Introduction, Classification of Numbers

1) what value will replace the question mark in each of the following questions?

i) $? - 1936248 = 1635773$

$x - 1936248 = 1635773$

$x = 1635773 + 1936248$

$\boxed{x = 3572021}$

ii) $9587 - ? = 4358$

$9587 - x = 4358$

$9587 - 4358 = x$

$\boxed{x = 5229}$

iii) $9587 - ? = 7429 - 4358$

$9587 - x = 3071$

$9587 - 3071 = x$

$\boxed{x = 6516}$

2) what will be the maximum value of Q in the following equation?

$5P9 + 3R7 + 2Q8 = 1114$

$$\begin{array}{r} \text{①②} \\ 5P9 \\ 3R7 \\ 2Q8 \\ \hline 1114 \end{array}$$

$2 + P + R + Q = 11$

$P + R + Q = 11 - 2$

$P + R + Q = 9$

if the value of P & $R = 0$ then the maximum value of Q is 9 $\boxed{Q = 9}$

3) A is counting the numbers from 1 to 31 and B from 31 to 1. A is counting the odd numbers only. The speed of both is the same. what will be the number which will be pronounced by A & B together?

A	1	3	5	7	9	11	13	15	17	19	21
B	31	30	29	28	27	26	25	24	23	22	21

A & B pronounce the number 21 together

4) If Δ stands for the operation "adding first number to twice the second number" then find the value of

$(1 \Delta 2) \Delta 3$

$= (1 + (2 \times 2)) \Delta 3$

$= (1 + 4) \Delta 3$

$= (5 \Delta 3)$

$= [5 + (2 \times 3)]$

$= 5 + 6$

$= 11$

5) P and Q are two positive integers such that $PQ = 64$. Which of the following cannot be the value of $P+Q$?

a) 16 b) 20 c) 35 d) 65

$PQ = 64$

P	Q
64 × 1 = 64	
32 × 2 = 64	
16 × 4 = 64	
8 × 8 = 64	

$P+Q =$

$64 + 1 = 65$

$32 + 2 = 34$

$16 + 4 = 20$

$8 + 8 = 16$

$\boxed{P+Q = 35}$ cannot be the value

6) If n is an integer between 20 and 80, then any of the following could be $n+7$ except

a) 47 b) 58 c) 84 d) 88

$n+7 = 47$ $\quad n = 47-7 = 40$

$n+7 = 58$ $\quad n = 58-7 = 51$

$n+7 = 84$ $\quad n = 84-7 = 77$

$n+7 = 88$ $\quad n = 88-7 = 81$

81 is not b/w 20 to 80

hence the answer is 88

7) Which of the following digits will replace the H in the following equation?

$9H + H8 + H6 = 230$

a) 3 b) 4 c) 5 d) 9

e) None of these

$$\begin{array}{r} 9H \\ H8 \\ H6 \\ \hline 230 \end{array}$$

$H + 8 + 6 = 20$

$H + 14 = 20$

$\boxed{H = 6}$

$$\begin{array}{r} 96 \\ 68 \\ 66 \\ \hline 230 \end{array}$$

ans: None of those

) If * means adding 6 times the second number to the first number, then (1 * 2) * 3 equals

(1 * 2) * 3

[1 + (2 × 6)] = 1 + 12 = 13

(13 * 3)

13 + (3 × 6) = 13 + 18 = 31

$\boxed{(1 * 2) * 3 = 31}$

Formation of Numbers

Place Value & Face Value

↳ Local value

) The sum of the place value of 3 in the number 503535 is

503535

ten — 3 × 10 = 30

thousand — 3 × 1000 = 3000

Sum = 3000 + 30 = 3030

) The difference between the place values of 7 and 3 in the number 527435 is

527435

ten — 3 × 10 = 30

thousand — 7 × 1000 = 7000

difference = 7000 ~ 30 = 6970

3) The difference between the local value and the face value of 7 in the numeral 32675149 is

local value

32675149

ten thousand

7 × 10,000 = 70,000

Face value

32675149

7 Face value

difference = 70,000 ~ 7

difference = 69993

Divisibility Rule:

1) what least value must be assigned to * so that the number 197*5462 is divisible by 9?

1 + 9 + 7 + x + 5 + 4 + 6 + 2

= (34 + x) least value = 2

34 + 2 = 36 ÷ 9.

$\boxed{\text{ans: } 2}$

2) Which digits should come in place of * and $ if the number 62684*$ is divisible by both 8 & 5?

÷ by 5 means the last digit should be 0 (or) 5 but the Number ending with 5 is Never divisible by 8

hence $ = 0

÷ by 8 means the last 3 digit should be ÷ by 8

4*0

400 ÷ 8
440 ÷ 8
480 ÷ 8

hence (0, 4, 8)
these are all the values will replace *

ans: $ = 0
* = 0 (or) 4 (or) 8

ans: 6268400
6268440
6268480

3) Is 52563744 divisible by 24?

24 = 3×8 (Co-Prime)

÷ by 3 & 8 is always ÷ by 24

÷ 3

5+2+5+6+3+7+4+4 = 36 ÷ 3

÷ 8 means take last 3 digit

```
     93
8 ) 744
    72
    ---
     24
     24
     --
      0
```

hence the Number is divisible by 24

4) If the number 3422213pq is divisible by 99, find the missing digits p & q

99 = 9 × 11 (Co-prime)

if the given number is divisible by 9 & 11 then it is divisible by 99

÷ 9

(3+4+2+2+2+1+3+p+q) = divisible by 9

17+(p+q) = divisible by 9

$17 + (p+q) =$ divisible by 9

possible values are

$17 + 1 = 18$

$17 + 10 = 27$

hence $p+q = 1$ --- ① } possible values
$p+q = 10$ --- ②

$\div 11$

(Sum of odd) ~ (Sum of even) = 0 (or) $\div$ by 11

$(q+3+2+2+3) - (p+1+2+4) = 0$

$(q+10) - (p+7) = 0$ (or) 11

$q+10-p-7 = 0$ (or) 11

$q-p+3 = 0$ (or) 11

$q-p = -3$ (or) 8

$q-p = 3$ (or) 8

$-p+q = 3$ --- ⑤

$-p+q = 8$ --- ④

Solving ② & ④

$\cancel{p}+q = 10$

$-\cancel{p}+q = 8$

$2q = 18$

$q = \frac{18}{2} = 9$ $\boxed{q=9}$

$p+q = 10$

$p+q = 10$ $p = 10-9$ $\boxed{p=1}$

Sub the value in ④

$-p+q = 8$

$-1+9 = 8$

$\cancel{8} = \cancel{8}$

ans: $p = 1$

$q = 9$

5) find the number which is nearest to 3105 and is exactly divisible by 21

$$\begin{array}{r} 147 \\ 21\overline{)3105} \\ 21 \\ \hline 100 \\ 84 \\ \hline 165 \\ 147 \\ \hline 18 \end{array}$$

$21 \sim 18 = 3$

Hence the required number

$= 3105 + 3 = 3108$

6) find the smallest number of five digits which is exactly divisible by 476.

Smallest 5 digit number = 10,000

$$\begin{array}{r} 21 \\ 476\overline{)10000} \\ 972 \\ \hline 480 \\ 476 \\ \hline 4 \end{array}$$

$476 \sim 4 = 472$

Hence the required number

$= 10000 + 472 = 10,472$

7) Find the greatest number of five digits which is exactly divisible by 47.

Greatest 5 digit Number

= 99999

```
     2127
47 | 99999
     94
     ----
      59
      47
     ----
      129
       94
     ----
       359
       329
       ----
         30
```

Hence the required Number is

(99999 − 30) = 99969

8) when a certain number is multiplied by 13, the product consists entirely of fives. find the smallest such number.

```
     42735
13 | 555555
     52
     ----
      35
      26
     ----
      95
      91
     ----
       45
       39
      ----
        65
        65
       ----
         0
```

hence the required Number is

$\boxed{42735}$

9) when a certain Number is multiplied by 18, the Product consists entirely of 2's. what is the minimum Number of 2's in the Product?

```
        12345679
18 | 222222222
     18
     ----
      42
      36
     ----
       62
       54
      ----
        82
        72
       ----
         102
          90
        ----
         122
         108
        ----
          142
          126
         ----
           162
           162
          ----
             0
```

minimum Number of 2's in the product

= 9

10) If x and y are positive integers such that $(3x+7y)$ is a multiple of 11, then which of the following will also divisible by 11?

a) $4x+6y$ b) $x+y+4$

c) $9x+4y$ d) $4x-9y$

$(3x+7y)$ is a multiple of 11

putting $x=5$ $y=1$.

$(3\times5+7\times1) = (15+7) = 22$ which is divisible by 11

$4x+6y$

$(4\times5+6\times1) = (20+7) = 27$

$x+y+4$

$5+1+4 = 10$

$9x+4y$

$(9\times5+4\times1) = (45+4) = 49$

$4x-9y$

$(4\times5-9\times1) = (20-9) = 11$

ans: $4x-9y$

11) A 3-digit number $4a3$ is added to another 3-digit number 984 to give the four digit number 1·3b7 which is divisible by 11 Then $(a+b)$ is

a) 10 b) 11 c) 12 d) 15

$$\begin{array}{r} 4a3 \\ 984 \\ \hline 1\,3\,b\,7 \end{array}$$

$a+8=b$

$b-a=8$

also 13b7 is divisible by 11

1 3 b 7

$(7+3) - (b+1) = 0$

odd digit even digit

$(7+3)-(b+1)=0$

$10-b-1=0$

$9-b=0$

$+b=+9$ $\boxed{b=9}$

$b-a=8$

$9-a=8$

$-a=8-9$ $\boxed{a=1}$

ans: $(a+b)=10$

12) If x is a whole number, then $x^2(x^2-1)$ is always divisible by

a) 12 b) 24 c) $12-x$ d) multiple of 12

Put $x = 2$

$2^2(2^2-1)$

$= 4(4-1)$

$= 4(3)$

$= 12$

ans: $x^2(x^2-1)$ is always divisible by 12

Unit digit

1) find the unit's digit in the product $(2467)^{153} \times (341)^{72}$

$= 7^{153} \times 1^{72}$

$= 7 \times 1$

$\boxed{\text{ans} = 7}$

7 cyclicity 4

$$\begin{array}{r} 13 \\ 4\overline{)53} \\ 4 \\ \hline 13 \\ 12 \\ \hline 1 \end{array}$$

$7^1 = 7$

2) find the unit's digit in $(264)^{102} + (264)^{103}$

$4^{102} + 4^{103}$

4 cyclicity 2

$= 4 + 6$

$= 10$

$\boxed{\text{ans}: 0}$

3) If the unit digit in the product $(459 \times 46 \times 28* \times 484)$ is 2, the digit in place of * is

a) 3 b) 5 c) 7 d) None of these

$(459)(46)(28*)(484)$

$(9 \times 6 \times 4) = 216$

In order to obtain 2 at the unit place, we must multiply 216 by 2 (or) 7

$\boxed{\text{ans}: 7}$

4) The digit in the unit's place of the number represented by $(7^{95} - 3^{58})$ is:

a) 0 b) 4 c) 6 d) 7

7^{95} = unit digit = 3

3^{58} = unit digit = 9

7 (cyclicity 4)

$$\begin{array}{r} 23 \\ 4\overline{)95} \\ 8 \\ \hline 15 \\ 12 \\ \hline 3 \end{array}$$

3 cyclicity 4

$$\begin{array}{r} 14 \\ 4\overline{)58} \\ 4 \\ \hline 18 \\ 16 \\ \hline 2 \end{array}$$

$3 - 9$

ans = 4

5) If x is an even number, then x^{4n}, where n is a +ve integer, will always have:

a) zero in the unit's place

b) 6 in the unit's place

c) either 0 or 6 in the unit's place

d) none of these

$(x^4)^n = x^{4n}$

x is an even number

$x = 2, 4, 6, 8 - - -$

$(2^4)^n$ (or) $(4^4)^n$ (or) $(6^4)^n$ (or) $(8^4)^n$

2^4 = Unit digit = 6	2 cyclicity 4
4^4 = Unit digit = 6	4 cyclicity 2
6^4 = unit digit = 6	6 cyclicity 1
8^4 = unit digit 6	8 cyclicity 4

ans: 6 in the unit's place

Division algorithm

1) on dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. find the divisor.

$$\text{Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{quotient}}$$

$$= \frac{15968 - 37}{89} = 179$$

2) A number when divided by 342 gives a remainder 47. when the same number is divided by 19, what would be the remainder?

Dividend = (Divisor × quotient) + Remainder

divisor = 342

quotient = Q

Remainder = 47

Dividend = N

$N = (342 \times Q) + 47$

$N = 342Q + 47$ $\quad N \div 19$, $R = ?$

$= 19 \times 18Q + 19 \times 2 + 9$

$= 19(18Q + 2) + 9$

Remainder = 9

3) A number being successively divided by 3, 5 and 8 leaves remainders 1, 4 & 7 respectively. find the respective remainders if the order of divisor be reversed.

$$\begin{array}{r|l} 3 & x \\ \hline 5 & y - 1 \\ \hline 8 & z - 4 \\ \hline & 1 - 7 \end{array}$$

$z = (8 \times 1 + 7) = 15$

$y = (5 \times 15 + 4) = 79$

$x = (3 \times 79 + 1) = 238$

$$\begin{array}{r|l} 8 & 238 \\ \hline 5 & 29 - 6 \\ \hline 3 & 5 - 4 \\ \hline & 1 - 2 \end{array}$$

Respective remainders are 6, 4, 2

4) A Number when divided by the Sum of 555 and 445 gives two times their difference as quotient and 30 as the remainder. The number is

Divisior = Sum of 555 & 445

$$\begin{array}{r} 555 \\ +\ 445 \\ \hline 1000 \end{array}$$

quotient = two times the difference

$$\begin{array}{r} 555 \\ (-)\ 445 \\ \hline 110 \\ \hline \end{array}$$

2 times the difference

$2 \times 110 = 220$

Dividend = (Divisior × quotient) + Remainder

$= (1000 \times 220) + 30$

$= 220030$

5) A four-digit number is divisible by 7 becomes divisible by 3, when 10 is added to it. The largest such number is

largest 4 digit Number 9999

$$\begin{array}{r|l} & 1428 \\ \hline 7 & 9999 \\ & 7 \\ \hline & 29 \\ & 28 \\ \hline & 19 \\ & 14 \\ \hline & 59 \\ & 56 \\ \hline & 3 \end{array}$$

$9999 - 3 = 9996$

$(9996 + 10)$ be divisible by 3

$(9996 - x) + 10$ $\quad x = 7$

$(9996 - 7) + 10$

$9989 + 10 = 9999 \div$ by 3

ans: 9989

6) A Number when divided by 296 gives a remainder 75. when the same Number is divided by 37, then the remainder will be?

$N = (296 \times Q) + 75$

$= (37 \times 8Q) + (37 \times 2) + 1$

$= 37(8Q + 2) + 1$

Remainder = 1

7) In a division sum, the divisor is 10 times the quotient and 5 times the remainder. If the remainder is 46, the dividend is

Divisor = 10 times the quotient

$= (10 \times Q)$

(or)

= 5 times the remainder

$= 5 \times 46$

Divisor = 230

Divisor = $10 \times Q$

$230 = 10 \times Q$

$Q = \frac{23\cancel{0}}{1\cancel{0}} = 23$

Divisor = 230
Quotient = 23
Remainder = 46

Dividend = (Divisor × quotient) + Remainder

$= (230 \times 23) + 46$

Dividend = 5336

Prime factors:

1) find the total number of Prime factors in the expression

$4^{11} \times 7^5 \times 11^2$

4 is not a Prime Number hence convert it into Prime Number

$= (2 \times 2)^{11} \times 7^5 \times 11^2$

$= 2^{11} \times 2^{11} \times 7^5 \times 11^2$

$= 2^{22} \times 7^5 \times 11^2$

take the power values and add togather to get Prime factors

$= (22 + 5 + 2)$

$\boxed{\text{ans} = 29}$

2) The Number of prime factors of

$(3 \times 5)^{12} (2 \times 7)^{10} (10)^{25}$

10 is not a Prime Number hence convert it into Prime Number

$(3 \times 5)^{12} (2 \times 7)^{10} (2 \times 5)^{25}$

$= 3^{12} \times 5^{12} \times 2^{10} \times 7^{10} \times 2^{25} \times 5^{25}$

$= 2^{35} \times 3^{12} \times 5^{37} \times 7^{10}$

add all the Power values to get Prime factors.

$= 35 + 12 + 37 + 10$

$\boxed{\text{ans} = 94}$

HCF and LCM

1) Reduce $\frac{391}{667}$ to lowest terms

HCF for 391, 667

```
        1
391 | 667
      391   1
      276 | 391
            276   2
            115 | 276
                  230   2
                   46 | 115
                        92   2
                        23 | 46
                           46
                            0
```

HCF = 23

on dividing by 23 on Num & Den we get the lowest terms

$= \frac{391/23}{667/23}$ $\boxed{\text{ans} = \frac{17}{29}}$

2) find the HCF and LCM of

$\frac{2}{3}, \frac{8}{9}, \frac{16}{81}$ & $\frac{10}{27}$

$HCF = \frac{\text{HCF of } 2, 8, 16, 10}{\text{LCM of } 3, 9, 81, 27} = \frac{2}{81}$

$LCM = \frac{\text{LCM of } 2, 8, 16, 10}{\text{HCF of } 3, 9, 81, 27} = \frac{80}{3}$

HCF of NM

```
      4          8          5
2 | 8      2 | 16     2 | 10
    8          16         10
    0           0          0
```

HCF of DN

```
      3          27         9
3 | 9      3 | 81     3 | 27
    9           6         27
    0          21          0
              21
               0
```

Lcm of NM

```
2 | 2, 8, 16, 10
2 | 1, 4, 8, 5
2 | 1, 2, 4, 5
2 | 1, 1, 2, 5
5 | 1, 1, 1, 5
    1, 1, 1, 1
```

$2 \times 2 \times 2 \times 2 \times 5$

$= 80$

Lcm of DN

```
3 | 3, 9, 81, 27
3 | 1, 3, 27, 9
3 | 1, 1, 9, 3
3 | 1, 1, 3, 1
    1, 1, 1, 1
```

$3 \times 3 \times 3 \times 3$

$= 81$

3) find the HCF and LCM of 0.63, 1.05 and 2.1

first make all decimal's equal

0.63 1.05 2.10

multiply by 100

63 105 210

HCF

63 | 105 (1)
63
42 | 63 (1)
42
21 | 42 (2)
42
0

21 | 210 (10)
210
0

HCF = 21

Divide by 100

$\frac{21}{100} = 0.21$

$\boxed{\text{HCF ans: } 0.21}$

LCM

3 | 63 105 210
7 | 21 35 70
3 | 3 5 10
5 | 1 5 10
2 | 1 1 2
1 1 1

$3 \times 7 \times 3 \times 5 \times 2 = 630$

LCM = 630

Divide by 100

$\frac{630}{100} = 6.3$

$\boxed{\text{LCM ans: } 6.30}$

4) Two Numbers are in the ratio of 15 : 11. If their HCF is 13 find the numbers

Required numbers $15x$ & $11x$

HCF is x $\quad x = 13$

$= 15 \times 13 \quad 11 \times 13$

$195 \quad 143$

$\boxed{\text{ans } 195, 143}$

5) The HCF of two numbers is 11 and their LCM is 693. If one of the Numbers is 77. find the other.

Product of Two number = Product of LCM & HCF of those Numbers.

$x \times 77 = 11 \times 693$

$x = \frac{11 \times 693}{77} = 99$

$\boxed{\text{ans: } 99}$

6) find the greatest Possible length which can be used to measure exactly the lengths 4m 95cm, 9m and 16m 65 cm.

first Convert all units into a single Unit $\boxed{\text{CM}}$

1m = 100cm

4m 95cm = 495cm

9m = 900cm

16m 65cm = 1665cm

Clue word greatest (HCF) highest

495, 900, 1665

HCF

```
      1
495 | 900
     495   1
     405 | 495
           405   4
            90 | 405
                360   2
                 45 | 90
                      90
                       0
```

HCF = 45

```
      37
45 | 1665
     135
      315
      315
        0
```

$\boxed{\text{HCF} = 45}$

The required length = 45 cm

7) Find the greatest number which on dividing 1657 and 2037 leaves remainders 6 & 5

Clue word greatest (HCF)

1657 − 6 = 1651
2037 − 5 = 2032 } find HCF

```
      1
1651 | 2032
       1651   4
        381 | 1651
              1524   3
               127 | 381
                     381
                       0
```

HCF = 127

The greatest Number = 127

8) Find the largest number which divides 62, 132, 237 to leave the same remainder in each case

132 − 62 = 70

237 − 132 = 105

237 − 62 = 175

HCF

70, 105, 175

```
      1
70 | 105
     70   2
     35 | 70
          70
           0
```

```
      5
35 | 175
     175
       0
```

HCF = 35

The largest Number = 35

7) The traffic lights at 3 different road crossings change after every 48 sec, 72 sec and 108 sec, respectively. If they all change simultaneously at 8:20:00 hours then at what time will they again change simultaneously?

LCM = 48, 72, 108

$$\begin{array}{r|l} 2 & 48, 72, 108 \\ \hline 2 & 24, 36, 54 \\ \hline 2 & 12, 18, 27 \\ \hline 3 & 6, 9, 27 \\ \hline 3 & 2, 3, 9 \\ \hline 3 & 2, 1, 3 \\ \hline 2 & 2, 1, 1 \\ \hline & 1, 1, 1 \end{array}$$

$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 = 432$

Total = 432 sec

1 min = 60 sec

$$\begin{array}{r|l} & 7 \\ \hline 60 & 432 \\ & 420 \\ \hline & 12 \end{array}$$

$\frac{432}{60} =$

7 min 12 sec

First change happened 8:20:00 hrs

7:12

Next change 8:27:12 hrs

10) Arrange the fractions $\frac{17}{18}, \frac{31}{36}, \frac{43}{45}, \frac{59}{60}$ in the ascending order.

LCM = 18, 36, 45, 60

$$\begin{array}{r|l} 2 & 18, 36, 45, 60 \\ \hline 3 & 9, 18, 45, 30 \\ \hline 3 & 3, 6, 15, 10 \\ \hline 2 & 1, 2, 5, 10 \\ \hline 5 & 1, 1, 5, 5 \\ \hline & 1, 1, 1, 1 \end{array}$$

$2 \times 3 \times 3 \times 2 \times 5 = 180$

$\frac{17}{18} = \frac{17}{18} \times \frac{10}{10} = \frac{170}{180}$

$\frac{31}{36} = \frac{31}{36} \times \frac{5}{5} = \frac{155}{180}$

$\frac{43}{45} = \frac{43}{45} \times \frac{4}{4} = \frac{172}{180}$

$\frac{59}{60} = \frac{59}{60} \times \frac{3}{3} = \frac{177}{180}$

$\frac{170}{180}, \frac{155}{180}, \frac{172}{180}, \frac{177}{180}$

arrange in ascending order

$\frac{155}{180} < \frac{170}{180} < \frac{172}{180} < \frac{177}{180}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$\frac{31}{36} < \frac{17}{18} < \frac{43}{45} < \frac{59}{60}$

11) Three different containers contain 496 litres, 403 litres and 713 litres of mixtures of milk and water respectively. What biggest measure can measure all the different quantities exactly?

Clue word biggest measure = HCF

HCF = 496, 403, 713

```
      1
403 | 496
      403     4
      ---------
       93 | 403
            372    3
            ----------
             31 | 93
                  93
                  --
                   0
```

= 31

```
      23
31 | 713
     62
     ---
      93
      93
      --
       0
```

HCF = 31

The biggest measure can measure all the different quantities exactly is 31 litres

12) The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is:

Clue word maximum = HCF

HCF = 1001, 910

```
       1
910 | 1001
      910     10
      ----------
       91 | 910
            910
            ---
              0
```

HCF = 91

ans: maximum number of students 91

13) The LCM of two numbers is 45 times their HCF. If one of the number is 125 and the sum of HCF and LCM is 1150, the other number is:

HCF be H

LCM be L

L = 45H

Sum of LCM & HCF = L + H = 1150

L + H = 1150

Sub the value of $L = 45H$

$$45H + H = 1150$$
$$46H = 1150$$
$$H = \frac{1150}{46} = 25$$

$$L + H = 1150$$
$$L + 25 = 1150$$
$$L = 1150 - 25 = 1125$$

Product of two Numbers = Product of LCM & HCF of that Number

$$x \times 125 = 25 \times 1125$$
$$x = \frac{25 \times 1125}{125} = 225$$

$\boxed{\text{The other number} = 225}$

14) If the sum of two numbers is 55 and the HCF and LCM of these Numbers are 5 & 120 respectively, then the sum of the reciprocals of the Numbers is equal to

Let the Numbers be A & B

$$A + B = 55$$

Product of 2 Numbers = Product of LCM & HCF of that Numbers

$$AB = 5 \times 120$$
$$AB = 600$$

The sum of reciprocal of the Numbers

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{55}{600} = \frac{11}{120}$$

15) A, B & C Start at the same time in the same direction to run around a circular stadium. A completes a round in 252 sec, B in 308 sec & C in 198 sec. All starting at the same point. After what time will they meet again at the starting point?

LCM = 252, 308, 198

$$\begin{array}{r|l} 2 & 252, 308, 198 \\ \hline 2 & 126, 154, 99 \\ \hline 11 & 63, 77, 99 \\ \hline 7 & 63, 7, 9 \\ \hline 9 & 9, 1, 9 \\ \hline & 1, 1, 1 \end{array}$$

$$2 \times 2 \times 11 \times 7 \times 9 = 2772$$

Total time = 2772 sec

1 min = 60 sec

$$\begin{array}{r|l} & 46 \\ \hline 60 & 2772 \\ & 240 \\ \hline & 372 \\ & 360 \\ \hline & 12 \end{array}$$

46 min 12 sec

A, B & C will again meet at starting point in 2772 sec

(or)

46 min 12 sec

(6) Six bells commence toling together and toll at intervals of 2, 4, 6, 8, 10 and 12 seconds respectively. In 30 minutes how many times do they toll together

LCM = 2, 4, 6, 8, 10, 12

2	2, 4, 6, 8, 10, 12
2	1, 2, 3, 4, 5, 6
2	1, 1, 3, 2, 5, 3
3	1, 1, 3, 1, 5, 3
5	1, 1, 1, 1, 5, 1
	1, 1, 1, 1, 1, 1

$2 \times 2 \times 2 \times 3 \times 5 = 120$

The bells will toll together after 120 sec

1 min = 60 sec $\quad \frac{120}{60}$

$60 \overline{)120}$ = 2 min

In 30 min how many times do they toll together

$\frac{30}{2} = 15 + 1 = 16$ times

always add 1

$2 \overline{)30}$ = 15

(7) Four different electronic devices make a beep after every 30 min, 1 hour, 1½ hour and 1 hour 45 min respectively. All the devices beeped together at 12 noon. They will again beep together at

find amount all take one unit

30 min, 60 min, 90 min, 105 min

LCM = 30, 60, 90, 105

3	30, 60, 90, 105
5	10, 20, 30, 35
2	2, 4, 6, 7
2	1, 2, 3, 7
3	1, 1, 3, 7
7	1, 1, 1, 7
	1, 1, 1, 1

$3 \times 5 \times 2 \times 2 \times 3 \times 7 = 1260$

The devices will beep again together after 1260 min

1 hr = 60 min

$60 \overline{)1260}$ = 21 hrs

first beep @ 12:00 pm

add 21 hrs with it

$\boxed{\text{ans: } 9:00 \text{ AM}}$

18) A rectangular court yard 3.78 metres long and 5.25 metres wide is to be paved exactly with square tiles, all of the same size. What is the largest size of the tile which could be used for the purpose?

Convert it into cm

HCF = 378 cm, 525 cm

```
        1
378 | 525
      378    2
      ---
  147 | 378
        294    1
        ---
   84 | 147
        84     1
        ---
   63 | 84
        63     3
        ---
   21 | 63
        63
        ---
         0
```

HCF = 21

The largest size of the tile which could be used for the purpose = 21 cm

19) Find the least number which when divided by 4, 9 & 12 will leave in each case a remainder 3

LCM = 4, 9, 12

```
2 | 4, 9, 12
  ---------
2 | 2, 9, 6
  ---------
3 | 1, 9, 3
  ---------
3 | 1, 3, 1
  ---------
    1, 1, 1
```

LCM = 36

Remainder = 3

The least number = 36 + 3 = 39

20) Find the least number which when divided by 4, 6, 8, 12, 16 will leave in each case a remainder 2

```
2 | 4, 6, 8, 12, 16
  ----------------
2 | 2, 3, 4, 6, 8
  ----------------
2 | 1, 3, 2, 3, 4
  ----------------
2 | 1, 3, 1, 3, 2
  ----------------
3 | 1, 3, 1, 3, 1
  ----------------
    1, 1, 1, 1, 1
```

2 × 2 × 2 × 2 × 3 = 48 = LCM

remainder = 2

The least number = 48 + 2 = 50

21) find the least Number which when divided by 12, 15, 20 454 will leave in each case a remainder of 4

$$
\begin{array}{r|l}
2 & 12, 15, 20, 54 \\
\hline
2 & 6, 15, 10, 27 \\
\hline
3 & 3, 15, 5, 27 \\
\hline
3 & 1, 5, 5, 9 \\
\hline
3 & 1, 5, 5, 3 \\
\hline
5 & 1, 5, 5, 1 \\
\hline
 & 1, 1, 1, 1
\end{array}
$$

$2 \times 2 \times 3 \times 3 \times 3 \times 5 = 540$ = LCM

remainder = 4

least Number = 540 + 4 = 544

22) find the least Number which when divided by 10, 9, 8 leaves a remainders 9, 8 & 7 respectively in each case

LCM

$$
\begin{array}{r|l}
2 & 10, 9, 8 \\
\hline
2 & 5, 9, 4 \\
\hline
2 & 5, 9, 2 \\
\hline
5 & 5, 9, 2 \\
\hline
3 & 1, 9, 1 \\
\hline
3 & 1, 3, 1 \\
\hline
 & 1, 1, 1
\end{array}
$$

LCM = 360

$2 \times 2 \times 2 \times 5 \times 3 \times 3$

if the remainder is different then take the difference b/w the divisor and remainder then subtract the value from the LCM

$10 \overline{}$ $\overline{9}$ $\quad 9 \overline{}$ $\overline{8}$ $\quad 8 \overline{}$ $\overline{7}$

$36 \overline{}$ $\overline{25}$ $\quad 48 \overline{}$ $\overline{37}$ $\quad 64 \overline{}$ $\overline{53}$

$10 \sim 9 = 1$
$9 \sim 8 = 1$
$8 \sim 7 = 1$ } 1 difference b/w the divisor & remainder

LCM = 360

The least Number = 360 − 1 = 359

---

23) find the least Number which when divided by 36, 48 & 64 leaves a remainder 25, 37 & 53 respectively in each case

LCM
2 | 36, 48, 64
2 | 18, 24, 32
2 | 9, 12, 16
2 | 9, 6, 8
2 | 9, 3, 4
2 | 9, 3, 2
3 | 9, 3, 1
3 | 3, 1, 1
1, 1, 1

LCM = 576

Different remainder

$36 \sim 25 = 11$
$48 \sim 37 = 11$
$64 \sim 53 = 11$

The least Number = 576 − 11

$\boxed{\text{ans} = 565}$

24) find the least number of four digit which is divisible by 4, 6, 8, 10

Least four digit number = 1000

LCM
2 | 4, 6, 8, 10
2 | 2, 3, 4, 5
2 | 1, 3, 2, 5
3 | 1, 3, 1, 5
5 | 1, 1, 1, 5
1, 1, 1, 1

LCM = 120

find the least four digit multiples of 120.

120×9
1080

the least four digit Number divisible by 4, 6, 8, 10 = 1080

(or)

Least four digit Number = 1000

8
120 | 1000
960
40

= 1000 + (120 − 40) = 1080

25) find the least number of 4 digit which is divisible by 18, 24, 32

Least four digit number = 1000

2 | 18, 24, 32
2 | 9, 12, 16
2 | 9, 6, 8
2 | 9, 3, 4
2 | 9, 3, 2
3 | 9, 3, 1
3 | 3, 1, 1
1, 1, 1

LCM = 288

288×4
1152

$\boxed{\text{ans: } 1152}$

26) Find the least five digit number which is divisible by 16, 18, 24 & 30

2 | 16, 18, 24, 30
2 | 8, 9, 12, 15
2 | 4, 9, 6, 15
2 | 2, 9, 3, 15
3 | 1, 9, 3, 15
3 | 1, 3, 1, 5
5 | 1, 1, 1, 5
1, 1, 1, 1

LCM = 720

720 × 14

ans: 10080

27) Find the smallest 4 digit number such that when divided by 12, 18, 21 & 28 it leaves remainder 3 in each case

Least Number 1000

2 | 12, 18, 21, 28
2 | 6, 9, 21, 14
3 | 3, 9, 7, 7
3 | 1, 3, 7, 7
7 | 1, 1, 7, 7
1, 1, 1, 1

LCM = 252

252 × 4

1008

remainder − 3

1008 + 3 = 1011

ans: 1011

28) Find the greatest 4 digit number which when divided by 12, 15, 20 & 35 leaves no remainder

greatest 4 digit Number = 9999

2 | 12, 15, 20, 35
2 | 6, 15, 10, 35
3 | 3, 15, 5, 35
5 | 1, 5, 5, 35
7 | 1, 1, 1, 7
1, 1, 1, 1

LCM = 420

420 × 23

ans: 9660

29) Find the greatest 4 digit number which when divided by 10, 15 & 20 leaves no remainder

greatest 4 digit Number = 9999

2 | 10, 15, 20
2 | 5, 15, 10
5 | 5, 15, 5
3 | 1, 3, 1
1, 1, 1

LCM = 60

```
      166
60 | 9999
     60
     399
     360
     399
     360
      39
```

Subtract

9999 − 39

ans: 9960

30) Find the greatest 5 digit number which when divided by 8, 9, 10 leaves 3 as a remainder in each case

99999

2 | 8, 9, 10
2 | 4, 9, 5
2 | 2, 9, 5
3 | 1, 9, 5
3 | 1, 3, 5
5 | 1, 1, 5
1, 1, 1

LCM = 360

```
      277
360 | 99999
      720
      2799
      2520
      2799
      2520
       279
```

99999 − 279 = 99720

remainder = 3 given in question.

