EE - 2703 Applied Programming Lab

Assignment -7

The Laplace Transform

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Contents

1 Introduction

This assignment is based on analysis Linear-Time invariant systems. Using python signals package of scipy module in python signals impulse response can be analyzed. It is a very powerful package, it allows various operations such as construction transfer function, obtaining its response for any time domain signal and much more.

2 Problem -1 : Time Response to spring

The position of the spring it defined by

$$\frac{d^2x}{dt^2} + 2.25x = f(t)$$

Here f(t) is the forcing function . For this problem f(t) is

$$f(t) = cos(1.5t)e^{-0.5t}u_o(t)$$

Its Laplace transform is given by

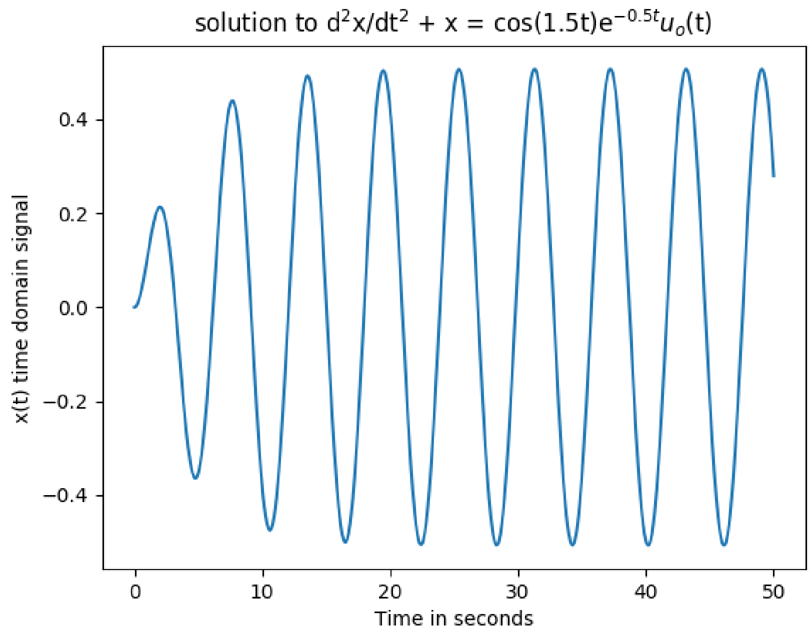
$$F(s) = \frac{s + 0.5}{(s + 0.5)^2 + 2.25}$$

so from the above equations and the given initial conditions as x(0) = 0 and $\frac{dx}{dt} = 0$. X(s) can be written as

$$X(s) = \frac{s + 0.5}{(s^2 + s + 2.5)(s^2 + 2.25)}$$

Inverse Laplace transform of the above expression will time domain position . Using python function impulse from scipy.signal , impulse response of the system can be obtained.

```
p1 = np.poly1d([1,0.5]) #polynomial s + 1/2
                p2 =: np.poly1d([2,1,2.5]) #polynomial s^2 + s + 5/2
                p3 = np.poly1d([2,0,2.25]) #polynomial s^2 + 2.25
                p2p3 = np.polymul(p2,p3) # multiply p2,p3 (the denominator)
                hs = sp.lti(p1,p2p3)     #numerator divided by denominator (defining t
                t,x = sp.impulse(hs,None,np.linspace(0,50,1000)) #comuting impulse re
                plt.plot(t,x) #plotting time domain signal with x
                plt.xlabel("Time in seconds")
                plt.ylabel("x(t) time domain signal")
                plt.title("solution to d$^{2}$x/dt$^{2}$ + x = cos(1.5t)e$^{-0.5t}u_{
                plt.show()
```



