DIGITAL COMMUNICATION

MODULE 3: DIGITAL MODULATION TECHNIQUES

> PSK using coherent detection
   i) generation, detection & error probabilities of BPSK & QPSK
   ii) M-ary PSK
   iii) M-ary QAM

> FSK using coherent detection
   i) BFSK generation, detection & error probability

> Non coherent orthogonal Modulation Techniques
   i) BFSK
   ii) DPSK Symbol representation
   iii) Block diagrams treatment of Transmitter & receiver
   iv) Probability of error.

DIGITAL MODULATION

- The different types of basic modulation schemes used in transmission of digital data are i) Amplitude Modulation (AM) ii) Phase Modulation (PM) and frequency modulation (FM).
- when the data is binary sequence, the amplitude in AM or phase in PM or frequency in FM switches between 2 distinct values corresponding to symbol 0 and symbol 1.
- These special cases are called as
  i) Amplitude Shift keying (ASK)
  ii) Phase Shift keying (PSK)
  iii) frequency shift keying.

DIGITAL MODULATION FORMATS

- In digital communications, the digital data is usually in the form of binary. It can also be a block of n binary bits encoded into $M = 2^n$ possible discrete phases or amplitudes, or M discrete phases or M discrete frequencies. Such schemes are generally called as M-ary Signalling. M=2 corresponds to binary symbols 0 & 1.

The three basic binary modulation Schemes are

1) Amplitude Shift Keying (ASK)

- In ASK, the amplitude of the carrier will have a value say A volts for binary Symbol 1 & a value zero for Symbol 0.
- It is equivalent to switching a sinusoidal Oscillator on and off.
- Therefore this modulation process is also called as on-off keying.

2) Phase Shift Keying (PSK / BPSK)

- If the carrier amplitude and frequency are fixed but the phase has zero radians for Symbol 1 & π radians for Symbol 0, the modulation process is called PSK or binary PSK (BPSK).

3) Frequency Shift Keying (FSK)

- In FSK, the carrier amplitude is fixed, but the frequency has a value f_0 Hz for Symbol 0 and a different value f_1 Hz for Symbol 1.
- The instantaneous frequency of the carrier switches b/w f_0Hz and f_1Hz corresponding to Symbols 0 & 1.

Fig (1) gives ASK, PSK and FSK waveforms for the binary data 1001011.

