Assignment 10 - Windowed Fourier Spectra

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1 INTRODUCTION

This assignment is about analysing frequency spectra on non periodic signals.

DFT of a function is given by

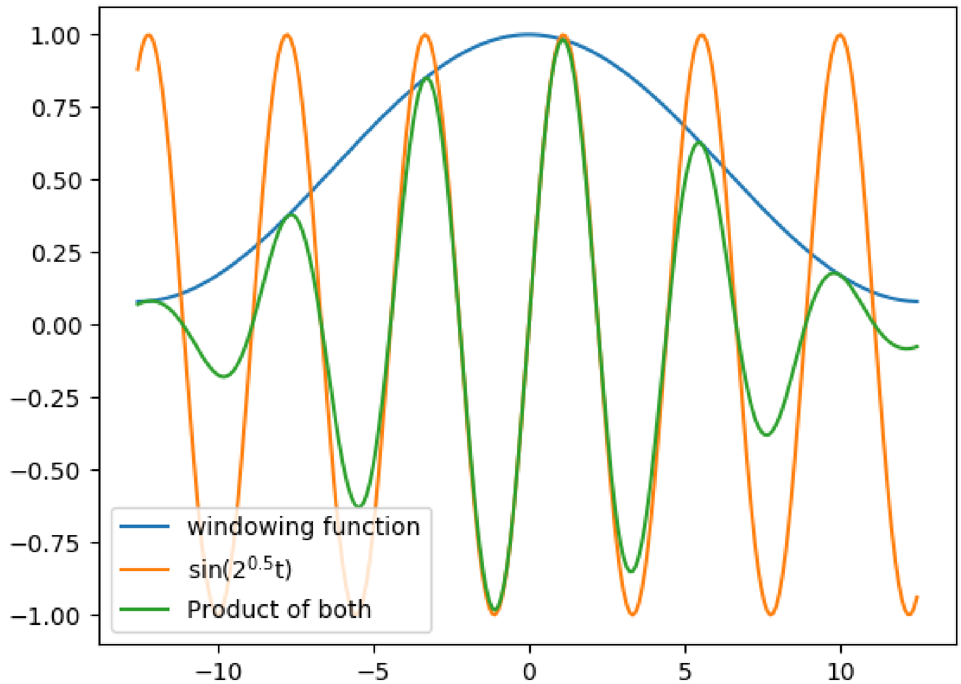
$$X\left[k\right] = \sum_{n=0}^{N-1} x[n]e^{\frac{2\pi jkn}{N}}$$

In this assignment we will try of obtain DFT on non periodic function. Last assignment DFT of periodic function was evaluated. To obtain DFT of non periodic functions , we will take a part of the function and make it periodic. And then evaluate its DFT but since it non periodic and if its repeated on a specific interval then at those points function will become discontinuous.To solve this We will multiply our input function with windowing function to make it continous. In this assignment Hamming Windowing Function is used.

$$w\left[n\right] = \begin{cases} 0 & |n| > \frac{N-1}{2} \\ 0.54 + 0.46cos(\frac{2\pi n}{N-1}) & |n| \leq \frac{N-1}{2} \end{cases}$$