99720 + 3

ans: 99723

Simplification VBODMAS

1) $\dfrac{2^5 \times 9^2}{8^2 \times 3^5}$

$= \dfrac{(\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}\times\cancel{2}) \times (\cancel{9}\times\cancel{9})}{(\cancel{8}\times\cancel{8}) \times (\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}\times\cancel{3}\times 3)} = \dfrac{1}{2\times 3}$

2

$\boxed{ans = \dfrac{1}{6}}$

2) $\dfrac{(6.5 \times 4.7) + (6.5 \times 5.3)}{1.3 \times 7.9 - 1.3 \times 6.9}$

$= \dfrac{6.5\,[4.7 + 5.3]}{1.3\,[7.9 - 6.9]} = \dfrac{6.5\,[10]}{1.3\,[1]}$

Cancel the Decimal

$= \dfrac{650}{13}$ 50

3) $\dfrac{(81)^{0.81}(9)}{(81)^{0.99}(9)^{0.64}}$

$= \dfrac{(9^2)^{0.81}(9)}{(9^2)^{0.99}\,9^{0.64}}$

$= \dfrac{9^{1.62} \times 9^1}{9^{1.98} \times 9^{0.64}} = \dfrac{9^{2.62}}{9^{2.62}}$

$\boxed{ans = 1}$

4) $1 + \dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{9}}}$

$1 + \dfrac{1}{9} = \dfrac{10}{9}$

$= 1 + \dfrac{1}{1 + \dfrac{1}{\frac{10}{9}}} = 1 + \dfrac{1}{1 + \frac{9}{10}}$

$= 1 + \dfrac{1}{\frac{19}{10}} = 1 + \dfrac{10}{19}$

$\boxed{ans = \dfrac{29}{19}}$

5) $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$

$= \sqrt{248 + \sqrt{51 + 13}}$

$= \sqrt{248 + \sqrt{64}}$

$= \sqrt{248 + 8}$

$= \sqrt{256}$

$\boxed{ans: 16}$

$\sqrt{169} = 13$

$\begin{array}{r} 51 \\ 13 \\ \hline 64 \end{array}$

$\sqrt{64} = 8$

$\sqrt{256} = 16$

6) $\dfrac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$

$= \dfrac{\sqrt{8\times4}+\sqrt{12\times4}}{\sqrt{8}+\sqrt{12}}$

$= \dfrac{2\sqrt{8}+2\sqrt{12}}{\sqrt{8}+\sqrt{12}}$

$= \dfrac{2\left[\sqrt{8}+\sqrt{12}\right]}{\left[\sqrt{8}+\sqrt{12}\right]}$

$\boxed{\text{ans} = 2}$

7) $\dfrac{\sqrt{1.21\times0.9}}{\sqrt{1.1\times0.11}}$

Cancel all Decimal

$= \dfrac{\sqrt{121\times9}}{\sqrt{11\times11}}$

$11\times11=121$

$= \sqrt{\dfrac{121\times9}{11\times11}}$

$= \sqrt{9}$

$\boxed{\text{ans} = 3}$

8) $(3)^4+(2)^4+2\times25 = ?$

$= (3\times3\times3\times3)+(2\times2\times2\times2)+2\times25$

$= 81+16+50$

$\boxed{\text{ans} = 147}$

9) $\sqrt{\dfrac{0.289}{0.00121}} = ?$

$= \sqrt{\dfrac{\frac{289}{1000}}{\frac{121}{100000}}}$

$\sqrt{289} = 17$

$\sqrt{100} = 10$

$\sqrt{121} = 11$

$= \sqrt{\dfrac{289\times100}{121}}$

$= \dfrac{17\times10}{11} = \dfrac{170}{11}$

$\boxed{\text{ans} = \dfrac{170}{11}}$

10) $\dfrac{\sqrt{24}+\sqrt{216}}{\sqrt{96}}$

$= \dfrac{\sqrt{6\times4}+\sqrt{36\times6}}{\sqrt{16\times6}}$

$= \dfrac{2\sqrt{6}+6\sqrt{6}}{4\sqrt{6}}$

$= \dfrac{8\sqrt{6}}{4\sqrt{6}} = \dfrac{8}{4}$

$\boxed{\text{ans}: 2}$

11) $\sqrt{43+\sqrt{31+\sqrt{21+\sqrt{11+\sqrt{25}}}}}$

$\sqrt{25}=5$

$=\sqrt{43+\sqrt{31+\sqrt{21+\sqrt{11+5}}}}$

$=\sqrt{43+\sqrt{31+\sqrt{21+\sqrt{16}}}}$ $\quad \sqrt{16}=4$

$=\sqrt{43+\sqrt{31+\sqrt{21+4}}}$

$=\sqrt{43+\sqrt{31+5}}$ $\quad \sqrt{25}=5$

$=\sqrt{43+6}$ $\quad \sqrt{36}=6$

$=\sqrt{49}$ $\quad \sqrt{49}=7$

$\boxed{\text{ans}=7}$

12) $3\frac{1}{2}+5\frac{1}{5}+6\frac{1}{4}=?$

$\frac{7}{2}+\frac{26}{5}+\frac{25}{4}=?$

LCM
2 | 2, 5, 4
2 | 1, 5, 2
5 | 1, 5, 1
1, 1, 1

LCM = 20

26×4
104

$=\frac{70}{20}+\frac{104}{20}+\frac{125}{20}$

70
104
125
299

$=\frac{70+104+125}{20}$

$\boxed{\text{ans}=\frac{299}{20}}$

13) $\frac{5}{10}\times\frac{8}{7}\times\frac{100}{50}\times\frac{35}{40}\times\frac{?}{16}=\frac{1}{8}$

$\frac{\cancel{5}}{\cancel{10}_{\cancel{8}}}\times\frac{\cancel{8}}{\cancel{7}}\times\frac{\cancel{100}^{\cancel{2}}}{\cancel{50}}\times\frac{\cancel{35}^{\cancel{7}}}{\cancel{40}_{\cancel{8}}}\times\frac{?}{16}=\frac{1}{8}$

$\frac{?}{16}=\frac{1}{8}$

$?=\frac{\cancel{16}^{2}}{\cancel{8}}$

$?=2$

$\boxed{\text{ans}:2}$

14) $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\times\frac{3}{4}\left[\frac{1}{3}+\frac{4}{5}\right]=?$

$=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\times\frac{3}{4}\left[\frac{5+12}{15}\right]=?$

$=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\times\frac{\cancel{3}^{1}}{4}\left[\frac{17}{\cancel{15}_{5}}\right]$

$=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\times\frac{17}{20}$

$=\frac{1}{2}+\frac{1}{3}+\frac{17}{80}$

2 | 2, 3, 80
2 | 1, 3, 40
2 | 1, 3, 20
2 | 1, 3, 10
3 | 1, 3, 5
5 | 1, 1, 5
1, 1, 1

LCM = 240

$=\frac{120+80+51}{240}$

$\boxed{\text{ans}=\frac{251}{240}}$

15) $\frac{\sqrt{225}}{45} \times \frac{9}{\sqrt{169}} \times \frac{26}{2} = ?$

$= \frac{\cancel{15}^{3}}{\cancel{45}_{\cancel{5}}} \times \frac{\cancel{9}}{\cancel{13}} \times \frac{\cancel{26}^{\cancel{2}}}{\cancel{2}}$

$\sqrt{225} = 15$

$\sqrt{169} = 13$

$\boxed{\text{ans: } 3}$

16) 2 of $\frac{3}{4} \div \frac{3}{4} + \frac{1}{4} = ?$

$\cancel{2} \times \frac{3}{\cancel{4}_{2}} \div \frac{3}{4} + \frac{1}{4}$

$\frac{3}{2} \div \frac{3}{4} + \frac{1}{4}$

$\frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{4}^{2}}{\cancel{3}} + \frac{1}{4}$

$= 2 + \frac{1}{4}$

convert ÷ into × take reciprocal

$\boxed{\text{ans} = \frac{9}{4}}$

17) $\frac{\frac{1}{4} + \frac{1}{4} \div 1\frac{1}{4}}{\frac{1}{4} \times \frac{1}{4} + 2\frac{1}{4}} = ?$

$= \frac{\frac{1}{4} + \frac{1}{4} \div \frac{5}{4}}{\frac{1}{4} \times \frac{1}{4} + \frac{9}{4}} = ?$

$= \frac{\frac{1}{4} + \frac{1}{\cancel{4}} \times \frac{\cancel{4}}{5}}{\frac{1}{16} + \frac{9}{4}} = \frac{\frac{1}{4} + \frac{1}{5}}{\frac{1}{16} + \frac{9}{4}}$

$2 \,|\, 4, 5$
$2 \,|\, 2, 5$
$5 \,|\, 1, 5$
$1, 1, 1$

LCM = 20

$2 \,|\, 16, 4$
$2 \,|\, 8, 2$
$2 \,|\, 4, 1$
$2 \,|\, 2, 1$
$1, 1$

LCM = 16

$= \frac{\frac{5+4}{20}}{\frac{1+36}{16}}$

$= \frac{9}{\cancel{20}_{\cancel{10}\,5}} \times \frac{\cancel{16}^{\cancel{8}\,4}}{37} = \frac{36}{185}$

$\boxed{\text{ans} = \frac{36}{185}}$

18) $\frac{1}{3 + \frac{2}{2 + \frac{1}{2}}}$

$2 + \frac{1}{2} = \frac{5}{2}$

$= \frac{1}{3 + \frac{(2 \times 2)}{5}} = \frac{1}{3 + \frac{4}{5}} = \frac{1}{\frac{19}{5}}$

$\boxed{\text{ans} = \frac{5}{19}}$

19) $15\frac{2}{3} \times 3\frac{1}{6} + 6\frac{1}{3} = 11\frac{7}{18} + ?$

$\frac{47}{3} \times \frac{19}{6} + \frac{19}{3} = \frac{205}{18} + ?$

$\frac{47 \times 19}{18} + \frac{19}{3} - \frac{205}{18} = ?$

$$\frac{893}{18} + \frac{19}{3} - \frac{205}{18} = ?$$

LCM $2 \mid 18, 3$
$3 \mid 9, 3$
$3 \mid 3, 1$
$1, 1$

LCM = 18

$$= \frac{893 + 114 - 205}{18}$$

$$= \frac{\cancel{802}}{\cancel{18}} = \frac{401}{9}$$

$\boxed{\text{ans}: \frac{401}{9}}$

20) $48 \div 12 \left(\frac{9}{8} \text{ of } \frac{4}{3} \div \frac{3}{4} \text{ of } \frac{2}{3}\right)$

$$= 48 \div 12 \left(\frac{\cancel{9}^{3}}{\cancel{8}_{2}} \times \frac{\cancel{4}}{\cancel{3}_{1}} \div \frac{\cancel{3}}{\cancel{4}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{3}}\right)$$

$$= 48 \div 12 \left(\frac{3}{2} \div \frac{1}{2}\right)$$

Convert ÷ into ×

$$= 48 \div 12 \left[\frac{3}{\cancel{2}} \times \frac{\cancel{2}}{1}\right]$$

$$= 48 \div 12 (3)$$

$$= 48 \div 36$$

$$\frac{\cancel{48}^{4}}{\cancel{36}_{3}}$$

$\boxed{\text{ans} = \frac{4}{3}}$

21) $\frac{7}{8} - \frac{7}{x} + \frac{9}{16} = 1$

$$= \frac{7}{8} + \frac{9}{16} - 1 = \frac{7}{x}$$

LCM $2 \mid 8, 16$
$2 \mid 4, 8$
$2 \mid 2, 4$
$2 \mid 1, 2$
$1, 1$

LCM = 16

$$\frac{14 + 9 - 16}{16} = \frac{7}{x}$$

$$\frac{\cancel{7}}{16} = \frac{\cancel{7}}{x}$$

$x = 16$

$\boxed{\text{ans}: 16}$

22) 35% of $\frac{9}{21}$

$$= \frac{\cancel{35}^{\cancel{5}}}{\cancel{100}_{20}} \times \frac{\cancel{9}^{3}}{\cancel{21}_{\cancel{3}}} = \frac{3}{20}$$

$\boxed{\text{ans}: \frac{3}{20}}$

23) $4\frac{1}{3} + 3\frac{1}{6} - 5\frac{1}{9} = ?$

$$\frac{13}{3} + \frac{19}{6} - \frac{46}{9} = ?$$

LCM $3 \mid 3, 6, 9$
$2 \mid 1, 2, 3$
$3 \mid \ , 1, 3$
$1, 1, 1, 1$

LCM = 18

$$\frac{(13 \times 6) + (19 \times 3) - (46 \times 2)}{18} = ?$$

$$\frac{78 + 57 - 92}{18} = \frac{135 - 92}{18}$$

$\boxed{\text{ans}: \frac{43}{18}}$

24) $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

Conjugate

$= \dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} \times \dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}}$

$(a^2-b^2) = (a+b)(a-b)$

$(a+b)^2 = a^2+b^2+2ab$

$= \dfrac{(\sqrt{7}+\sqrt{5})^2}{7-5}$

$= \dfrac{7+5+2\sqrt{7}\times\sqrt{5}}{2}$

$= \dfrac{12+2\sqrt{35}}{2}$

$= \dfrac{\cancel{2}(6+1\sqrt{35})}{\cancel{2}}$

$\boxed{\text{ans}: 6+1\sqrt{35}}$

25) $5\sqrt{2}+\sqrt{32}$

$= 5\sqrt{2}+\sqrt{16\times 2}$

$= 5\sqrt{2}+4\sqrt{2}$

$\boxed{\text{ans} = 9\sqrt{2}}$

26) $\dfrac{(5+\sqrt{7})}{(5-\sqrt{7})} + \dfrac{(5-\sqrt{7})}{(5+\sqrt{7})} = ?$

$= \dfrac{(5+\sqrt{7})^2+(5-\sqrt{7})^2}{(5-\sqrt{7})(5+\sqrt{7})}$

$= \dfrac{25+7+\cancel{2(5)(\sqrt{7})}+25+7-\cancel{2(5)(\sqrt{7})}}{25-7}$

$a=5$
$b=\sqrt{7}$

$(5-\sqrt{7})(5+\sqrt{7})$ $(a^2-b^2) = (a+b)(a-b)$

$= \dfrac{\cancel{64}}{\cancel{18}} = \dfrac{32}{9}$

$\boxed{\text{ans} = \dfrac{32}{9}}$

27) If $\dfrac{x}{y} = \dfrac{6}{5}$ find the value of

$\dfrac{x^2+y^2}{x^2-y^2}$

$\dfrac{x^2+y^2}{x^2-y^2} = \dfrac{\dfrac{x^2}{y^2}+1}{\dfrac{x^2}{y^2}-1} = \dfrac{\left(\frac{6}{5}\right)^2+1}{\left(\frac{6}{5}\right)^2-1}$

$= \dfrac{\frac{36}{25}+1}{\frac{36}{25}-1} = \dfrac{\frac{61}{25}}{\frac{11}{25}}$

$= \dfrac{61}{\cancel{25}} \times \dfrac{\cancel{25}}{11} = \dfrac{61}{11}$

$\boxed{\text{ans} = \dfrac{61}{11}}$

28) If $\dfrac{2x}{1+\dfrac{1}{1+\dfrac{x}{1-x}}} = 1$, then find the value of x

$$1+\frac{x}{(1-x)} = \frac{(1-x)+x}{(1-x)}$$

$$= \frac{2x}{1+\dfrac{1}{\dfrac{(1-\cancel{x})+\cancel{x}}{(1-x)}}} = \frac{2x}{1+\dfrac{1}{\dfrac{1}{(1-x)}}}$$

$$= \frac{2x}{1+(1-x)} = \frac{2x}{2-x}$$

$$2x = 2-x$$

$$2x+x = 2$$

$$3x = 2$$

$$\boxed{x = \frac{2}{3}}$$

29) If $\frac{a}{b} = \frac{3}{4}$ and $8a+5b=22$ then find the value of a

$$\frac{a}{b} = \frac{3}{4}$$

$$b = \frac{4}{3}a$$

$$8a+5b = 22$$

$$8a+5\times\frac{4}{3}a = 22$$

$$8a+\frac{20}{3}a = 22$$

$$\frac{24a+20a}{3} = 22$$

$$44a = 22\times 3$$

$$44a = 66$$

$$a = \frac{\cancel{66}}{\cancel{44}} \quad \frac{3}{2}$$

$$\boxed{\text{ans: } a = \frac{3}{2}}$$

30) If $\frac{x}{4} - \frac{x-3}{6} = 1$ then find the value of x

$$\begin{array}{r|l} 2 & 4, 6 \\ 2 & 2, 3 \\ 3 & 1, 3 \\ & 1, 1 \end{array}$$

Lcm = 12

$$\frac{3x-2(x-3)}{12} = 1$$

$$3x-2x+6 = 12$$

$$x+6 = 12$$

$$x = 12-6$$

$$\boxed{x = 6}$$

31) If $2x+3y=34$ and $\frac{x+y}{y}=\frac{13}{8}$ then find the value of $5y+7x$

$2x+3y=34$ --- ①

$\frac{x+y}{y}=\frac{13}{8}$

$8(x+y)=13y$

$8x+8y=13y$

$8x+8y-13y=0$

$8x-5y=0$ --- ②

× by 5 in eqn ①

× by 3 in eqn ②

$10x+\cancel{15y}=170$

$24x-\cancel{15y}=0$

$34x=170$

$x=\frac{170}{34}$ 5

$x=5$

$10x+15y=170$

$10(5)+15y=170$

$50+15y=170$

$15y=170-50$

$15y=120$

$y=\frac{120}{15}$ 8

$y=8$

$5y+7x$

$x=5 \quad y=8$

$5\times8+7\times5$

$40+35=75$

$\boxed{\text{ans: } 75}$

32) A third of Arun's marks in mathematics exceeds a half of his marks in English by 30. If he got 240 marks in the two subjects together, how many marks did he get in english?

maths mark x

English marks y

$\frac{1}{3}x-\frac{1}{2}y=30$

$\frac{2x-3y}{6}=30$

$2x-3y=30\times6$

$2x-3y=180$ --- ①

$x+y=240$ --- ② given

× eqn ② by 3

$2x-\cancel{3y}=180$

$3x+\cancel{3y}=720$

$5x=900$

$x=\frac{900}{5}$ 180

$x=180$

$2x - 3y = 180$

$2 \times 180 - 3y = 180$

$360 - 180 = 3y$

$180 = 3y$

$y = \frac{180}{3} = 60$

$y = 60$

maths mark = 180
English mark = 60

33) A tin of oil was $\frac{4}{5}$ full. When 6 bottles of oil were taken out and four bottles of oil were poured into it, it was $\frac{3}{4}$ full. How many bottles of oil can the tin contain?

$\frac{4}{5}x - \frac{3}{4}x = (6-4)$

$\frac{4x - 3x}{20} = (6-4)$

$\frac{x}{20} = 2$

$\boxed{x = 40}$

$\boxed{\text{40 bottles of oil}}$

34) A man has Rs. 480 in the denominations of one-rupee notes, five-rupee notes and ten-rupee notes. The number of notes of each denomination is equal. What is the total number of notes that he has?

number of notes of each denominations be x

$x + 5x + 10x = 480$

$16x = 480$

$x = \frac{480}{16} = 30$

Hence total number of notes $= 3x = 90$

35) There are 2 examination rooms A & B. If 10 students are sent from A to B, then the number of students in each room is the same. If 20 candidates are sent from B to A, then the number of students in A is double the number of students in B. The number of students in room A is

No. of students in rooms A & B are x & y

$x - 10 = y + 10 \rightarrow x - y = 20$

$x + 20 = 2(y - 20) \rightarrow x - 2y = -60$

$\cancel{x} - y = 20$

$\cancel{x} - 2y = -60$

$(-)\ (+)\ (+)$

$y = 80$

$x - y = 20$

$x - 80 = 20$

$x = 20 + 80$

$\boxed{x = 100}$

No. of Students in Room. A. = 100

36) The Price of 10 chairs is equal to that of 4 tables. The price of 15 chairs and 2 tables together is Rs: 4000. The total price of 12 chairs and 3 tables is

The cost of chair & table is x & y

$10x = 4y \rightarrow y = \frac{10x}{4} = \frac{5x}{2}$

$15x + 2y = 4000$

$15x + 2\left(\frac{5x}{2}\right) = 4000$

$20x = 4000$

$x = \frac{4000}{20} = 200$

$y = \frac{5x}{2} = \frac{5 \times 200}{2} = 500$

$12x + 3y = ?$

$12 \times 200 + 3(500) =$

$2400 + 1500 = 3900$

37) The Price of 2 Sarees and 4 shirts is Rs 1600. With the same money one can buy 1 Saree and 6 shirts. If one wants to buy 12 shirts. how much shall he have to pay?

Saree & shirt be x & y

$2x + 4y = 1600$ --- ①

$1x + 6y = 1600$ --- ②

÷ by 2 eqn ①

$x + 2y = 800$

$x + 6y = 1600$

$-4y = -800$

$y = \frac{800}{4} = 200$

$x + 2y = 800$

$x + 2(200) = 800$

$x = 800 - 400$

$x = 400$

12 shirts $= 12 \times 200$

$=$ Rs: 2400/-

38) A man has some hens and cows. If the number of heads be 48 and the number of feet equals 140, then the number of hens will be

hens & cows be x & y

$x + y = 48$ - ①

$2x + 4y = 140$ - ②

÷ eqn ② by ②

$$\cancel{x} + y = 48$$
$$x + 2y = 70$$
$$(-) \quad \cancel{x} \quad (-)$$

$$\cancel{+2y = +22}$$
$$\cancel{y = \frac{22}{2} = 11}$$

$$\cancel{x + y = 48}$$
$$\cancel{x + 11 = 48}$$
$$\cancel{x = 48 - 11}$$
$$\cancel{x = 37}$$

$$\cancel{x} + y = 48$$
$$x + 2y = 70$$
$$(-) \quad (-) \quad (-)$$

$$+y = +22$$
$$y = 22$$

$$x + y = 48$$
$$x + 22 = 48$$
$$x = 48 - 22$$
$$x = 26$$

x = hens

$\boxed{\text{ans}: 26}$

39) In a regular week, there are 5 working days and for each day, the working hours are 8. A man gets Rs:240 per hour for regular and Rs:320 per hour for overtime. If he earns Rs. 432 in 4 weeks then how many hours does he work for?

man works overtime for x hours

working hours in 4 weeks $= (5 \times 8 \times 4)$
$= 160$

$$160 \times 2.40 + x(3.20) = 432$$
$$384 + x(3.20) = 432$$
$$x(3.20) = 432 - 384$$
$$x = \frac{48}{3.2} = 15$$

total hours of work $= (160 + 15)$

$\boxed{\text{ans} = 175}$

40) David gets on the elevator at the 11th floor of a building and rides up at the rate of 57 floors per min. At the same time, Albert gets on an elevator at the 51st floor of the same building and rides down at the rate of 63 floors per min. If they continue travelling at these rates then at which floor will their paths cross?

their paths cross after x min

$$11 + 57x = 51 - 63x$$
$$57x + 63x = 51 - 11$$
$$120x = 40$$
$$x = \frac{\cancel{40}}{\cancel{120}} = \frac{1}{3}$$

No of floors covered by david

$$57 \times \frac{1}{3} = 19$$

So their path cross at $(11+19) = 30$

$\boxed{\text{ans: } 30^{th} \text{ floor}}$

41) $\sqrt[3]{-67 - \sqrt[3]{-25 + \sqrt[3]{-8}}}$

$= \sqrt[3]{-67 - \sqrt[3]{-25 - 2}}$ $\qquad \sqrt[3]{-8} = -2$

$= \sqrt[3]{-67 - (-3)}$ $\qquad \sqrt[3]{-27} = -3$

$= \sqrt[3]{-64}$ $\qquad \sqrt[3]{-64} = -4$

$\boxed{\text{ans} = -4}$

42) $\sqrt[3]{8x^3 \times 27x^3 \times 64x^3}$

$= (2x) \times (3x) \times (4x)$

$\boxed{\text{ans} = 24x^3}$

43) $\sqrt{x} : 16 = 49 : x^{\frac{3}{2}}$

$\frac{\sqrt{x}}{16} = \frac{49}{x^{\frac{3}{2}}}$ $\qquad a:b = \frac{a}{b}$

$x^{\frac{3}{2}} \times x^{\frac{1}{2}} = 49 \times 16$

$x^{\frac{4}{2}} = 49 \times 16$

$x^{\frac{1}{2}} = 49 \times 16$

$x = \sqrt{49 \times 16}$

$x = 7 \times 4$

$\boxed{x = 28}$

44) $\sqrt{a^{-1}b} \times \sqrt{b^{-1}c} \times \sqrt{c^{-1}a}$

$= \sqrt{\frac{1}{a} \times b} \times \sqrt{\frac{1}{b} \times c} \times \sqrt{\frac{1}{c} \times a}$

$= \sqrt{\frac{1}{\cancel{a}} \times \cancel{b} \times \frac{1}{\cancel{b}} \times \cancel{c} \times \frac{1}{\cancel{c}} \times \cancel{a}}$

$= \sqrt{1}$

$\boxed{\text{ans: } 1}$

45) $\frac{(2^3)^2}{5^3} \times \frac{(125)^{\frac{2}{3}}}{24} = ?$

$= \frac{2^6}{5^3} \times \frac{(5^3)^{\frac{2}{3}}}{24}$ $\qquad 5^{\cancel{3} \times \frac{2}{\cancel{3}}}$

$= \frac{\cancel{2^6}}{\cancel{5^3}} \times \frac{\cancel{5^2}}{\cancel{24}} = \frac{2^2}{5^1} = \frac{4}{5}$

$\boxed{\text{ans: } \frac{4}{5}}$

~~46) Simplify~~

$\cancel{b - [b + (a+b) - \{b - (b - a - b)\} + 2a]}$

Percentage

If the price of a commodity increases by R% then the reduction in consumption so as not to increase the expenditure is

$$\left[\frac{R}{100+R} \times 100\right] \%$$

If the price of a commodity decreases by R% then the increase in consumption so as not to decrease the expenditure is

$$\left[\frac{R}{100-R} \times 100\right] \%$$

Results on Population

Population of a town P increase at the rate of R% per annum

1) Population after n years $= P\left(1+\frac{R}{100}\right)^n$

2) Population n years ago $= \dfrac{P}{\left(1+\frac{R}{100}\right)^n}$

Results on Depreciation:

Present value of machine P depreciates at the rate of R% per annum

1) Value of machine after n years

$$= P\left(1-\frac{R}{100}\right)^n$$

2) Value of machine n years ago

$$= \frac{P}{\left(1-\frac{R}{100}\right)^n}$$

If A is R% more than B, then B is less than A by

$$\left[\frac{R}{100+R} \times 100\right] \%$$

If A is R% less than B, then B is more than A by

$$\left[\frac{R}{100-R} \times 100\right] \%$$

1) Evaluate:

28% of 450 + 45% of 280

$$= \frac{28}{100} \times 450 + \frac{45}{100} \times 280$$

$$= 126 + 126 \quad \boxed{\text{ans} = 252}$$

2) $16\frac{2}{3}$% of 600 gm − $33\frac{1}{3}$% of 180 gm

$$\left(\frac{50}{3} \times \frac{1}{100} \times 600\right) - \left(\frac{100}{3} \times \frac{1}{100} \times 180\right)$$

$$(100 - 60)$$

$$\boxed{\text{ans} = 40 \text{ gm}}$$

3) find the missing figures

i) ?% of 25 = 2.125

ii) 9% of ? = 6.3

iii) 0.25% of ? = 0.04

i) $\frac{x}{\cancel{100}_{4}} \times \cancel{25}^{1} = 2.125$

$x = (2.125 \times 4) = 8.5$

ii) $\frac{9}{100} \times x = 6.3$

$x = \left(\frac{6.3 \times 100}{9}\right) = 70$

iii) $\frac{0.25}{100} \times x = 0.04$

$x = \left(\frac{0.04 \times 100}{0.25}\right) = 16$

4) If the sales tax be reduced from $3\frac{1}{2}$% to $3\frac{1}{3}$% then what difference does it make to a person who purchases an article with marked price of Rs. 8400?

difference = $\left(3\frac{1}{2}\% \text{ of } 8400\right) - \left(3\frac{1}{3}\% \text{ of } 8400\right)$

$= \left[\frac{7}{2} - \frac{10}{3}\right]\%$ of Rs. 8400

$= \frac{1}{6}\%$ of 8400

$= \left[\frac{1}{6} \times \frac{1}{100} \times 8400\right] = \text{Rs. } 14$

5) Difference of two numbers is 1660. If 7.5% of one number is 12.5% of the other number, find the two numbers.

7.5% of x = 12.5% of y

$\frac{7.5}{100} \times x = \frac{12.5}{100} \times y$

$x = \frac{12.5}{100} \times \frac{100}{7.5} \times y$

$x = \frac{125}{75} y$

$x = \frac{5}{3} y$

$x - y = 1660$

$\frac{5}{3} y - y = 1660$

$\frac{2}{3} y = 1660$

$y = \frac{1660 \times 3}{2}$

$\boxed{y = 2490}$

$x = \frac{5}{3}(2490)$

$\boxed{x = 4150}$

6) Shobha's mathematics test had 75 problems i.e 10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of algebra and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right. How many more questions she would have needed to answer correctly to earn a 60% passing grade?

No. of questions attempted correctly } 70% of 10 + 40% of 30 + 60% of 35

$$\left[\frac{70}{100}\times 10 + \frac{40}{100}\times 30 + \frac{\cancel{60}^{3}}{\cancel{100}_{\cancel{2}}}\times \cancel{35}^{7}\right]$$

$$[7+12+21] = 40$$

Questions to be answered correctly for 60% grade = 60% of 75

$$\frac{\cancel{60}^{3}}{\cancel{100}_{\cancel{2}}}\times \cancel{75}^{15} = 45$$

Required No. of questions = (45 − 40)

$\boxed{\text{ans} = 5}$

7) If 50% of $(x-y)$ = 30% of $(x+y)$ then what percent of x is y?

$$\frac{50}{100}(x-y) = \frac{30}{100}(x+y)$$

$$5(x-y) = 3(x+y)$$

$$5x-5y = 3x+3y$$

$$5x = 3x+3y+5y$$

$$5x-3x = 8y$$

$$2x = 8y$$

$$x = \frac{\cancel{8}^{4}y}{\cancel{2}}$$

$$x = 4y$$

Required percentage $= \left(\frac{y}{x}\times 100\right)\%$

$$= \left[\frac{\cancel{y}}{\cancel{4y}}\times \cancel{100}^{25}\right]\%$$

$\boxed{\text{ans} = 25\%}$

8) Mr. Jones gave 40% of the money he had, to his wife. He also gave 20% of the remaining amount to each of his three sons. Half of the amount now left was spent on miscellaneous items and the remaining amount of Rs. 12000/- was deposited in the bank. How much money did Mr. Jones have initially?

initial amount with Mr. Jones be Rs: x

money given to wife $= \frac{\cancel{40}^{2}}{\cancel{100}_{5}}x$

$$= \frac{2x}{5}$$

Balance $= x - \frac{2x}{5}$

$= \frac{3x}{5}$

money given to 3 sons

$= \left[3 \times \left[\frac{\cancel{20}^{1}}{\cancel{100}_{5}} \times \frac{3x}{5}\right]\right]$

$= \frac{9x}{25}$

Balance $= \frac{3x}{5} - \frac{9x}{25}$

$= \frac{6x}{25}$

Amount deposited in bank

$= \left(\frac{1}{2} \times \frac{\cancel{6}^{3}x}{25}\right) = \frac{3x}{25}$

$\frac{3x}{25} = 12000 \qquad x = \frac{\cancel{12000}^{4000} \times 25}{\cancel{3}}$

$x = 1,00,000.$

Mr. Jones initially had Rs: 1,00,000/-

9) In an election between two candidates, 75% of the voters cast their votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. find the total number of votes enrolled in that election.

Total No. of. Votes x

No. of. Votes cast $= 75\%$ of x

Valid Votes $= 98\%$ of $(75\%$ of $x)$

75% of $[98\%$ of $(75\%$ of $x)] = 9261$

$\left[\frac{75}{100} \times \frac{98}{100} \times \frac{75}{100} \times x\right] = 9261$

$x = \frac{9261 \times 100 \times 100 \times 100}{75 \times 98 \times 75}$

$\boxed{x = 16800}$

10) A Salesman's commission is 5% on all sales upto Rs: 10,000 and 4% on all sales exceeding this. the remits Rs. 31,100 to his parent company after deducting his commission. find the total sales.

