre of mass of the system	Velocity of centre of mass of the system	Acceleration of Centre of mass of the system
$\vec{r}_{cm} = \dfrac{\sum m_i \vec{r}_i}{\sum m_i}$	$\vec{v}_{cm} = \dfrac{\sum m_i \vec{v}_i}{\sum m_i}$	$\vec{a}_{cm} = \dfrac{\sum m_i \vec{a}_i}{\sum m_i}$

ANGULAR MOMENTUM

$\vec{L}_o = \vec{r}_{OA} \times \vec{P}$ (angular momentum about point O)

$= \vec{r}_{OA} \times (m\vec{v}) = m\vec{r}_{OA} \times \vec{v}$

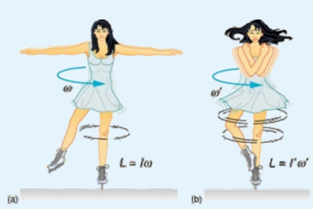
$= m|\vec{r}_{OA}||\vec{v}|\sin\theta$



Law of Conservation of Angular Momentum

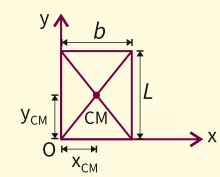
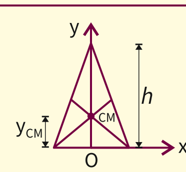
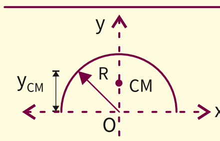
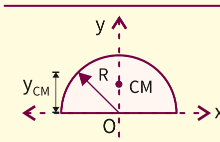
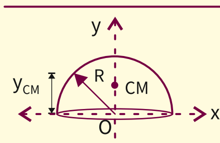
Angular momentum of a system is conserved as long as there is no net external torque acting on the system.

$$I_1\omega_1 = I_2\omega_2$$



Ex: Dancer rotates faster with closed arms

Center of Mass of some Symmetric Bodies

	Rectangular plate (By symmetry) $x_{CM} = \frac{b}{2}$ $y_{CM} = \frac{L}{2}$
	Triangular plate (By qualitative argument) At the centroid, $x_{CM} = \frac{h}{3}$; $y_{CM} = 0$
	Semi - circular ring $x_{CM} = 0$; $y_{CM} = \frac{2R}{\pi}$
	Semi - circular disc $x_{CM} = 0$; $y_{CM} = \frac{4R}{3\pi}$
	Hemispherical shell $x_{CM} = 0$; $y_{CM} = \frac{R}{2}$

MOMENT OF INERTIA

Inertia of Rotational motion

where r is distance perpendicular to the axis of Rotation.

$$I = \sum_{i=1}^{n} m_i r_i^2$$

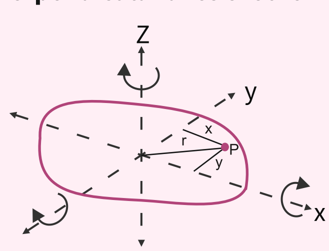
Radius of gyration

$$K = \sqrt{\frac{r_1^2 + r_2^2 \ldots\ldots\ldots r_n^2}{n}} = \sqrt{\frac{I}{M}}$$

$I = MK^2$

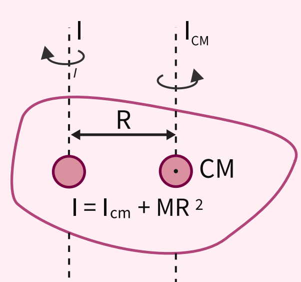
Theorem of moment of Inertia

Perpendicular axes theorem



$$I_Z = I_X + I_Y$$

Parallel – axes theorem

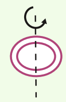
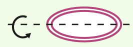
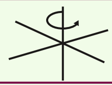
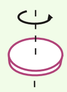
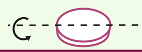
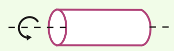
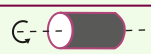
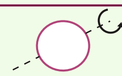


$$I = I_{cm} + MR^2$$

Factors on radius of gyration depend on

(1) Position & configuration of the axis of rotation

(2) distribution of mass about the axis of Rotation.

Body	Axis	Figure	*I*
Thin circular ring of radius *R*	Perpendicular to the plane at centre		MR^2
Thin circular ring of radius *R*	Diameter		$\frac{MR^2}{2}$
Thin rod of lenght *L*	Perpendicular to rod & at mid point		$\frac{ML^2}{12}$
Circular disc of radius *R*	Perpendicular to the plane of disc and through centre		$\frac{MR^2}{2}$
Circular disc of radius *R*	Diameter		$\frac{MR^2}{4}$
Hollow cylinder of radius *R*	Axis of cylinder		MR^2
Solid cylinder of radius *R*	Axis of cylinder		$\frac{MR^2}{2}$
Solid sphere of radius R	Diameter		$\frac{2MR^2}{5}$

ANGULAR MOMENTUM CONSERVATION

Relation between Torque & Ang ular momentum:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Unit of Torque = N. m

Dimensional formula = $[M^1L^2T^{-2}]$

Angular Impulse: $J = \int \tau.dt$, $J_{net} = L_f - L_i$,

Unit: Nm·s

$$\vec{\tau} = \vec{r} \times \vec{F} \quad ; \quad \vec{\tau}_{net} = \frac{d\vec{L}}{dt} \text{ if } \vec{\tau}_{net} = 0 \Rightarrow \vec{L} = \text{constant}$$

$$L_{system} = \sum_{i=1}^{n} L_i \qquad \left[\frac{d\vec{L}}{dt} = 0\right]$$

Angular momentum of rigid body performing pure rotation about fixed axis $(L_{sys}) = I_{AOR}\omega$

Analogy between Linear & Rotational Motion

Linear motion	Rotational Motion
Velocity $V = \frac{ds}{dt}$	**Angular velocity** $\omega = \frac{d\theta}{dt}$
Acceleration $a = \frac{dv}{dt}$	**Angular acceleration** $\alpha = \frac{d\omega}{dt}$
Force $F = ma = \frac{mdv}{dt}$	**Torque** $\tau = \frac{d}{dt}(I\omega) = I\frac{d\omega}{dt} = I\alpha$
Work done $W = F.S$	**Work done** $w = \tau.\theta$
Linear K.E $= \frac{1}{2}mv$	**Rotational K.E** $= \frac{1}{2}I\omega^2$
Power $P = F.V$	**Power** $P = \tau.\omega$
Linear momentum $P = mv$	**Angular momentum** $L = I\omega$
Impulse $F\Delta t = mv - mu$	**Angular Impulse** $\tau\Delta t = I\omega_f - I\omega_i$

Motion of System of Particles & Rigid Body

Pure Rotational Motion:-

(1)distance between two particles of a rigid body remains constant, So the relative motion of one particle w.r.t other particle is circular motion.

(2) Angular velocity of all the particles about a given point of a Rigid body is same. $\mathbf{S = R\theta, V = R\omega;}$

(3) If = Constant (angular acceleration), $\omega_f = \omega_i + \alpha t$

$\theta = \omega_i t + \quad \alpha t^2$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$\theta = \left[\frac{\omega_f + \omega_i}{2}\right]t$