2.1 Problem -2 : Response of spring with slower decaying forcing function

In the second problem which is a subproblem to the first part. The forcing function in this problem is

$$f(t) = cos(1.5t)e^{-0.05t}u_o(t)$$

In s domain the forcing function is given by

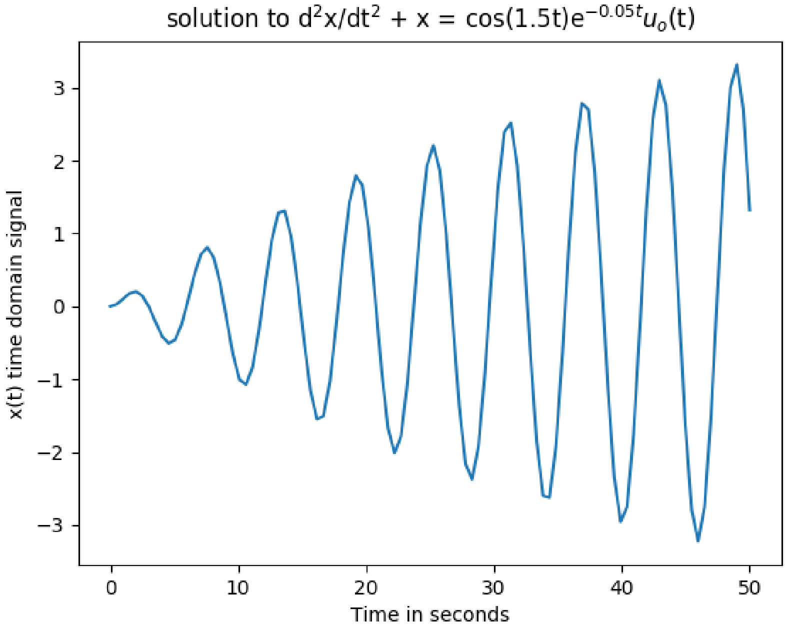
$$F(s) = \frac{s + 0.05}{(s + 0.05)^2 + 2.25}$$

This gives X(s) as

$$X(s) = \frac{s + 0.05}{(s^2 + 0.1s + 2.2525)(s^2 + 2.25)}$$

Similarly as in previous problem inverse Laplace transform of the above expression will time domain position . Using python function impulse from scipy.signal , impulse response of the system can be obtained.

```
p1 = np.poly1d([1,0.05]) #polynomial s + 1/2
                p2 = np.poly1d([2,0.1,2.25+0.0025]) #polynomial s + 1/2
                p3 = np.poly1d([2,0,2.25]) #polynomial s^2 + 1/2
                p2p3 = np.polymul(p2,p3) # multiply p2,p3 (the denominator)
                hs = sp.lti(p1,p2p3) #numerator divided by denominator (defining tran
                t,x = sp.impulse(hs,None,np.linspace(0,50,100)) #comuting impulse resp
                plt.plot(t,x) #plotting time domain signal with x
                plt.xlabel("Time in seconds")
                plt.ylabel("x(t)")
                plt.title("solution to d$^{2}$x/dt$^{2}$ + x = cos(1.5t)e$^{-0.05t}u_
                plt.show()
```



2.2 Problem 3: Response of spring to varying frequency of the forcing function

In this problem the frequency of the forcing function is varied between 1.4 to 1.6 in steps 0.05.