Total Sales $= x$

Total Sales − commission $= 31,100$

$x - [5\%$ of $10,000 + \cancel{4\%}\ 4\%$ of $(x - 10000)] = 31100$

$x - \left[\frac{5}{100} \times 10000 + \frac{\cancel{4}^{1}}{\cancel{100}_{25}} \times (x - 10000)\right] = 31100$

$x - 500 - \frac{(x - 10000)}{25} = 31100$

$$x - 500 - \frac{x}{25} + \frac{\cancel{10000}^{400}}{\cancel{25}} = 31100$$

$$x - \frac{x}{25} = 31100 + 100$$

$$\frac{24x}{25} = 31200$$

$$x = \frac{31200 \times 25}{24}$$

$$\boxed{x = 32{,}500}$$

Total Sales = Rs: 32,500/—

11) Raman's Salary was decreased by 50% and subsequently increased by 50%. How much percent does he lose?

original Salary = Rs: 100/—

New final Salary = 150% of (50% of 100)

$$\left(\frac{150}{100} \times \frac{50}{100} \times 100\right) = \text{Rs: } 75/-$$

$$\boxed{\text{Decrease} = 25\%}$$

12) If the numerator of a fraction be increased by 15% and its denominator be diminished by 8%. the value of the fraction is $\frac{15}{16}$. find the original fraction.

Assume original fraction $= \frac{x}{y}$

$$\frac{115\% \text{ of } x}{92\% \text{ of } y} = \frac{15}{16}$$

$$\frac{115x}{92y} = \frac{15}{16}$$

$$\frac{x}{y} = \left(\frac{15}{16} \times \frac{92}{115}\right) = \frac{3}{4}$$

13) In the new budget the price of kerosene oil rose by 25%. By how much percent must a person reduce his consumption so that his expenditure on it does not increase?

Reduction in consumption

$$= \left[\frac{R}{100+R} \times 100\right] \%$$

$$= \left[\frac{25}{100+25} \times 100\right] \%$$

$$\boxed{\text{ans} = 20\%}$$

14) The population of a town is 1,76,400. If it increases at the rate of 5% per annum, what will be its population 2 years hence? What was it 2 years ago?

Population after 2 years $= 176,400 \times \left(1 + \frac{5}{100}\right)^2$

$= 176400 \left[\frac{\cancel{105}^{21}}{\cancel{100}_{20}}\right]^2$

$= 1764\cancel{00} \times \frac{21}{2\cancel{0}} \times \frac{21}{2\cancel{0}}$

$= 1,94,481$

Population 2 years ago $= \frac{176400}{\left[1 + \frac{5}{100}\right]^2}$

$= \frac{176400}{\left[\frac{\cancel{105}^{21}}{\cancel{100}_{20}}\right]^2}$

$= 176400 \times \frac{20}{21} \times \frac{20}{21}$

$= 1,60,000$

15) The value of a machine depreciates at the rate of 10% per annum. If its present value is Rs. 1,62,000. What will be its worth after 2 years? What was the value of the machine 2 years ago?

Value of machine after 2 years $= 162000 \left[1 - \frac{10}{100}\right]^2$

$= 162000 \left[\frac{9\cancel{0}}{10\cancel{0}}\right]^2$

$= 1620\cancel{00} \times \frac{9}{1\cancel{0}} \times \frac{9}{1\cancel{0}}$

Rs $= 131220$

Value of machine 2 years ago $= \frac{162000}{\left[1 - \frac{10}{100}\right]^2}$

$= \frac{162000}{\left[\frac{9\cancel{0}}{10\cancel{0}}\right]^2}$

$= 162000 \times \frac{10}{9} \times \frac{10}{9}$

Rs $= 2,00,000$

16) During one year, the population of a town increased by 5% and during the next year, the population decreased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year?

$$= \frac{9975}{\left[1+\frac{5}{100}\right]\left[1-\frac{5}{100}\right]}$$

$$= \frac{9975}{\left[\frac{\cancel{105}^{21}}{\cancel{100}_{20}}\right]\left[\frac{\cancel{95}^{19}}{\cancel{100}_{20}}\right]}$$

$$= 9975 \times \frac{20}{21} \times \frac{20}{19}$$

$$= 10{,}000$$

17) If A earns $33\frac{1}{3}$% more than B, how much percent does B earns less than A?

$$\text{Required percentage} = \left[\frac{33\frac{1}{3}}{100+33\frac{1}{3}} \times 100\right]\%$$

$$= \left[\frac{\frac{100}{3}}{100+\frac{100}{3}} \times 100\right]\%$$

$$= \left[\frac{\frac{100}{\cancel{3}}}{\frac{400}{\cancel{3}}} \times 100\right]\%$$

$$= \left[\frac{\cancel{100}^{25}}{\cancel{400}} \times \cancel{100}\right]\%$$

$$\boxed{\text{ans} = 25\%}$$

18) If A's Salary is 20% less than B's Salary, by how much percentage B's Salary more than A's?

$$\text{Required percentage} = \left[\frac{20}{100-20} \times 100\right]\%$$

$$= \left[\frac{\cancel{20}^{1}}{\cancel{80}_{4}} \times \cancel{100}^{25}\right]\%$$

$$\boxed{\text{ans} = 25\%}$$

19) How many kg of pure salt must be added to 30 kg of 2% solution of salt and water to increase it to a 10% solution?

Amount of salt in 30 kg solution

$$= \left(\frac{2}{100} \times 30\right) kg = 0.6 \text{ kg}$$

x kg of pure salt be added

Then, $\frac{0.6 + x}{30 + x} = \frac{10}{100}$

$$60 + 100x = 300 + 10x$$

$$100x - 10x = 300 - 60$$

$$90x = 240$$

$$x = \frac{\cancel{240}^{8}}{\cancel{90}_{3}}$$

$$x = \frac{8}{3} \text{ (or) } 2\frac{2}{3}$$

20) Due to a reduction of $6\frac{1}{4}$% in the price of sugar, a man is able to buy 1 kg more for Rs: 120/- find the original and reduced rate of sugar.

original rate be x per kg

Reduced rate $= \text{Rs}\left[\left(100 - \frac{25}{4}\right)\frac{1}{100}x\right]$

$$= \frac{15x}{16} \text{ per kg}$$

$$\frac{120}{\frac{15x}{16}} - \frac{120}{x} = 1$$

$$\frac{128}{x} - \frac{120}{x} = 1$$

$$\therefore x = 8$$

original rate = Rs: 8 /kg

Reduced rate $= \left(\frac{15}{\cancel{16}_{2}} \times \cancel{8}^{1}\right) = \frac{15}{2}$

$\boxed{\text{Reduced rate} = \text{Rs: } 7.50 / \text{kg}}$

21) In an examination, 80% of the students passed in English, 85% of the students passed in maths and 75% in both English & maths. If 40 students failed in both the subjects, find the total number of students.

total NO. of. students be x

$$= \left(\frac{80x}{100} + \frac{85x}{100} - \frac{75x}{100}\right)$$

$$= \frac{90x}{100} = \frac{9x}{10}$$

Students who failed in both subjects

$$= \left[x - \frac{9x}{10}\right]$$

$= \frac{x}{10}$

$\frac{x}{10} = 40$

$x = 400$

Hence total No. of. Students = 400

22) In an examination 35% of total Students failed in hindi, 45% failed in English and 20% in both. find the Percentage of those who passed in both the Subjects.

$(35 + 45 - 20) = 60$

percentage failed in hindi (or) English (or) both = 60%

Percentage Passed = (100 - 60)

$\boxed{\text{ans} = 40\%}$

23) what percent of a day is 3 hours?

$= \left(\frac{\cancel{3}^{1}}{\cancel{24}_{\cancel{8}_{2}}} \times \cancel{100}^{25}\right) = \frac{25}{2}\%$ (or)

$12\frac{1}{2}\%$

24) what percentage of numbers from 1 to 70 have unit digit 1 & 9.

1, 9, 11, 19, 21, 29, 31, 39, 41, 49, 51, 59, 61, 69

No. of. Such numbers = 14

Required percentage $= \left(\frac{\cancel{14}^{\cancel{2}}}{\cancel{70}_{\cancel{10}_{\cancel{5}}}} \times \cancel{100}^{20}\right)\%$

$\boxed{\text{ans} = 20\%}$

25) $33\frac{1}{3}\%$ of a man's daily output is equal to 50% of a second man's daily output. If the second man turns out 1500 screws daily. then the first mans output in terms of making screws is

$33\frac{1}{3}\%$ of x = 50% of 1500

$33\frac{1}{3} \times \frac{1}{100} \times x = \frac{50}{100} \times 1500$

$\frac{\cancel{100}}{3} \times \frac{1}{\cancel{100}} \times x = \frac{50}{\cancel{100}} \times 1500$

$x = 50 \times 15 \times 3$

$\boxed{x = 2250}$

26) A bag contains 600 coins of 25 p denomination and 1200 coins of 50 p denomination. If 12% of 25p coins and 24% of 50 p. coins are removed, the percentage of money removed from the bag is nearly:

Total money $= Rs\left(600 \times \frac{25}{100} + 1200 \times \frac{50}{100}\right)$

$= Rs: 750$

25 paise coins removed $= \left(\frac{12}{100} \times 600\right) = 72$

50 paise coins removed $= \left(\frac{24}{100} \times 1200\right) = 288$

Money removed $= \left(72 \times \frac{25}{100} + 288 \times \frac{50}{100}\right)$

$= Rs: 162$

Required percentage $= \left[\frac{162}{\cancel{750}_{15}} \times \cancel{100}^{2}\right]\%$

$= \left[\frac{324}{15}\right]\%$

$\boxed{ans = 21.6\%}$

PROFIT & LOSS

Profit = SP − C.P

$\text{profit\%} = \left[\frac{SP - CP}{CP} \times 100\right]\%$

Loss = CP − S.P

$\text{Loss\%} = \left[\frac{CP - S.P}{CP} \times 100\right]\%$

1) A man buys an article for Rs: 27.50 and sells it for Rs: 28.60. find his gain percent

CP = 27.50 SP = 28.60

$\text{profit\%} = \left[\frac{28.60 - 27.50}{27.50} \times 100\right]\%$

$= \left[\frac{1.10}{27.50} \times 100\right]\%$

ans = 4 %

2) find SP when

i) CP = RS 56.25 gain = 20%

ii) CP = RS 80.40 loss = 15%

i) SP = 120% of 56.25

$= \left[\frac{120}{100} \times 56.25\right]$

$= RS: 67.50$

ii) $SP = 85\%$ of 80.40

$= \left[\frac{85}{100} \times 80.40\right]$

$= RS: 68.34$

3) find CP when

i) $SP = RS: 40.60$ Gain $= 16\%$

ii) $SP = RS\ 51.70$ Loss $= 12\%$

i) $CP = RS\left(\frac{100}{116} \times 40.60\right)$

$= RS: 35$

ii) $CP = RS\left(\frac{100}{88} \times 51.70\right)$

$= 58.75$

4) A person incurs 5% loss by selling a watch for RS. 1140. At what price should the watch be sold to earn 5% profit.

Let new S.P = RS. x

$(100 - \text{loss}\%) : (1st\ SP) =$

$(100 + \text{gain}\%) : (2nd\ S.P)$

$\left(\frac{100-5}{1140}\right) = \left(\frac{100+5}{x}\right)$

$\frac{95}{1140} = \frac{105}{x}$

$x = \frac{105 \times 1140}{95}$

$x = 1260$

$\boxed{\text{New } SP = 1260}$

5) A vendor bought bananas at 6 for RS. 10 and sold them at 4 for RS. 6. find his gain or loss percent

$LCM = 2\,|\,6, 4$ $\quad LCM = 12$

$2\,|\,3, 2$

$3\,|\,3, 1$

$1, 1$

$CP = RS\left(\frac{10}{6} \times 12\right) = RS. 20$

$SP = RS\left(\frac{6}{4} \times 12\right) = RS. 18$

$\text{Loss}\% = \left(\frac{2}{20} \times 100\right)\% = 10\%$

6) A grocer purchased 80kg of sugar at RS. 13.50 per kg and mixed it with 120kg sugar at RS. 16 per kg. At what rate should he sell the mixture to gain 16%?

CP of 200kg $= [(80 \times 13.5) + (120 \times 6)]$

$= Rs: 3000$

SP = 116% of Rs 3000

$= \left(\frac{116}{100} \times 3000\right) = Rs.3480$

Rate of SP of the mixture $= \left(\frac{3480}{200}\right) = Rs.17.40/kg$

$\boxed{\text{ans: } 17.40/kg}$

7) A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg weight. find his gain percent.

$$\text{Gain\%} = \left[\frac{\text{Error}}{\text{True value} - \text{Error}} \times 100\right]\%$$

$$= \left[\frac{40}{960} \times 100\right]\%$$

$$= \frac{25}{6}\% \text{ (or) } 4\frac{1}{6}\%$$

8) If the manufacturer gains 110% the wholesale dealer 115% and the retailer 125%. then find the cost of production of a table. the retail price of which is Rs. 1265?

table production cost $= x$

$$\frac{125}{100} \times \frac{115}{100} \times \frac{110}{100} \times x = 1265$$

$$\boxed{x = 800}$$

9) A man sold two flats for Rs. 6,75,958 each on one he gains 16% while on the other he loses 16%. How much does he gain (or) lose in the whole transaction?

$$\text{Loss\%} = \left(\frac{\text{Common loss \& gain \%}}{100}\right)^2$$

$$= \left(\frac{16}{100}\right)^2 \% = \left(\frac{256}{100}\right)\%$$

$$\boxed{\text{Loss} = 2.56\%}$$

10) find the single discount equivalent to a series discount of 20%, 10% and 5%

marked price be Rs: 100

S.P = 95% of 90% of 80% of Rs.100

$= \left(\frac{95}{100} \times \frac{90}{100} \times \frac{80}{100} \times 100\right) = Rs.68.40$

Required discount = (100 − 68.40)%

$\boxed{ans = 31.6\%}$

11) After getting two successive discounts a shirt with a list price of Rs.150 is available at Rs.105. If the second discount is 12.5%. find the first discount.

Let the first discount be X%

87.5% (100 − X)% of 150 = 105

$= \frac{87.5}{100} \times \frac{100X}{100} \times 150 = 105$

$100 - X = \left(\frac{105 \times 100 \times 100}{150 \times 87.5}\right)$

$100 - X = 80$

$100 - 80 = X$

first discount $\boxed{X = 20}$ %

SIMPLE INTEREST

$S.I = \left(\frac{P \times R \times T}{100}\right)$

S.I - simple interest

P - principal

R - Rate of interest / annum

T - Time in years

$P = \left(\frac{100 \times SI}{R \times T}\right)$

$R = \left(\frac{100 \times S.I}{P \times T}\right)$

$T = \left(\frac{100 \times S.I}{P \times R}\right)$

1) find the simple interest on Rs. 68,000 at $16\frac{2}{3}$% per annum for 9 months

$S.I = \left(68{,}000 \times 16\frac{2}{3} \times \frac{\cancel{9}^{3}}{\cancel{12}_{4}} \times \frac{1}{100}\right)$

$= \left(68000 \times \frac{50}{3} \times \frac{3}{4} \times \frac{1}{100}\right)$

= Rs.8500/—

2) find the simple interest on RS.3000 at $6\frac{1}{4}$% per annum for the period from 4th feb 2005 to 18th April 2005

Time $= (24 + 31 + 18) = 73$ days

$= \frac{73}{365} = \frac{1}{5}$ year

$S.I = \left(3000 \times 6\frac{1}{4} \times \frac{1}{5} \times \frac{1}{100}\right)$

$= \left(3000 \times \frac{25}{4} \times \frac{1}{5} \times \frac{1}{100}\right)$

$S.I = RS.37.50$

3) A sum at simple interest at $13\frac{1}{2}$% per annum amounts to RS. 2502.50 after 4 years find the sum (Sum - Principal)

$S.I = \left(x \times 13\frac{1}{2} \times 4 \times \frac{1}{100}\right)$

$= \left(x \times \frac{27}{2} \times 4 \times \frac{1}{100}\right)$

$SI = \frac{27x}{50}$

Amount $= \left(x + \frac{27x}{50}\right) = \frac{77x}{50}$

$\frac{77x}{50} = 2502.50$

$x = \frac{2502.50 \times 50}{77}$

$x = 1625$

Hence Sum $= 1625$

4) A sum of RS. 800 amounts to RS. 920 in 3 years at simple interest. If the interest rate is increased by 3% it would amount to how much?

$SI = (920 - 800) = 120$

$P = 800$

$T = 3$ years

$R = \left(\frac{100 \times 120}{800 \times 3}\right) = 5\%$

New rate $= (5 + 3) = 8\%$

New $SI = \left(\frac{800 \times 8 \times 3}{100}\right) = 192$

New amount $= 800 + 192$

$\boxed{\text{ans} = 992}$

5) Adam borrowed some money at the rate of 6% Pa for the first two years, at the rate of 9% Pa for the next three years and at the rate of 14% Pa for the period beyond five years. If he pays a total interest of RS.11400/- at the end of nine years. How much money did he borrow?

$$\left[\frac{x \times 6 \times 2}{100}\right] + \left[\frac{x \times 9 \times 3}{100}\right] + \left(\frac{x \times 14 \times 4}{100}\right) = 11400$$

$$\left[\frac{12x}{100} + \frac{27x}{100} + \frac{56x}{100}\right] = 11400$$

$$\frac{95x}{100} = 11400$$

$$x = \frac{11400 \times 100}{95}$$

$$x = 12000$$

Sum borrowed = Rs: 12,000/-

6) At what rate percent per annum will a sum of money double in 16 years?

$P = P$ $S.I = P$ $T = 16$ yrs

$$\text{Rate} = \left[\frac{100 \times P}{P \times 16}\right]\% = 6\frac{1}{4}\% \text{ Pa}$$

7) The simple interest on a sum of money is $\frac{4}{9}$ of the principal find the rate percent and time. if both are numerically equal.

Let sum $= x$ $SI = \frac{4x}{9}$

rate $= R\%$ time $= R$ years.

$$\left(\frac{x \times R \times R}{100}\right) = \frac{4x}{9}$$

$$R^2 = \frac{400}{9}$$

$$R = \sqrt{\frac{400}{9}} = \frac{20}{3} = 6\frac{2}{3}$$

Rate $= 6\frac{2}{3}\%$ and time $= 6\frac{2}{3}$ years

$=$ 6 years 8 months

7) The simple interest on a certain sum of money for $2\frac{1}{2}$ years at 12% per annum is Rs. 40 less than the simple interest on the same sum for $3\frac{1}{2}$ years at 10% per annum. find the sum

Sum be x

$$\left(x \times 10 \times \frac{7}{2} \times \frac{1}{100}\right) - \left(x \times 12 \times \frac{5}{2} \times \frac{1}{100}\right)$$

$$\frac{7x}{20} - \frac{3x}{10} = 40$$

$$\begin{array}{r|l} \text{LCM } 2 & 20, 10 \\ 2 & 10, 5 \\ 5 & 5, 5 \\ & 1, 1 \end{array} = \text{LCM} = 20$$

$$\frac{7x - 6x}{20} = 40$$

$$x = 40 \times 20 = 800$$

Hence the sum is Rs: 800/-

8) A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate it would have fetched Rs. 360 more. find the sum.

Sum = P original rate = R

$$\left[\frac{P \times (R+2) \times 3}{100}\right] - \left[\frac{P \times R \times 3}{100}\right] = 360$$

$$\frac{\cancel{3PR} + 6P - \cancel{3PR}}{100} = 360$$

$$6P = 360 \times 100$$

$$P = \frac{\overset{60}{\cancel{360}} \times 100}{\cancel{6}}$$

$$P = 6000$$

hence the sum is Rs: 6000/-

9) A sum of RS. 1550 is lent out into two parts one at 8%. and another one at 6%. If the total annual income is RS. 106. find the money lent at each rate

let the sum lent at 8% be x

that at 6% be RS $(1550 - x)$

$$\left[\frac{x \times 8 \times 1}{100}\right] + \left[\frac{(1550-x) \times 6 \times 1}{100}\right] = 106$$

$$\frac{8x + (1550 \times 6) - 6x}{100} = 106$$

$$\frac{2x + (1550 \times 6)}{100} = 106$$

$$2x = (106 \times 100) - (1550 \times 6)$$

$$2x = 10600 - 9300$$

$$2x = 1300$$

$$x = \frac{1300}{2}$$

$$x = 650$$

Money lent at 8% = RS. 650

Money lent at 6% = RS $(1550 - 650)$

= RS. 900

10) A person invested in all RS. 2600 at 4%, 6% and 8% per annum simple interest. At the end of the year he got the same interest in all the three cases. The money invested at 4% is

Let the parts be x y and $(2600 - (x+y))$

$$\left[\frac{x \times 4 \times 1}{100}\right] = \left[\frac{y \times 6 \times 1}{100}\right] = \left[\frac{(2600 - (x+y)) \times 8 \times 1}{100}\right]$$

$$\frac{4x}{100} = \frac{6y}{100} = \left[\frac{(2600 - x - y)8}{100}\right]$$

$$\frac{4x}{\cancel{100}} = \frac{6y}{\cancel{100}}$$

$$y = \frac{4x}{6} = \frac{2x}{3}$$

$$\frac{4x}{\cancel{100}} = \frac{\left(2600 - x - \frac{2x}{3}\right)8}{\cancel{100}}$$

$$4x = \left(2600 - \frac{3x - 2x}{3}\right)8$$

$$4x = \left(2600 - \frac{5x}{3}\right)8$$

$$4x = \frac{(7800 - 5x)8}{3}$$

$$12x = (7800 \times 8) - 40x$$

$$40x + 12x = 7800 \times 8$$

$$52x = 7800 \times 8$$

$$x = \left(\frac{7800 \times 8}{52}\right)$$

$$x = 1200$$

Money invested at 4% = RS. 1200

Profit & Loss

If a trader professes to sell his goods at a profit of x % but uses false weight which is y% less that the actual weight, then

$$\text{Gain \%} = \left\{\left(\frac{x+y}{100-y}\right) \times 100\right\}\%$$

If a trader professes to sell his goods at a loss of x % but uses false weight which is y % less than the actual weight, then

$$\text{Gain\% or Loss\%} = \left\{\left[\frac{y-x}{100-y}\right] \times 100\right\}\%$$

+ve value Gain%
-ve value loss%

A trader cheats while buying and selling by using false weights

$$\text{Total Gain\%} = \left[\frac{(100 + \text{Common gain\%})^2}{100} - 100\right]\%$$

12) A Shopkeeper cheats to the extent of 10% while buying as well as selling by using false weights. H is total gain is?

$$\text{Gain\%} = \left[\frac{(100+10)^2}{100} - 100\right]\%$$

$$= \left[\frac{(100+10)^2}{100} - 100\right]\%$$

$$= \left[\frac{110^2}{100} - 100\right]\%$$

$$= \left[\frac{12100 - 10000}{100}\right]\%$$

$$= \left[\frac{2100}{100}\right]\%$$

Gain% = 21%

13) A grocer sells rice at a profit of 10% and uses weights which are 20% less than the market weight. The total gain earned by him will be?

Consider 1 kg of rice (or) 1000 gm

20% less weight = 80% of 1000

$= \frac{80}{100} \times 100$

CP = 800 gm

SP of this Packet = 110%

C.P of 1kg $= \left(\frac{110}{100} \times 1000\right) = 1100$

Gain% $= \left(\frac{300}{800} \times 100\right)\% = 37.5\%$

Short Cut:

Gain% $= \left\{\left(\frac{10+20}{100-20}\right) \times 100\right\}\%$

$= \left[\frac{30}{80} \times 100\right]\%$

Gain% = 37.5%

14) A dishonest dealer sells the goods at 20% loss on cost price but uses 15% less weight. What is his percentage profit or loss?

Gain% (or) loss% $= \left[\left[\frac{15-20}{100-15}\right] \times 100\right]\%$

$= \left[\frac{-5}{85} \times 100\right]\%$

$= \frac{-100}{17}\% \text{ (or) } -5\frac{15}{17}\%$

The value is -ve hence it is loss percentage $\boxed{\text{Loss\%} = \frac{-100}{17}\%}$

15) A merchant professes to lose 4% on a certain tea but he uses a weight equal to 840g instead of 1kg find his real loss or gain percent.

Percentage reduction in weight =

1000 gm − 840 gm = 160

$\left[\frac{160}{1000} \times 100\right]\% = 16\%$

$$\text{Gain\%} \text{ (or) Loss\%} = \left(\left|\frac{16-4}{100-16}\right| \times 100\right)\%$$

$$= \left(\frac{12}{84} \times 100\right)\%$$

$$= \frac{100}{7}\% \text{ (or) } 14\frac{2}{7}\%$$

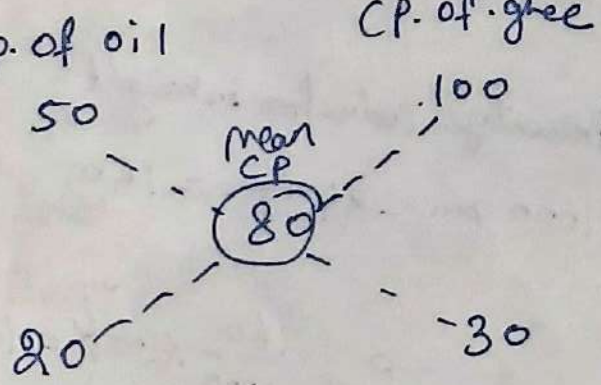
The value is +ve hence gain%.

$$\boxed{\text{Gain\%} = \frac{100}{7}\%}$$

16) Pure ghee cost Rs. 100/kg. after adulterating it with vegetable oil costing Rs. 50/kg, a shop keeper sells the mixture at the rate of Rs. 96/kg there by making a profit of 20%. In what ratio does he mix the two

$$\text{mean cost price} = \left(\frac{100}{120} \times 96\right) = 80/\text{kg}$$

rule of alligation.



$$\text{Required ratio} = 20 : 30$$

$$= 2 : 3$$

COMPOUND INTEREST

1) when interest is compound Annually

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$$

2) when interest is compounded half-yearly

$$\text{Amount} = P\left[1 + \frac{(R/2)}{100}\right]^{2n}$$

3) when interest is compounded quarterly:

$$\text{Amount} = P\left[1 + \frac{(R/4)}{100}\right]^{4n}$$

4) when interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P\left[1 + \frac{R}{100}\right]^3 \times \left[1 + \frac{\frac{2}{5}R}{100}\right]$$

5) when rates are different for different years say $R_1\%, R_2\%, R_3\%$

$$\text{Amount} = P\left[1 + \frac{R_1}{100}\right]\left[1 + \frac{R_2}{100}\right]\left[1 + \frac{R_3}{100}\right]$$

6) present worth of Rs. x due n years hence is given by

$$\text{present worth} = \frac{x}{\left[1 + \frac{R}{100}\right]^n}$$

1) find Compound interest on Rs. 7500 at 4% per annum for 2 years, Compounded annually.

$$\text{Amount} = \left[7500 \times \left[1 + \frac{4}{100}\right]^2\right]$$

$$\text{Rs} = 8112$$

$$CI = (8112 - 7500) = \text{Rs. } 612$$

2) find Compound interest on Rs. 8000/- at 15% per annum for 2 years 4 months, Compounded annually.

$$\text{Time} = 2 \text{ years } 4 \text{ months}$$

$$= 2\ \frac{4}{12} \text{ years}$$

$$= 2\ \frac{1}{3} \text{ years.}$$

$$\text{Amount} = \left[8000\left[1 + \frac{15}{100}\right]^2 \times \left[1 + \frac{\frac{1}{3} \times 15}{100}\right]\right]$$

$$= \left(8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20}\right)$$

$$\text{Rs} = 11109$$

$$CI = (11109 - 8000) = \text{Rs. } 3109$$

3) find the Compound interest on Rs 10000 in 2 years at 4% per annum the interest being Compounded half-yearly

$$\text{Amount} = \left[10000\left(1 + \frac{\frac{4}{2}}{100}\right)^{2 \times 2}\right]$$

$$= 10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50}$$

$$\text{Rs. } 10824.32$$

$$C.I = (10824.32 - 10000)$$

$$= \text{Rs. } 824.32$$

4) find the Compound interest on Rs 16000 at 20% per annum for 9 months, compounded quarterly

$$\text{Amount} = 16000 \times \left[1 + \frac{\frac{20}{4}}{100}\right]^{4 \times \frac{9}{12}}$$

$$= 16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$= 18522$$

$$CI = \text{Rs}(18522 - 16000)$$

$$= \text{Rs. } 2522$$

5) If the Simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200. find the Compound interest on the same sum for the same period at the same rate.

$$P = \left(\frac{100 \times SI}{R \times N}\right) = \left(\frac{100 \times 1200}{3 \times 5}\right) = 8000$$

$$\text{Amount} = \left[8000 \times \left(1 + \frac{5}{100}\right)^3\right] = 9261$$

$$CI = 9261 - 8000 = 1261$$

6) In what time will Rs. 1000 become Rs. 1331 at 10% per annum compounded annually?

$$CI = \left[1000\left(1+\frac{10}{100}\right)^n\right] = 1331$$

$$= 1000 \times \left(\frac{11\cancel{0}}{10\cancel{0}}\right)^n = 1331$$

$$= \left(\frac{11}{10}\right)^n = \frac{1331}{1000} = \left(\frac{11}{10}\right)^3$$

$n = 3$ years.

7. If Rs. 500 amounts to Rs. 583.20 in two years compounded annually, find the rate of interest per annum.

$$\left[500\left(1+\frac{R}{100}\right)^2\right] = 583.20$$

$$\left[1+\frac{R}{100}\right]^2 = \frac{583.20}{500} = \frac{5832}{5000}$$

$$\frac{5832}{5000} \times \frac{2}{2} = \frac{11664}{10000}$$

$$\left(1+\frac{R}{100}\right)^{\cancel{2}} = \left[\frac{108}{100}\right]^{\cancel{2}}$$

$$1+\frac{R}{100} = \frac{108}{100}$$

$$\frac{R}{100} = \frac{108}{100} - 1$$

$$\frac{R}{100} = \frac{108-100}{100}$$

$$\frac{R}{\cancel{100}} = \frac{8}{\cancel{100}}$$

$$\boxed{R = 8\%} \text{ p.a}$$

8) If compound interest on a certain sum at $16\frac{2}{3}\%$ for 3 years is Rs. 1270, find the simple interest on the same sum at the same rate and for the same period.

$$C.I = \left[x \times \left(1+\frac{\overset{1}{\cancel{50}}}{3\times\underset{2}{\cancel{100}}}\right)^3 - x\right]$$

$$= x\left(\frac{\overset{7}{\cancel{35}}}{\underset{6}{\cancel{30}}}\right)^3 - x$$

$$= x\left(\frac{7}{6}\right)^3 - x$$

$$= \frac{343x}{216} - x$$

$$= \frac{343x - 216x}{216}$$

$$C.I = \frac{127x}{216}$$

$$\frac{127x}{216} = 1270$$

$$x = \frac{\overset{10}{\cancel{1270}} \times 216}{\cancel{127}}$$

Sum $\boxed{x = 2160}$

$$S.I = \left(2160 \times \frac{50}{3} \times 3 \times \frac{1}{100}\right) = Rs.1080$$

9) The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. find the sum.

$$CI = x\left[1+\frac{\cancel{10}^{1}}{\cancel{100}_{10}}\right]^2 - x$$

$$= x\left[\frac{11}{10}\right]^2 - x$$

$$= x\left[\frac{121}{100}\right] - x$$

$$= \frac{121x}{100} - x$$

$$= \frac{121x - 100x}{100}$$

$$CI = \frac{21x}{100}$$

$$SI = \left[\frac{x \times \cancel{10} \times \cancel{2}}{\cancel{10}_{5}}\right] = \frac{x}{5}$$

$$CI - SI = 631$$

$$\left[\frac{21x}{100} - \frac{x}{5}\right] = 631$$

$$LCM \; 2 \,|\, 100, 5 \quad 2 \,|\, 50, 5 \quad 5 \,|\, 25, 5 \quad 5 \,|\, 5, 1 \quad 1, 1 = 100$$

$$\frac{21x - 20x}{100} = 631$$

$$\frac{x}{100} = 631$$

$$x = 63100$$

Hence, the sum is Rs. 63,100/—

10) The difference between the compound interest and the simple interest accrued on an amount of Rs. 18000 in 2 years was Rs. 405. What was the rate of interest per annum?

$$\left[18000\left(1+\frac{R}{100}\right)^2 - 18000\right] - \left[\frac{18000 \times R \times 2}{100}\right] = 405$$

$$\left[18000\,\frac{(100+R)^2}{10000} - 18000\right] - 360R = 405$$

11) Divide RS. 1301 between A & B so that the amount of A after 7 years is equal to the amount of B after 9 years, the interest being compounded at 4% per annum.

Let the two parts be RS. x and $(1301-x)$

$$x\left(1+\frac{4}{100}\right)^7 = (1301-x)\left(1+\frac{4}{100}\right)^9$$

$$\frac{x}{(1301-x)} = \frac{\left(1+\frac{4}{100}\right)^9}{\left(1+\frac{4}{100}\right)^7}$$

$$\frac{x}{(1301-x)} = \left(1+\frac{4}{100}\right)^2$$

$$\frac{x}{(1301-x)} = \left(\frac{\cancel{104}^{26}}{\cancel{100}_{25}}\right)^2$$

$$\frac{x}{1301-x} = \left[\frac{676}{625}\right]$$

$$625x = 676(1301-x)$$

$$625x = (676 \times 1301) - 676x$$

$$625x + 676x = 676 \times 1301$$

$$1301x = 676 \times 1301$$

$$x = \frac{676 \times 1301}{1301}$$

$$x = 676$$

The two parts are

$x = 676$

RS. $(1301 - 676)$

Rs. 625 & Rs 676

12) A sum of money doubles itself at compound interest in 15 years. In how many years will it become eight times?

$$P\left[1+\frac{R}{100}\right]^{15} = 2P$$

$$\left[1+\frac{R}{100}\right]^{15} = \frac{2\cancel{P}}{\cancel{P}}$$

$$\left[1+\frac{R}{100}\right]^{15} = 2$$

$$P\left[1+\frac{R}{100}\right]^{n} = 8P$$

$$\left[1+\frac{R}{100}\right]^{n} = \frac{8\cancel{P}}{\cancel{P}}$$

$$\left[1+\frac{R}{100}\right]^{n} = 8$$

$2^3 = 8$

$$\left[1+\frac{R}{100}\right]^n = 2^3$$

$$\left[1+\frac{R}{100}\right]^n = \left\{\left[1+\frac{R}{100}\right]^{15}\right\}^3$$

$$\left[1+\frac{R}{100}\right]^n = \left[1+\frac{R}{100}\right]^{45}$$

$$n = 45$$

Thus the required time = 45 years.