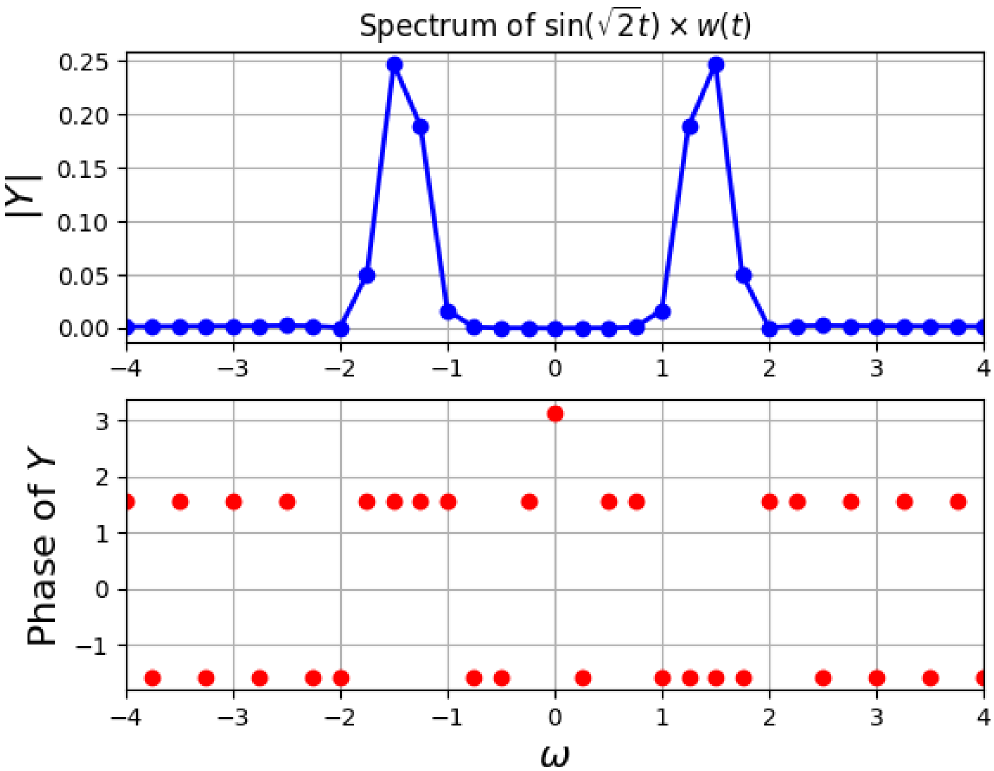
2 Problem 1

```
t = linspace(-4*pi,4*pi,257)
t = t[:-1]
dt = t[1]-t[0]
fmax=1/dt
n = arange(256)
wnd = fftshift(0.54+0.46*cos(2*pi*n/256))
y = sin(sqrt(2)*t)
y=sin(1.25*t)
plot(t,wnd)
plot(t,y)
plot(t,y*wnd)
legend(["windowing function","sin($2^{0.5}$t)","Product of both"])
show()
```




```
y = y * wnd y[0] = 0
y = fftshift(y)
Y = fftshift(fft(y))/256.0
w = linspace(-pi*fmax, pi*fmax, 257)
w=w[:-1]
plot(w,wnd)
show() #plotting results
figure()
subplot(2,1,1)
plot(w,abs(Y),'b',w,abs(Y),'bo',lw=2)
xlim([-4,4])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\sin\left(\sqrt{2}t\right)\times w(t)$")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'ro',lw=2)
```

```
xlim([-4,4])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
show()
```



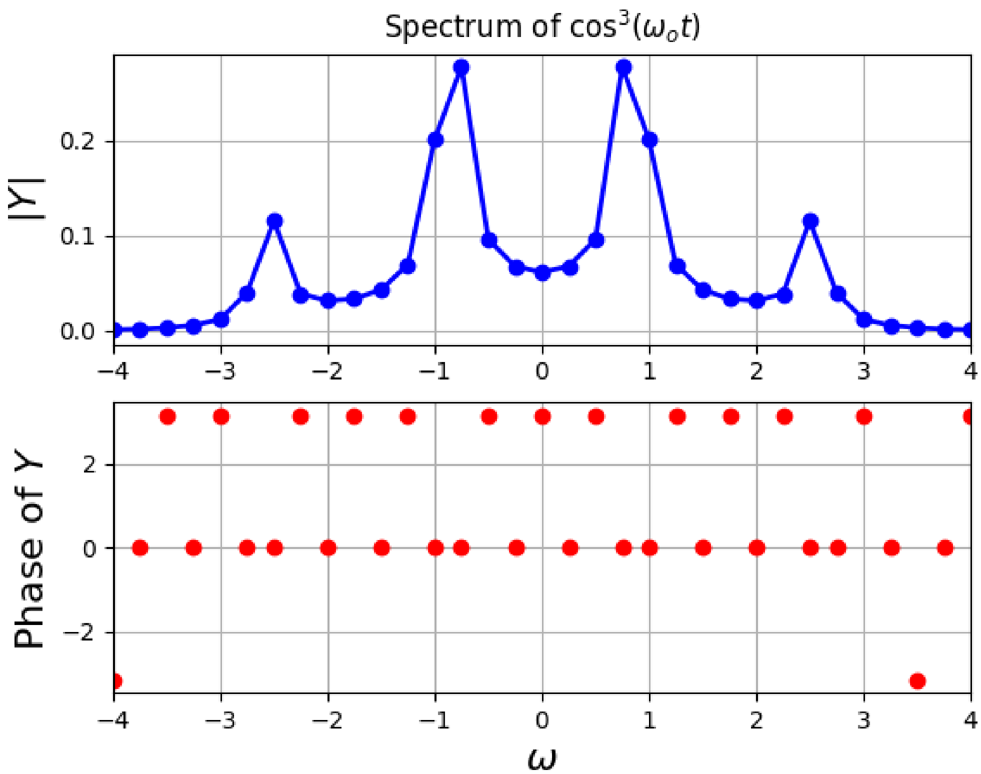
As expected since DTFT of $\sin(\sqrt{2}t)$ has two peaks at $\pm\sqrt{2}$ similarly the peaks observed in the graph are close to the value. Phase corresponding to the maximum peak should be $\mp\frac{\pi}{2}$ for respective peaks which is consistent with obtained output. The peak is less sharp because of the effect cause by the windowing function.

