3. MOTION IN A PLANE

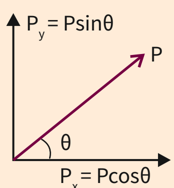
SCALAR PHYSICAL QUANTITY

Quantities having only magnitude but no direction are called scalar quantities.

VECTOR PHYSICAL QUANTITY

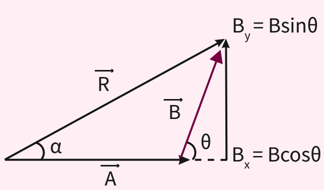
Quantities having both magnitude as well as direction and follows triangular law of vector addition are called vector quantities.

Resolution of a Vector

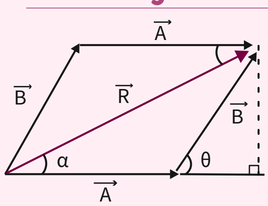


VECTOR LAW'S

Triangle law:



Parallelogram law:

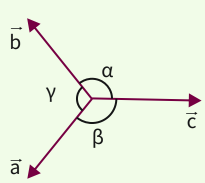


$\vec{R} = (A + B\cos\theta)\hat{i} + B\sin\theta\,\hat{j}$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\tan\alpha = \dfrac{|\vec{B}|\sin\theta}{|\vec{A}| + |\vec{B}|\cos\theta}$

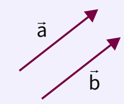
Lami's Theorem



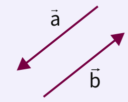
$\Rightarrow \dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

TYPES OF VECTOR

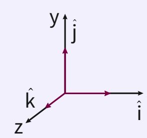
PARALLEL VECTORS : Vectors having same direction, i.e., angle between them is 0°.



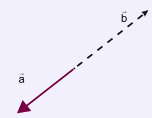
ANTI PARALLEL VECTORS : Vectors having opposite direction, i.e., angle between them is 180°.



UNIT VECTOR : $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$; $\hat{i}, \hat{j}, \hat{k}$, are unit vectors along x, y, z direction.



COLLINEAR VECTORS : Vectors lie on the same line or parallel line. $\vec{a} = \pm\lambda\vec{b}$



Mathematical (Arithmetic) Operations of Vectors

Addition

$\vec{A} + \vec{B} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}$

Subtraction

$\vec{A} - \vec{B} = (a_x - b_x)\hat{i} + (a_y - b_y)\hat{j} + (a_z - b_z)\hat{k}$

Multiplication

Dot product (Scalar product)

1) $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$

2) $\vec{A}\cdot\vec{B} = a_xb_x + a_yb_y + a_zb_z$

3) $\hat{i}\cdot\hat{i} = 1$, $\hat{j}\cdot\hat{j} = 1$, $\hat{k}\cdot\hat{k} = 1$ | $\hat{i}\cdot\hat{j} = 0$, $\hat{j}\cdot\hat{k} = 0$, $\hat{k}\cdot\hat{i} = 0$

Cross product (vector product)

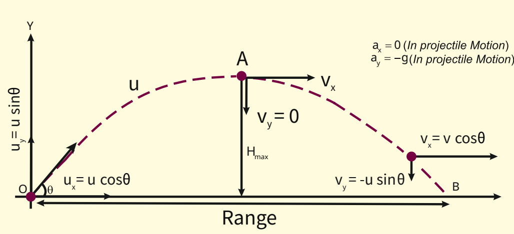
1) $\vec{A}\times\vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta\,\hat{n}$

2) $\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$

$= \hat{i}(a_yb_z - a_zb_y) - \hat{j}(a_xb_z - a_zb_x) + \hat{k}(a_xb_y - a_yb_x)$

3) $\hat{i}\times\hat{j} = \hat{k}$, $\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$ | $\hat{i}\times\hat{i} = 0$, $\hat{j}\times\hat{j} = 0$, $\hat{k}\times\hat{k} = 0$

Projectile motion - Oblique projectile



Time of flight (T) = $\dfrac{2u\sin\theta}{g}$

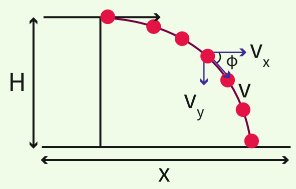
Range (R) = $u_xT = \dfrac{u^2\sin 2\theta}{g}$