$$f(t) = cos(\omega * t)e^{-0.05t}u_o(t) \qquad 1.4 < \omega < 1.6$$

In s domain the forcing function is given by

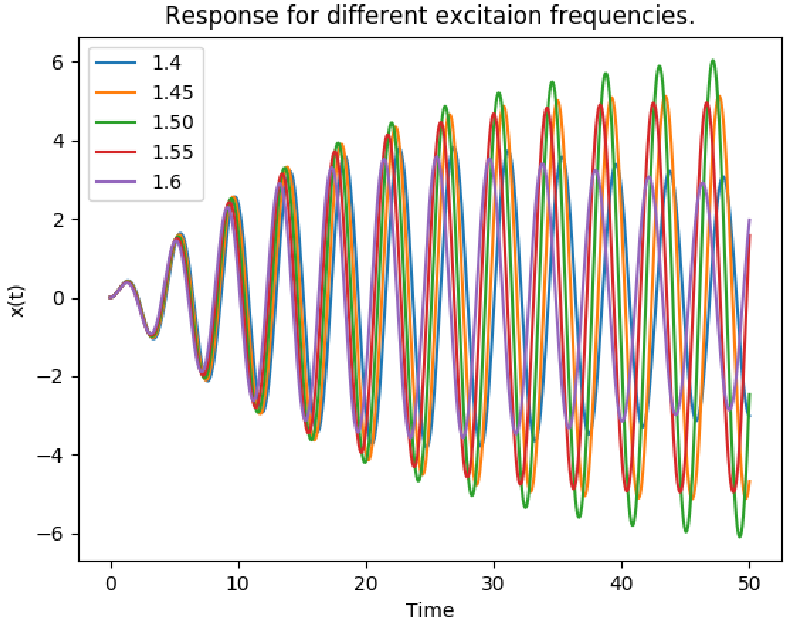
$$F(s) = \frac{s + 0.05}{(s + 0.05)^2 + \omega^2} \qquad 1.4 < \omega < 1.6$$

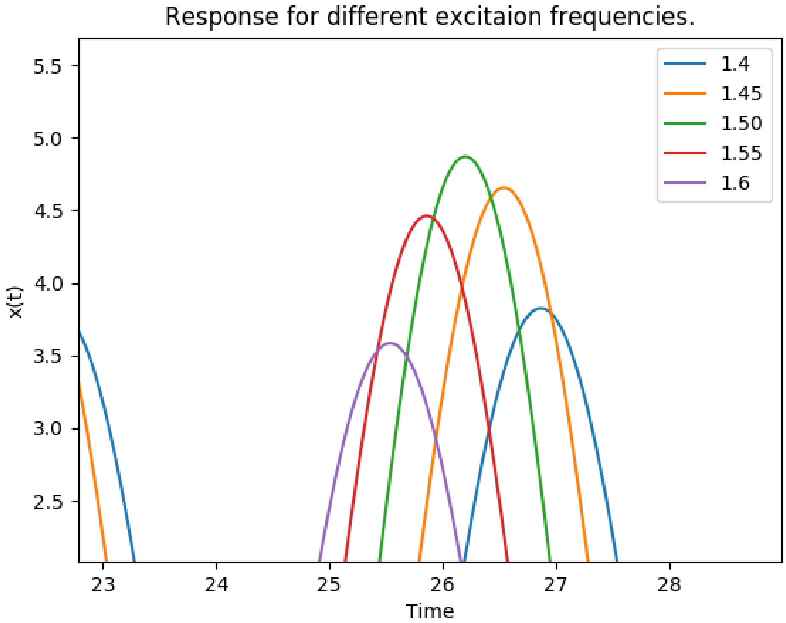
for this problem scipy.signal's lsim function is used to compute the response from spring system whose transfer function is

$$H(s) = \frac{1}{s^2 + 2.25}$$

lsim takes the transfer function in s domain and input in time domain and returns the time domain output of the system.

```
t = np.linspace(0,50,10000) #calculating for 50 sec
                p1 = np.poly1d([1]) # numerator 1
                p2 = np.poly1d([1,0,2.25])      #denominator polynomial s^2 + 2.25
                H = sp.lti(p1,p2) #numerator by denominator
                for var in np.arange(1.4, 1.6,0.05): #changing frequency of the cosine
                        ft = np.cos(var*t)*np.exp(-0.05*t) # input excitation
                        t,x,svec = sp.lsim(H,ft,t)  #response of the input excitation
                        plt.plot(t,x) #plotting its response
                plt.legend(["1.4","1.45","1.50","1.55","1.6"])  #plot of input respon
                plt.title("Response for different excitaion frequencies.")
                plt.xlabel("Time")
                plt.ylabel("x(t)")
                plt.show()
```