3 Problem 2

In this Problem we need to obtain DFT of $\cos^3(0.86t)$. DTFT of $\cos^3(\omega_o t)$ is

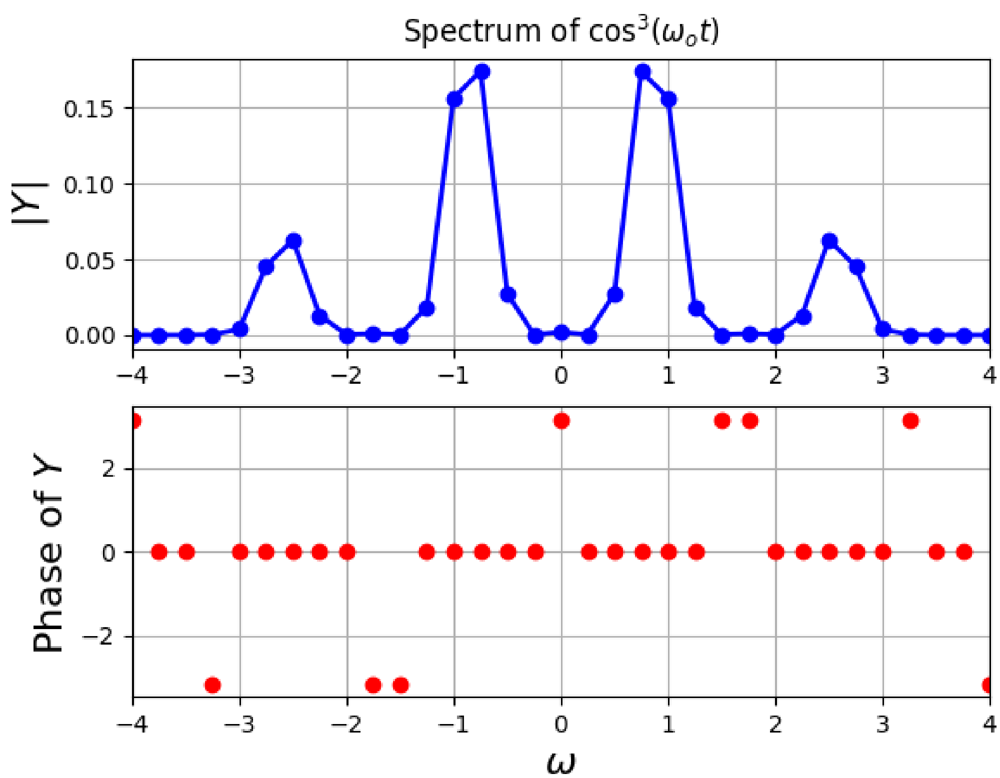
$$X(\exp^{j\omega}) = \frac{3\pi}{4}(\delta(\omega - \omega_o) + \delta(\omega + \omega_o)) + \frac{\pi}{4}(\delta(\omega - 3\omega_o) + \delta(\omega + 3\omega_o))$$

```
#Without Windowing
t=linspace(-4*pi,4*pi,257)
t=t[:-1]
dt=t[1]-t[0]
fmax=1/dt
n=arange(256)
y=cos(0.86*t)**3
y[0]=0
y=fftshift(y)
Y=fftshift(fft(y))/256.0
w=linspace(-pi*fmax,pi*fmax,257)
w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),'b',w,abs(Y),'bo',lw=2)
xlim([-4,4])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of cos$^{3}(\omega_{o}t)$")
grid(True) subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'ro',lw=2)
xlim([-4,4])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
show()
```




```
t=linspace(-4*pi,4*pi,513)
t=t[:-1]
dt=t[1]-t[0]
fmax=1/dt
n=arange(512)
wnd=fftshift(0.54+0.46*cos(2*pi*n/512))
y=cos(0.86*t)**3
y=y*wnd
y[0]=0
y=fftshift(y)
Y=fftshift(fft(y))/512.0
w=linspace(-pi*fmax,pi*fmax,513)
w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),'b',w,abs(Y),'bo',lw=2)
xlim([-4,4])
```

```
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of cos$^{3}(\omega_{o}t)$")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'ro',lw=2)
xlim([-4,4])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
show()
```