RATIO & PROPORTION

1) If $a:b = 5:9$ & $b:c = 4:7$ find $a:b:c$

A	B	C
5	9	9
4	4	7
20	36	63

ans: $20:36:63$

2) find

i) The fourth proportional to 4, 9, 12.

ii) The third proportional to 16 & 36

iii) The mean proportional b/w 0.08 & 0.18

i) Fourth proportional

$4:9::12:x$

$$x = \frac{9\times12}{4} = 27$$

ii) Third proportional

$16:36::36:x$

$$x = \frac{36\times36}{16} = 81$$

iii) mean proportional

$$= \sqrt{0.08\times0.18}$$

$$= \sqrt{\frac{8}{100}\times\frac{18}{100}} = \sqrt{\frac{144}{100\times100}} = \frac{12}{100} = 0.12$$

3) If $x:y = 3:4$ find $(4x+5y):(5x-2y)$

$$\frac{x}{y} = \frac{3}{4} \qquad \frac{4x+5y}{5x-2y}$$

÷ by y in NR & DR

$$= \frac{4\frac{x}{y}+5}{5\frac{x}{y}-2}$$

$$= \frac{4\left(\frac{3}{4}\right)+5}{5\left(\frac{3}{4}\right)-2} = \frac{(3+5)}{\frac{7}{4}} = \frac{32}{7}$$

4) A sum of RS. 53 is divided among A, B, C in such a way that A gets RS. 7 more than what B gets and B gets RS. 8 more than what C gets. The ratio of their shares is:

C gets RS: x

$(x+15) : (x+8) : x$

$x + (x+8) + (x+15) = 53$

$x = 10$

A : B : C

$(10+15) : (10+8) : 10$

$25 : 18 : 10$

5) Divide RS. 672 in the ratio 5:3

Sum of ratio terms $= (5+3) = 8$

First Part $= \left(672 \times \frac{5}{8}\right) =$ RS: 420/-

Second Part $= \left(672 \times \frac{3}{8}\right) =$ RS. 252/-

6) Divide RS. 1162 among A, B, C in the ratio 35 : 28 : 20

Sum of ratio terms $= (35+28+20) = 83$

A Share $= \left(1162 \times \frac{35}{83}\right) =$ RS. 490/-

B Share $= \left(1162 \times \frac{28}{83}\right) =$ RS. 392/-

C Share $= \left(1162 \times \frac{20}{83}\right) =$ RS. 280/-

7) A bag contains 50P, 25P and 10P coins in the ratio 5 : 9 : 4, amounting to RS. 206. Find the number of coins of each type

$$\frac{5x}{2} + \frac{9x}{4} + \frac{4x}{10} = 206$$

LCM

2	2, 4, 10	
2	1, 2, 5	
5	1, 1, 5	
	1, 1, 1	

LCM = 20

$$\frac{50x + 45x + 8x}{20} = 206$$

$50x + 45x + 8x = 206 \times 20$

$103x = 4120$

$x = 40$

Number of 50 p coins $= (5 \times 40) = 200$

Number of 25 p coins $= (9 \times 40) = 360$

Number of 10 p coins $= (4 \times 40) = 160$

8) A mixture contains milk and water in the ratio 4 : 3. If 5 litres of water is added to the mixture, the ratio becomes 4 : 5. Find the quantity of ~~ots~~ milk in the given mixture.

$\frac{4x}{3x}$ 4 : 3

↓ 5 l

4 : 5

$$\frac{4x}{(3x+5)} = \frac{4}{5}$$

$$20x = 12x + 20$$

$$20x - 12x = 20$$

$$8x = 20$$

$$x = 2.5$$

quantity of milk $= 4x$

$= 4 \times 2.5$

$= 10$ litres.

9) If $(a+b) : (b+c) : (c+a) = 6:7:8$ and $(a+b+c) = 14$ then the value of c is ?

$(a+b) = 6x$

$(b+c) = 7x$

$(c+a) = 8x$

$2(a+b+c) = 21x$

$2 \times 14 = 21x$

$28 = 21x$

$x = \frac{\cancel{28}}{\cancel{21}} \frac{4}{3}$

$(a+b) = 6x = 6 \times \frac{4}{3} = 8$

$c = (a+b+c) - (a+b)$

$= 14 - 8$

$\boxed{c = 6}$

10) A sum of RS. 1300 is divided amongst P, Q, R and S such that

$$\frac{P\text{'s share}}{Q\text{'s share}} = \frac{Q\text{'s share}}{R\text{'s share}} = \frac{R\text{'s share}}{S\text{'s share}} = \frac{2}{3}$$

then P's share is ?

Let $P = 2x$ $\quad Q = 3x$

$\frac{Q}{R} = \frac{2}{3}$ $\quad R = \frac{3}{2} Q$ $\quad R = \frac{9x}{2}$

$R = \frac{3}{2}(3x)$

$\frac{R}{S} = \frac{2}{3}$ $\quad S = \frac{3}{2} R$ $\quad S = \frac{27x}{4}$

$S = \frac{3}{2}\left(\frac{9x}{2}\right)$

$(P + Q + R + S) = 1300$

$\left(2x + 3x + \frac{9x}{2} + \frac{27x}{4}\right) = 1300$

LCM $2 \mid 2, 4$ $\quad$ LCM $= 4$

$2 \mid 1, 2$

$1, 1$

$$\frac{(8x + 12x + 18x + 27x)}{4} = 1300$$

$8x + 12x + 18x + 27x = 1300 \times 4$

$65x = 5200$

$x = \frac{5200}{65}$

$\boxed{x = 80}$

P's share $= 2x = 2 \times 80 =$ RS. 160

11) Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is?

Let the third number be x

first number $= 120\%$ of $x = \frac{120x}{100} = \frac{6x}{5}$

second number $= 150\%$ of $x = \frac{150x}{100} = \frac{3x}{2}$

ratio of first two numbers $= \frac{6x}{5} : \frac{3x}{2}$

$12x : 15x = \boxed{4:5}$

12) Seats for mathematics, physics and Biology in a school are in the ratio 5:7:8. There is a proposal to increase these seats by 40%, 50% and 75%, respectively. what will be the ratio of increased seats?

$5x : 7x : 8x$

$\frac{140 \times 5x}{100} = 7x$

$\frac{150 \times 7x}{100} = \frac{21x}{2}$

$\frac{175 \times 8x}{100} = 14x$

$7x : \frac{21x}{2} : 14x$

Lcm = 2 $\quad 14x : 21x : 28x$

$\boxed{2 : 3 : 4}$

13) The ratio of number of boys and girls in a college is 7:8. If the percentage increase in the number of boys and girls be 20% & 10% respectively. What will be the new ratio?

$\frac{120 \times 7x}{100} : \frac{110 \times 8x}{100}$

$\frac{42x}{5} : \frac{44x}{5}$

$\boxed{21 : 22}$

14) A sum of money is to be distributed among A, B, C, D in the proportion of 5:2:4:3. If C gets Rs. 1000 more than D what is B's share?

$5x : 2x : 4x : 3x$

$4x - 3x = 1000$

$x = 1000$

B's share $= 2x$

$= 2 \times 1000$

$\boxed{\text{ans} = 2000}$

15) If 40% of a number is equal to two-third of another number. What is the ratio of first number to the second number?

$$\frac{40 \times A}{100} = \frac{2}{3} B$$

$$\frac{2A}{5} = \frac{2}{3} B \qquad \frac{A}{B} = \frac{\cancel{2}}{3} \times \frac{5}{\cancel{2}}$$

$$\frac{A}{B} = \frac{5}{3} \qquad \boxed{A : B = 5 : 3}$$

16) Ratio of the earnings of A & B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25% the new ratio of their earnings becomes 8 : 7. what are A's earnings?

a) Rs. 21000
b) Rs. 26000
c) Rs. 28000
d) Data inadequate

$4x : 7x$

$$\frac{150 \times 4x}{100} : \frac{75 \times 7x}{100}$$

$$6x : \frac{21x}{4} = 8 : 7$$

Can't able to find x so the given data is inadequate.

17) What least number must be subtracted from each of the numbers 14, 17, 34 and 42 so that the remainders may be proportional?

Let the required number be x

$14 : 17 :: 34 : 42$

$(14-x) : (17-x) :: (34-x) : (42-x)$

$$\frac{14-x}{17-x} = \frac{34-x}{42-x}$$

$(14-x)(42-x) = (34-x)(17-x)$

$x^2 - 56x + 588 = x^2 - 51x + 578$

$5x = 10$

$\boxed{x = 2}$

Required number = 2

18) In a mixture of 60 litres, the ratio of milk and water is 2 : 1. If this ratio is to be 1 : 2 then the quantity of water to be further added is?

Sum of ratio terms $= (2+1) = 3$

quantity of milk $= \left(60 \times \frac{2}{3}\right) = 40\,l$

quantity of water $= (60 - 40) = 20\,l$

New ratio 1 : 2

quantity of water to be added is x

$80 = 20 + x$

$$\frac{40}{(20+x)} = \frac{1}{2}$$

$\boxed{x = 60}$

quantity of water added = 60 l

19) The fourth proportional to 5, 8, 15 is

$5 : 8 :: 15 : x$

$5x = (8 \times 15) \quad x = \frac{8 \times \cancel{15}^{3}}{\cancel{5}} = 24$

20) The mean proportional between 234 and 104 is

mean proportional $= \sqrt{234 \times 104}$

$= \sqrt{13 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 13}$

$= (13 \times 3 \times 2 \times 2)$

$= 156$

21) The third proportional to 0.36 and 0.48 is

$0.36 : 0.48 :: 0.48 : x$

$\frac{36}{100} : \frac{48}{100} :: \frac{48}{100} : x$

$\frac{x}{\cancel{100}} = \frac{\frac{\cancel{48}^{4} \times \cancel{48}^{16}}{\cancel{100}}}{\frac{\cancel{36}^{\cancel{3}}}{\cancel{100}}}$

$x = \frac{64}{100}$

$\boxed{x = 0.64}$

22) The third proportional to $(x^2 - y^2)$ and $(x - y)$ is

$(x^2 - y^2) : (x - y) :: (x - y) : z$

$(x^2 - y^2) \times z = (x - y)^2$

$z = \frac{(x - y)^2}{(x^2 - y^2)}$

23) The ratio of third proportional to 12 and 30 and the mean proportional between 9 and 25 is)

Third proportional

$12 : 30 :: 30 : x$

$x = \frac{30 \times 30}{12} = 75$

mean proportional $= \sqrt{9 \times 25}$

$= 3 \times 5 = 15$

Required ratio $= 75 : 15$

$\boxed{5 : 1}$

24) An amount of RS. 735 was divided between A, B & C. If each of them had Received RS. 25 less, their shares would have been in the ratio of 1:3:2. The money received by C was?

Remainder $= 735 - (25 \times 3)$
$= 660$

Money received by C

Sum of ratio terms $= 1+3+2 = 6$

$C = \left(660 \times \frac{2}{6}\right) + 25$

C = RS. 200

25) The ratio between two numbers is 3:4 and their LCM is 180. The first number is?

$3x : 4x$

LCM is 12x $\quad 12x = 180$

$x = \frac{180}{12}$ 15

Hence the first number is

$3x$

3×15

$\boxed{\text{ans} = 45}$

26) In a mixture of three varieties of tea, the ratio of their weights is 4:5:8. If 5 kg tea of the first variety, 10 kg tea of the second variety and some quantity of tea of the third variety are added to the mixture, the ratio of the weights of three varieties of tea becomes 5:7:9. Find the quantity of the third variety of tea in the final mixture.

$4 : 5 : 8 \qquad 5 : 7 : 9$

$4x : 5x : 8x$

$\frac{4x+5}{5x+10} = \frac{5}{7} \qquad 7(4x+5) = 5(5x+10)$

$\boxed{x = 5}$

$\frac{5x+10}{8x+y} = \frac{7}{9}$

$\frac{5(5)+10}{8(5)+y} = \frac{7}{9} \qquad 9(25+10) = 7(40+y)$

$\boxed{y = 5}$

third variety in the final mixture

$= 8x + 5y$

$= 8(5) + 5$

$\boxed{\text{ans} = 45 \text{ kg}}$

Partnership

Ratio of Division of gains

i) Simple partnership
ii) Compound partnership

i) Simple partnership (Same time period)

Gain (or) loss distributed among the partners in the ratio of their investments

A & B Invest RS.x & RS.y for one year at the end of the year

(A' share of Profit) : (B' share of Profit)

$$x : y$$

ii) Compound Partnership (Different time period)

A invest RS.x for p months
B invest RS.y for q months

(A' share of Profit) : (B' share of Profit)

$$xp : yq$$

Working and Sleeping Partners:

A partner who manages the business is known as a working partner.

The one who simply invests the money is a sleeping partner.

1) A, B and C started a business by investing RS. 1,20,000/-, RS. 1,35,000 and RS. 1,50,000. Find the share of each out of an annual profit of RS. 56,700.

120000 : 135000 : 150000

8 : 9 : 10

Sum of ratio terms = 8+9+10 = 27

A's share = $\left(56700 \times \frac{8}{27}\right)$ = RS. 16800.

B's share = $\left(56700 \times \frac{9}{27}\right)$ = RS. 18900

C's share = $\left(56700 \times \frac{10}{27}\right)$ = RS. 21000

2) Alfred started a business investing Rs. 45,000. after 3 months peter joined him with a capital of Rs. 60,000. after another 6 months Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. find the share of each.

Alfred	Peter	Ronald
45000	3m	
	60,000	9m
		90,000
12m	9m	3m

Ratio of their Capitals.

$(45000 \times 12) : (60,000 \times 9) : (90000 \times 3)$

$540000 : 540000 : 27000$

$2 : 2 : 1$

$2+2+1=5$

Alfred share $= \left(16500 \times \frac{2}{5}\right) =$ Rs. 6600

peter share $= \left(16500 \times \frac{2}{5}\right) =$ Rs. 6600

Ronald share $= \left(16500 \times \frac{1}{5}\right) =$ Rs. 3300

3) A, B & C started a business each investing Rs. 20,000. after 5 months A withdrew Rs. 5000, B withdrew Rs. 4000 & C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. find the share of each.

A	B	C
20,000	20,000	20,000
-5000 5m	-4000 5m	+6000 5m
15000	16,000	26,000
7m	7m	7m

$(20000 \times 5) + (15000 \times 7) = A$

$(20000 \times 5) + (16000 \times 7) = B$

$(20000 \times 5) + (26000 \times 7) = C$

A : B : C

$205000 : 212000 : 282000$

$205 : 212 : 282$

$205 + 212 + 282 = 699$

A share $=$ Rs. $\left(69900 \times \frac{205}{699}\right) = 20500$

B share $=$ Rs. $\left(69900 \times \frac{212}{699}\right) = 21200$

C share $=$ Rs. $\left(69900 \times \frac{282}{699}\right) = 28200$

4) A, B & C enter into partnership A invests 3 times as much as B invests & B invests two-third of what C invests. At the end of the year the profit earned is RS. 6600. what is the share of B?

$C = x \qquad B = \frac{2}{3}x$

$A = 3 \times \frac{2}{3}x = 2x$

Ratio of their capitals $= 2x : \frac{2}{3}x : x$

$= 6 : 2 : 3 \qquad 6+2+3=11$

Hence B's share $= \left(6600 \times \frac{2}{11}\right)$

$= RS. 1200$

5) Four milk men rented a pasture. A grazed 24 cows for 3 months B 10 cows for 5 months, C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is RS. 720. find the total rent of the field.

A : B : C : D

$(24\times3) : (10\times5) : (35\times4) : (21\times3)$

$72 : 50 : 140 : 63$

$72 + 50 + 140 + 63 = 325$

Let total rent be x

$\frac{72x}{325} = 720$

$x = \frac{720\times325}{72} = 3250$

Hence total rent of the field is RS. 3250.

6) A invested RS 76,000 in a business. After few months, B joined him with RS 57000. At the end of the year, the total profit was divided between them in the ratio 2:1 after how many months did B join

B joined after x months

$(12-x)$

$\frac{76000\times12}{57000\times(12-x)} = \frac{2}{1}$

$912000 = 114000(12-x)$

$12-x = \frac{912}{114}$

$12-x = 8$

$x = 4$

B joined after 4 months

7) A, B and C enter into a partnership by investing in the ratio of 3:2:4 after one year, B invests another Rs. 2,70,000 and C at the end of 2 years also invests Rs. 2,70,000. At the end of three years, profits are shared in the ratio of 3:4:5. find the initial investment of each.

Initial investment

$$3x : 2x : 4x$$

$$(3x \times 36) : [(2x \times 12) + ((2x + 270000) \times 24)] :$$

$$[(4x \times 24) + [(4x + 270,000) \times 12]]$$

$$= 3 : 4 : 5$$

$$\frac{108x}{72x + 6480000} = \frac{3}{4}$$

$$432x = 216x + 19440000$$

$$x = 90000$$

A' initial investment $= 3x =$ Rs. 2,70,000

B' initial investment $= 2x =$ Rs. 1,80,000

C' initial investment $= 4x =$ Rs. 3,60,000

8) A & B started a business with investments of Rs. 5000 and Rs. 4500 respectively. After 4 months A takes out half of his capital. After two more months B takes out one third of his capital while C joins them with a capital of Rs. 7000. At the end of a year, they earn a profit of Rs. 5080. find the share of each member in the profit

A : B : C

$$[(5000 \times 4) + (2500 \times 8)] = A$$

$$\left[(4500 \times 6) + \left[4500 \times \frac{1}{3} \times 6\right]\right] = B$$

$$[7000 \times 6] = C$$

$$40 : 45 : 42 \qquad 40 + 45 + 42 = 127$$

A' share $= \left(5080 \times \frac{40}{127}\right) =$ Rs. 1600

B' share $= \left(5080 \times \frac{45}{127}\right) =$ Rs. 1800

C' share $= \left(5080 \times \frac{42}{127}\right) =$ Rs. 1680

PROBLEMS ON AGES

1) Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev?

Rajeev's present age $= x$

after 15 years $= (x+15)$

5 years back $= (x-5)$

$$x+15 = 5(x-5)$$

$$x+15 = 5x-25$$

$$x = 10$$

Rajeev present age $= 10$

2) The ages of two persons differ by 16 years. If 6 years ago, the elder one be 3 times as old as younger one. find their present ages.

age of younger one be x

age of elder $= (x+16)$

6 years ago

$$3(x-6) = (x+16-6)$$

$$x = 14$$

present age of younger $= 14$

elder $= 30$

3) The product of the ages of ankit and Nikita is 240. If twice the age of Nikita is more than ankit's age by 4 years, what is Nikita's age?

Ankit age be x

Then Nikita age $= \frac{240}{x}$

$$2 \times \frac{240}{x} - x = 4$$

$$480 - x^2 = 4x$$

$$x^2 + 4x - 480 = 0$$

$$(x+24)(x-20) = 0$$

$$x = 20$$

Nikita age $= \frac{240}{20} = 12$ years

4) The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. find the present age of the father?

Son's present age $= x$

present age of father $= (3x+3)$

$$(3x+3+3) = 2(x+3)+10$$

$$3x+3+3 = 2x+6+10$$

$$\boxed{x = 10}$$

Hence father present age

$= 3x+3$

$= 3(10)+3$

$= 33$ years.

5) Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages?

Sons age 8 years ago be x

Rohit age 8 years ago $= 4x$

Sons age after 8 years $= (x+8)+8$

$= x+16$

Rohit age after 8 years $= (4x+8+8)$

$= (4x+16)$

$2(x+16) = (4x+16)$

$x = 8$

Son's age (present) $= (x+8) = 16$

Rohit age (present) $= (4x+8) = 40$

6) one year ago the ratio of Gaurav's and sachins age was 6:7 respectively. Four years hence, this ratio would become 7:8. How old is sachin?

one year ago the ratio

$6x : 7x$

Gaurav's age 4 years hence $= (6x+1)+4$

$= 6x+5$

Sachins age 4 years hence $= (7x+1)+4$

$= 7x+5$

$\frac{6x+5}{7x+5} = \frac{7}{8}$

$8(6x+5) = 7(7x+5)$

$x = 5$

Hence sachins age $= (7x+1)$

$= 7(5)+1$

$= 36$ years.

7) Abhay's age after 6 years will be three-seventh of his fathers age. Ten years ago, the ratio of their ages was 1:5. what is Abhay's father's age at present?

10 years ago ages be $x : 5x$

Abhay's age after 6 years

$= (x+10)+6$

$= x+16$

Father's age after 6 years

$= (5x+10)+6$

$= 5x+16$

$(x+16) = \frac{3}{7}(5x+16)$ $x = 8$

Fathers age $= (5x+10) = 50$ years

8) The sum of the ages of a father and his son is 45 years five years ago, the product of their ages was 34. The ages of the son and the father are respectively:

age of father x

age of son $= 45 - x$

five years ago

$$(x-5)(45-x-5) = 34$$

$$(x-5)(40-x) = 34$$

$$x^2 - 45x + 234 = 0$$

$$(x-39)(x-6) = 0$$

$$x = 39 \text{ (or) } x = 6$$

father's age $= 39$

son's age $= 6$

9) The difference b/w the present ages of Arun and deepak is 14 years. Seven years ago, the ratio of their ages was 5:7 respectively what is deepak's present age?

$$5x : 7x$$

Seven years ago

present age

$$(5x+7) : (7x+7)$$

Difference b/w present age

$$(7x+7) - (5x+7) = 14$$

$$2x = 14$$

$$x = 7$$

Deepak's present age

$$= 7x + 7$$

$$= (7 \times 7) + 7$$

$$= 56 \text{ years.}$$

10) The sum of the ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child?

Let the ages of children be

$x, (x+3), (x+6), (x+9), (x+12)$

$$x + x+3 + x+6 + x+9 + x+12 = 50$$

$$5x = 20$$

$$x = 4$$

Age of youngest child $= 4$ years

11) A man was asked to state his age in years. His reply was "Take my age 3 years hence, multiply it by 3 and then subtract 3 times my age 3 years ago and you will know how old I am" What is the age of the man?

Let the present age of man be x

$$3(x+3) - 3(x-3) = x$$
$$3x+9-(3x-9) = x$$
$$\cancel{3x}+9-\cancel{3x}+9 = x$$
$$x = 18$$

The present age of the man is 18 years

12) The sum of the ages of Jayant, Prem and Poras is 93 years. Ten years ago, the ratio of their ages was 2 : 3 : 4. What is the present age of Poras?

$$(2x+10)+(3x+10)+(4x+10) = 93$$
$$9x+30 = 93$$
$$9x = 93-30$$
$$9x = 63$$
$$x = \frac{63}{9} = 7$$

Present age of Poras $= 4x+10$
$= 4(7)+10$
$= 38$ years

13) The ratio b/w the ages of Neelam and Shiny is 5 : 6 respectively. If the ratio b/w the one-third age of Neelam and half of Shiny's age is 5 : 9 then what is Shiny's age?

a) 25 years b) 30 years. c) 36 years
d) Cannot be determined e) none of these

$5x : 6x$

$$\frac{1}{3}5x : \frac{1\times 6x}{2} = \frac{5}{9}$$

$$\frac{5x}{3} : \frac{\overset{3}{\cancel{6}}x}{\cancel{2}} = 5:9$$

$$\frac{5x}{3\times 3x} = \frac{5}{9}$$

Shiny's age cannot be determined

14) 18 years ago, a man was three times as old as his son. Now, the man is twice as old as his son. The sum of the present ages of man and his son is

18 years ago son's age be x
18 years ago man age be $3x$
present age

$$(3x+18) = 2(x+18)$$

$$\boxed{x = 18}$$

Sum of present age of man and son

$3x + 18 + x + 18$

$= (4x + 36)$

$= 4(18) + 36$

$\boxed{\text{ans} = 108 \text{ years.}}$

15) Sneh's age is $\frac{1}{6}$th of her father's age. Sneh's father's age will be twice of vimal's age after 10 years. If Vimal's 8th birthday was celebrated 2 years ago, then what is Sneh's present age?

Vimal's present age $= (8+2) = 10$

Sneh's father age $= 2(10+10) = 40$ years

Sneh's age $= \left(\frac{1}{6} \times 40\right) = \frac{20}{3} = 6\frac{2}{3}$ years

Partnership

9) Two persons A and B take a field on rent. A put on it 21 horses for 3 months and 15 cows for 2 months. B puts 15 cows for 6 months and 40 sheep for $7\frac{1}{2}$ months. If in one day 3 horses eat as much as 5 cows eat and 6 cows as much as 10 sheep, what part of the rent should A pay?

6 cows = 10 sheep

1 cows $= \frac{\cancel{10}^{5}}{\cancel{6}_{3}}$ sheep $= \frac{5}{3}$ sheep

3 horses = 5 cows

1 horse $= \frac{5}{3}$ cows $= \frac{5}{3} \times \frac{5}{3}$ sheep

1 horse $= \frac{25}{9}$ sheep

Ratio of shares of A & B

$$\left[\left[21 \times \frac{25}{9} \times 3\right] + \left[15 \times \frac{5}{3} \times 2\right]\right] = A$$

$$\left[\left[15 \times \frac{5}{3} \times 6\right] + \left[40 \times \frac{15}{2}\right]\right] = B$$

A : B

225 : 450

1 : 2 $\quad 1+2=3$

Hence part of rent paid by

$\boxed{A = \frac{1}{3}}$

10) A, B & C took a house on rent for one year for Rs. 13824. They remained together for 4 months and then C left the house. After 5 more months, B also left the house. How much rent should each pay?

monthly rent $= \frac{13824}{12} = 1152$

Rent for first 4 months $= (1152 \times 4)$
$= 4608$

Share of each (A+B+C) $= \frac{4608}{3} = 1536$

Rent for next 5 months $= (1152 \times 5)$
$= 5760$

Share of each A & B $= \frac{5760}{2}$
$= 2880$

Rent for next 3 months $= (1152 \times 3)$
It is to be paid by A only $= 3456$

Total rent paid by A $= 3456 + 2880 + 1536$
$= 7872$

Total rent paid by B $= 1536 + 2880$
$= 4416$

Total rent paid by C $= 1536$.

11) A, B and C enter into a partnership. Their capital contribution is in the ratio 21:18:14. At the end of the business term they share profits in the ratio 15:8:9. Find the ratio of time for which they invest their capitals.

$21 : 18 : 14$

$21x : 18x : 14x$

Let the month of investments be p, q & r for A, B & C

$21x \times p : 18x \times q : 14x \times r = 15:8:9$

$\frac{21p}{18q} = \frac{15}{8}$ $\quad p = \left(\frac{15}{8} \times \frac{28}{21}\right) q = \frac{45}{28} q$

$\frac{18q}{14r} = \frac{8}{9}$ $\quad q = \left(\frac{8}{9} \times \frac{14}{18}\right) r = \frac{56}{81} r$

$p = \frac{45}{28} q = \left[\frac{45}{28} \times \frac{56}{81}\right] r = \frac{10}{9} r$

So required ratio $p : q : r$

$\frac{10}{9} r : \frac{56}{81} r : r$

$\frac{10}{9} : \frac{56}{81} : 1$

$\boxed{\text{Ans} = 90:56:81}$

12) A, B & C enter into a Partnership with Capitals in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$ after 4 months A increases his share of capital by 50%. If at the end of the year the total profit earned is RS. 2430. find the share of each in the profit.

Ratio of Capitals $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$

LCM of 2, 3, 5 = 30

$$\left(\frac{7}{2} \times 30\right) : \left(\frac{4}{3} \times 30\right) : \left(\frac{6}{5} \times 30\right)$$

105 : 40 : 36

The initial capitals

$105x : 40x : 36x$

after 4 months A increases capital by 50%

$$\left[(105x \times 4) + \left[\left[\frac{150}{100} \times 105x\right] 8\right]\right] = A$$

$(40x \times 12) = B$

$(36x \times 12)$ C

1680 : 480 : 432

35 : 10 : 9

A's share $= \left(2430 \times \frac{35}{54}\right) = 1575$

B's share $= \left(2430 \times \frac{10}{54}\right) = 450$

C's share $= \left(2430 \times \frac{9}{54}\right) = 405$

13) A, B and C are Partners in a business. A whose money has been used for 4 months, claims $\frac{1}{8}$ of the profit. B, whose money has been used for 6 months claims $\frac{1}{3}$ of the profit. C had invested RS. 1560 for 8 months. How much money did A & B contribute?

Let the total profit be x

A's share $= \frac{x}{8}$

B's share $= \frac{x}{3}$

C's share $= \left(x - \left(\frac{x}{8} + \frac{x}{3}\right)\right)$

$$= \left(x - \frac{11x}{24}\right)$$

$$C = \frac{13x}{24}$$

Ratio of share A : B : C

$$\frac{x}{8} : \frac{x}{3} : \frac{13x}{24}$$

$$3 : 8 : 13$$

Lcm 24

Suppose A invested Rs. y for 4 months
B invested Rs. z for 6 months

then

$$\frac{y \times 4}{1560 \times 8} = \frac{3}{13} \quad y = 720$$

$$\frac{z \times 6}{1560 \times 8} = \frac{8}{13} \quad z = 1280$$

Hence A's Contribution = Rs. 720
B's Contribution = Rs. 1280

14) The ratio of investments of two partners A & B is 11:12 and the ratio of their profits is 2:3. If A invested the money for 8 months, then for how much time B invested his money?

investment ratio 11 : 12

$$11x : 12x$$

$$\frac{11x \times 8}{12x \times y} = \frac{2}{3}$$

$$24y = 264$$

$$y = 11$$

Hence B invested the money for 11 months.

15) In a partnership A invests $\frac{1}{6}$ of the capital for $\frac{1}{6}$ of the time B invests $\frac{1}{3}$ of the capital for $\frac{1}{3}$ of the time and C, the rest of the capital for the whole time. out of a profit of Rs. 4600 B's share is?

Suppose A invests $\frac{x}{6}$ for $\frac{y}{6}$ months

x - investment money
y - investment months (total month)

B invests $\frac{x}{3}$ for $\frac{y}{3}$ months

C invests $\left[x - \left(\frac{x}{6} + \frac{x}{3}\right)\right]$

$= \frac{x}{2}$ for y months.

A : B : C

$$\left(\frac{x}{6}\times\frac{y}{6}\right) : \left(\frac{x}{3}\times\frac{y}{3}\right) : \left(\frac{x}{2}\times y\right)$$

$$\frac{1}{36} : \frac{1}{9} : \frac{1}{2}$$

LCM 36 $\quad 1 : 4 : 18 \quad \boxed{18+4+1 = 23}$

Hence B's share $= \left(4600 \times \frac{4}{23}\right)$

$= 800/-$

Compound interest

Instalments model

1) A TV set is available for Rs. 19650 cash payment (or) for Rs. 3100 cash down payment and three equal annual instalments. If the shopkeeper charges interest at the rate of 10% per annum compounded annually. Calculate the amount of each instalment.

Cost of TV = 19650

Down payment = 3100

Balance = 19650 − 3100

= 16550

The value of each instalment be x then

(P.W of x due 1 year hence) +

(P.W of x due 2 year hence) +

(P.W of x due 3 year hence) = 16550

$$P.W = \frac{x}{\left[1+\frac{R}{100}\right]^n}$$

$$\frac{x}{\left[1+\frac{10}{100}\right]^1} + \frac{x}{\left[1+\frac{10}{100}\right]^2} + \frac{x}{\left[1+\frac{10}{100}\right]^3} = 16550$$

$$\frac{10x}{11} + \frac{100x}{121} + \frac{1000x}{1331} = 16550$$

$$\frac{1210x + 1100x + 1000x}{1331} = 16550$$

$x = 6655$

Hence the value of each instalment

= Rs. 6655/-

2) A sum of money is borrowed and paid back in two annual instalments of RS. 882 each allowing 5% compound interest. The sum borrowed was?

principle = (pw of 882 due 1 year) + (pw of 882 due 2 year)

$$= \frac{882}{\left(1+\frac{5}{100}\right)} + \frac{882}{\left(1+\frac{5}{100}\right)^2}$$

principle = 1640.