Height (H) = $\dfrac{u^2\sin^2\theta}{2g}$

Equation of Trajectory

$y = x\tan\theta - \dfrac{1}{2}\dfrac{gx^2}{u^2\cos^2\theta} = x\tan\theta\left(1 - \dfrac{x}{R}\right)$

Horizontal projectile



$u_y = 0,\ u_x = u$

$x = u_x t = ut,\ t = x/u$

$y = \frac{1}{2}gt^2$

Equation of Trajectory (y) $= \frac{1}{2}gt^2 = \frac{gx^2}{2u^2}$

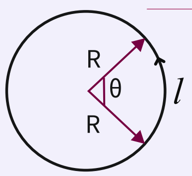
Time of flight (T) $= \sqrt{\frac{2h}{g}}$

Range (R) $= u_x T = u\sqrt{\frac{2h}{g}}$

$$v_{ins} = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + g^2t^2} = \sqrt{u^2 + 2gy}$$

tanϕ $= \frac{v_y}{v_x} = \frac{gt}{u}$

CIRCULAR MOTION



Angular displacement (θ):

$l = R\theta$

Angular velocity (ω):

T= Time period

f = frequency

$v = r\omega$

linear velocity (ms^{-1})

Angular Acceleration (α):

$\alpha = \frac{d\omega}{dt}(rads^{-2})$

$a = R\,\alpha$

linear acceleration (ms^{-2})

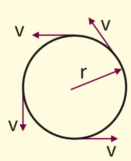
Equation of motion on Circular track:

$$\omega_f = \omega_i + \alpha t \mid \theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 - \omega_i^2 = 2\alpha\theta$$

Types of Circular Motion

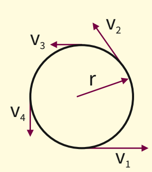
Uniform circular motion



1) a_T (Tangential acceleration) = 0

2) a_r (Radial acceleration)

Non – uniform Circular motion



1) $v_1 \neq v_2 \neq v_3 \neq v_4$

2) $a_T \neq 0$

3) $a_r = \frac{v_{ins}^2}{R}$

4) $a_{net} = \sqrt{a_T^2 + a_r^2}$

RELATIVE MOTION ON 2 D – PLANE

Relative velocity of one body w.r.t. other:

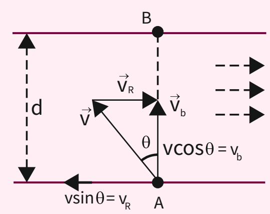
$\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$ $\quad$ $\vec{v}_{PQ}$ = relative velocity of P w.r.t.Q

RIVER-BOAT PROBLEM

Time taken, $t = \frac{d}{v\cos\theta}$

Drift, $x = (v_R - v\sin\theta)t = (v_R - v\sin\theta)\frac{d}{v\cos\theta}$

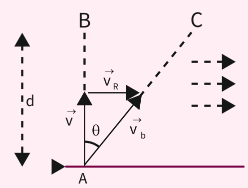
Case I: Shortest path



$v_R = v\sin\theta$

$t = \frac{d}{v\cos\theta} = \frac{d}{\sqrt{v^2 - v_R^2}}$

Case 2: Minimum Time

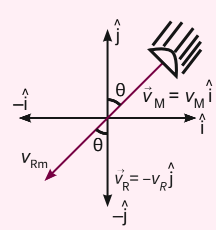


$t_{min} = \frac{d}{v}$

$BC = v_R t_{min} = \frac{v_R d}{v}$

RAIN-MAN PROBLEM

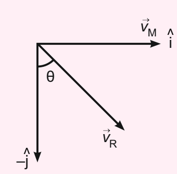
Case I: Rain is falling vertically



$v_{RM} = \sqrt{v_R^2 + v_m^2}$

$\theta = \tan^{-1}\left(\frac{v_M}{v_R}\right)$

Case 2: Rain is falling at angle θ with vertical



$v_R\sin\theta - v_m = 0 \Rightarrow \sin\theta = \frac{v_M}{v_R}$

$v_{RM} = v_R\cos\theta = \sqrt{v_R^2 + v_m^2}$