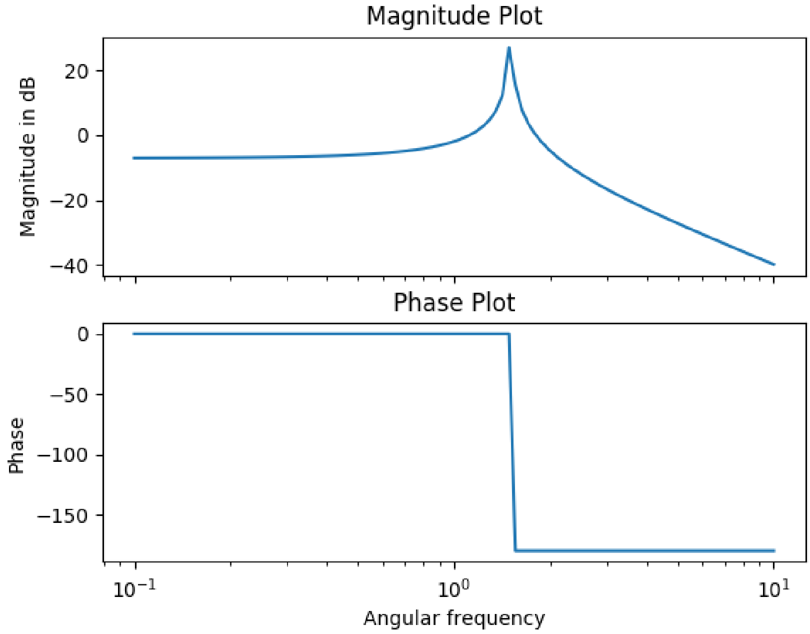


Plot of Magnitude and phase response of the spring system which is given by

$$H(s) = \frac{1}{s^2 + 2.25}$$

.

```
omega,s,phi = H.bode() #plotting phase and magnitude response
                f, axarr = plt.subplots(2,sharex = True)
                axarr[0].set_title("Magnitude Plot")
                axarr.flat[0].set(ylabel = "Magnitude")
                axarr[0].semilogx(omega,s)
                axarr[1].set_title("Phase Plot")
                axarr.flat[1].set(xlabel ="Angular frequency", ylabel = "Phase")
                axarr[1].semilogx(omega,phi)
                plt.show()
```



From the Magnitude response of the spring mass system the peak occurs at $\omega = 1.5$. Which matches with observations from the plot of position of the block for different excitation frequencies. Maximum amplitude of oscillation is for $\omega = 1.5$ which is the natural frequency of the system.