3.1 OBSERVATIONS

In both case three distinguished peaks are visible at the expected position phase corresponding to the peaks is phase is 0 . Which is correct for cosines. In second is a more correct approximation

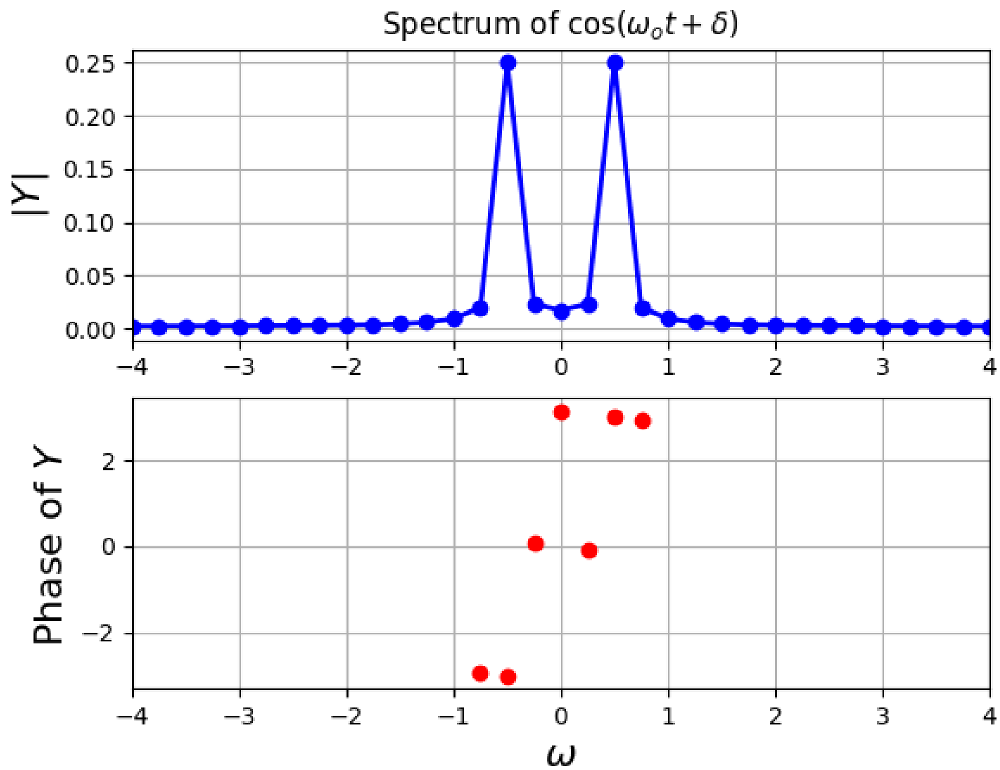
without windowing function it slowly falls to 0, and has broader peaks. Sharper peaks are observed by using windowing function.