3) A man borrows RS. 2550 to be paid back with compound interest at the rate of 4% p.a by the end of 2 years in two equal yearly instalments. How much will each instalment be?

$$\left[\frac{x}{1+\frac{4}{100}}\right] + \left[\frac{x}{\left(1+\frac{4}{100}\right)^2}\right] = 2550$$

$$\frac{25x}{26} + \frac{625x}{676} = 2550$$

$$1275x = 2550 \times 676$$

$$x = 1352$$

Value of each instalment = RS. 1352

4) under the Rural housing scheme the Delhi Development Authority allotted a house to Kamal raj for RS. 1,26,100. This Payment is to be made in three equal annual instalments If the money is reckoned at 5% p.a compound interest. then how much is to be paid by Kamal raj in each instalment?

$$\left[\frac{x}{\left(1+\frac{5}{100}\right)^1}\right] + \frac{x}{\left(1+\frac{5}{100}\right)^2} + \frac{x}{\left(1+\frac{5}{100}\right)^3} = 126100$$

$$\frac{20x}{21} + \frac{400x}{441} + \frac{8000x}{9261} = 126100$$

$$\frac{882x + 8400x + 8000x}{9261} = 126100$$

$$x = 46305$$

$$\boxed{\text{instalment} = \text{RS. } 46305}$$

5) A tape recorder is sold for RS. 3500 cash (or) RS. 1000 cash down payment and the balance in three equal easy instalments. If $12\frac{1}{2}$% is the rate of interest compounded annually. find the amount of instalment.

Total cost = 3500

Down payment = 1000

Balance = (3500 − 1000)

= 2500

$$\left[\frac{x}{1+\frac{25}{2\times100}}\right] + \frac{x}{\left[1+\frac{25}{2\times100}\right]^2} + \frac{x}{\left[1+\frac{25}{2\times100}\right]^3} = 2500$$

$$\frac{8x}{9} + \frac{64x}{81} + \frac{512x}{729} = 2500$$

$$x = \frac{2500\times729}{1736} = 1049.83$$

$$\boxed{\text{instalment} = \text{RS. } 1049.83}$$

6) one can purchase a flat from a house building Society for RS. 55000 Cash (or) on the terms that he should pay RS. 4275 as cash down payment and the rest in three equal instalments. The Society charges interest at the rate of 16% p.a Compounded half-yearly. If the flat is purchased under instalment plan find the value of each instalment

Total cost of flat = 55000

Down payment = 4275

Balance = 55000 − 4275

= 50725

Rate of interest = 16% pa

Rate of interest half-yearly

$= \frac{16}{2} = 8\%$

$$\frac{x}{\left[1+\frac{8}{100}\right]} + \frac{x}{\left[1+\frac{8}{100}\right]^2} + \frac{x}{\left[1+\frac{8}{100}\right]^3} = 50725$$

$$\frac{25x}{27} + \frac{625x}{729} + \frac{15625x}{19683} = 50725$$

$$\frac{50725x}{19683} = 50725$$

$$x = 19683$$

The value of each instalment = RS. 19683/-

7) what annual payment will discharge a debt of RS. 1025 due in 2 years at the rate of 5% Compound interest?

$$\frac{x}{\left[1+\frac{5}{100}\right]} + \frac{x}{\left[1+\frac{5}{100}\right]^2} = 1025$$

$$\frac{20x}{21} + \frac{400x}{441} = 1025$$

$$\frac{820x}{441} = 1025$$

$$x = 551.25$$

Value of each instalment = RS. 551.25

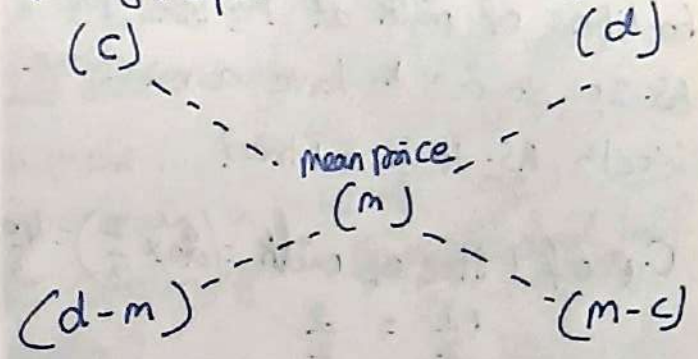
ALLIGATION (or) MIXTURE

1) **Alligation:** It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

2) **Mean Price:**
The cost price of a unit quantity of the mixture is called the mean price.

3) **Rule of Alligation**
If two ingredients are mixed then

$$\left[\frac{\text{quantity of cheaper}}{\text{quantity of dearer}}\right] = \frac{\text{C.P of dearer} - \text{mean price}}{\text{mean price} - \text{CP of cheaper}}$$



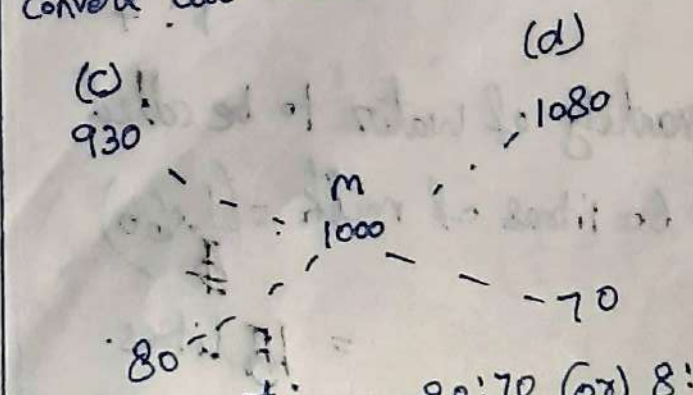
cheaper quantity : Dearer quantity

$(d-m) : (m-c)$

4) Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid

$$= \left[x\left(1-\frac{y}{x}\right)^n\right] \text{ units}$$

1) In what ratio must rice at RS. 9.30 per kg be mixed with rice at RS. 10.80 per kg so that the mixture be worth RS. 10 per kg?
Convert all amount into paise



Required ratio = 80:70 (or) 8:7

2) How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs. 20. So as to have a mixture worth Rs. $10\frac{2}{3}$ a litre?

C.P of 1 litre of milk $= \left(20 \times \frac{2}{3}\right) = \frac{40}{3}$

$1\frac{1}{2} = \frac{3}{2}$

Mean price $= 10\frac{2}{3} = \frac{32}{3}$

C.P. of 1 l water		CP of 1 l of milk
0		$\frac{40}{3}$
	$\frac{32}{3}$	
$\left(\frac{40}{3} - \frac{32}{3}\right)$		$\left(\frac{32}{3} - 0\right)$
$\frac{8}{3}$		$\frac{32}{3}$

Ratio of water & milk is

$\frac{8}{3} : \frac{32}{3}$ (or) $8 : 32$

(or)

$1 : 4$

Quantity of water to be added to 60 litres of milk $= \left(\frac{1}{4} \times 60\right)$

$= 15$ litres.

3) In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price?

CP of milk be Rs.1 per litre.

SP of 1 litre of mixture = 1

Gain = 20%.

CP of 1 litre of mixture $= \left(\frac{100}{120} \times 1\right) = \frac{5}{6}$

Mean price $= \frac{5}{6}$

CP of 1 l water		CP of 1 l milk
0		1
	$\frac{5}{6}$	
$\left(1 - \frac{5}{6}\right)$		$\left(\frac{5}{6} - 0\right)$
$\frac{1}{6}$		$\left(\frac{5}{6}\right)$

Ratio of water & milk

$\frac{1}{6} : \frac{5}{6} = 1 : 5$

4) How many kgs. of wheat costing Rs. 8 per kg must be mixed with 36 kg of rice costing Rs. 5.40 per kg so that 20% gain may be obtained by selling the mixture at Rs. 7.20 per kg?

$= \frac{SP}{100+\text{Gain\%}} \times 100$

Cost price 1 kg mixture $= \left(\frac{100}{120} \times 7.20\right)$

$= 6$

Convert all into paise

mean price = 600 P

CP of wheat

540

CP of wheat

800

600

(800-600)

200

600-540

60

200 : 60

$10 : 3 = 36 : x$

$\frac{10}{3} = \frac{36}{x}$

$x = \frac{36 \times 3}{10} = 10.8$ kg

5) The milk and water in two vessels A & B are in the ratio 4:3 and 2:3 respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessels C containing half milk and half water?

Cost price of milk be 1 per litre.

milk in 1 litre mixture of A $= \frac{4}{7}$ litre

milk in 1 litre mixture of B $= \frac{2}{5}$ litre

milk in 1 litre mixture of C $= \frac{1}{2}$ litre

CP of 1 litre mixture in A $= \frac{4}{7}$

CP of 1 litre mixture in B $= \frac{2}{5}$

mean price $= \frac{1}{2}$

B

$\frac{2}{5}$

A

$\frac{4}{7}$

$\frac{1}{2}$

$\left(\frac{4}{7} - \frac{1}{2}\right)$

$\left(\frac{1}{2} - \frac{2}{5}\right)$

$\frac{1}{14}$

$\frac{1}{10}$

Required ratio $= \frac{1}{14} : \frac{1}{10}$ (or) 5:7

6) In what ratio must a grocer mix two varieties of pulses costing Rs. 15 & Rs. 20 per kg respectively. So as to get a mixture worth Rs. 16.50 per kg?

CP of cheaper | CP of dearer

15 | 20

16.50

3.50 | 1.50

Required Rate = 3.50 : 1.50

= 35 : 15

= 7 : 3

7) A milk man mixed some water with milk to gain 25% by selling the mixture at the cost price. the ratio of water and milk is respectively?

8) 20 litres of a mixture contains milk and water in the ratio 3:1. Then the amount of milk to be added to the mixture so as to have milk and water in ratio 4:1 is

3 : 1 $\quad (3+1)=4$

quantity of milk $= \left(\frac{3}{4} \times 20\right) = 15$ liters.

quantity of water $= \left(\frac{1}{4} \times 20\right) = 5$ liters.

Let the quantity of milk added be x liters.

$$\frac{(15+x)}{5} = \frac{4}{1}$$

$$(15+x) = 4 \times 5$$

$$x = 20 - 15$$

$$x = 5$$

The amount of milk to be added to the mixture is 5 liters.

9) How many liters of water should be added to a 30 litre mixture of milk and water containing milk and water in the ratio of 7:3 such that the resultant mixture has 40% water in it?

The quantity of mixture = 30 l

7 : 3 $\quad 7+3=10$

quantity of milk = $\left(\frac{7}{10}\times 30\right) = 21\ l$

quantity of water = $\left(\frac{3}{10}\times 30\right) = 9\ l$

Let water to be mixed x litre

then

$$(30+x)\times\frac{40}{100} = 9+x$$

$$120+4x = 90+10x$$

$$120-90 = 10x-4x$$

$$30 = 6x$$

$$x = \frac{30}{6} = 5$$

Hence 5 l of water is added to the mixture

10) 35 kg of type A Sandal powder which cost RS. 614/kg was mixed with a certain amount of type B Sandal powder which costs RS 695/kg Then the mixture was sold at the rate of RS. 767/kg and 18% profit was earned. What was the amount (in kg) of type B Sandal powder in the mixture? 3

Mean price of mixture (or) CP of mixture $\Big\} = \frac{\text{Sale price}\times 100}{(100+\text{gain\%})}$

$$= \frac{767}{118}\times 100 = \text{RS. } 650$$ 3

Type A 614 $\qquad$ Type B 695

650

(695 − 650) 45 $\qquad$ (650 − 614) 36

Ratio 45 : 36

5 : 4

quantity of A type Sandal is 35 kg.

$$5x = 35$$

$$x = \frac{35}{5} = 7 \text{ Kg}$$

Thus B type Sandal $= 35 - 7$
$= 28$ kg

11) The respective ratio of milk and water in the mixture is 4 : 3. If 6 litres of water is added to this mixture the respective ratio of milk and water becomes 8 : 7. what is the quality of milk in the original mixture?

$$4 : 3$$

$$4x : 3x$$

$$\frac{4x}{3x+6} = \frac{8}{7}$$

$$28x = 24x + 48$$

$$28x - 24x = 48$$

$$4x = 48$$

$$x = \frac{48}{4} = 12$$

required quantity of milk $= 4x = 4 \times 12$
$= 48$ litre

12) A vessel contains a mixture of Grape, Pineapple and Banana Juices in the respective ratio [4 : 6 : 5]. 15 litres of this mixture is taken out and 8 litres of grape juice and 2 litres of Pineapple juice is added to the vessel. If the resultant quantity of grape juice is 10 l less than the resultant quantity of Pineapple juice, what was the initial quantity of mixture in the vessel? (in litres)

13) A Container Contains 40 l of milk from this Container 4 l of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the Container?

Amount of milk left after 3 operations

$$= \left[40\left(1-\frac{4}{40}\right)^3\right]$$

$$= \left[40\left(\frac{36}{40}\right)^3\right]$$

$$\frac{\cancel{36}}{\cancel{40}} \quad \frac{9}{10}$$

$$= \left[40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right]$$

$$= 29.16 \text{ litres.}$$

14) 8 litres are drawn from a Container full of milk and is then filled with water. This operation is performed three more times. The ratio of the quantity of milk now left in Container to that of the water is 16:65. How much milk did the Container hold originally?

Let the quantity of milk in the container originally be x litres

quantity of milk left in Container after 4 operations.

$$= \left[x\left(1-\frac{8}{x}\right)^4\right]$$

TIME & WORK

1) Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same work. How long should it take both A & B, working together but independently to do the same job?

A's 1 hour work $= \frac{1}{8}$

B's 1 hour work $= \frac{1}{10}$

$(A+B) = \left[\frac{1}{8} + \frac{1}{10}\right] = \frac{9}{40}$

Both A & B finish the work in

$\frac{40}{9} = 4\frac{4}{9}$ days

2) A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?

$(A+B) = \frac{1}{4}$

A's 1 day work $= \frac{1}{12}$

B's 1 day work $= \left[\frac{1}{4} - \frac{1}{12}\right] = \frac{1}{6}$

Hence B alone can complete the work in 6 days.

3) A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together $8\frac{2}{5}$ hours a day?

A can complete work in $7 \times 9 = 63$ hours

B can complete work in $6 \times 7 = 42$ hours

A's 1 hour work $= \frac{1}{63}$

B's 1 hour work $= \frac{1}{42}$

(A+B)'s 1 hour work $= \frac{1}{63} + \frac{1}{42} = \frac{5}{126}$

Both will finish the work in $\left(\frac{126}{5}\right)$ hrs

Total work = Per day work × No. of. days

$8\frac{2}{5} = \frac{42}{5}$ per day work

$\frac{126}{5}$ = total work

$\frac{126}{5} = \frac{42}{5} \times$ No. of. days

No. of. days $= \frac{126}{5} \times \frac{5}{42} = 3$ days

4) A & B can do a piece of work in 18 days. B & C can do it in 24 days. A & C can do it in 36 days. In how many days will A, B & C finish it, working together and separately?

(A+B)'s 1 day work $= \frac{1}{18}$

(B+C)'s 1 day work $= \frac{1}{24}$

(A+C)'s 1 day work $= \frac{1}{36}$

adding all three

2 (A+B+C)'s 1 day work $= \left[\frac{1}{18} + \frac{1}{24} + \frac{1}{36}\right]$

$2(A+B+C) = \frac{1}{8}$

$A+B+C = \frac{1}{16}$

Thus if A, B & C together can finish work in 16 days.

A's 1 day work $= (A+B+C) - (B+C)$

$= \left(\frac{1}{16} - \frac{1}{24}\right)$

$= \frac{1}{48}$

A alone can finish the work in 48 days.

B's 1 day work $= \left[\frac{1}{16} - \frac{1}{36}\right]$ $(A+B+C) - [A+C]$

$= \frac{5}{144}$

B alone can finish the work in $\frac{144}{5} = 28\frac{4}{5}$ days

C's 1 day work $= [A+B+C] - [A+B]$

$= \left[\frac{1}{16} - \frac{1}{18}\right]$

$= \frac{1}{144}$

C alone can finish the work in 144 days.

5) A is twice as good a workman B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

A's 1 day's work : B's 1 day's work

2 : 1

(A+B)'s 1 day's work $= \frac{1}{18}$

Divide $\frac{1}{18}$ in the ratio 2 : 1

total 3 part (2+1)

A's 1 day work $= \left[\frac{1}{18} \times \frac{2}{3}\right] = \frac{1}{27}$

Hence A alone can finish the work in 27 days.

6) A can do a piece of work in 80 days. He works at it for 10 days and then B alone finishes the remaining work in 42 days. In how much time will A & B working together, finish the work?

work done by A in 10 days $= \left[\frac{1}{80} \times 10\right]$

$= \frac{1}{8}$

Remaining work $= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Now $\frac{7}{8}$ work is done by B in 42 days.

whole work will be done by B in

$\left[42 \times \frac{8}{7}\right] = \frac{1}{48}$ one days work

A's 1 day work $= \frac{1}{80}$

B's 1 day work $= \frac{1}{48}$

(A+B)'s 1 day's work $= \left[\frac{1}{80} + \frac{1}{48}\right]$

$= \frac{8}{240}$

$= \frac{1}{30}$

Hence both will finish the work in 30 days

7) A can do a certain job in 12 days. A is 60% more efficient than B. How many days does B alone take to do the same job?

Ratio of times taken by A & B

160 : 100

8 : 5

Suppose B alone takes x days to do the job

Then $8 : 5 :: 12 : x$

$8x = 5 \times 12$

$x = 7\frac{1}{2}$ days.

8) A & B undertake to do a piece of work for Rs: 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. find the share of each.

C's 1 day's work $= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right)$

$= \frac{1}{24}$

A : B : C = Ratio of their 1 day's work

$= \frac{1}{6} : \frac{1}{8} : \frac{1}{24}$

$= 4 : 3 : 1$

4) A & B can do a piece of work in 18 days. B & C can do it in 24 days. A & C can do it in 36 days. In how many days will A, B & C finish it, working together and separately?

$(A+B)$'s 1 day work $= \frac{1}{18}$

$(B+C)$'s 1 day work $= \frac{1}{24}$

$(A+C)$'s 1 day work $= \frac{1}{36}$

adding all three

$2\,(A+B+C)$'s 1 day work $= \left[\frac{1}{18} + \frac{1}{24} + \frac{1}{36}\right]$

$2\,(A+B+C) = \frac{1}{8}$

$A+B+C = \frac{1}{16}$

Thus if A, B & C together can finish work in 16 days.

A's 1 day work $= (A+B+C) - (B+C)$

$= \left(\frac{1}{16} - \frac{1}{24}\right)$

$= \frac{1}{48}$

A alone can finish the work in 48 days.

B's 1 day work $= \left[\frac{1}{16} - \frac{1}{36}\right]$ $(A+B+C) - [A+C]$

$= \frac{5}{144}$

B alone can finish the work in $\frac{144}{5} = 28\frac{4}{5}$ days

C's 1 day work $= [A+B+C] - [A+B]$

$= \left[\frac{1}{16} - \frac{1}{18}\right]$

$= \frac{1}{144}$

C alone can finish the work in 144 days.

5) A is twice as good a workman and together they finish a piece of work in 18 days. In how many days will A alone finish the work?

A's 1 day's work : B's 1 day's work

2 : 1

$(A+B)$'s 1 day's work $= \frac{1}{18}$

Divide $\frac{1}{18}$ in the ratio 2 : 1

total 3 part $(2+1)$

A's 1 day work $= \left[\frac{1}{18} \times \frac{2}{3}\right] = \frac{1}{27}$

Hence A alone can finish the work in 27 days.

6) A can do a piece of work in 80 days. He works at it for 10 days and then B alone finishes the remaining work in 42 days. In how much time will A & B working together, finish the work?

work done by A in 10 days $= \left[\frac{1}{80} \times 10\right]$

$= \frac{1}{8}$

Remaining work $= 1 - \frac{1}{8}$

$= \frac{7}{8}$

Now $\frac{7}{8}$ work is done by B in 42 days.

whole work will be done by B in

$\left[42 \times \frac{8}{7}\right] = 48$ days

$\frac{1}{48}$ one days work

A's 1 day work $= \frac{1}{80}$

B's 1 day work $= \frac{1}{48}$

(A+B)'s 1 day's work $= \left[\frac{1}{80} + \frac{1}{48}\right]$

$= \frac{8}{240}$

$= \frac{1}{30}$

Hence both will finish the work in 30 days.

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Ratio of times taken by A & B

160 : 100

8 : 5

Suppose B alone takes x days to do the job

Then $8 : 5 :: 12 : x$

$8x = 5 \times 12$

$x = 7\frac{1}{2}$ days.

8) A & B undertake to do a piece of work for Rs: 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.

C's 1 day's work $= \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right)$

$= \frac{1}{24}$

A : B : C = Ratio of their 1 day's work

$= \frac{1}{6} : \frac{1}{8} : \frac{1}{24}$

$= 4 : 3 : 1$

A's share = RS. $\left(600 \times \frac{4}{8}\right)$ = RS. 300

B's share = RS. $\left(600 \times \frac{3}{8}\right)$ = RS. 225

C's share = RS. 600 − (300 + 225)

= RS. 75

9) A & B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately A beginning in how many days, the work will be completed?

(A+B)'s 2 days work = $\left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{36}$

$$\begin{array}{r} 5 \\ 7\overline{)36} \\ \underline{35} \\ 1 \end{array}$$

work done in 5 pairs of days = $\left(5 \times \frac{7}{36}\right)$

$\boxed{5 \times 2 = 10 \text{ days}}$ $= \frac{35}{36}$

Remaining work = $1 - \frac{35}{36}$

$= \frac{1}{36}$

On 11th day it is A's turn. $\frac{1}{9}$ work is done by him in 1 day.

Total work = Per day work × No. of days.

$\frac{1}{36} = \frac{1}{9} \times$ No. of days.

$9 \times \frac{1}{36} = D$ $D = \frac{1}{4}$ days.

∴ Total time taken = $10\frac{1}{4}$ days.

10) 45 men can complete a work in 16 days. Six days after they started working 30 more men joined them. How many days will they now take to complete the remaining work?

(45×16) men can complete the work

1 man's 1 day's work = $\frac{1}{720}$

45 men's 6 days work = $\left(\frac{1}{16} \times 6\right) = \frac{3}{8}$

Remaining work = $1 - \frac{3}{8}$

$= \frac{5}{8}$

75 men's 1 day's work = $\frac{75}{720} = \frac{5}{48}$

Now $\frac{5}{48}$ work is done by them in 1 day

$\frac{5}{8}$ work is done by them in $\left(\frac{48}{5} \times \frac{5}{8}\right)$

= 6 days.

11) 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

1 man's 1 day's work $= x$

1 boy's 1 day's work $= y$

Then $2x + 3y = \frac{1}{10}$

$3x + 2y = \frac{1}{8}$

Solving we get

$x = \frac{7}{200}$ $y = \frac{1}{100}$

(2 men + 1 boy's)'s 1 day's work

$= \left(2 \times \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}$

So 2 men & 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days.

12) 3 men and 4 women can earn RS. 3780 in 7 days. 11 men and 13 women can earn RS. 15040 in 8 days. In what time will 7 men and 9 women earn RS. 12400?

1 man's 1 day's earning be RS. x

1 women's 1 day's earning be RS. y

$3x + 4y = \frac{3780}{7} = 540$

$11x + 13y = \frac{15040}{8} = 1880$

$3x + 4y = 540$ --- (1)

$11x + 13y = 1880$ --- (2)

Solving the equation

$x = 100$ $y = 60$

(7 men + 9 women's) 1 day's earning

$= [7 \times 100 + 9(60)] = 1240$

hence required time $= \frac{12400}{1240}$

$= 10$ days.

13) 9 children can complete a piece of work in 360 days. 18 men can complete the same piece of work in 72 days and 12 women can complete it in 162 days. In how many days can 4 men, 12 women and 10 children together complete the piece of work?

1 man's 1 day's work $= \frac{1}{72 \times 18} = \frac{1}{1296}$

1 woman's 1 day's work $= \frac{1}{162 \times 12} = \frac{1}{1944}$

1 child's 1 day's work $= \frac{1}{360 \times 9} = \frac{1}{3240}$

(4 men + 12 women + 10 children) 1 day's work =

$$\left[4 \times \frac{1}{1296} + 12 \times \frac{1}{1944} + 10 \times \frac{1}{3240}\right] = \frac{4}{324}$$

$$= \frac{1}{81} = 81 \text{ days.}$$

hence 4 men, 12 women & 10 children can complete the work in 81 days

14) 10 persons begin to work together on a job but after some days 4 persons leave. As a result the job which could have been completed in 40 days is completed in 50 days. How many days after the commencement of the work did the 4 persons leave?

10 persons can complete the work in 40 days.

1 person's 1 day's work $= \frac{1}{40 \times 10} = \frac{1}{400}$

Suppose 4 person's left after x days

Then

$$\frac{1}{400} \times 10 \times x + \frac{1}{400} \times 6 \times (50 - x) = 1$$

$$\frac{1}{40}x + \frac{3}{200}(50 - x) = 1$$

$$\frac{1}{40}x + \frac{3}{4} - \frac{3x}{200} = 1$$

$$\frac{1}{100}x = \frac{1}{4} \qquad x = 25$$

Hence 4 person's left 25 days after the commencement of the work.

15). A and B can do a piece of work in 12 days. B and C together can do it in 15 days. If A is twice as good a workman as C find in what time B alone can do it?

A's 1 day work = C's 2 day work

(A+B)'s 1 day work =

(B's 1 days work) + (C's 2 day work)

$= \frac{1}{12}$ — (1)

(B's 1 days work) + (C's 1 day's work) $= \frac{1}{15}$ (2)

Subtracting (ii) from (i)

we get C's 1 days work

C's 1 day work $= \left(\frac{1}{12} - \frac{1}{15}\right) = \frac{1}{60}$

B's 1 day work $= \left(\frac{1}{15} - \frac{1}{60}\right) = \frac{3}{60}$

$= \frac{1}{20}$ = 20 days.

B alone can finish the work in 20 days

16) A can do a piece of work in 120 days and B can do it in 150 days. They work together for 20 days. Then B leaves and A alone continues the work. 12 days after that C joins A and the work is completed in 48 days more. In how many days can C do it, if he works alone?

((A+B)'s 20 days work) + (A's 12 days work)

$= 20\left(\frac{1}{120} + \frac{1}{150}\right) + \left(12 \times \frac{1}{120}\right) = \frac{2}{5}$

Remaining work $= 1 - \frac{2}{5}$

$= \frac{3}{5}$

(A+C)'s 48 days work.

(A+C)'s 1 days work $= \left(\frac{3}{5} \times \frac{1}{48}\right) = \frac{1}{80}$

C's 1 days work $= \left(\frac{1}{80} - \frac{1}{120}\right)$

$= \frac{1}{240}$

Hence C alone can finish the work in 240 days.

17) A & B can do a piece of work in 45 and 40 days respectively. They began the work together but A leaves after some days and B finished the remaining work in 23 days. After how many days did A leave?

B's 23 days work $= \left(23 \times \frac{1}{40}\right)$

$= \frac{23}{40}$

Remaining work $= 1 - \frac{23}{40}$

$= \frac{17}{40}$

(A+B)'s 1 day's work $= \left(\frac{1}{45} + \frac{1}{40}\right) = \frac{17}{360}$

Total work = per day work × No. of days.

$\frac{17}{40} = \frac{17}{360} \times$ No. of days.

$\frac{360}{17} \times \frac{17}{40}$ = No. of days. = 9 days.

Hence A left after 9 days

18) A can complete a work in 10 days B in 12 days and C in 15 days. All of them began the work together, but A had to leave the work after 2 days of the start and B, 3 days before the completion of the work. How long did the work last?

(A+B+C)'s 2 day's work $= 2\left(\frac{1}{10} + \frac{1}{12} + \frac{1}{15}\right)$

$= \frac{1}{2}$

C's 3 day's work $= \left(3 \times \frac{1}{15}\right) = \frac{1}{5}$

Remaining work $= 1 - \left(\frac{1}{2} + \frac{1}{5}\right) = \frac{3}{10}$

(B+C)'s 1 day work $= \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{27}{180} = \frac{3}{20}$

Total work = per day work × No. of days.

$\frac{3}{10} = \frac{3}{20} \times$ No. of days.

$\frac{20}{3} \times \frac{3}{10}$ = No. of days.

No. of days = 2 days

Total time taken $= (2 + 3 + 2)$

= 7 days

19) A can do a piece of work in 10 days and B in 20 days. They work together but 2 days before the completion of the work, A leaves. In how many days was the work completed?

B's 2 day's work $= \left(\frac{1}{20} \times 2\right) = \frac{1}{10}$

Remaining work $= 1 - \frac{1}{10} = \frac{9}{10}$

(A+B)'s 1 day work $= \left(\frac{1}{10} + \frac{1}{20}\right) = \frac{3}{20}$

Total work = Per day work × No. of days.

$\frac{9}{10} = \frac{3}{20} \times$ No. of days.

No. of days $= \frac{9}{10} \times \frac{20}{3} = 6$ days.

Total time taken $= 2 + 6 = 8$ days

20) A & B can do a piece of work in 9 days. B & C can do it 12 days. A & C can do it in 18 days. In how many days will A, B & C finish it working together and separately?

(A+B)'s 1 day's work $= \frac{1}{9}$ — (1)

(B+C)'s 1 day's work $= \frac{1}{12}$ — (2)

(A+C)'s 1 day's work $= \frac{1}{18}$ — (3)

adding 1, 2 & 3

& (A+B+C)'s 1 day work $= \left(\frac{1}{9} + \frac{1}{12} + \frac{1}{18}\right)$

$= \frac{9}{36} = \frac{1}{4}$

(A+B+C)'s 1 day work $= \frac{1}{4 \times 2} = \frac{1}{8}$

hence all together can finish the work in 8 days.

A's 1 day work = (A+B+C)'s 1 day work − (B+C)' 1 day work

$= \left(\frac{1}{8} - \frac{1}{12}\right) = \frac{1}{24}$

A alone can finish the work in 24 days

B's 1 day work = (A+B+C) − (A+C)

$= \left(\frac{1}{8} - \frac{1}{18}\right) = \frac{5}{72}$

B alone can finish the work in $\frac{72}{5}$ (or) $14\frac{2}{5}$ days

C's 1 day work = (A+B+C) − (A+B)

$= \left(\frac{1}{8} - \frac{1}{9}\right) = \frac{1}{72}$

C alone can finish the work in 72 days.

Q1) Rahul takes twice as much time as manick and thrice as much as Sachin to complete a job. If working together, they can complete the job in 4 days. find the time taken by each of them separately to complete the work.

Rahul takes x hours to compete the job

manick takes $\frac{x}{2}$ hours to do the job

Sachin takes $\frac{x}{3}$ hours to do the job

Rahul's 1 hour's work $= \frac{1}{x}$

manick's 1 hour's work $= \frac{2}{x}$

Sachin 1 hour's work $= \frac{3}{x}$

$$4\left(\frac{1}{x}+\frac{2}{x}+\frac{3}{x}\right)=1$$

$$\frac{6}{x}=\frac{1}{4} = 24=x$$

Rahul takes 24 hours.

manick takes 12 hours.

Sachin takes 8 hours to complete the job

PIPES AND CISTERNS

INLET: A pipe connected with a tank (or) a cistern (or) a reservoir, that fills it, is known as an inlet.

OUTLET: A pipe connected with a tank (or) a cistern (or) a reservoir, emptying it, is known as an outlet.

i) If a pipe can fill a tank in x hours then part filled in 1 hour $= \frac{1}{x}$

ii) If a pipe can empty a full tank in y hours then part emptied in 1 hour $= \frac{1}{y}$

iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours [where $y > x$]

then on opening both the pipes the net part filled in 1 hour $= \left[\frac{1}{x}-\frac{1}{y}\right]$

iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours [where $x > y$]

then on opening both the pipes the net part filled in 1 hour $= \left[\frac{1}{y}-\frac{1}{x}\right]$

1) A tank 9ft by 5ft by 2ft is fitted with an inlet pipe and an exhaust pipe. The inlet pipe pours in 576 cu. inch of water per minute and the exhaust pipe can empty the full tank in 3 hours. If the tank is full and both pipes are open, how many hours will it take to empty it?

Volume of the tank $= (9 \times 5 \times 2)$ cu.ft

$= 90$ cu.ft

1ft = 12 inch

$= (90 \times 12 \times 12 \times 12)$ cu.inch

Volume of water drained by exhaust pipe in one minute $= \dfrac{90 \times 12 \times 12 \times 12}{3 \times 60}$

$= 864$ cu.inch

Net volume drained in one minute, when both the pipes are opened $=$

$(864 - 576) = 288$ cu.inch

Required time $= \left[\dfrac{90 \times 12 \times 12 \times 12}{288 \times 60}\right]$ hrs

$= 9$ hrs

2) Pipe A can fill a tank in 30 hrs and pipe B in 45 hours. If both the pipes are opened in an empty tank how much time will they take to fill it?

Part filled by A in 1 hour $= \dfrac{1}{30}$

Part filled by B in 1 hour $= \dfrac{1}{45}$

Part filled by (A+B) in 1 hour $= \left(\dfrac{1}{30} + \dfrac{1}{45}\right) = \dfrac{10}{180}$

$= \dfrac{1}{18}$

Hence pipes A & B together will fill the tank in 18 hrs.

3) A cistern can be filled by pipes A & B in 4 hours and 6 hours respectively. When full, the cistern can be emptied by pipe C in 8 hours. If all the pipes were turned on at the same time, in how much time will the cistern be filled?

Net part filled in 1 hour $= \left(\dfrac{1}{4} + \dfrac{1}{6} - \dfrac{1}{8}\right)$

$= \dfrac{7}{24}$

The cistern will be full in $\dfrac{24}{7}$ hrs

i.e $3\dfrac{3}{7}$ hrs.

4) A pipe can empty a tank in 40 minutes. A second pipe with diameter twice as much as that of the first is also attached with the tank to empty it. How much time will the two pipes together take to empty the tank?

∴ Diameters of pipes be d & $2d$

time taken to fill (or) empty the tank is inversely proportional to the square of the diameter.

