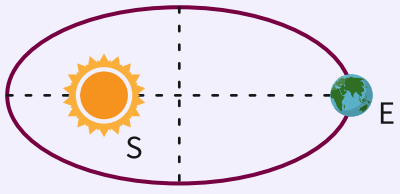
7. GRAVITATION

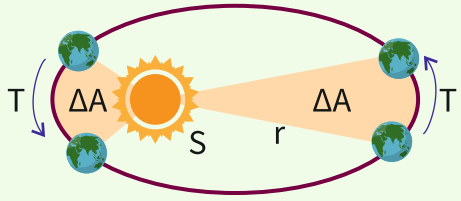
KEPLER's LAWS OF PLANETARY MOTION

LAW OF ORBIT



Every planet revolves around the sun in an elliptical orbit and sun is at it's one of the foci.

LAW OF AREA

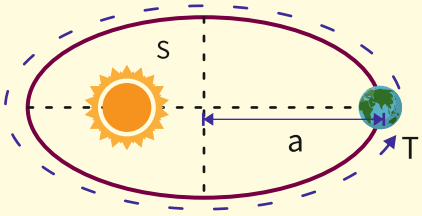


(i) The line joins any planet to the sun sweeps equal area in equal intervals of time

(ii) $\frac{dA}{dt} = \frac{L}{2m}$

(iii) Areal velocity is constant.

LAW OF PERIODS



(i) The square of time period of revolution of a planet is proportional to cube of semi – major axis of its elliptical orbit.

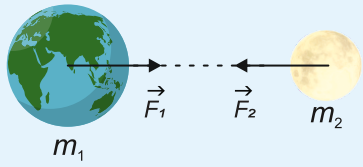
(ii) $T^2 \propto R^3$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Superposition Principle In Vector Form

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots\ldots + \vec{F}_{1N}$$

NEWTON'S LAW OF GRAVITATION

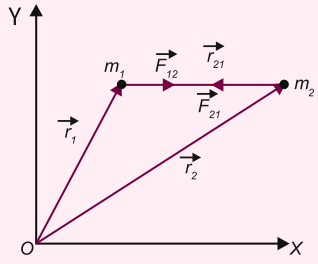


The gravitational Force acting between two bodies separated by distance 'r' is directly proportional to product of their masses and inversely proportional to square of distance between them.

$$F = \frac{Gm_1m_2}{r^2}$$

$\Rightarrow G = 6.67 \times 10^{-11}\ Nm^2Kg^{-2}$

Newton's Law in Vector Form



$\vec{r}_1$ = position of first particle

$\vec{r}_2$ = position of second particle

$\vec{r}_{12}$ = Force between them.

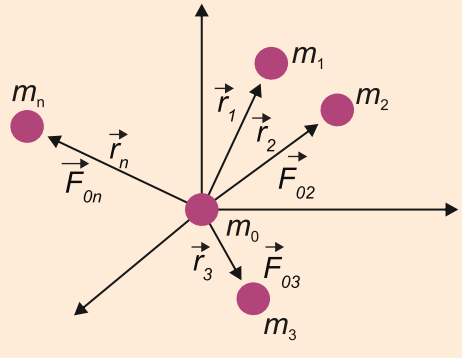
$$\vec{F}_{12} = \frac{Gm_1m_2}{|\vec{r}_1 - \vec{r}_2|^2}\hat{r}_{12}$$

$$\vec{F}_{12} = \frac{Gm_1m_2(\vec{r}_1 - \vec{r}_2)}{|r_1 - r_2|^3}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Superposition Principle

Resultant force acting on a particle due to other particles is vector sum of forces exerted by individual particle in it.



GRAVITATIONAL ACCELERATION

At surface of earth, $F_{gravitational}$ = Weight

$$Mg = \frac{GM_eM}{R_e^2}$$

$$g = \frac{GM_e}{R_e^2}$$

Variation of 'g' at height 'h'

$$g_h = g_s\left(\frac{R_E}{R_E + h}\right)^2$$

if $h <<<< R_E$

$$g_h = g_s\left(1 - \frac{2h}{R_E}\right)^{-2}$$

Variation of 'g' with depth

$$g_d = g_s\left(1 - \frac{d}{R_E}\right)$$

Variation of 'g' with latitude

$$g' = g - R\omega^2\cos^2\lambda$$

ESCAPE SPEED & ENERGY CONSERVATION

Minimum speed required by an object to escape Gravitational Field of Earth.

$$V_e = \sqrt{\frac{2Gm}{R_E}} = \sqrt{2gR_E} = 11.2\text{Km/s}$$

GRAVITATIONAL POTENTIAL ENERGY & GRAVITATIONAL POTENTIAL

Gravitational Potential Energy

Energy required to bring a mass from an infinite position to point under gravitational field of earth with constant velocity.

$$U = \frac{Gm_1m_2}{r^2}$$

Generally, infinite is reference point.

Gravitational Potential

Amount of work done in moving a unit test mass from ∞ - position to point under gravitational field of earth.

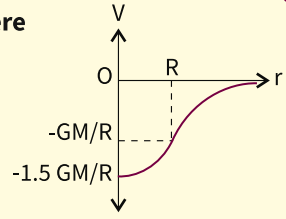
$$V = -\frac{Gm}{r} = \frac{U}{M}$$

Gravitational Potential For solid sphere

(i) $r < R \rightarrow V = -GM\frac{(3R^2 - r^2)}{2R^3}$

(ii) $r = R \rightarrow V = -\frac{Gm}{R}$

(iii) $r > R \rightarrow V = -\frac{Gm}{r}$



Gravitational Field intensity

Strength of Gravitational field applied per unit test mass is defined as Gravitational Field Intensity.

$$\vec{E} = \frac{-Gm}{r^2}\hat{r}$$

Relation between Gravitational potential & Intensity

(i) $E = -\frac{dv}{dr}$ (ii) $\Delta V = \int \vec{E}.d\vec{r}$

WEIGHTLESSNESS

(1) During Free fall under gravity inside a spacecraft or satellite, body is weightless.
(2) Effective weight of body becomes Zero.

Orbital velocity of satellite

(i) orbital velocity = $\sqrt{\frac{GM}{r}}$; $r = R + h$

(ii) Total energy of satellite = Constant = K.E + P.E

(iii) Total energy = $\frac{-GMm}{2r}$