4 Problem 3 and 4

4.1 Problem 3

In this problem we are selecting a function cos(ωt+δ) where ω ranges from 0.5 and 1.5 and δ varies from 0 to π . Considering them unformly distributed random numbers ,we can select a function and then estimate value of ω and δ.

```
omega = np.random.uniform(0.5,1.5)#random
delta = np.random.uniform(-1,1)*pi #random
print omega
print delta
t = linspace(-pi,pi,257)
dt = t[1] - t[0]
fmax = 1/dt
t = t[:-1]
n=arange(256)
wnd= fftshift(0.54+0.46*cos(2*pi*n/256))
y = cos( omega * t + delta )
y[0] = 0
y = fftshift(y)
Y = fftshift(fft(y))/512.0
w=linspace(-pi*fmax,pi*fmax,257)
w=w[:-1]
```




```
figure()
subplot(2,1,1)
plot(w,abs(Y),'b',w,abs(Y),'bo',lw=2)
xlim([-4,4])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of cos$(\omega_{o}t + \delta)$")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'ro',lw=2)
xlim([-4,4])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
show()
maximum = max(abs(Y))
jj = where(abs(Y) == maximum)
```

```
print "omega estimation",w[jj]
print "delta estimation",angle(Y[jj])
```

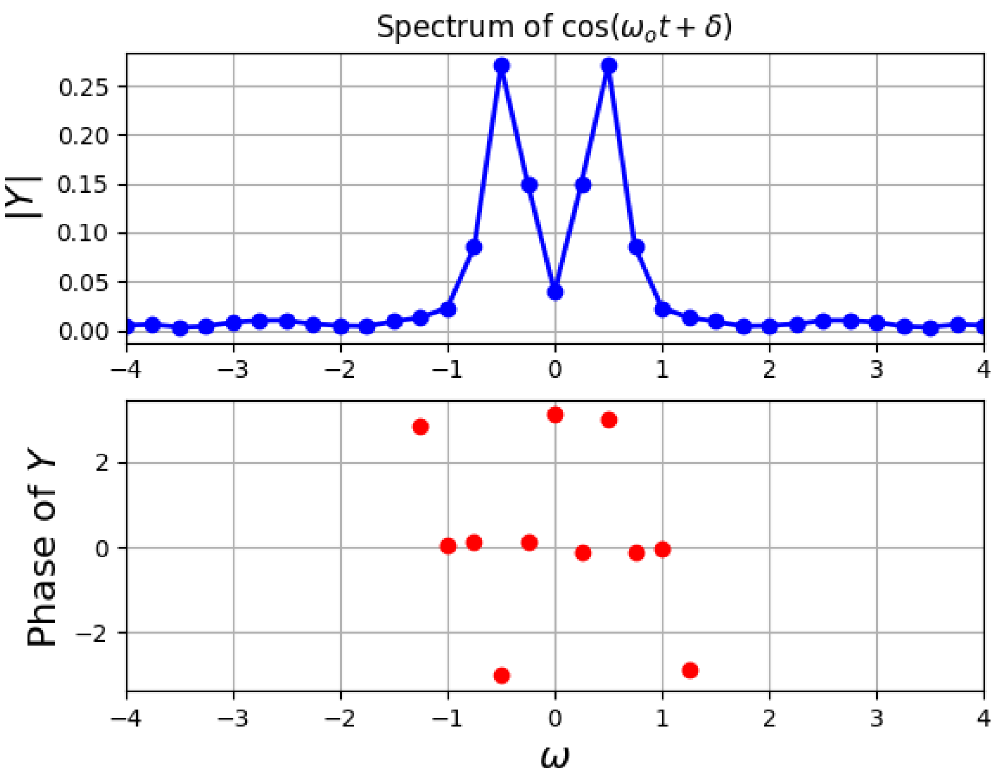
4.1.1 Output

```
omeaga 0.976881673062
delta -0.270480948569
omega estimation [-1. 1.]
delta estimation [ 0.27857661 -0.27857661]
```

4.1.2 Observations

The output of the program returns both the peaks. Value of the omega closely resembles the actual value used and other output is the phase corresponding to both the peaks. Estimated delta value also closely resembles to the actual value of the δ used.