The times taken by them to empty the tank be t and T minutes respectively.

$$t \propto \frac{1}{d^2} = \frac{K}{d^2} = 40$$

$$T \propto \frac{1}{(2d)^2} = \frac{1}{4d^2} = T = \frac{1}{4} \times \frac{K}{d^2}$$

$$= \frac{1}{4} \times 40 = 10$$

net part emptied in 1 min $= \left[\frac{1}{40} + \frac{1}{10}\right]$

$$= \left[\frac{5}{40}\right] = \frac{1}{8}$$

2 pipes together will empty the tank in 8 minutes.

5) A tap can fill a tank in 10 min and another can empty it in 6 min. If the tank is already two fifths full and both the taps are opened together, will the tank be filled (or) emptied? How long will it take before the tank is either filled completely or emptied completely, as the case may be?

outlet pipe is faster than the inlet pipe and so the tank will be emptied.

Part to be emptied $= \frac{2}{5}$

Net part emptied in 1 min $= \left[\frac{1}{6} - \frac{1}{10}\right]$

$$= \frac{2}{30} = \frac{1}{15}$$

$$\frac{1}{15} : \frac{2}{5} :: 1 : x$$

$$x = \left(\frac{2}{5} \times 1 \times 15\right)$$

$$x = 6 \text{ min}$$

The tank will be emptied in 6 min

6) A cistern has two taps which fill it in 12 min & 15 min. There is also a wate pipe in the cistern. When all the three are open, the empty cistern is full in 20 min. How long will the waste pipe take to empty the full cistern?

work done by the waste pipe in 1 min

$$= \frac{1}{20} - \left(\frac{1}{12} + \frac{1}{15}\right)$$

$$= \frac{1}{10}$$ [-ve sign means emptying]

waste pipe will empty the full cistern in 10 min.

7) An electric pump can fill a tank in 3 hours. Because of a leak in the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

work done by the leak in 1 hour

$$= \left[\frac{1}{3} - \frac{1}{\left(\frac{7}{2}\right)}\right] = \left(\frac{1}{3} - \frac{2}{7}\right) = \frac{1}{21}$$

The leak will empty the tank in 21 hours

8) Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 min more to fill the cistern. When the cistern is full, in what time will the leak empty it?

work done by two pipes in 1 hour

$$= \left[\frac{1}{14} + \frac{1}{16}\right] = \frac{15}{112}$$

time taken to these pipes to fill the tank $= \frac{112}{15} = 7$ hrs 28 min

due to leakage time taken = 7 hrs 28 min + 32 min = 8 hrs.

work done by (two pipes + leak) in 1 hour

$$= \frac{1}{8}$$

work done by the leak in 1 hour

$$= \left[\frac{15}{112} - \frac{1}{8}\right] = \frac{1}{112}$$

hence the leak will empty the full cistern in 112 hours.

9) If two pipes function simultaneously the reservoir will be filled in 12 hours. The second pipe fills the reservoir 10 hours faster than the first. How many hours does it take the second pipe to fill the reservoir?

The reservoir be filled by first pipe in x hours

The second pipe will fill it in

$\cancel{(x+10)}$ $(x-10)$ faster means -ve
slower means +ve

$$\frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$$

$$\frac{x-10+x}{x(x-10)} = \frac{1}{12}$$

$$x^2 - 34x + 120 = 0$$

$$(x-30)(x-4) = 0 \quad x = 30$$

(Neglect -ve sign)

The second pipe will take $(30-10)$ 20 hrs to fill the reservoir.

10) A tank is fitted with two inlet pipes A and B and an outlet pipe C. A is twice as efficient as B which in turn is twice as efficient as C. The empty tank gets filled in 16 hours. When all the three pipes are opened. How many hours will be taken to fill the empty tank if B is plugged and the other two pipes are opened?

A is alone takes x hours to fill the tank. B takes $2x$ C takes $4x$

time taken to fill (or) empty the tank is inversely proportional to the efficiency of pipe

$$\left[\frac{1}{x} + \frac{1}{2x} - \frac{1}{4x}\right] = \frac{1}{16}$$

$$\frac{8+4-2}{8x} = \frac{1}{16}$$

$$x = 20$$

Net part filled by A and C in 1 hour

$$= \left[\frac{1}{x} - \frac{1}{4x}\right] = \frac{3}{4x} = \frac{3}{80}$$

Hence A & C will fill the tank in $\frac{80}{3}$ hrs (or) $26\frac{2}{3}$ hrs (or) 26 hrs 40 min

11) Two pipes A & B can fill a tank in 12 min. and 15 min respectively while a third pipe C can empty the full tank in 20 min. All the three pipes are opened in the beginning. However, pipe C is closed 6 min before the tank is filled. In what time will the tank be full?

Let the tank be full in x min

(A+B) worked for x minutes while pipe C worked for $(x-6)$ min

$$\frac{x}{12}+\frac{x}{15}-\frac{(x-6)}{20}=1$$

$$\frac{5x+4x-3(x-6)}{60}=1$$

$$6x+18=60 \quad x=7$$

Hence the tank will be full in 7 min

12) Pipes A & B can completely fill a water tank in 4 hours and 5 hours respectively. A pipe C can empty a tank filled completely with water in 3 hours. Initially the tank is empty and all pipes are closed. Pipe A is opened first at time $t=0$ and pipe C is opened at the instant when the tank is exactly half filled with water. Pipe B is opened after pipe C and at the instant when the tank is exactly one-fourth filled with water. Find the total time taken to fill the tank completely counting from $t=0$.

Pipe A alone fills half the tank

Let time taken be t_1 then $t_1=2$ hrs.

pipe A & C together empty $\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{1}{4}$ of the tank Let time taken be t_2

work alone by A & C in 1 hour

$=\left(\frac{1}{4}-\frac{1}{3}\right)=-\frac{1}{12}$ [-ve sign means emptying]

$\frac{1}{12}:\frac{1}{4}::1:t_2 \quad t_2=\frac{1}{4}\times 12=3$ hrs.

Pipes A, B & C together fill $\left(1-\frac{1}{4}\right)=\frac{3}{4}$ of the tank

Let time taken be t_3

work done by A, B & C in 1 hour

$$\left(\frac{1}{4}+\frac{1}{5}-\frac{1}{3}\right)=\frac{7}{60}$$

$$\frac{7}{60}:\frac{3}{4}::1:t_3 \quad t_3=\frac{3}{4}\times\frac{60}{7}=\frac{45}{7}$$

$$t_3=6\frac{3}{7} \text{ hrs.}$$

Hence total time taken $=(t_1+t_2+t_3)$

$$=\left(2+3+6\frac{3}{7}\right)=11\frac{3}{7} \text{ hrs.}$$

13) A cistern has three pipes A, B & C. A & B can fill it in 3 hours and 4 hrs respectively while C can empty the completely filled cistern in 1 hour. If the pipes are opened in order at 3, 4 and 5 pm respectively at what time will the cistern be empty?

Let the cistern be empty t hours after 3 pm.

work done by pipe A in t hours + work done by pipe B in (t-1) hours + work done by pipe C in (t-2) hours = 0

$$\frac{t}{3} + \frac{(t-1)}{4} - \frac{(t-2)}{1} = 0$$

$$4t + 3(t-1) - 12(t-2) = 0$$

$$-5t + 21 = 0 \quad 5t = 21$$

$$t = \frac{21}{5}$$

$$t = 4\frac{1}{5} \text{ hrs}$$

(or)

$$t = 4 \text{ hrs } 12 \text{ min}$$

14) Three taps A, B & C can fill a tank in 12, 15 & 20 hours respectively. If A is open all the time and B & C are open for one hour each alternately. the tank will be full in

$$(A+B)\text{'s 1 hour work} = \left(\frac{1}{12} + \frac{1}{15}\right) = \frac{3}{20}$$

$$(A+C)\text{'s 1 hour work} = \left(\frac{1}{12} + \frac{1}{20}\right) = \frac{2}{15}$$

$$\text{Part filled in 2 hours} = \left(\frac{3}{20} + \frac{2}{15}\right) = \frac{17}{60}$$

$$17 \overline{)60} = 3, \quad 51, \quad 9$$

3×2 hours = 6 hours

$$\text{Remaining} = \frac{9}{60} = \frac{3}{20}$$

Next its (A+B) turn remaining part $\frac{3}{20}$ will be filled in 1 hour

Total time taken = (6+1) = 7 hours.

15) Two pipes can fill a tank in 20 and 24 minutes respectively and a waste pipe can empty 3 gallons per minute. All the three pipes working together can fill the tank in 15 minutes. The capacity of the tank is:

work done by the waste pipe in 1 min

$= \frac{1}{15} - \left(\frac{1}{20} + \frac{1}{24}\right) = -\frac{1}{40}$

-ve sign means emptying

Volume of $\frac{1}{40}$ part = 3 gallons.

Volume of whole = (3×40) = 120 gallons.

16) Two pipes A and B can fill a cistern in $37\frac{1}{2}$ min and 45 min respectively. Both pipes are opened. The cistern will be filled in just half an hour, if the pipe B is turned off after.

If B be turned off after x minutes then

part filled by (A+B) in x min + part filled by A in $(30-x)$ min = 1

$$x\left[\frac{2}{75} + \frac{1}{45}\right] + (30-x)\frac{2}{75} = 1$$

$$\frac{11x}{225} + \frac{60-2x}{75} = 1$$

$$11x + 180 - 6x = 225$$

$$x = 9$$

B be turned after 9 min

17) Three pipes A, B & C can fill a tank in 6 hours. After working at it together 2 hours. C is closed and A & B can fill the remaining part in 7 hours. The number of hours taken by C alone to fill the tank is

Part filled by 2 hours $= \frac{2}{6} = \frac{1}{3}$

Remaining part $= \left(1 - \frac{1}{3}\right) = \frac{2}{3}$

(A+B)'s 7 hours work $= \frac{2}{3}$

Total work = per hour work × No. of hours.

$\frac{2}{3}$ = per hour work × 7

(A+B)'s 1 hour work $= \frac{3 \times 7}{2} = \frac{21}{2}$

$= \frac{2}{21}$

C's one hour work = (A+B+C) − (A+B)

$= \left(\frac{1}{6} - \frac{2}{21}\right) = \frac{1}{14}$

14 hours

C alone can fill the tank in 14 hours.

18) A large tanker can be filled by two pipes A and B in 60 min and 40 min respectively. How many minutes will it take to fill the tanker from empty state if B is used for half the time and A & B fill it together for the other half?

Part filled by (A+B) in 1 min $= \left(\frac{1}{60} + \frac{1}{40}\right)$

$= \frac{1}{24}$

Suppose the tank is filled in x min

$$\frac{x}{2}\left(\frac{1}{24} + \frac{1}{40}\right) = 1$$

$$\frac{x}{2} \times \frac{1}{15} = 1$$

$$x = 30 \text{ min}$$

19) Two pipes A & B can fill a tank in 15 min and 20 min respectively. Both the pipes are opened together but after 4 min. pipe A is turned off. What is the total time required to fill the tank?

Part filled in 4 min $= 4\left(\frac{1}{15} + \frac{1}{20}\right) = \frac{7}{15}$

Remaining part $= 1 - \frac{7}{15} = \frac{8}{15}$

Part filled by B in 1 min $= \frac{1}{20}$

$\frac{8}{15} \times 20 = x \quad x = 10\frac{2}{3}$ min

(or)

10 min 40 sec

20) A tank is filled by three pipes with uniform flow. The first two pipes operating simultaneously fill the tank in the same time during which the tank is filled by the third pipe alone. The second pipe fills the tank 5 hours faster than the first pipe and 4 hours slower than the third pipe. The time required by the first pipe is:

21) Three pipes A, B & C are attached to a tank A & B can fill it in 20 and 30 minutes respectively while C can empty it in 15 min. If A, B & C are kept open successively for 1 min each how soon will the tank be filled?

$$(A+B+C)\text{'s } 3 \text{ min} = \left(\frac{1}{20} + \frac{1}{30} - \frac{1}{15}\right)$$

$$= \frac{1}{60}$$

TIME AND DISTANCE

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Distance} = \text{Time} \times \text{Speed}$$

$$x \text{ km/hr} = \left(x \times \frac{5}{18}\right) \text{ m/sec}$$

$$x \text{ m/sec} = \left(x \times \frac{18}{5}\right) \text{ Km/hr}$$

i) If the ratio of the speeds of A and B is $a:b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ (or) $b:a$

ii) Suppose a man covers a certain distance at x Km/hr and an equal distance at y Km/hr. then the average speed during the whole journey is $\left[\frac{2xy}{x+y}\right]$ Km/hr

iii) Suppose two men are moving in the same direction at u m/s and v m/s respectively where $u > v$ then their relative speed $= (u-v)$ m/s

iv) Suppose two men are moving in opposite directions at u m/s and v m/s respectively. then their relative speed $= (u+v)$ m/s

v) If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journeys in a and b hours respectively, then.

A's speed : B's speed $= \sqrt{b} : \sqrt{a}$

---

1) A train travels 82.6 Km/hr. How any Meters will it travel in 15 minutes?

Distance travelled in 1 min $= \left(\frac{82.6}{60}\right)$ Km

Distance travelled in 15 min $= \left(\frac{82.6}{60} \times 15\right)$ Km

$= 20.65$ Km $= (20.65 \times 1000)$ m

$= 20650$ m

2) A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Distance covered by dog in 1 leap is x

Distance covered by hare in 1 leap is y

$3x = 4y \qquad x = \frac{4}{3}y$

$\Rightarrow$ 4 leap $= 4x = \frac{16}{3}y$

ratio of speeds of dog and hare

Ratio of distance covered by them in the same time

$4x : 5y = \frac{16}{3}y : 5y = 16:15$

3) A trip to a destination is made in the following way: 900 km by train at an average speed of 60 km/hr, 3000 km by plane at an average speed of 500 km/hr, 400 km by boat at an average speed of 25 km/hr, 15 km by taxi at an average speed of 45 km/hr. What is the average speed for the entire journey?

Total time taken $= \left(\frac{900}{60} + \frac{3000}{500} + \frac{400}{25} + \frac{15}{45}\right)$ hr

$= \frac{112}{3}$ hr

Total distance travelled $= 900 + 3000 + 400 + 15$

$= 4315$ km

Average speed for the whole journey

$= \left(4315 \times \frac{3}{112}\right)$ km/hr

$= 115\left(\frac{65}{12}\right)$ km/hr

$= 115\frac{65}{12}$ km/hr.

4) one third of a certain journey was covered at the speed of 20 km/hr, one-fourth at 30 km/hr and the rest at the speed of 50 km/hr. find the average speed per hour for the whole journey.

Let the total distance be x

Distance covered at 20 km/hr $= \frac{x}{3}$ km

Distance covered at 30 km/hr $= \frac{x}{4}$ km

Distance covered at 50 km/hr $= x - \left(\frac{x}{3} + \frac{x}{4}\right)$

$= \frac{5x}{12}$ km

$\therefore$ Total time taken $= \frac{D}{S}$

$$= \left[\frac{(x/3)}{20} + \frac{(x/4)}{30} + \frac{[5x/12]}{50}\right] \text{hrs.}$$

$$= \left[\frac{x}{60} + \frac{x}{120} + \frac{x}{120}\right] \text{hrs}$$

$$= \frac{4x}{120} \text{ hrs} = \frac{x}{30} \text{ hrs.}$$

Average Speed for the whole journey $= \left(x \times \frac{30}{x}\right)$ Km/hr

$= 30$ Km/hr.

5) An aeroplane flies along the four sides of a square at the speeds of 100, 200, 300 and 400 Km/hr. find the average speed of the plane around the field.

ANS: Let each side of the square be x km and the average speed of the plane around the field be y Km/hr.

$$\frac{x}{100} + \frac{x}{200} + \frac{x}{300} + \frac{x}{400} = \frac{4x}{y}$$

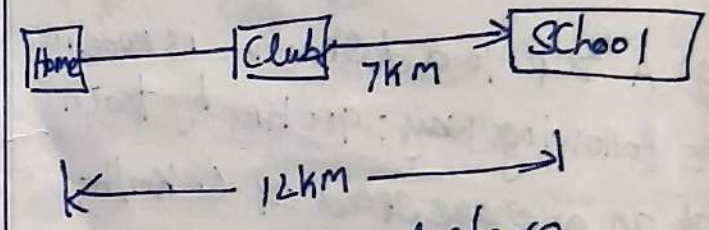
$$\frac{4x}{y} = \frac{25x}{1200} \qquad y = \frac{1200 \times 4}{25} = 192$$

Avg Speed $= 192$ Km/hr

6) Akash leaves home for school which is 12 km from his house. After the school he goes to his Club which is 7 Km from his school. If his house, school and Club all fall in a line, then what is the minimum distance he has to travel to get back home?

a) 5 Km b) 7 Km c) 12 km d) 17 km e) 19

For the distance to be minimum the Club must lie b/w home & School



Hence required distance

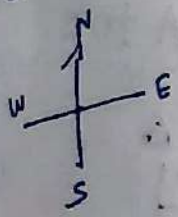
$= (12 - 7)$

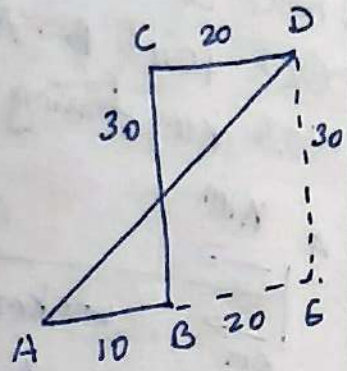
$= 5$ Km

7) A is 10 miles west to B. C is 30 miles north of B. D is 20 miles east of C. What is the distance from A to D?

a) 10 miles b) 30 miles c) $10\sqrt{10}$ miles

d) $10\sqrt{13}$ miles e) $30\sqrt{2}$ miles





$AD^2 = AE^2 + ED^2$

$= \sqrt{AE^2 + ED^2}$

$= \sqrt{(30)^2 + (30)^2}$

$= \sqrt{900 + 900}$

$= \sqrt{1800}$

$= 30\sqrt{2}$ miles

8) A plane flying north at 500 mph passes over a city at 12 noon. A plane flying east at the same altitude passes over the same city at 12:30 PM. The plane is flying east at 400 mph. To the nearest hundred miles how far apart are the two planes at 2 P.M?

a) 600 miles b) 1000 miles

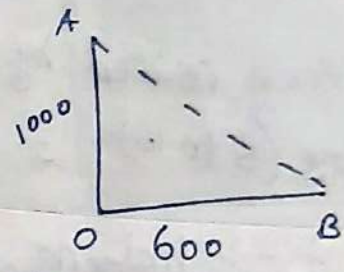
c) 1100 miles d) 1200 miles e) 1300 miles

Distance covered by the first plane till 2 P.M ie in 2 hrs

$= (500 \times 2)$ miles $= 1000$ miles

Distance covered by the second plane till 2 P.M i.e in $\frac{1}{2}$ hrs

$= \left(400 \times \frac{3}{2}\right)$ miles $= 600$ miles



Required distance

$= \sqrt{(1000)^2 + (600)^2}$

$= \sqrt{1000000 + 360000}$

$= 200\sqrt{34}$

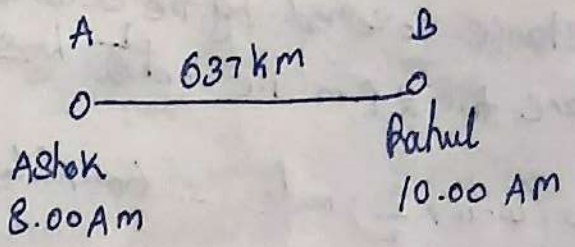
$= 1166$ miles $\simeq 1200$ miles

9) Ashok left from place A for place B at 8 a.m. and Rahul left place B for place A at 10.00 AM the distance between place A and B is 637 km. If Ashok and Rahul are travelling at a uniform speed of 39 kmph and 47 kmph respectively at what time will they meet?

Speed of Ashok = 39 km/ph

Speed of Rahul = 47 km/ph

Distance between place A and B = 637 km



Distance covered by ashok in 2 hrs (8 to 10) = 2 × 39 = 78 km

Remaining distance = 637 − 78
= 559

Relative Speed = 39 + 47 = 86 km/ph

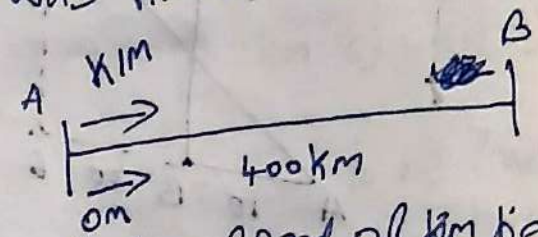
time taken to travel 559 km

$= \frac{559}{86} = 6.5$ hours.

So they meet at (10 AM + 6.5 hours)

= 4:30 pm

10) KIM and OM are travelling from points A to B, which are 400 km apart. Travelling at a certain speed KIM takes one hour more than OM to reach point B. If KIM doubles her speed she will take 1 hour 30 mins less than OM to reach point B. At what speed was KIM driving from A to B?



Let the speed of KIM be a and that of OM be b

Distance b/w point A & B = 400 km.

Then $\frac{400}{a} - \frac{400}{b} = 1$

$\frac{1}{a} = x \quad \frac{1}{b} = y$

$400x - 400y = 1$ — (1)

Speed of KIM doubles and she will take lets time i.e again

$\frac{400}{b} - \frac{400}{2a} = \frac{3}{2}$

$400y - 200x = \frac{3}{2}$

$800y - 400x = 3$ --- (2)

Solving eqn (1) & (2)

$400x - 400y = 1$

$-400x + 800y = 3$

$400y = 4$

$y = \frac{4}{400} = \frac{1}{100}$ km

$b = 100$ km

Now $\frac{400}{a} = \frac{400}{100} = 4$

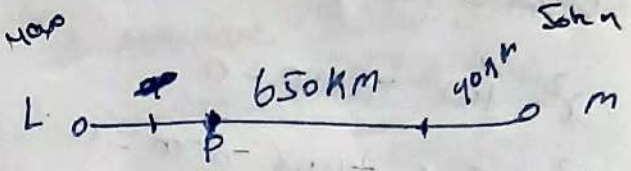
(or) $\frac{400}{a}$ $a = 80$ kmph

11) The speeds of john and max are 30km/h and 40 km/h initially max is at a place L and john is at a place M. The distance b/w L and M is 650km. John started his journey 3 hours earlier than max to meet each other. If they meet each other at a place P somewhere between L and M, then the distance between P & M is?

Speed of john = 30km/h

Speed of max = 40km/h

Distance between L & M = 650km



Distance travelled by john in 3 hrs = $30 \times 3 = 90$ km

Time taken by max and john to travel remaining $650 - 90 = 560$

$= \frac{560}{40+30} = 8$ hours.

Distance travelled by john to reach point $P = 8 \times 30 = 240$ km

Distance b/w P & M = $240 + 90 = 330$ km

12) A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than that of the fast train, then find the speeds of the two trains.

Let the speed of fast train be x km/hr

The speed of slow train $= (x-10)$ km/hr

$$\frac{600}{(x-10)} - \frac{600}{x} = 3$$

$$600x - 600(x-10) = 3x(x-10)$$

$$x^2 - 10x - 2000 = 0$$

$$x^2 - 50x + 40x - 2000 = 0$$

$$(x-50)(x+40) = 0$$

$$x = 50$$

Hence the speed of first train = 50 km/hr and speed of slow train = 40 km/hr.

13) By walking at $\frac{3}{4}$ of his usual speed, a man reaches his office 20 minutes later than his usual time. Find the usual time taken by him to reach his office.

New speed = $\frac{3}{4}$ of usual speed.

New time taken = $\frac{4}{3}$ of usual time

($\frac{4}{3}$ of the usual time) − (usual time) = 20 min

$\frac{1}{3}$ of usual time = 20 min ⇒ usual time = 60 min = 1 hour.

14) A person reaches his destination 40 minutes late if his speed is 3 km/hr and reaches 30 minutes before time if his speed is 4 km/hr. Find the distance of his destination from his starting point.

Let the required distance be x

Difference in the time taken at two speeds = 70 min = $\frac{7}{6}$ hr

$$\frac{x}{3} - \frac{x}{4} = \frac{7}{6}$$

$$4x - 3x = 14 \quad x = 14$$

Hence the required distance is 14 km

15) A train after travelling 150 km meets with an accident and then proceeds at $\frac{3}{5}$ of its former speed and arrives at its destination 8 hours late. Had the accident occurred 360 km further it would have reached the destination 4 hours late. What is the total distance travelled by the train?

Let the original speed of the train be x km/hr

Then $\frac{360}{\left(\frac{3}{5}x\right)} - \frac{360}{x} = 4$

$\frac{600}{x} - \frac{360}{x} = 4$

$\frac{240}{x} = 4 \qquad x = 60$

Let the distance travelled by the train be y km

Then

$\left[\frac{150}{60} + \frac{(y-150)}{\left(\frac{3}{5}\times 60\right)}\right] - \frac{y}{60} = 8$

$\frac{5}{2} + \frac{y-150}{36} - \frac{y}{60} = 8$

$2x = 1740$

$x = 870$

Hence required distance = 870 km.

16) Excluding the stoppages, the speed of a bus is 64 km/hr and including the stoppages the speed of the bus is 48 km/hr. For how many minutes does the bus stop per hour?

Due to stoppage the bus covers $(64-48) = 16$ km less per hour.

Time taken to cover 16 km $= \left(\frac{16}{64}\times 60\right)$

$= 15$ min

Hence stoppage time per hour = 15 min

17) A Aeroplane started 30 minutes later than the scheduled time from a place 1500 km away from its destination. To reach the destination at the scheduled time the pilot had to increase the speed by 250 km/hr. What was the speed of the aeroplane per hour during the journey?

Let the original speed of the aeroplane be x km/hr

$\frac{1500}{x} - \frac{1500}{(x+250)} = \frac{1}{2}$

$3000(x+250) - 3000x = x(x+250)$

$x^2 + 250x - 750000 = 0$

$x(x+1000) - 750(x+1000) = 0$

$(x+1000)(x-750) = 0$

$x = 750$

Hence the speed of the aeroplane during the journey $= (750+250)$ km/hr

$= 1000$ km/hr.

18) Two boys A and B start at the same time to ride from Delhi to meerut 60 km away. A travels 4 km an hour slower than B. B reaches meerut and at once turns back meeting A 12 km from meerut. find A's speed.

Let A's speed $= x$ km/hr

B's speed $= (x+4)$ km/hr

time taken by B to cover $(60+12)$ 72 km = time taken by A to cover $(60-12)$ i.e. 48 km

$$\frac{72}{(x+4)} = \frac{48}{x} \qquad 72x = 48x + 192$$

$$24x = 192 \qquad x = 8$$

Hence A's speed = 8 km/hr.

19) A and B are two stations 390 km apart. A train starts from A at 10 a.m and travels towards B at 65 kmph. another train starts from B at 11 A.M and travels towards A at 35 kmph. At what time do they meet?

Suppose they meet x hours after 10. a.m

[Distance moved by first in x hrs] + [Distance moved by second in $(x-1)$ hrs] = 390

$$65x + 35(x-1) = 390$$

$$100x = 425$$

$$x = 4\tfrac{1}{4}$$

So they meet 4 hrs 15 min after 10. a.m

i.e. at 02.15 PM

20) Two places A and B are 80 km apart from each other on a highway. A car starts from A and another from B at the same time. If they move in the same direction they meet each other in 8 hours. If they move in opposite directions towards each other, they meet in 1 hour 20 min. Determine the speed of the cars.

Let their speeds be x kmph and y kmph

then $\frac{80}{x-y} = 8 \qquad x-y=10$ — (1)

$\frac{80}{x+y} = 1\frac{1}{3} = \frac{4}{3} \qquad x+y=60$ — (2)

adding ① & ②

$2x = 70$ $x = 35$

Putting $x = 35$ in eqn ① we get $y = 25$

Hence the speeds of the two cars are 35 kmph and 25 kmph.

21) A man takes 6 hours 30 min in going by a cycle and coming back by scooter. He would have lost 2 hours 10 min by going on cycle both ways. How long would it take him to go by scooter both ways?

Let the distance be x km

[Time taken to cover x km by cycle] + [Time taken to cover x km by scooter] = 6 hours 30 min

[Time taken to cover $2x$ by cycle] + [Time taken to cover $2x$ by scooter] = 13 hrs.

$$\begin{array}{r} 6.30 \\ 2.10 \\ \hline 8.40 \end{array}$$

time taken to cover $2x$ km by cycle = 8 hr 40 min

Time taken to cover $2x$ km by scooter = 13 hrs - 8 hr 40 min

= 4 hr 20 min

Hence time required = 4 hr 20 min

22) Rani goes to school from her house in 30 minutes Raja takes 45 minutes in covering the same distance. Find the ratio between time taken by Rani and Raja.

Rani goes to school from her house = 30 min

Raja goes to school from her house = 45 min

Ratio required = 30 : 45

2 : 3

23) Arun had ridden one-third the total distance of his trip when his scooter got punctured. He finished the journey on foot, spending twenty times as long walking as he had spent riding. What was the ratio of his riding speed to his walking speed?

Let the total distance be x km and time spent in riding be y hours.

Distance covered by riding = $\left(\frac{x}{3}\right)$ km

Distance covered by walking

$$= \left(x - \frac{x}{3}\right) km = \frac{2x}{3} km$$

Time spent in walking $= (20y)$ hrs.

Riding speed $= \left(\frac{\frac{x}{3}}{y}\right) km/hr$

$$= \left(\frac{x}{3y}\right) km/hr.$$

Walking speed $= \left(\frac{\frac{2x}{3}}{20y}\right) km/hr$

$$= \left(\frac{x}{30y}\right) km/hr.$$

Required ratio $= \frac{x}{3y} : \frac{x}{30y}$

$$= 1 : \frac{1}{10}$$

$$= 10 : 1$$

Q4) Three persons are walking from a place A to another place B. Their speeds are in the ratio of 4:3:5. The time ratio to reach B by these persons will be

Ratio of speeds $= 4 : 3 : 5$

Ratio of times taken $= \frac{1}{4} : \frac{1}{3} : \frac{1}{5}$

LCM (60) $= 15 : 20 : 12$

4, 3, 5

25) The speeds of three cars are in the ratio 2:3:4. The ratio of the times taken by these cars to travel the same distance is?

Ratio of speeds $= 2 : 3 : 4$

Ratio of times taken $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

LCM (12) $= 6 : 4 : 3$

2, 3, 4

26) The speeds of A and B are in the ratio 3:4. A takes 20 min more than B to reach a destination. In what time does A reach the destination?

Ratio of speeds = 3 : 4

Ratio of times taken = $\frac{1}{3} : \frac{1}{4}$

= 4 : 3

Let A & B take 4x & 3x min to reach the destination

$4x - 3x = 20$

$x = 20$

time taken by A = 4x = 4 × 20

= 80 min

= $1\frac{1}{3}$ hr.

27) A truck covers a distance of 550 metres in 1 minute whereas a bus covers a distance of 33 kms in 45 minutes. The ratio of their speeds is?

Ratio of speeds = $\left(\frac{550}{60} \times \frac{18}{5}\right) : \left(\frac{33}{45} \times 60\right)$

= 33 : 44 = 3 : 4

28) The ratio b/w the speeds of two trains is 7:8. If the second train runs 400 kms in 4 hours, then the speed of the first train is?

Let the speed of train be

7x & 8x km/hr.

then

$8x = \frac{400}{4} = 100 \quad x = \left(\frac{100}{8}\right)$

$x = 12.5$

Speed of first train = 7x

= 7 × 12.5

= 87.5 km/hr.

29) Sneha is picked up by her father by car from college every day. The college gets over at 4 PM daily. One day the college got over an hour earlier than usual. Sneha started walking towards her house. Her father unaware of this fact leaves his house as usual, meets his daughter on the way, picks her up and they reach the house 15 min earlier than usual. What is the

ratio of the fathers driving speed to Sneha's walking speed?

15 min was saved.
It means that Sneha's father drives from the meeting point to the college and back to the meeting point in 15 min. i.e he can drive from the meeting point to college in $\left(\frac{15}{2}\right)$

$= 7.5$ min

But he reaches the college daily at 4 PM. So Sneha and her father meet on the way at

$3:52\frac{1}{2}$ PM.

Thus Sneha walked for 52.5 min and covered the same distance as covered by her father in 7.5 min

Since speed varies inversely as time taken to cover a distance we have

$$\frac{\text{Father driving speed}}{\text{Sneha walking speed}} = \frac{52.5}{7.5} = \frac{7}{1}$$

Hence ratio required $= 7:1$

30) A train travels at the speed of 65 km/hr and halts at 8 junctions for a certain time. It covers a distance of 1300 km in 1 day (24 hrs). How long does the train stop at each junction, if it halts for the same period of time at all the junctions?

Time taken to cover 1300 km $= \frac{1300}{65}$

$= 20$ hrs.

Halt time $= (24 - 20) = 4$ hrs

Halting time at each junction

$$= \left(\frac{4 \times 60}{8}\right) = 30 \text{ min}$$

31) The mileage of a motorbike A & B is 42 km per litre and 52 km per litre. Motor bike A covered 294 km and Motor bike B covered 208 km. If the cost of 1 litre of petrol is Rs. 48. How much amount would be spent on petrol to cover the total distance by both the motor bikes together?

quantity of petrol consumed by both the bikes.