3 Problem 4 :Coupled Spring Problem

Positions are described by the given pair of equations

$$\frac{d^2x}{dt^2} + (x - y) = 0$$

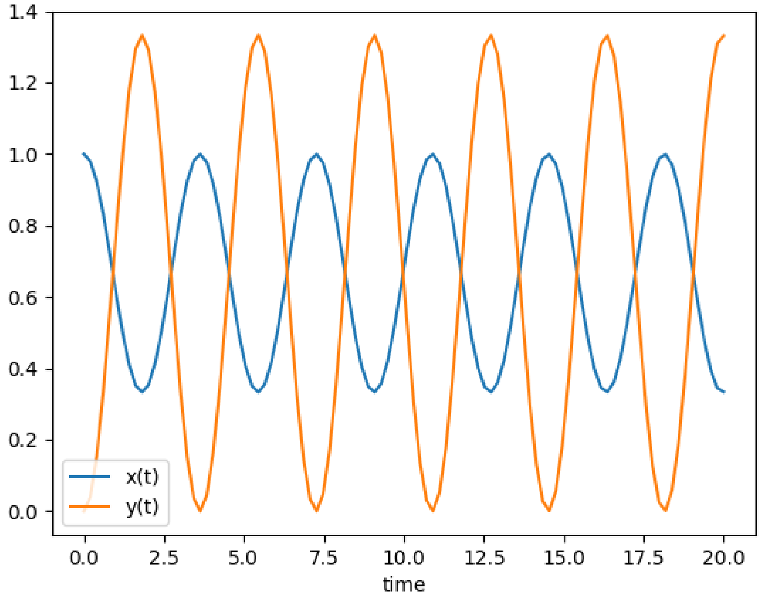
$$\frac{d^2y}{dt^2} + 2(y - x) = 0$$

With initial conditions described as $\frac{dx}{dt}_{x=0} = 1$, $\frac{dy}{dt}_{x=0} = 1$, x(0) = 0 and y(0) = 0 From the following conditions above X(s) and Y(s) are given by

$$X(s) = \frac{s^2 + 2}{s^3 + 3s}$$

$$Y(s) = \frac{s^2}{s^3 + 3s}$$

```
#solving coupled equations
        px1 = np.poly1d([1,0,2]) #polynomial s^2 + 2
        px2 = np.poly1d([1,0,3,0]) #polynomial s^3 + 3s
        X = sp.lti(px1,px2) #transfer function for x
        t1,x = sp.impulse(X,None,np.linspace(0,20,100)) #for 20 sec
        plt.plot(t1,x)
        py1 = np.poly1d([2])     #polynomial 2
        py2 = np.poly1d([1,0,3,0]) #polynomial s^3 + 3s
        Y = sp.lti(py1,py2)
        t2,y = sp.impulse(Y,None,np.linspace(0,20,100))#for 20 sec
        plt.plot(t2,y)
        plt.xlabel("time")
        plt.legend(["x(t)","y(t)"])
        plt.show()
```



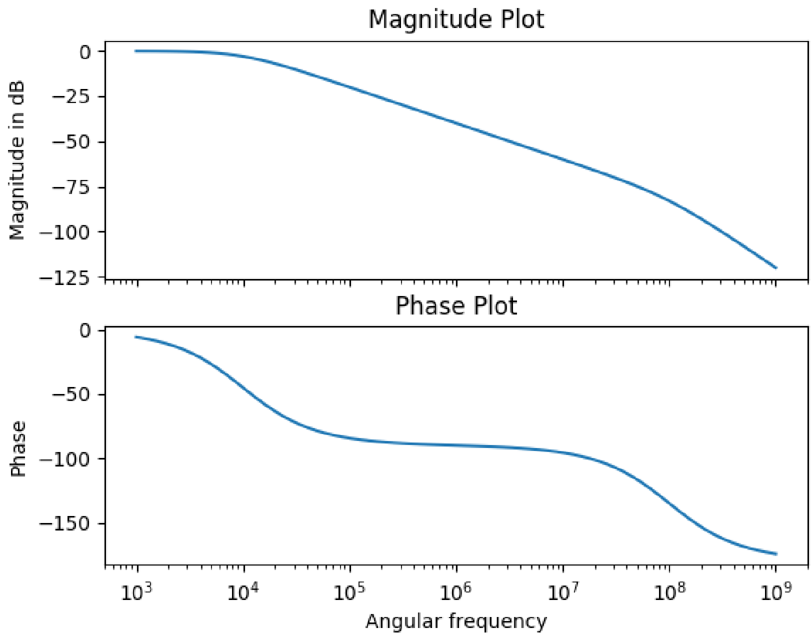
4 Problem 5: Magnitude and phase response of RLC circuit