Binary Data

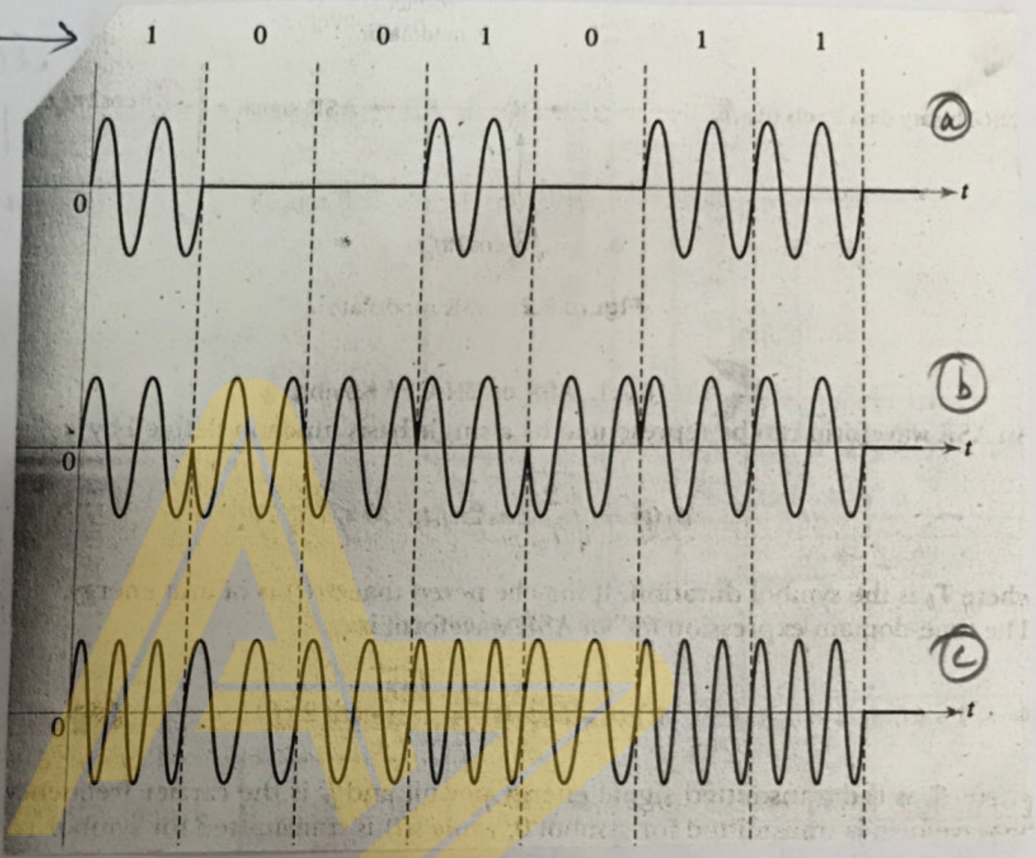


fig (1) (a) : ASK
(b) : PSK
(c) : FSK.

- Demodulation is the inverse process of recovering the original binary waveform from the received modulated signal whereas detection refers to the process of symbol decision.

- Detection & Decision are used interchang

- Demodulation can be done using either a coherent receiver or a non-coherent receiver. Then a demodulation is respectively called as coherent detection & non-coherent detection.

<u>Coherent Detection</u>: Synchronized local reference of the transmitted signals must be available. That is the receiver should have the exact knowledge of the carrier wave's phase reference. The receiver is then said to be phase locked to the transmitter.

- In coherent detection, the received signal is cross correlated with each one of the reference signals and decision is based on the correlator o/p.

<u>Non-coherent Detection</u>: The knowledge of phase of carrier wave is not needed. These the complexity of receiver is reduced.

- The performance of non-coherent receivers in the presence of noise is inferior as compared to coherent receiver.

Phase Shift Keying (PSK) Techniques using coherent detection.

Binary Phase Shift Keying (BPSK)

- In binary PSK System, the pair of Signals $S_1(t)$ and $S_2(t)$ used to represent binary Symbols 1 & 0, respectively is defined by

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t \quad ; \; 0 \leq t \leq T_b \longrightarrow (1)$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} (\cos 2\pi f_c + \pi)$$

$$= -\sqrt{\frac{2E_b}{T_b}} (\cos 2\pi f_c t) \quad ; \quad 0 \leq t \leq T_b \longrightarrow (2)$$

- where $T_b \rightarrow$ bit duration

  $E_b \rightarrow$ Transmitted signal Energy per bit

- The carrier frequency f_c is chosen such that $f_c >>> \frac{1}{T_b}$ and f_c is equal to some integer multiple of $1/T_b$.

- From eqns (1) & (2), we see that $S_1(t)$ & $S_2(t)$ are out of phase by $180°$.

- A pair of signals which differ only in phase by 180° are referred to as antipodal signals.

- From eqn ① & ②, it is clear that in case of BPSK, there is only one Basis function of unit energy

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b \rightarrow ③$$

Then we may express the transmitted signals $s_1(t)$ and $s_2(t)$ in terms of $\phi_1(t)$ as