4.2 Problem 4



The above Plot is obtained after adding noise to the input. Plot closely resembles with the acctual plot obtained without noice. Using the same value of omega and delta (Randomly Generated) , However there estimate is now more erroneous still very close to the actual value.

4.2.1 Output

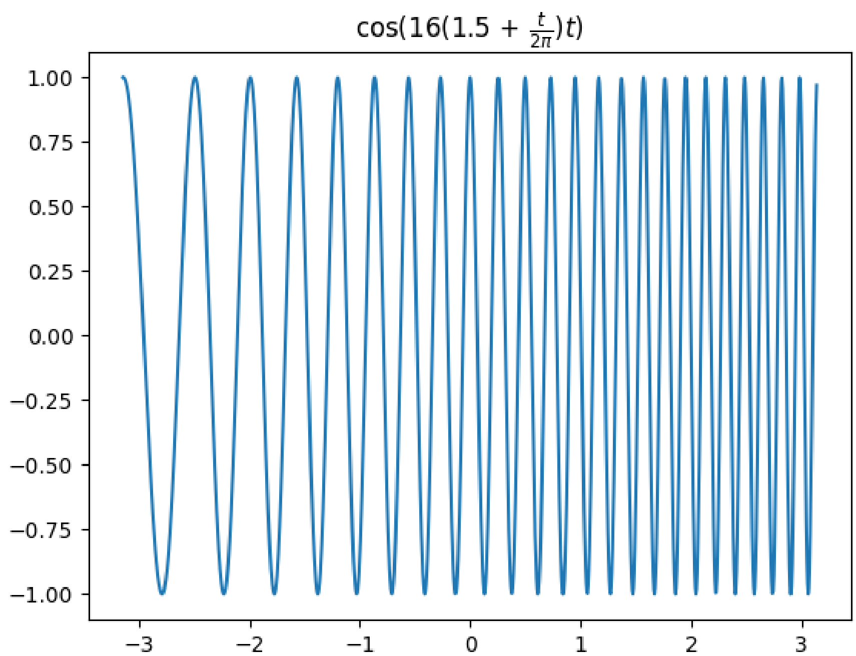
```
omeaga 0.976881673062
delta -0.270480948569
omega estimation [-1.  1.]
delta estimation [ 0.29102405 -0.29102405]
```