$\left(\frac{294}{42} + \frac{208}{52}\right) = 11$ litres

$\therefore$ Total amount spent on petrol $= RS(48 \times 11) = RS.528$

PROBLEMS ON TRAINS

i) x km/hr $= \left(x \times \frac{5}{18}\right)$ m/sec

ii) x m/sec $= \left(x \times \frac{18}{5}\right)$ km/hr

iii) Time taken by a train of length l meters to pass a pole (or) a standing man (or) a signal post is equal to the time taken by the train to cover l meters.

iv) Time taken by a train of length l meters to pass a stationary object of length x metres is the time taken by the train to cover $(l+x)$ meters

v) Suppose two trains (or) two bodies are moving in the same direction at x m/s and y m/s where $x > y$. then their relative speed $= (x-y)$ m/s

vi) Suppose two trains (or) two bodies are moving in opposite directions at x m/s and y m/s then their relative speed $= (x+y)$ m/s

vii) If two trains of length a meter and b meter are moving in opposite directions at x m/s and y m/s then time taken by the trains to cross each other $= \frac{(a+b)}{(x+y)}$ sec

viii) If two trains of length a meter and b meter are moving in same direction at x m/s and y m/s then the time taken by the faster train to cross the slower train $= \frac{(a+b)}{(x-y)}$ sec

ix) If two trains (or) bodies start at the same time from points A & B towards each other and after crossing they take a and b sec in reaching B & A.

(A's speed) : (B's speed) $= (\sqrt{b} : \sqrt{a})$

1) A 160-m long train crosses a 160-m long platform in 16 seconds. find the speed of the train.

Distance covered in passing the platform $= (160 + 160) = 320\,m$

Speed of train $= \left(\frac{320}{16}\right)$ m/sec

$= 20$ m/sec

$= \left(20 \times \frac{18}{5}\right)$

$= 72$ Km/hr

2) A person standing on a railway platform noticed that a train took 21 seconds to completely pass through the platform which was 84m long and it took 9 seconds in passing him. find the speed of the train in km/hr

length of the train be x metres

The train covers x metres in 9 seconds and $(x+84)$ metres in 21 seconds.

$$\frac{x}{9} = \frac{x+84}{21}$$

$$21x = 9x + 756$$

$$21x - 9x = 756$$

$$12x = 756$$

$$x = \frac{756}{12} = 63$$

length of the train $= 63\,m$

Speed of the train $= \left(\frac{63}{9}\right)$ m/sec

$= 7$ m/sec

$= \left(7 \times \frac{18}{5}\right)$

$= 25.2$ Km/hr

(or)

$= \left(\frac{126}{5}\right)$ Km/hr.

3) A train travelling with constant speed crosses a 90m long platform in 12 seconds and a 120m long platform in 15 seconds. find the length of the train and its speed.

Let the length of the train be x metres

$$\frac{x+90}{12} = \frac{x+120}{15}$$

$$15(x+90) = 12(x+120)$$

$$15x + 1350 = 12x + 1440$$

$$3x = 90 \quad x = 30$$

speed of the train $= \left(\frac{30+90}{12}\right)$ m/sec

$= 10$ m/sec

$= \left(10 \times \frac{18}{5}\right)$ km/hr

$= 36$ km/hr.

length of the train $= 30$m

Speed of train $= 36$ km/hr

4) Two trains 240 m & 270m in length are running towards each other on parallel lines, one at the rate of 60 kmph and another at 48 kmph. How much time will they take to cross each other?

Relative speed of the two trains

$= (60 + 48)$ kmph

$= 108$ kmph

$= \left(108 \times \frac{5}{18}\right) = 30$ m/sec

Time taken by the train to pass each other

$= (240 + 270)$ m at 30 m/sec

$\frac{51\cancel{0}}{3\cancel{0}} = 17$ sec

5) A 300-m long train passed a man walking along the line in the same direction at the rate of 3km/hr in 33 seconds. find the speed of the train in km/hr.

Speed of the train relative to man

$= \left(\frac{300}{33}\right) = \left(\frac{100}{11}\right)$ m/sec

$= \left(\frac{100}{11} \times \frac{18}{5}\right) = \left(\frac{360}{11}\right)$ km/hr.

Let the speed of the train be x km/hr

then, relative speed $= (x-3)$ km/hr

$x - 3 = \frac{360}{11}$

$x = \frac{360}{11} + 3$ $\quad x = \frac{393}{11} = 35\frac{8}{11}$

Hence speed of train $= 35\frac{8}{11}$ km/hr.

6) A 100m long train takes $7\frac{1}{5}$ seconds to cross a man walking at the rate of 5km/hr in the direction opposite to that of the train. What is the speed of the train?

Let the speed of the train be x km/hr

speed of the train relative to man

$(x+5)$ km/hr $= \left((x+5) \times \frac{5}{18}\right)$

$$\frac{100}{(x+5)\times\frac{5}{18}} = \frac{36}{5}$$

$$10x+50 = 500$$

$$x = 45$$

Hence the speed of the train = 45 km/hr

7) A train 100m long travelling at 60 km/hr passes another train, twice as fast as this train and travelling in opposite direction in 10 seconds. Find the length of the second train.

Relative speed = (60+120)

= 180

$= \left(180 \times \frac{5}{18}\right) = 50$ m/sec

Let the length of the train be x m

$$\frac{x+100}{10} = 50$$

$$x = 400$$

Hence length of the second train

= 400 m

8) A moving train 66 metres long, overtakes another train 88 metres long, moving in the same direction, in 0.168 minutes. If the second train is moving at 30 km per hour, at what speed is the first train moving?

Let the speed of the first train be x km/hr

Sum of lengths of train = (66+88)m

= 154 m

Relative speed of two trains = (x-30)

$\left((x-30)\times\frac{5}{18}\right)$ m/sec

$$\frac{154}{(x-30)\times\frac{5}{18}} = 0.168 \times 60$$

$$5(x-30) = \frac{154\times 18}{10.08}$$

$$5(x-30) = 275$$

$$x-30 = 55$$

$$x = 85$$

Hence the speed of the first train

= 85 km/hr

9) A man sitting in a train is counting the pillars of electricity. The distance between two pillars is 60 m and the speed of the train is 42 Km/hr. In 5 hours how many pillars will he count?

Distance covered by the train in 5 hours $= (42 \times 5) = 210$ Km

$= 210000$ m

No of. Pillars counted by the man

$= \frac{210000}{60} + 1 = 3500 + 1$

$= 3501$

10) A train takes 5 minutes to cross a telegraphic post. then the time taken by another train whose length is just double of the first train and moving with same speed to cross a platform of its own length is

length of the train be x metres

time taken to cover x m $= 5$ min

$= (5 \times 60) = 300$ sec

speed of train $= \left(\frac{x}{300}\right)$ m/sec

length of second train $= 2x$ meter

length of platform $= 2x$ meter

Required time $= \left[\frac{2x + 2x}{\frac{x}{300}}\right] = \frac{4x \times 300}{x}$

$= 1200$ sec $= \frac{1200}{60} = 20$ min

11) Train A crosses a stationary train B in 50 seconds and a pole in 20 seconds with the same speed. The length of the train A is 240 m. What is the length of the stationary train B?

Speed of train A $= \left(\frac{240}{20}\right) = 12$ m/sec

Let the length of the train be x m

$\frac{240 + x}{12} = 50$

$240 + x = 50 \times 12$

$240 + x = 600$

$x = 360$ m

Hence the length of the stationary train B is 360 m

12) A train with 90 km/hr crosses a bridge in 36 seconds. Another train 100 metres shorter crosses the same bridge at 45 km/hr. What is the time taken by the second train to cross the bridge?

Let the length of the train and the bridge be x meters and y meters

Speed of the first train = 90 km/hr

$$= \left(90 \times \frac{5}{18}\right)$$

$$= 25 \text{ m/sec}$$

Speed of the second train = 45 km/hr

$$= \left(45 \times \frac{5}{18}\right) = \frac{25}{2} \text{ m/sec}$$

$$\frac{x+y}{36} = 25 \qquad x+y = 900$$

$$\text{Required time} = \frac{(x-100)+y}{\frac{25}{2}}$$

$$= \frac{x+y-100}{\frac{25}{2}}$$

$$= \left(800 \times \frac{2}{25}\right) = 64 \text{ sec}$$

13) Two trains of lengths 120m and 90m are running with speeds of 80 km/hr and 55 km/hr respectively towards each other on parallel lines. If they are 90m apart, after how many seconds they will cross each other?

Relative speed = (80+55)

= 135 km/hr

$$\left(135 \times \frac{5}{18}\right) = \left(\frac{75}{2}\right) \text{ m/sec}$$

Distance covered = (120+90+90)

= 300 m

$$\text{Required time} = \left(300 \times \frac{2}{75}\right)$$

= 8 sec.

14) Two trains are coming from opposite directions with speeds of 75 km/hr and 100 km/hr on two parallel tracks. At some ~~moment~~ moment the distance between them is 100 km. After T hours, distance between them is again 100 km. T is equal to?

Relative speed = (75+100)

= 175 km/hr

Time taken to cover 175 km at relative speed = 1 hr

T = time taken to cover 200 km

$$\left(\frac{1}{175} \times 200\right) = \frac{8}{7} \text{ hr} = 1\frac{1}{7} \text{ hr}$$

15) Two identical trains A & B running in opposite directions at same speed take 2 minutes to cross each other completely. The number of bogies of A are increased from 12 to 16. How much more time would they now require to cross each other?

Let the length of each train be x metres and let the speed of each train be y m/sec

$$\frac{2x}{2y} = 120 \quad \frac{x}{y} = 120$$

New length of train A $= \left(\frac{16}{12}x\right) = \left(\frac{4x}{3}\right)$ m

time taken by trains to cross each other

$$\frac{x}{y} = 120$$

$$\left(\frac{x + \frac{4x}{3}}{2y}\right) = \frac{7x}{6y} = \frac{7}{6} \times \frac{x}{y}$$

$$\left(\frac{7}{6} \times 120\right) = 140 \text{ sec}$$

Hence difference in times taken

$$= (140 - 120) = 20 \text{ sec.}$$

BOATS AND STREAMS

i) In water the direction along the stream is called downstream. and the direction against the stream is called upstream.

ii) If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr then

Speed downstream = (u+v) km/hr

Speed upstream = (u-v) km/hr

iii) If the speed downstream is a km/hr and the speed upstream is b km/hr then:

Speed in still water $= \frac{1}{2}(a+b)$ km/hr

Rate of stream $= \frac{1}{2}(a-b)$ km/hr

iv) Suppose a man can swim in still water at the rate of u km/hr the speed of current / stream is v km/hr and the man wishes to cross the stream (of width x m)

straight along its width, then the time taken to cross the river is the same as time taken to swim x metres at u km/hr.

NOTE: This is because the stream sways the man such that both the distance and the effective velocity increase and the time taken to cross the river remains unaffected.

v) A man can swim directly across a stream of width x km in t hours when there is no current and in t' hours when there is a current. Then, the rate of the current is

$$= \left[x \sqrt{\frac{1}{t^2} - \frac{1}{t'^2}} \right] \text{ km/hr}$$

vi) Average speed $= \dfrac{2xy}{x+y}$

1) The speed of a boat when travelling downstream is 32 km/hr, whereas when travelling upstream it is 28 km/hr. What is the speed of the boat in still water and the speed of the stream?

Speed of boat in still water

$$= \frac{1}{2}(32+28) \text{ km/hr}$$

$$= 30 \text{ km/hr}$$

Speed of stream $= \frac{1}{2}(32-28)$ km/hr

$$= 2 \text{ km/hr}$$

2) The speed of a motor boat is that of the current of water as 36:5. The boat goes along with the current in 5 hours 10 minutes. How much time will it take to come back?

the speed of the motor boat and that of the current be $36x$ km/hr and $5x$ km/hr

Speed down stream $= (36x + 5x)$
$= 41x$ km/hr

Speed up stream $= (36x - 5x)$
$= 31x$ km/hr

Let the distance be d km

$5\frac{14}{60}$

$\frac{d}{41x} = 5\frac{10}{60}$ $\quad 5\frac{1}{6}$

$\frac{31}{6}$

$\frac{d}{41x} = \frac{31}{6}$

$d = \frac{31 \times 41x}{6} = \frac{1271x}{6}$

time taken while coming back

$= \frac{d}{31x} = \left[\frac{1271x}{6} \times \frac{1}{31x}\right]$

$= \frac{41}{6}$ hrs

$= 6\frac{5}{6}$ hrs

3) A man can row 6 km/hr in still water. It takes him twice as long to row up as to row down the river. find the rate of stream.

man's rate up stream be x km/hr
man's rate down stream be $2x$ km/hr

Rate in still water $= \frac{1}{2}(x + 2x) = \frac{3x}{2}$

$\frac{3x}{2} = 6 \qquad x = 4$

Rate up stream $= 4$ km/hr
Rate down stream $= 8$ km/hr (4×2)

Hence rate of stream $= \frac{1}{2}(8 - 4)$
$= 2$ km/hr.

4) A man can row $7\frac{1}{2}$ kmph in still water. If in a river running at 1.5 km an hour. It takes him 50 minutes to row to a place and back, how far off is the place?

Speed down stream $= (7.5 + 1.5)$
$= 9$ kmph

Speed up stream $= (7.5 - 1.5)$
$= 6$ kmph

the distance be x km

$$\frac{x}{9} + \frac{x}{6} = \frac{5}{6}$$

$$\frac{2x+3x}{18} = \frac{5}{6}$$

$$2x+3x = \frac{5 \times 18}{6}$$

$$5x = 15$$

$$x = 3$$

Hence the required distance is 3 km

5) A boat goes 8 km upstream and then returns. The total time taken is 4 hrs 16 minutes. If the velocity of current is 1 km/hr, find the actual velocity of the boat?

actual velocity of boat be x km/hr

Speed down stream $= (x+1)$ km/hr

Speed up stream $= (x-1)$ km/hr

$$\frac{8}{x+1} + \frac{8}{x-1} = 4\frac{16}{60}$$

$$\frac{8(x-1) + 8(x+1)}{(x+1)(x-1)} = \frac{64}{15}$$

$$\frac{8[(x-1)+(x+1)]}{(x+1)(x-1)} = \frac{64}{15}$$

$$\frac{(x-1)+(x+1)}{(x+1)(x-1)} = \frac{64}{15 \times 8}$$

$$30x = 8(x^2-1)$$

$$4x^2 - 15x - 4 = 0$$

$$4x^2 - 16x + x - 4 = 0$$

$$4x(x-4) + (x-4) = 0$$

$$(x-4)(4x+1) = 0$$

$$x = 4$$

Hence the actual velocity of the boat $= 4$ km/hr.

6) A boat man rows to a place 45 km distant and back in 20 hours. He finds that he can row 12 km with the stream in the same time as 4 km against the stream. find the speed of the stream.

Suppose he moves 12 km downstream in x hours, then

Speed downstream $= \left(\frac{12}{x}\right)$ km/hr

Speed upstream $= \left(\frac{4}{x}\right)$ km/hr

$$\frac{45}{\frac{12}{x}} + \frac{45}{\frac{4}{x}} = 20$$

$$45\left[\frac{x}{12} + \frac{x}{4}\right] = 20$$

$$\frac{x}{12} + \frac{x}{4} = \frac{\cancel{20}\,4}{\cancel{45}\,9}$$

$$\frac{4x + 12x}{48} = \frac{4}{9}$$

$$16x = \frac{4 \times \cancel{48}^{16}}{\cancel{9}_{3}}$$

$$x = \frac{4 \times \cancel{16}}{3 \times \cancel{16}}$$

$$x = \frac{4}{3}$$

Speed downstream $= \dfrac{12}{\frac{4}{3}}$

$= \left(12 \times \frac{3}{4}\right)$ km/hr

$= 9$ km/hr

Speed upstream $= \dfrac{4}{\frac{4}{3}}$

$= \left(\frac{4 \times 3}{4}\right)$ km/hr

$= 3$ km/hr

Speed of the stream $= \frac{1}{2}(9-3)$

$= 3$ km/hr.

7) A boat takes half time in moving a certain distance downstream than upstream. What is the ratio between the rate in still water and the rate of current?

Ratio of time taken

Downstream : upstream

1 : 2

Speed downstream : upstream

2 : 1

Required ratio $= \dfrac{\text{Rate of Still water}}{\text{Rate of current}}$

$$= \frac{\frac{1}{2}(2x + x)}{\frac{1}{2}(2x - x)}$$

$= 3 : 1$

8) A boat while going downstream in a river covered a distance of 50 miles at an average speed of 60 miles per hour. While returning because of the water resistance, it took 1 hour 15 minutes to cover the same distance. What was the average speed during the whole journey?

Time taken to cover 50 miles downstream $= \left(\frac{50}{60}\right) = \frac{5}{6}$ hr

Time taken to cover 50 miles upstream $= 1$ hr 15 min $= 1\frac{1}{4}$ hr

$= \frac{5}{4}$ hr.

Total time taken to cover 100 miles

$= \left(\frac{5}{6} + \frac{5}{4}\right) = \frac{25}{12}$ hrs.

Average speed

$= \frac{100}{\frac{25}{12}} = \frac{100 \times 12}{25}$

Avg speed $= 48$ mph

9) A man can swim in still water at a rate of 4 km/hr. The width of the river is 1 km. How long will he take to cross the river straight. If the speed of the current is 3 km/hr?

Required time = Time taken to cover 1 km @ 4 kmph

$= \left(\frac{1}{4} \times 60\right) = 15$ min

10) A man wishes to cross a river perpendicularly. In still water he takes 4 minutes to cross the river, but in flowing river he takes 5 minutes. If the river is 100 metres wide, the velocity of the flowing water of the river is ?

Velocity of river $= 100\sqrt{\frac{1}{4^2} - \frac{1}{5^2}}$

$= 100\sqrt{\frac{1}{16} - \frac{1}{25}}$

$= 100\sqrt{\frac{9}{400}}$

$= \left(100 \times \frac{3}{20}\right)$

$= 15$ m/min

11) Boat A travels downstream from point x to point y in 3 hours less than the time taken by Boat B to travel upstream from point y to z. The distance b/w x and y is 20 km, which is half of the distance b/w y & z. The speed of Boat B in still water is 10 km/hr and the speed of Boat A in still water is equal to the speed of Boat B upstream. What is the speed of Boat A in still water? (Consider the speed of the current to be the same).

Let the speed of current in water $= s$ km/hr

time taken by boat A $= t_a$

time taken by boat B $= t_b$

Distance b/w the point x & y = 20km

Distance b/w the point y & z = 40km

according to the question

$$\frac{40}{t_b} = 10+s \quad - (1)$$

$$t_a = t_b 3$$

The speed of boat A in still water = The speed of boat B in upstream

$= (10+s)$ km/h

So $t_a = 20/10 = 2$ hours.

Hence $t_b = t_a + 3$
$= 2+3$
$= 5$ hrs

By using the equn (1)

$$\frac{40}{5} = 10+s$$

$$s = 10-8$$

$$s = 2 \text{ km/hr}$$

The speed of boat A in still water $= 10-2$
$= 8$ km/hr.

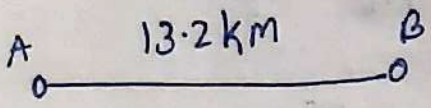
12) The speed of the boat in still water is 5 times that of the current it takes 1.1 hours to row to point B from point A downstream. The distance between point A and point B is 13.2 km. How much distance (in km) will it cover in 312 minutes upstream?

Let the speed of the current be x kmph

Then the speed of the boat in still water $= 5x$

Down stream speed $= (5x + x)$
$= 6x$ kmph

upstream speed $= (5x - x)$
$= 4x$ kmph.



according to the question

$1.1 \times 6x = 13.2$

$6.6x = 13.2$

$x = \frac{13.2}{6.6}$

$x = 2$ kmph

upstream speed $= 4x = 4 \times 2 = 8$ kmph

312 minutes $= 5\frac{1}{5}$ hours $= \frac{26}{5}$ hours.

Required distance travelled upstream = speed × time

$= 8 \times \frac{26}{5} = 41.6$ km

13) There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12 km/hr. while the other sails on a boat at a speed of 10 km/hr. If the river flows at the speed of 4 km/hr, which of the two friends will return to place A first?

the cyclist moves both ways at a speed of 12 km/hr

average speed of cyclist = 12 km/hr

boat sailor moves

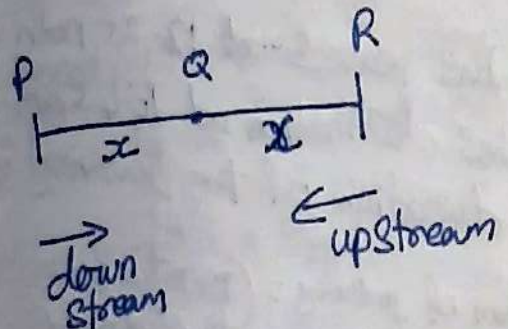
down stream $= 10 + 4$
$= 14$ km/hr

upstream $= 10 - 4$
$= 6$ km/hr

Average speed $= \left(\frac{2 \times 14 \times 6}{14 + 6}\right) = \frac{42}{5}$ km/hr
$= 8.4$ km/hr.

the avg speed of cyclist is greater, He will return to A first.

14) P, Q and R are three towns on a river which flows uniformly. Q is equidistant from P and R. I row from P to Q and back in 10 hours and I can row from P to R in 4 hours. Compare the speed of my boat in still water with that of the river.



Let $PQ = QR = x$ km

Speed downstream = a km/hr

Speed upstream = b km/hr

$\frac{x}{a} + \frac{x}{b} = 10$ $\quad x = \frac{10ab}{a+b}$ — (i)

$\frac{2x}{a} = 4$ $\quad x = \frac{4a}{2} = 2a$ (ii)

from (i) & (ii)

$2a = \frac{10ab}{a+b}$ $\quad 5b = a+b = a = 4b$

Required ratio = $\frac{\text{Speed in still water}}{\text{Speed of river}}$

$= \frac{\frac{1}{2}(a+b)}{\frac{1}{2}(a-b)} = \frac{a+b}{a-b} = \frac{4b+b}{4b-b}$

$= \frac{5}{3}$ $\quad \boxed{5:3}$

15) A boat running upstream takes 8 hours 48 minutes to cover a certain distance while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively?

upstream be x kmph

down stream be y kmph

Distance by upstream = 8 hrs 48 min

Distance by downstream = 4 hrs

$x \times \left(8\frac{4}{5}\right) = (y \times 4)$

$\frac{44}{5}x = 4y$ $\quad y = \frac{11}{5}x$

Required ratio

$= \left(\frac{y+x}{2}\right) : \left(\frac{y-x}{2}\right)$

$= \frac{1}{2}\left(\frac{11x}{5}+x\right) : \frac{1}{2}\left(\frac{11x}{5}-x\right)$

$\frac{16x}{10} : \frac{6x}{10}$

$8 : 3$

CLOCKS

The face (or) dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

A clock has two hands, the smaller one is called the hour hand (or) short hand while the larger one is called the minute hand or long hand.

i) In 60 min the minute hand gains 55 minutes on the hour hand.

ii) In every hour, both the hands coincide once.

iii) The hands are in the same straight line when they are coincident or opposite to each other

iv) When the two hands are at right angles they are 15 minute space apart

v) When the hands are in opposite directions, they are 30 min space apart

vi) Angle traced by hour hand in 12 hrs = 360°

vii) Angle traced by min hand in 60 min = 360°

1) London time is five and a half hours behind Delhi time. What time is it in London if it is 02:35 in Delhi?

2:35 in Delhi
5.30 behind
9.05 (or) 21.05 hrs.

2) A bus leaves at 12.25 noon and reaches destination at 10:45 am. The duration of the journey is?

Duration of journey = (Duration from 12:25 noon to mid night) + (Duration from mid night to 10:45 am)

= 11 hr 35 min + 10 hr 45 min

= 22 hrs 20 min

3) How many rotations will the hour hand of a clock complete in 72 hours?

No. of rotations $= \frac{72}{12} = 6$

4) Through what angle does the minute hand of a clock turn in 5 minutes?

Angle traced by minute hand in 5 min

$= \left[\frac{360}{60} \times 5\right] = 30^\circ$

5) At 9:38 AM through how many degrees the hour hand of a clock moved since noon the previous day?

Time from 12 noon to 9:38 AM

$= 12$ hrs $+ 9$ hrs 38 min

$= 21$ hrs 38 min

$= 21\frac{38}{60}$ hrs $= 21\frac{19}{30}$ hrs $= \frac{649}{30}$ hrs

Angle traced by the hour hand in 12 hrs $= 360°$

Angle traced by the hour hand in

$\frac{649}{30}$ hrs $= \left[\frac{360}{12} \times \frac{649}{30}\right] = 649°$

6) At 3.40 the hour hand and the minute hand of a clock form an angle of

Angle traced by hour hand in 12 hrs $= 360°$

Angle traced by it in $\frac{11}{3}$ hrs $= \left(\frac{360}{12} \times \frac{11}{3}\right)$

$= 110°$

Angle traced by minute hand in 60 min $= 360°$

Angle traced by it in 40 min $= \left(\frac{360}{60} \times 40\right)$

$= 240°$

Required angle $= (240 - 110) = 130°$

7) At what angle the hands of a clock are inclined at 15 minutes past 5?

angle traced by hour hand in $\frac{21}{4}$ hrs

$= \left(\frac{21}{4} \times \frac{360}{12}\right) = 157\frac{1}{2}°$

angle traced by min hand in 15 min

$= \left(\frac{360}{12} \times 15\right) = 90°$

Required angle $\left(157\frac{1}{2}\right) - 90 = 67\frac{1}{2}°$

8) what is the area of the face of a clock described by its minute hand between 9 a.m and 9.35 am if the minute hand is 10 cm long?

Angle swept by the minute hand in 35 min

$\left(\frac{360}{60} \times 35\right) = 210°$

Required area = area of a sector of a circle with radius 10 cm and central angle 210°

$= \frac{\pi r^2 \theta}{360} = \left[\frac{22}{7} \times 10 \times 10 \times \frac{210}{360}\right] cm^2$

$= \frac{550}{3} cm^2 = 183\frac{1}{3} cm^2$

9) The hands of a clock are 10 cm and 7 cm respectively. The difference between the distance traversed by their extremities in 3 days 5 hours is?

No. of. rounds completed by the minute hand in 3 days 5 hrs

$= (3 \times 24 + 5) = 77$

No. of. rounds completed by the hour hand in 3 days 5 hrs.

$= \left(3 \times 2 + \frac{5}{12}\right) = 6\frac{5}{12}$

Difference b/w the distance traversed

$= \left[\pi\left(2 \times \frac{22}{7} \times 10\right) - 6\frac{5}{12}\left(2 \times \frac{22}{7} \times 7\right)\right]$ cm

$= (4840 - 282.33)$ cm

$= 4557.67$ cm

10) At which time number of minutes elapsed since midnight is nine times the number of minutes before noon?

Let the required time be x minutes after midnight

Duration from 12 midnight to 12 noon

$= 12$ hrs $= (12 \times 60)$ min $= 720$ min

so $x = 9(720 - x) = 10x = 6480$

$x = 648$

required time $= 648$ min after midnight

$= 10$ hrs 48 min after midnight

$= 10.48$ AM.

11) How many times in a day, the hands of a clock are straight?

In 12 hours the hand coincide (or) are in opposite direction 22 times.

In 24 hours 44 times

12) How many times in a day, are the hands of a clock in straight line but opposite in directions?

11 times in every 12 hours.

in a day 24 hrs = 22 times.

13) How many times are the hour hand and the minute hand of a clock at right angles during their motion from 1.00 pm to 10.00 pm?

duration 1.00 pm to 10.00 pm

$= 9$ hrs. 1 hrs - 2 times

required number $= 9 \times 2 = 18$

14) A clock strikes once at 1 o'clock, twice at 2'o clock, thrice at 3'o clock and so on. what is the total number of strikings in a day?

Total No. of Strikings

$= 2(1+2+3+4+ \cdots 12)$

$= 2(78)$

$= 156$

15) A wall-clock takes 9 seconds in tringing at 9'o clock. The time it will take in tringing at 11'o clock is?

There are 8 intervals in 9 trings and 10 intervals in 11 trings.

Time duration of 8 intervals = 9 sec

Required time = Duration of 10 intervals.

$= \left(\frac{9}{8} \times 10\right)$ sec

$= 11.25$ sec

16) In every 30 minutes the time of a watch increases by 3 minutes. After showing the correct time at 5. AM. what time will the watch show after 6 hours?

Time gained in 1 hour = 6 min

Time gained in 6 hours = $(6 \times 6) = 36$ min

after 6 hours the correct time is 11:00 AM and the watch will show 11:36 AM.

17) A watch becomes fast by 5 minutes everyday. by what percent does it become fast?

No. of minutes in a day $= (24 \times 60)$

$= 1440$

Required percentage $= \left[\frac{5}{1440} \times 100\right]\%$

$= \frac{50}{144}\%$

18) A clock goes slow from midnight by 5 minutes at the end of the first hour, by 10 minutes at the end of the second hour, by 15 minutes at the end of the third hour and so on. what will be the time by this clock after 6 hours?

Time lost in 1 hour = 5 min

Time lost in 6 hours = $(5 \times 6) = 30$ min

after 6 hours the correct time will be 6:00 AM and the clock will show 5:30 AM.

19) It is between 3 pm and 4 pm and the distance between the hour and the minute hand of clock is 18 minute spaces. what time does the clock show?

3'o Clock the minute hand is 15 min spaces behind the hour hand.

Thus the minute hand has to gain

$(15+18) = 33$ minute spaces.

55 min are gained in 60 min

33 minutes are gained in $\left(\frac{60}{55} \times 33\right)$

$= 24$ min

The hands will be 18 min spaces apart at 3:36 pm

20) How much does a watch lose per day if its hands coincide every 64 minutes?

55 min. spaces are covered in 60 min

60 min spaces are covered in

$\left(\frac{60}{55} \times 60\right)$ min $= 65\frac{5}{11}$ min

Loss in 64 min $= \left(65\frac{5}{11} - 64\right)$

$= \frac{16}{11}$ min

Loss in 24 hrs $= \left(\frac{16}{11} \times \frac{1}{64} \times 24 \times 60\right)$ min

$= 32\frac{8}{11}$ min.

21) At what time, in minutes, between 3'o clock and 4'o clock both the needles will coincide each other?

At 3'o clock min hand is 15 min space apart from the hour hand.

To be coincident it must gain 15 min. spaces 55 min are gained in 60 min

15 min are gained in $\left(\frac{60}{55} \times 15\right)$ min

$= 16\frac{4}{11}$ min

The hands are coincident at $16\frac{4}{11}$ min Past 3.

22) At what time between 9 and 10'o clock will the hands of a watch be together?

9 to 10'o clock

the min hand has to gain 45 min

45 min spaces are gained in

$\left(\frac{60}{55} \times 45\right)$ min $= 49\frac{1}{11}$ min

The hands are together at

$49\frac{1}{11}$ min. past 9

23) At what time between 7 and 8'o clock will the hands of a clock be in the same straight line but, not together?

When the hands of the clock are in the same straight line but not together they are 30 minute spaces apart.

At 7'o clock they are 25 min apart minute hand will have to gain only 5 min

5 min spaces are gained $\left(\frac{60}{55} \times 5\right) = 5\frac{5}{11}$

Required time $= 5\frac{5}{11}$ min. past 7

24) Henry started a trip into the country between 8 AM and 9 AM. When the hands of clock were together. He arrived at his destination between 2 P.M and 3 P.M. When the hands of the clock were exactly 180° apart. How long did he travel?

8 AM to 9. AM

to be together minute hand has to gain 40 min

40 min spaces are gained in $\left(\frac{60}{55} \times 40\right)$

$= 43\frac{7}{11}$ min

Henry started the trip at $43\frac{7}{11}$ min past 8

Now to be 180° apart, the hands must be 30 min spaces apart

2 P.M they are 10 min spaces apart

The min hand will have to gain (10+30)

= 40 min spaces.

40 min spaces are gained in $\left(\frac{60}{55} \times 40\right)$

$= 43\frac{7}{11}$ min

Henry trip ended at $43\frac{7}{11}$ min past 2

Duration of travel = 8. $43\frac{7}{11}$ min AM to 2. $43\frac{7}{11}$ min PM.

= 6 hours.

25) Imagine that your watch was correct at noon, but then it began to lose 30 minutes each hour. It shows now 4 P.M but it stopped 5 hours ago. What is the correct time now?

The watch loose $\frac{1}{2}$ hour each hour.

So it must have taken 8 hours to show 4 P.M from 12.00 noon.

Thus it stopped at 8.00 PM. So the correct time is 5 hours ahead of 8. PM i.e 1 a.m

26) A mechanical grand father clock is at present showing 7hr 40min 6sec. Assuming that it loses 4 seconds in every hour, what time will it show after exactly $6\frac{1}{2}$ hours?

Time lost in $6\frac{1}{2}$ hours $= (6\frac{1}{2} \times 4)$
$= 26$ sec

Correct time after $6\frac{1}{2}$ hours
$= 7:40:06 + 6:30:00$
$= 14:10:06$

Time shown by the clock $= 14:10:06$
$- 26$ sec
$= 14$ hr 9 min 40 sec

27) A watch is 1 minute slow at 1 pm. on Tuesday and 2 minutes fast at 1pm on Thursday. When did it show the correct time?

Time from 1pm on Tuesday to 1pm on Thursday $= 48$ hrs.
The watch gains $(1+2) = 3$ min in 48 hrs
1 min is gained in $\left(\frac{48}{3}\right) = 16$ hrs.
The watch showed the correct time 16 hrs after 1pm on Tuesday
i.e 5AM on Wednesday.

28) Between 5 and 6, a lady looked at her watch and mistaking the hour hand for the minute hand, she thought that the time was 57 minutes earlier than the correct time. The correct time was?

time read by the lady was 57 minutes earlier than the correct time so the minute hand is $(60-57) = 3$ min space behind the hour hand
at 5'o clock the minute hand is 25 minute space behind the hour hand.
$(25-3) = 22$ min
22 min space are gained $= \left(\frac{60}{55} \times 22\right)$
$= 24$ min
Hence the correct time was 24 min past 5

29) How much does a watch lose per day if its hands coincide every 64 minutes?

