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Computer Algorithms Homework 8

24. Let G = (V, E) be a bipartite graph; suppose its nodes are partitioned into sets X and Y so that each edge has one end in X and the other in Y. We define an (a, b)-skeleton of G to be a set of edges $E' \subseteq$ E so that at most a nodes in X are incident to an edge in E', and at least b nodes in Y are incident to an edge in E'.

Show that, given a bipartite graph G and numbers a and b, it is NP-complete to decide whether G has an (a, b)-skeleton.

NP: Given a bipartite graph, set of edges E', and a (a,b)-skeleton you can go through each edge in E' and place nodes in two sets such that $X' \subseteq$ X and $Y' \subseteq$ Y. If $|X'| \leq$ a and $|Y'| \geq$ b then G has a valid (a,b)-skeleton.

Set Cover $\leq_p$ (a,b)-skeleton: Given the parameters of set cover; $S_1, S_2, ..., S_m$, set U of n elements. We define a bipartite graph where the nodes in X correspond to the sets $S_1, S_2, ..., S_m$, and the nodes in Y be the elements in U. We can create an edge between sets and elements if the set has that element in it. We set a = k and b = n. Put into (a,b)-skeleton blackbox, if yes then we have a set cover.

Proof: If G has a (a,b)-skeleton E', then the k nodes in X are incident to the edges in E' correspond to k sets which have all the elements. Therefore we have a set cover. If there is a set cover of size k, then taking E' to be the set of all edges incident to the corresponding set nodes gives you an (a,b)-skeleton.

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26. You and a friend have been trekking through various far-off parts of the world and have accumulated a big pile of souvenirs. At the time you weren't really thinking about which of these you were planning to keep and which your friend was going to keep, but now the time has come to divide everything up.

Here's a way you could go about doing this. Suppose there are n objects, labeled 1, 2, ..., n, and object i has an agreed-upon value x_i. (We could think of this, for example, as a monetary resale value; the case in which you and your friend don't agree on the value is something we won't pursue here.) One reasonable way to divide things would be to look for a partition of the objects into two sets, so that the total value of the objects in each set is the same.

This suggests solving the following Number Partitioning Problem. You are given positive integers x_1, ..., x_n; you want to decide whether the numbers can be partitioned into two sets S_1 and S_2 with the same sum:

$$\sum_{x_i \in S_1} x_i = \sum_{x_j \in S_2} x_j$$

Show that Number Partitioning is NP-complete.

NP: Given two sets S_1 and S_2 you can find the sum of all the objects values in each set, if the sums are equal then there is a number partitioning.

Subset Sum $\leq_p$ Number Partitioning: Given the parameters for subset sum (W = $\{w_1, w_2, ..., w_n\}$, $\bar{W}$) we can make each weight into a value for an object and we double the list. Now we make $2\bar{W}$ as a value for an object. Our list of values should look as follows, X = $\{w_1, w_1, w_2, w_2, ..., w_n, w_n, 2\bar{W}\}$. The construction of this list is 2n +1 which is polynomial. With this list we can input it into the Number Partitioning blackbox. If there is a number partitioning, then there is a subset sum.

Proof: If there is a subset sum, then there is a $W' \subseteq$ W where...

$$\sum_{w_i \in W'} w_i = \bar{W}$$

and using the construction we used for the reduction we can always make two sets S_1 and S_2 such that...

$$\sum_{w_i \in W'} w_i + \sum_{w_i \in W'} w_i + \sum_{w_j \notin W'} w_j = 2\bar{W} + \sum_{w_j \notin W'} w_j$$

where the left hand side objects are in S_1 and the right is S_2. A similar argument can be made for the other direction in that if there is a number partitioning, using the summation above we see that there is a subset sum of size $\bar{W}$