5 Problem 5

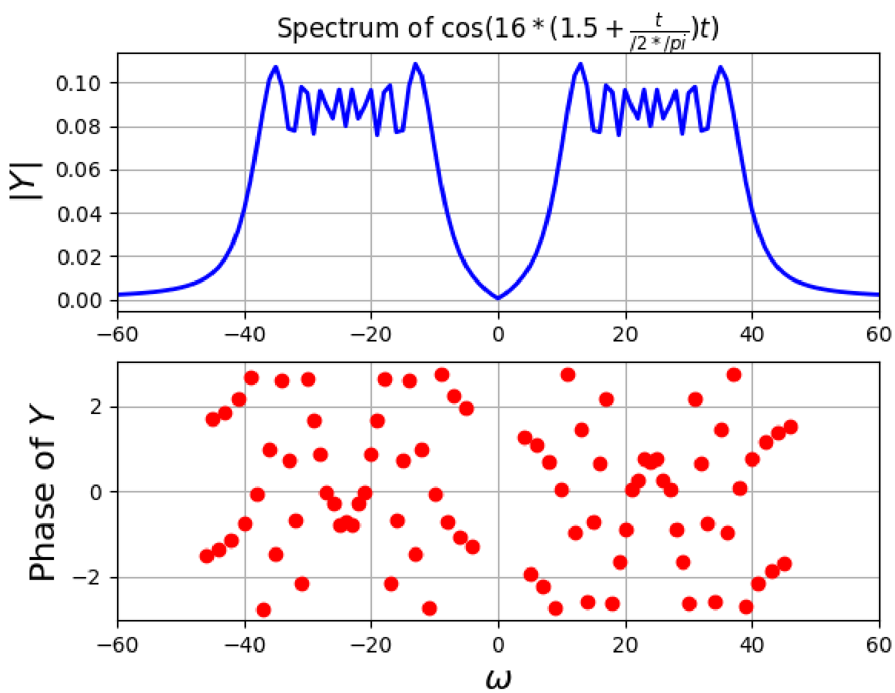
The given input signal is

$$cos(16(1.5 + \frac{t}{2\pi})t)$$

This is a called Chirped signal as its frequency varies with time. Time domain plot of this function is shown .



Its DFT for time going from -π to π is 1024 steps is evaluated. Its frequecy specOtrum thus obtained is .



5.1 Observation

In the duration $-\pi$ to π the chirping sgnal frequency changes from 16 to 32 . It can be seen from the spectrum first peak occur at ± 16 and ends at ± 32 .

6 Problem 6

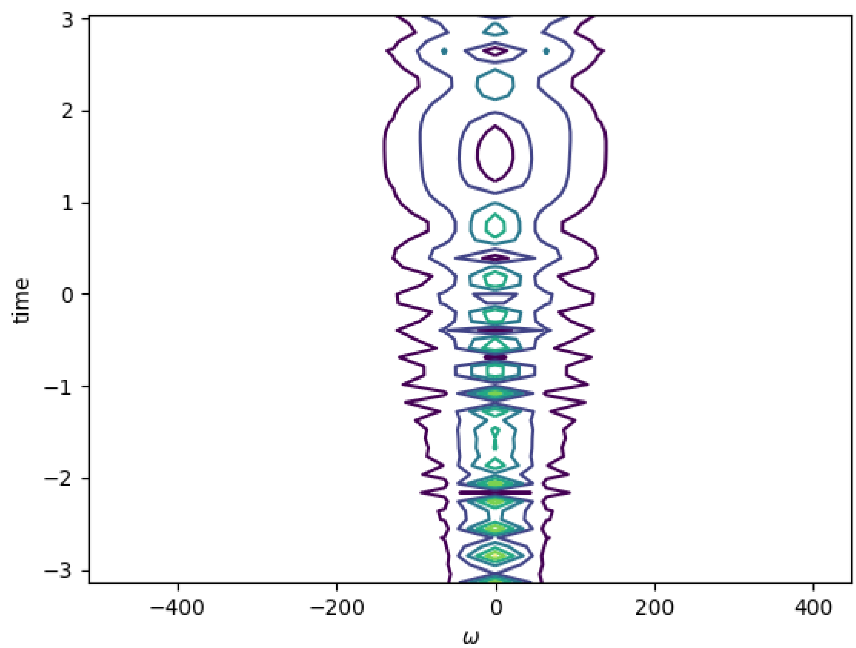
Using the same chirping signal as in last problem. The chirping signal

$$cos(16(1.5+\frac{t}{2\pi})t)$$

This signal has time dependent frequency term hence to know its exact spectrum DFT of this needs to be plotted aginst time . This is called Dynamic Spectrum. To do so we will break block of 1024 element going from -π to π into 64 of 62 elements each and then take transform.

6.1 Contour Plot

```
t = linspace(-pi, pi, 1025)
dt = t[1] - t[0]
fmax = 1/dt
t = t[:-1]
y = cos( 16*(1.5 + t/(2*pi))*t )
y = reshape(y,(-1,16))
y[:,0] = 0
y = fftshift(y)
Y = fftshift(fft(y))/16.0
w=linspace(-pi*fmax, pi*fmax, 17)
w=w[:-1]
plt.contour(w, t[::16], abs(Y)) plt.show()
```



6.2 Surface Plot

```
W,T = np.meshgrid(t[::16],w)
fig1 = plt.figure(1)
```

```
ax = p3.Axes3D(fig1)
plt.title('The 3-D surface plot')
surf = ax.plot_surface(T,W,abs(Y.T),rstride = 1,cstride = 1, cmap = plt.cm.jet)
plt.show()
```