60 min space are covered $= \left(\frac{60}{55} \times 60\right)$
$= 65\frac{5}{11}$ min

Loss in 64 minute $= 65\frac{5}{11} - 64$

$= \frac{16}{11}$ min

Loss in 24 hour $= \left(\frac{16}{11} \times \frac{1}{64} \times 24 \times 60\right)$

$= 32\frac{8}{11}$ min

30) Wall clock gains 2 minutes in 12 hours, while a table clock losses 2 minutes every 36 hours. Both are set right at 12 noon on Tuesday. The correct time when both show the same time next would be ?

31) The angle between the hands of a clock when the time is 4:25 am is ?

4:25 M - minute
H - Hour

Required angle $= 30\left[\frac{M}{5} - H\right] - \frac{M}{2}$

$= 30\left[\frac{25}{5} - 4\right] - \frac{25}{2}$

$= 17\frac{1}{2}^\circ$

32) At 8.30 the hour hand and the minute hand of clock form an angle of ?

In 1 hour, the hour hand make the angle of 30°

8.30 (or) $8\frac{1}{2}$.

$x = \left(30 \times 8\frac{1}{2}\right)$

$= 225^\circ$

The minute hand make the angle in 1 minute $= 6^\circ$

minute hand makes the angle in 30 min

$= (6 \times 30) = 180^\circ$

Required angle $= 225 - 180^\circ$

$= 75^\circ$

33) A clock is displaying correct time at 9 AM on monday. If the clock loses 12 minutes in 24 hours. then the actual time when the clock indicates 8.30PM on wednesday of the same week is ?

Time interval from 9am monday to 8:30 PM on wednesday

$= (24 \times 2.5) - 0:30$ hours

$= 60 - 0:30$

$= 59$ hours 30 min

$= 59\frac{1}{2}$ $\frac{119}{2}$ hours.

Also 24 hours -12 min

$= 23$ hours 48 minutes

$= 23 + \frac{48}{60} = 23\frac{4}{5}$ hours $= \frac{119}{5}$.

$\frac{119}{5}$ hours of this clock $=$ 24 hours of the correct clock

$\frac{119}{2}$ hours of this clock $= \frac{24 \times 5}{119} \times \frac{119}{2}$

$= 60$ hrs.

$\left(60 - \frac{119}{2}\right) = \frac{120 - 119}{2} = \frac{1}{2}$ hours

30 minutes

Hence the correct time is 30 minutes after 8:30pm i.e 9 PM

34) There are two clocks, both set to show 10 pm on 21'st January 2010. one clock gains 2 minutes in an hour and the other clock loses 5 minutes in an hour. Then by how many minutes do the two clocks differ at 4 pm on 22nd January 2010?

Time period 10 pm and 4 pm

$= 18$ hours.

one clock gains $= 2$ minutes

one clock loses $= 5$ minutes

Required difference $= (2 \times 18 + 5 \times 8)$ min

$= 126$ minutes.

CALENDAR

i) odd days:

In a given period, the number of days more than the complete weeks are called odd days.

ii) Leap year:

1) Every year divisible by 4 is a leap year, if it is not a century.

2) Every 4th century is a leap year and no other century is a leap year.

Note: A leap year has 366 days.

iii) ordinary year:

The year which is not a leap year is called an ordinary year. An ordinary year has 365 days.

iv) Counting of odd days:

1) one ordinary year = 365 days
(52 weeks + 1 day)

1 ordinary year has 1 odd day

2) 1 Leap year = 366 days
= (52 weeks + 2 day).

1 Leap year has 2 odd day

3) 100 years = 76 ordinary year + 24 leap year

$= (76 \times 1 + 24 \times 2) = 124$ odd days

= (17 weeks + 5 days)

100 years = 5 odd days.

Number of odd days

100 years = 5 odd days
200 years = 3 odd days
300 years = 1 odd day
400 years = 0 odd day

Similarly each one of 800, 1200, 1600, 2000, etc -- has 0 odd day.

v) Day of the week Related to odd days:

Day	No. of. days
Sunday	0
monday	1
Tuesday	2
Wednesday	3
Thursday	4
Friday	5
Saturday	6

1) What was the day of the week on 16th July, 1776?

∴ 16th July 1776 =

(1775 years + period from 1.1.1776 to 16.7.1776]

Counting of odd days

1600 years = 0

100 years = 5

1775 = (1600 + 100 + 75)

75 years =

[18 leap year + 57 ordinary year]

$$4\overline{)75}\ 18$$
4
35
32
3

75 − 18 = 57

= (18×2 + 57×1] = 93 odd days

= [13 weeks + 2 days]

= 2 odd days.

$$7\overline{)93}\ 13$$
7
23
21
2

1775 years = (1600 + 100 + 75)

1775 years = (0 + 5 + 2) = 7 odd days

= 0 odd days.

Jan feb mar April may Jun July

31 + 29 + 31 + 30 + 31 + 30 + 16

= 198 days

= (28 weeks + 2 days)

= 2 odd days.

$$7\overline{)198}\ 28$$
14
58
56
2

Total No. of. odd days

= (0 + 2)

= 2

Hence the required day is Tuesday.

2) What was the day of the week on 15th August 1947?

15th August 1947 = (1946 + 1.1.1947 to 15.8.1947)

1946 = (1600 + 300 + 46)

= 0 + 1 + 1

46 year = 35 ordinary year + 11 leap year

= (35×1 + 11×2)

= (35 + 22) = 57 odd day

= (8 weeks + 1 day)

= 1 day

$$7\overline{)57}\ 8$$
56
1

1946 = 2 odd day.

1.1.1947 to 15.8.1947

(31 + 28 + 31 + 30 + 31 + 30 + 31 + 15)

= 227 days

= (32 weeks + 3 days)

= 3 days.

$$7\overline{)227}\ 32$$
21
17
14
3

Total No. of. odd day = (2 + 3) = 5

Hence the required day is Friday.

3) Prove that the calendar for the year 2003 will serve for the year 2014?

We must have same day on 1.1.2003 and 1.1.2014

So, No. of odd days b/w 31.12.2002 and 31.12.2013 must be 0

This period has 3 leap years and 8 ordinary years.

Number of odd days $= (3+2+8\times1)$

$= 14$

$= 0$ odd day

Calendar for the year 2003 will serve for the year 2014.

4) On 8th feb, 2005 it was Tuesday what was the day of the week on 8th feb 2004

2004 is a leap year hence we need to add 2 days before that means Sunday.

5) For a certain month the dates of three of the Sundays are even numbers. Then the 15th of the that month falls on a?

The dates of three of the Sundays are even numbers is

2, 9, 16, 23, 30.

So on 16th of that month = Sunday

15th of that month falls on a Saturday

6) The calendar for the year 2009 will be the same as that of the year?

a) 2013 b) 2014 c) 2015 d) 2014

Suppose 2008 starts with monday.

Leap year 2009	wednesday
2010	Thursday
2011	Friday
2012	Saturday
Leap year 2013	monday
2014	Tuesday
2015	wednesday

2015 and 2009 was the same.

7) On what dates of April 2001 did wednesday fall?

a) 1, 8, 15, 22, 29

b) 2, 9, 16, 23, 30

c) 3, 10, 17, 24

d) 4, 11, 18, 25

1st April 2001 = (2000 + 1.1.2001 to 1.4.2001)

Jan feb mar April

31 + 28 + 31 + 1 = 91 days = 0 odd day

1st April 2001 = (0 + 0) = 0 odd days

hence 1st April 2001 is Sunday

1 - Sunday
2 - monday
3 - Tuesday
4 - wednesday.

Hence 4, 11, 18, 24 fall on wednesday.

8) January 1, 2007 was monday, what day of the week lies on jan 1 2008?

Tuesday.

9) January 1 2008 is Tuesday. what day of the week lies on jan 1 2009?

2008 is a leap year

2008 - ~~Thursday~~ Tuesday
2009 - ~~Thursday~~ Thursday

10) on 8th Dec, 2007 Saturday falls. what day of the week was it on 8th Dec. 2006.

2007 - Saturday
2006 - Friday.

11) on 6th march 2005 monday falls what was the day of the week on 6th march 2004.

2005 - monday
2004 - Saturday

12) Today is monday: After 61 days it will be

Today monday

$$7\overline{)61}\ 8,\ 56,\ 5$$

after 5 day Saturday

13) The last day of a century cannot be

a) monday b) wednesday c) Tuesday
D) friday Tuesday.

14) How many days are there in x weeks x days?

a) $7x^2$ b) $8x$ c) $14x$ d) 7

$7x + x = 8x$

PROBABILITY

i) **Experiment:** An operation which can produce some well-defined outcomes is called an experiment.

ii) **Random Experiment:** An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance is called a random experiment.

Example:

1) Rolling an unbiased dice
2) Tossing a fair coin
3) Drawing a card from a pack of well-shuffled cards.
4) Picking up a ball of certain colour from a bag containing balls of different colours.

Details:

1) When we throw a coin then either a head (H) (or) a Tail (T) appears
2) A Dice is a solid cube having 6 faces marked 1, 2, 3, 4, 5, 6. When we throw a die, the outcome is the number that appears on its upper face
3) A pack of cards has 52 cards

*) It has 13 cards of each suit, namely Spades, Clubs, Hearts and Diamonds.

*) Cards of Spades and Clubs are black

*) Cards of hearts and diamonds are Red

*) There are 4 honours of each suit. These are Aces, King, queen and Jack. These are face cards.

iii) **Sample space:**

When we perform an experiment then the set S of all possible outcomes is called the sample space.

Example:

1) In tossing a coin $S = [H, T]$
2) If two coins are tossed then $S = [HH, HT, TH, TT]$
3) In rolling a dice we have $S = [1, 2, 3, 4, 5, 6]$

iv) **Event:** Any subset of a sample space is called an event.

v) **Probability of occurrence of an event:**

Let S be the sample space and let E be an event. Then $E \subseteq S$

$$P(E) = \frac{n(E)}{n(S)}$$

vi) Results on probability:

i) $P(S)=1$ ii) $0 \le P(E) \le 1$

iii) $P(\phi)=0$

iv) for any events A and B, we have:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

v) If $\bar{A}$ denotes (not A) then

$P(\bar{A}) = 1 - P(A)$.

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1) A basket contains 4 red, 5 blue and 3 green marbles. If 2 marbles are drawn at random from the basket, what is the probability that both are red?

Total No. of balls $= 4+5+3 = 12$

Let E be the event of drawing 2 red balls.

Then $n(E) = {}^4C_2 = \frac{4\times3}{2\times1} = 6$

Also $n(S) = {}^{12}C_2 = \frac{12\times11}{2\times1} = 66$

$P(E) = \frac{n(E)}{n(S)} = \frac{6}{66} = \frac{1}{11}$

2) A bag contains 6 red, 4 blue, 2 green and 3 yellow marbles. If two marbles are drawn at random from the bag, what is the probability that both are red?

Total No. of balls $= 6+4+2+3 = 15$

E be the event of drawing 2 red ball

$n(E) = {}^6C_2 = \frac{6\times5}{2\times1} = 15$

$n(S) = {}^{15}C_2 = \frac{15\times14}{2\times1} = 105$

$P(E) = \frac{n(E)}{n(S)} = \frac{15}{105} = \frac{1}{7}$

3) A basket contains 6 blue, 2 red, 4 green and 3 yellow balls. If three balls are picked up at random. what is the probability that none is yellow?

Total No. of balls $= 6+2+4+3 = 15$

E be the event of drawing 3 non-yellow

$n(E) = {}^{12}C_3 = \frac{12\times11\times10}{3\times2\times1} = 220$

$n(S) = {}^{15}C_3 = \frac{15\times14\times13}{3\times2\times1} = 455$

$P(E) = \frac{n(E)}{n(S)} = \frac{220}{455} = \frac{44}{91}$

4) A bag contains 6 red, 4 blue, 2 green and 3 yellow marbles. If three marbles are picked up at random, what is the probability that 2 are blue and 1 is yellow?

Total No. of marbles = 6+4+2+3 = 15

E be the event of drawing 2 blue and 1 yellow marble.

$$n(E) = ({}^4C_2 \times {}^3C_1) = \frac{4\times3}{2\times1} \times 3 = 18$$

$$n(S) = {}^{15}C_3 = \frac{15\times14\times13}{3\times2\times1} = 455$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{18}{455}$$

5) A basket contains 6 red, 4 blue, 2 green and 3 yellow marbles. If four marbles are picked up at random, what is the probability that 1 is green, 2 are blue and 1 is red?

Total No. of marbles = 6+4+2+3 = 15

E be the event of drawing 1 Green, 2 blue and 1 red marble.

$$n(E) = ({}^2C_1 \times {}^4C_2 \times {}^6C_1) = 2 \times \frac{4\times3}{2\times1} \times 6 = 72$$

$$n(S) = {}^{15}C_4 = \frac{15\times14\times13\times12}{4\times3\times2\times1} = 1365$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{72}{1365} = \frac{24}{455}$$

6) A bag contains 6 red, 4 blue, 2 green and 3 yellow marbles. If two marbles are picked up at random, what is the probability that either both are green (or) both are yellow?

Total no. of marbles = 6+4+2+3 = 15

E be the event of drawing 2 marbles such that both are green (or) both are yellow.

$$n(E) = ({}^2C_2 + {}^3C_2) = \left(\frac{2\times1}{2\times1} + \frac{3\times2}{2\times1}\right)$$
$$= (1+3) = 4$$

$$n(S) = {}^{15}C_2 = \frac{15\times14}{2\times1} = 105$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{105}$$

7) A basket contains 6 blue, 2 red, 4 green and 3 yellow balls. If four balls are picked up at random what is the probability that 2 are red and 2 are green?

Total No. of balls = 6+2+4+3 = 15

E be the event of drawing 4 balls such that 2 are red and 2 are green.

$$n(E) = ({}^2C_2 \times {}^4C_2) = \left(1 \times \frac{4\times3}{2\times1}\right) = 6$$

$$n(S) = {}^{15}C_4 = \frac{15\times14\times13\times12}{4\times3\times2\times1} = 1365$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{1365} = \frac{2}{455}$$

8) A basket contains 4 red, 5 blue and 3 green marbles. If three marbles are picked up at random what is the probability that at least one is blue?

Total No. of. marbles $= (4+5+3) = 12$

E be the event of drawing 3 marbles such that none is blue.

r g

$4+3 = 7$

None blue

$n(E) =$

$$^7C_3 = \frac{7\times6\times5}{3\times2\times1} = 35$$

$$n(S) = {}^{12}C_3 = \frac{12\times11\times10}{3\times2\times1} = 220$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{35}{220} = \frac{7}{44}$$

Required probability $= 1 - P(E)$

$$= \left(1 - \frac{7}{44}\right) = \frac{37}{44}$$

9) A box contains 20 electric bulbs, out of which 4 are defective. Two bulbs are chosen at random from this box. The probability that at least one of them is defective, is?

Total No. of. bulb $= 20$

No. of. bulb in good condition $= 16$

No. of. defective bulb $= 4$

None defective

$$n(E) = {}^{16}C_2 = \frac{16\times15}{2\times1} =$$

$$n(S) = {}^{20}C_2 = \frac{20\times19}{2\times1}$$

$$P(E) = \left(\frac{16\times15}{2\times1} \times \frac{2\times1}{20\times19}\right) = \frac{12}{19}$$

$$P(\text{at least 1 is defective}) = \left(1 - \frac{12}{19}\right)$$

$$= \frac{7}{19}$$

10) In a class 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random what is the probability that he has offered English or Hindi?

$$P(E) = \frac{30}{100} = \frac{3}{10}$$

$$P(H) = \frac{20}{100} = \frac{1}{5}$$

$$P(E\cap H) = \frac{10}{100} = \frac{1}{10}$$

$$P(E \text{ or } H) = P(E\cup H)$$

$$= P(E) + P(H) - P(E\cap H)$$

$$= \left(\frac{3}{10} + \frac{1}{5} - \frac{1}{10}\right)$$

$$= \frac{4}{10}$$

$$= 2/5$$

11) A speaks truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in narrating the same incident?

E_1 - Event that A speaks the truth

E_2 - Event that B speaks the truth

$P(E_1) = \frac{75}{100} = \frac{3}{4}$

$P(E_2) = \frac{80}{100} = \frac{4}{5}$

$P(\bar{E_1}) = \left(1 - \frac{3}{4}\right) = \frac{1}{4}$

$P(\bar{E_2}) = \left(1 - \frac{4}{5}\right) = \frac{1}{5}$

P(A & B contradict each other)

P[A speak the truth and B tell lie) (or)

[A tell the lie and B speak the truth]

$= P(E_1 \cap \bar{E_2}) + P(\bar{E_1} \cap E_2)$

$= P(E_1) \cdot P(\bar{E_2}) + P(\bar{E_1}) \cdot P(E_2)$

$= \left(\frac{3}{4} \times \frac{1}{5}\right) + \left(\frac{1}{4} \times \frac{4}{5}\right)$

$= \left(\frac{3}{20} + \frac{1}{5}\right) = \frac{7}{20}$

$= \left(\frac{7}{20} \times 100\right)\% = 35\%$

12) A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $\frac{1}{7}$ and the probability of wife's selection is $\frac{1}{5}$. What is the probability that only one of them is selected?

E_1 - Event that husband is selected.

E_2 - Event that wife is selected.

Then

$P(E_1) = \frac{1}{7}$ $\quad P(E_2) = \frac{1}{5}$

$P(\bar{E_1}) = \left(1 - \frac{1}{7}\right) = \frac{6}{7}$ $\quad P(\bar{E_2}) = \left(1 - \frac{1}{5}\right) = \frac{4}{5}$

Required probability = P[(A and not B) (or) (B and not A)]

$= P[(E_1 \cap \bar{E_2}) \text{ (or) } (E_2 \cap \bar{E_1})]$

$= P(E_1) \cdot P(\bar{E_2}) + P(E_2) \cdot P(\bar{E_1})$

$= \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{1}{5} \times \frac{6}{7}\right) = \frac{10}{35} = \frac{2}{7}$

13) A bag contains 10 mangoes out of which 4 are taken out together. If one of them is found to be good, the probability that other is also good is?

out of 10 mangoes, 4 mangoes are rotten

$n(E) = {}^6C_2 = \frac{6\times5}{2\times1} = 15$

$n(S) = {}^{10}C_2 = \frac{10\times9}{2\times1} = 45$

$P(E) = \frac{n(E)}{n(S)} = \frac{15}{45} = \frac{1}{3}$

14) A bag contains 6 red balls 11 yellow balls and 5 pink balls. If two balls are drawn at random from the bag. one after another what is the probability that the first ball is red and second ball is yellow?

Total No. of. balls = 6+11+5 = 22

first ball red & second ball yellow

$n(E) = {}^6C_1 \times {}^{11}C_1 = 6\times11 = 66$

$n(S) = {}^{22}C_2 = \frac{22\times21}{2\times1}$ 231

$P(E) = \frac{n(E)}{n(S)} = \frac{66}{231} = \frac{2}{7}$

15) A bag contains 4 red balls, 6 blue balls and 8 pink balls. one ball is drawn at random and replace with 3 pink balls. A probability that the first ball drawn was either red or blue in colour and the second ball drawn was pink in colour?

Red - 4
Blue - 6
Pink - 8
} Total = 18

one red (or) blue is taken out and replaced with 3 pink.

Now the total No. of. ball is 20

Pink = 8+3 = 11

Required Probability $= \frac{11}{20}\left[\frac{4}{18}+\frac{6}{18}\right]$

$= \frac{11}{20} \times \frac{10}{18}$

$= \frac{11}{36}$

16) Dev can hit a target 3 times in 6 shots. Pawan can hit the target 2 times in 6 shots and Lakhan can hit the target 4 times in 4 shots. What is the probability that at least 2 shots hit the target?

Dev can hit target $= \frac{3}{6} = \frac{1}{2}$

Lakhan can hit target $= \frac{4}{4} = 1$

Pawan can hit target $= \frac{2}{6} = \frac{1}{3}$

Required probability that at least 2 shots hit target

$= \left(\frac{1}{2} \times \frac{3}{2}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right)$

$= \frac{1}{3} + \frac{1}{6} + \frac{1}{6}$

$= \frac{4}{6}$

$= \frac{2}{3}$

17) In a simultaneous throw of two coins the probability of getting at least one head is?

$S = (HH, HT, TH, TT) = 4$

$E = (HH, HT, TH] = 3$

$P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$

18) Three unbiased coins are tossed. What is the probability of getting at least 2 heads?

$S = [TTT, TTH, THT, THH, HTT, HTH, HHT, HHH]$

$E = [THH, HTH, HHT, HHH]$

$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

19) In a single throw of a die what is the probability of getting a number greater than 4?

$S = [1, 2, 3, 4, 5, 6]$

$E =$ greater than 4 $= [5, 6]$

$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

20) In a simultaneous throw of 2 dice, what is the probability of getting a total of 7?

Simultaneous throw of 2 dice

$n(S) = 6 \times 6 = 36$

$E =$ event of getting a total of 7

$\{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (6,1)\}$

$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

21) In a simultaneous throw of two dice, what is the probability of getting doublet?

$n(S) = 6 \times 6 = 36$

$E =$ event of getting doublet

$[(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)]$

$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

22) Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even?

$n(S) = 6 \times 6 = 36$

E = event of getting two numbers whose product is even.

(1,2) (1,4) (1,6) (2,1) (2,2) (2,3) (2,4)
(2,5) (2,6) (3,2) (3,4) (3,6) (4,1)
(4,2) (4,3) (4,4) (4,5) (4,6) (5,2)
(5,4) (5,6) (6,1) (6,2) (6,3) (6,4)
(6,5) (6,6)

$n(E) = 27$

$P(E) = \frac{n(E)}{n(S)} = \frac{27}{36} = \frac{3}{4}$

23) Tickets numbered 1 to 20 are mixed up and then a ticket is drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?

$n(S) = 20$

$n(E) =$ E = Event of getting multiple of 3 (or) 5

$= (3, 6, 9, 12, 18, 5, 10, 15, 20)$

$n(E) = 9$

$P(E) = \frac{n(E)}{n(S)} = \frac{9}{20} =$

24) In a lottery there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?

$n(S) = (10 + 25) = 35$

$n(E) = E$ = event of getting prize.

$P(E) = \frac{n(E)}{n(S)} = \frac{10}{35} = \frac{2}{7}$

25) One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card?

$n(S) = 52$

E = Event of getting face card

$n(E) = 16$

$P(E) = \frac{16}{52} = \frac{4}{13}$

26) A card is drawn from a pack of 52 cards. The probability of getting a queen of club (or) a king of heart is?

$n(S) = 52$

$n(E) = 2$

$P(E) = \frac{2}{52} = \frac{1}{26}$

27) one card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card (or) a king?

$n(S) = 52$

E - Event of getting red (or) a king

red card & 26 which includes & king + & king

$n(E) = 28$

$P(E) = \frac{28}{52} = \frac{7}{13}$

28) From a pack of 52 cards one card is drawn at random. What is the probablity that the card drawn is a ten (or) a spade?

$n(S) = 52$

E - Event of getting a ten (or) a spade

spade - 13 includes 1 ten + 3 ten

$n(E) = 16$

$P(E) = \frac{16}{52} = \frac{4}{13}$

29) The probability that a card drawn from a pack of 52 cards will be a diamond (or) a king is

$n(S) = 52$

E - Event of getting a king (or) diamond

Diamond - 13 includes 1 king + 3 king

$n(E) = 16$ $\quad P(E) = \frac{16}{52} = \frac{4}{13}$

30) From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings?

$n(S) = 52$

E - Event of getting both king

$n(E) = {}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$

$n(S) = {}^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 1326$

$P(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221}$

31) Two cards are drawn together from a pack of 52 cards. The probability that one is a spade and one is a heart is

$n(S) = {}^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 1326$

E - Event of getting 1 spade and 1 heart

$n(E) = ({}^{13}C_1 \times {}^{13}C_1) \quad (13 \times 13) = 169$

$P(E) = \frac{n(E)}{n(S)} = \frac{169}{1326} = \frac{13}{102}$

32) Two cards are drawn from a pack of 52 cards. The probablity that either both are red (or) both are kings?

$n(S) = {}^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 1326$

E - Event of getting 2 red (or) 2 king

Then E_1 = event of getting both red

E_2 - event of getting both kings

$E_1 \cap E_2$ - Event of 2 king red cards.

$n(E_1) = {}^{26}C_2 = \frac{26 \times 25}{2 \times 1} = 325$

$n(E_2) = {}^{4}C_2 = \frac{4 \times 3}{2 \times 1} = 6$

$n(E_1 \cap E_2) = {}^{2}C_2 = 1$

$P(E_1) = \frac{n(E_1)}{n(S)} = \frac{325}{1326}$

$P(E_2) = \frac{n(E_2)}{n(S)} = \frac{6}{1326}$

$P(E_1 \cap E_2) = \frac{1}{1326}$

P (both red (or) both king) = $P(E_1 \cup E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \left[\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}\right]$

$= \frac{330}{1326}$

$= \frac{55}{221}$

PERMUTATIONS & COMBINATIONS

Permutations:

The different arrangements of a given number of things by taking some or all at a time are called permutations.

Ex: 1 All permutations or arrangements made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb)

Ex: 2 All permutations made with the letters a, b, c by taking 3 at a time are (abc, acb, bac, bca, cab, cba)

i) Number of permutations of n things, taking r at a time is given by:

$${}^nP_r = n(n-1)(n-2)\cdots(n-r+1) = \frac{\lfloor n}{\lfloor n-r}$$

Ex: 1 ${}^8P_2 = (8 \times 7) = 56$

${}^7P_3 = (7 \times 6 \times 5) = 210$

ii) If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind

Such that $(P_1+P_2+P_3 - - + P_r) = n$

then No. of. Permutation $= \dfrac{\lfloor n}{\lfloor P_1 \lfloor P_2 \lfloor P_3 \cdots \lfloor P_r}$

Combinations:

Each of the different groups or selections which can be formed by taking some or all at a time, is called a combination.

Ex:1 out of three boys A, B, C we want to select two.
The possible selections are (AB, BC, CA)

Note: AB & BA represent the same combination.

Ex:2 The only combination of three letters A, B, C taken all at a time is ABC

Ex:3 Various groups of two out of 4 persons A, B, C, D are AB, AC, AD, BC, BD, CD

Important Note: AB and BA are two different permutations, But, they represent the same combination.

1) Number of all combinations of n things, taken r at a time is

$$^nC_r = \frac{\lfloor n}{(\lfloor r)(\lfloor n-r)} = \frac{n(n-1)(n-2)\cdots \text{to } r \text{ factors}}{\lfloor r}$$

ii) $^nC_n = 1$ and $^nC_0 = 1$

iii) $^nC_r = {}^nC_{(n-r)}$

Ex:1 $^8C_3 = \dfrac{8\times7\times6}{3\times2\times1} = 56$

Ex:2 $^{16}C_3 = {}^{16}C_{(16-13)} = {}^{16}C_3 = \dfrac{16\times15\times14}{3\times2\times1}$

$= 560$

1) Evaluate: $\dfrac{\lfloor 50}{\lfloor 48}$

$$\frac{\lfloor 50}{\lfloor 48} = \frac{50\times49\times\lfloor 48}{\lfloor 48} = 50\times49 = 2450$$

2) In how many different ways can the letters of the word "FIGHT" be arranged?

The given word contains 5 different letters

Required No. of. ways $= {}^5P_5 = \lfloor 5 =$

$5\times4\times3\times2\times1 = 120$

3) In how many different ways can the letters of the word "PRESENT" be arranged?

word PRESENT contains 7 letters & E and all other 5 different

$$\frac{\lfloor 7}{\lfloor 2} = \frac{7\times6\times5\times4\times3\times\lfloor 2}{\lfloor 2} = 2520$$

4) How many arrangements can be made out of the letters of the word "ENGINEERING"?

ENGINEERING contains 11 letters 3E, 2N, 2G, 2I & 3 other different

$$\text{No. of. Ways} = \frac{\lfloor 11}{\lfloor 3 \lfloor 3 \lfloor 2 \lfloor 2} =$$

$$= \frac{11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{(3\times2\times1)(3\times2\times1)(2\times1)(2\times1)}$$

$= 277200$

5) In how many different ways can the letters of the word "DESIGN" be arranged so that the vowels are at the two ends?

DESIGN contains 4 constants and 2 vowels.

At the two ends 2 vowels can be arranged in 2 ways.

4 constants can be arranged in $\lfloor 4$

$\lfloor 4 = (4\times3\times2\times1) = 24$ ways.

Total numbers of ways $= (24\times2) = 48$

No. of. ways $= 48$

6) In how many different ways can the letters of the word "DAUGHTER" be arranged so that the vowels always come together?

word contains 8 different letters

Vowels AUE are taken together, we may treat them as 1 letter

DGHTR [AUE]

$5 + 1 = 6$

$^6P_6 = \lfloor 6 = 6\times5\times4\times3\times2\times1$

$= 720$ ways.

Vowels AUE may be arranged in $\lfloor 3 = 6$ ways.

No. of. ways $= (720\times6) = 4320$ ways.

7) In how many different ways can the letters of the word "DIRECTOR" be arranged so that the vowels are always together?

Vowels I EO as letter

DRCTR (IEO)

$5 + 1 = 6$

2R and other different

$$\text{No. of. ways} = \frac{\lfloor 6}{\lfloor 2} = \frac{6\times5\times4\times3\times2\times1}{2\times1} = 360.$$

3 Vowels arranged in $\lfloor 3 = 3\times2\times1 = 6$

No. of ways $= (360\times6) = 2160$

8) In how many different ways can the letters of the word "DIGEST" be arranged so that the vowels are never together?

DGST (IE)

$4 + 1 = 5$

No. of ways $= \lfloor 5 = 5 \times 4 \times 3 \times 2 \times 1 = 120$

2 vowels arranged in $\lfloor 2 = 2 \times 1 = 2$

No. of ways $= (120 \times 2) = 240$ ways.

No. of ways of arranging all the letters of the given word.

total 6 letters $= \lfloor 6 = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= 720$ ways.

No. of ways arrangements so that the vowels are never together

$= (720 - 240) = 480$

9) In how many different ways can the letters of the word "DETAIL" be arranged so that the vowels occupy only the odd positions?

6 letters in the given word

3 constants & 3 vowels.

Let us mark these positions

1 2 3 4 5 6

3 vowels can be placed at any of 3 places, marked 1, 3, 5

No. of these arrangements $= {}^3P_3 = \lfloor 3 = 6$

3 constants can be placed at the remaining 3 places

No. of these arrangements $= {}^3P_3 = \lfloor 3 = 6$

Total No. of ways $= (6 \times 6) = 36$.

10) In how many ways can a cricket eleven be chosen out of 14 players?

No. of ways $= {}^{14}C_{11} = {}^{14}C_{(14-11)} = {}^{14}C_3$

$= \frac{14 \times 13 \times 12}{3 \times 2 \times 1} = 364$

11) In how many ways a committee of 6 members be selected from 7 men and 5 ladies, consisting of 4 men and 2 ladies?

We have to select 4 men out of 7

2 ladies out of 5

No. of ways $= {}^7C_4 \times {}^5C_2 = {}^7C_3 \times {}^5C_2$

$= \left[\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1}\right] = 350$

12) A select group of 4 is to be formed from 8 men and 6 women in such a way that the group must have atleast 1 woman. In how many different ways can it be done?

Required No. of ways

$$= ({}^6C_1 \times {}^8C_3) + ({}^6C_2 \times {}^8C_2) + ({}^6C_3 \times {}^8C_1) + ({}^6C_4 \times {}^8C_0)$$

$$= \left(6 \times \frac{8\times7\times6}{3\times2\times1}\right) + \left(\frac{6\times5}{2\times1} \times \frac{8\times7}{2\times1}\right) + \left(\frac{6\times5\times4}{3\times2\times1} \times 8\right) + \left(\frac{6\times5\times4\times3}{4\times3\times2\times1} \times 1\right)$$

$$= (336 + 420 + 160 + 15) = 931$$

13) In an examination there are three multiple choice questions and each questions has 4 choices. the number of ways in which a student can fail to get all answers correct is ?

No. of ways of attempting 1st, 2nd, 3rd questions are each

Total No. of ways $= 4^3 = 4\times4\times4 = 64$

No. of ways of getting all correct answers $= 1^3 = 1$

No. of ways of not getting all answer correct

$= 64 - 1 = 63$

14) There are six teachers. out of them two are primary teachers and two are secondary teachers. They are to stand in a row. So as the primary teachers, middle teachers and secondary teachers are always in a set. The number of ways in which they can do so is?

2 primary teachers.

They can stand in a row in $P(2,2) = 2!$

$= 2\times1 = 2$ ways.

2 middle teachers.

They can stand in a row in $P(2,2) = 2!$

$= 2\times1 = 2$ ways.

2 secondary teachers.

They can stand in a row in $P(2,2) = 2!$

$= 2\times1 = 2$ ways.

These 3 sets can be arranged them selves is $3!$

$= 3\times2\times1 = 6$ ways.

Hence the required No. of Ways.

$= 2 \times 2 \times 2 \times 6 = 48$ ways.

15) A committee of 5 members is to be formed out of 3 trainees, 4 Professors and 6 research associates. In how many different ways can this be done if the committee should have 2 trainees and 3 research associates?

Required No. of ways $= ({}^3C_2 \times {}^6C_3)$

$= ({}^3C_1 \times {}^6C_3) = \left(3 \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1}\right)$

$= 60$ ways.