$$s_1(t) = \sqrt{E_b}\,\phi_1(t) \longrightarrow ④ \qquad 0 \le t \le T_b$$

$$s_2(t) = -\sqrt{E_b}\,\phi_1(t) \longrightarrow ⑤ \qquad 0 \le t \le T_b.$$

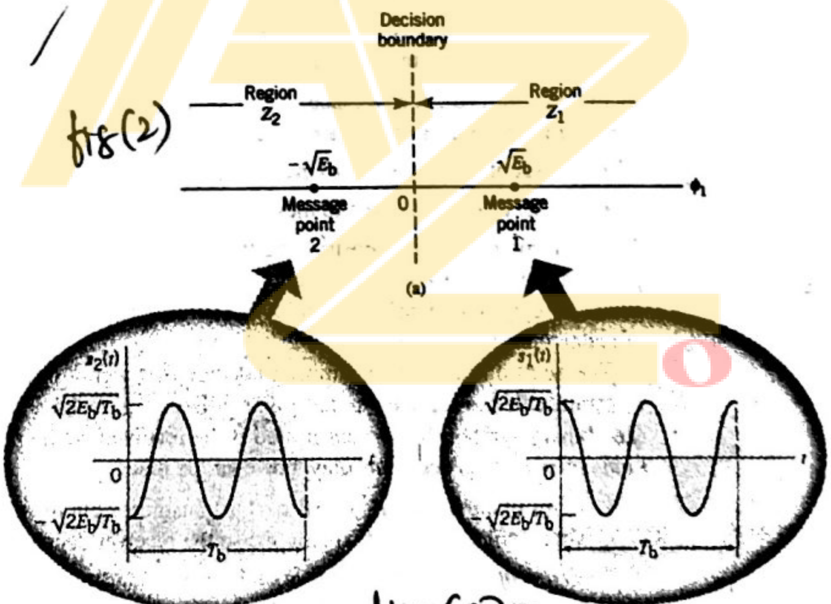
- A Binary PSK system is ∴ characterized by having a signal space that is 1-Dimensional (i.e, $N=1$) with signal constellation consisting of 2 message points $(M=2)$.
- The respective co-ordinates of the 2 message points are

$$s_{11} = \int_0^{T_b} s_1(t)\,\phi_1(t)\,dt = +\sqrt{E_b} \rightarrow ⑥$$

$$s_{21} = \int_0^{T_b} s_2(t)\,\phi_1(t)\,dt = -\sqrt{E_b} \rightarrow ⑦$$

- In otherwords, the message point corresponding to $s_1(t)$ is located at $s_{11} = +\sqrt{E_b}$ and the message point corresponding to $s_2(t)$ is located at $s_{21} = -\sqrt{E_b}$.
- fig(2) shows the signal space diagram for Binary PSK.
- fig(3) shows example waveforms of antipodal signals representing $s_1(t)$ & $s_2(t)$

fig(2)



fig(3)

fig(2) : Signal Space diagram

fig(3) : Waveforms depicting the transmitted Signals $S_1(t)$ & $S_2(t)$

Binary PSK Generation (Transmitter)

- The transmitter consists of 2 components as shown in fig (4).

① **Polar NRZ-level Encoder** :- which represents symbol 1 and 0 of the incoming binary sequence by amplitude levels $+\sqrt{E_b}$ & $-\sqrt{E_b}$ respectively.

② **Product Modulator** :- which multiplies the output of the polar NRZ encoder by the basis function $\phi_1(t)$. Sinusoidal $\phi_1(t)$ acts as the carrier of the Binary PSK signal.