The transfer function of the RLC network given is given by

$$H(s) = \frac{1}{s^2 10^{-12} + s10^{-4} + 1}$$

Using python function Bode to obtain phase and magnitude of any transfer function and plotting it in semi-log scale will give the bode plot of magnitude and phase.

```
px1 = np.poly1d([1]) #numerator 1
                                px2 = np.poly1d([1e-12,1e-4,1]) #deominator 10^-12s^2 + 10^-4 +1
                                X = sp.lti(px1,px2) #numerator by denomiantor
                                omega,s,phi = X.bode() #plotting frequency and magnitude response
                                f, axarr = plt.subplots(2,sharex = True)
                                axarr[0].set_title("Magnitude Plot")
                                axarr.flat[0].set(ylabel = "Magnitude")
                                axarr[0].semilogx(omega,s)
                                axarr[1].set_title("Phase Plot")
                                axarr.flat[1].set(xlabel ="Angular frequency", ylabel = "Phase")
                                axarr[1].semilogx(omega,phi)
                                plt.show()
```



4.1 Problem 6: response of RLC network to an input signal

For a given input voltage signal

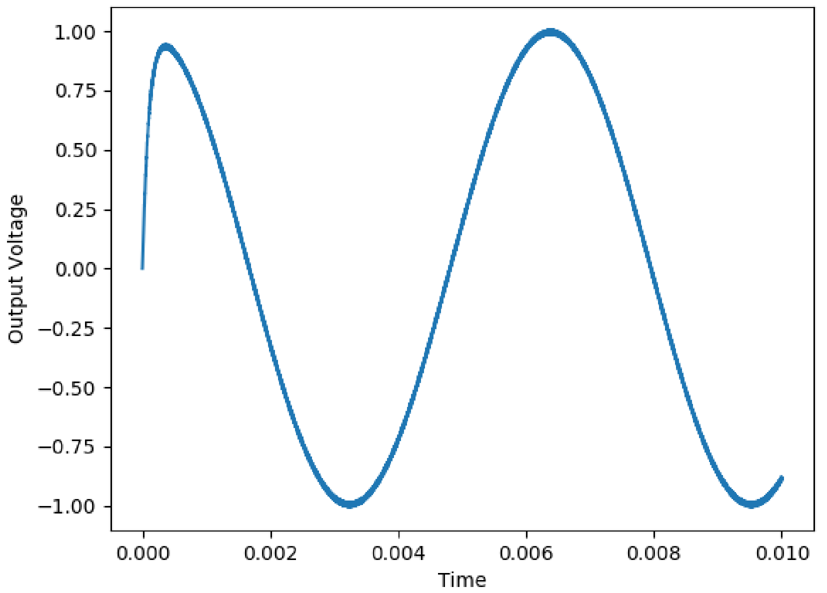
$$v_i(t) = cos(10^3t)u(t) - cos(10^6t)u(t)$$

The output voltage can be obtained by by scipy.signal's function lsim. It takes transfer function and time domain input and returns output.

```
#solving for response of rlc network for an input signal cos(10^3t)u(
px1 = np.poly1d([1])
px2 = np.poly1d([1e-12,1e-4,1])
X = sp.lti(px1,px2)
t1 = np.linspace(0,30e-6,10000) #for time 10ms
ft = np.cos(1e3*t1)-np.cos(1e6*t1)
t1,y,svec = sp.lsim(X,ft,t1)
plt.xlabel("Time")
plt.ylabel("Output Voltage")
plt.plot(t1,y)
plt.show()
```

4.1.1 For simulation up to 10ms

The given input to RLC network consist of two frequency components , 1K Rad/sec and 1M Rad/sec . Since the RLC network is a low pass filter the out put obtained looks line a sine wave. The Time period obtained from the output is 6.28 ms to which corresponding ω is 1K Rad/sec . Magnitude Response for this RLC network for ω = 1K Rad/sec is 0.9090 where as for ω = 1M Hz it is 0.00980. Hence 1M Hz component of the input is attenuated. The sharp rising is due the transient response and than it becomes close to 1K Rad/sec sine as transient response dies and steady state is reached.



4.1.2 For simulation up to 30us

The simulation from 0 - 30 μs shows ripples . These ripples are due to the 1M Rad/sec component of the input signal which gets attenuated by the low pass filter.