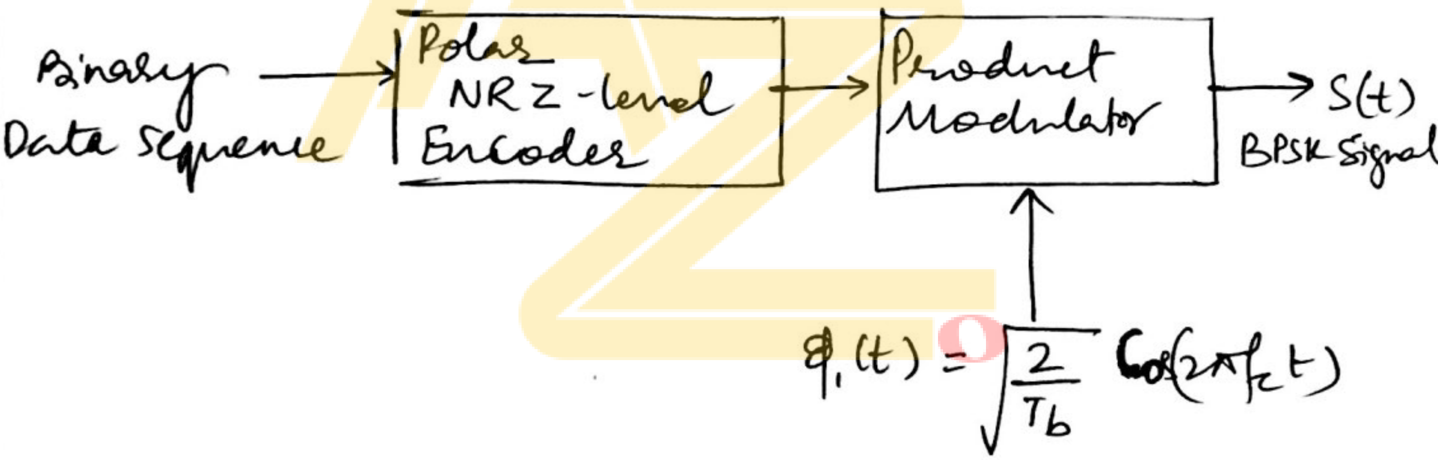


fig (4): Binary PSK Transmitter.

Binary PSK Detection (Receiver)

- To make an optimum decision on the received signal $x(t)$ in favor of symbol 0 or symbol 1, we assume that the receiver has access to locally generated replica of the basis function

$\phi_1(t)$.

- The receiver is synchronised with the Transmitter as shown in fig (5).

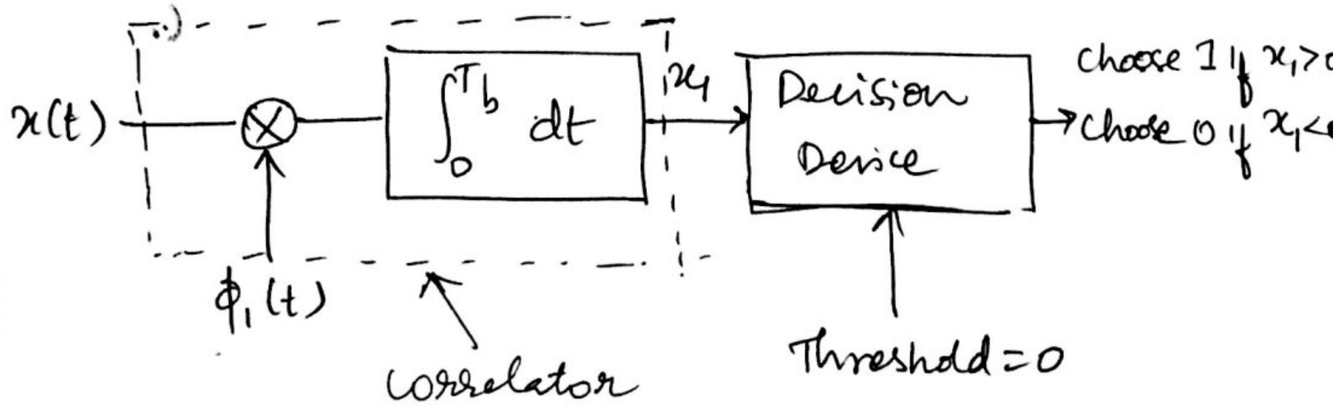
2 Basic components of the BPSK receiver are

① **Correlator** :- which correlates the received signal $x(t)$ with the basis function $\phi_1(t)$ on a bit by bit basis.

② **Decision Device** :- which compares the correlator o/p against a zero threshold assuming that binary symbols 1 & 0 are equiprobable.

(i) If the threshold is exceeded, a decision is made in favor of symbol 1.

(ii) if not, the decision is made in favor of symbol 0.



fig(5): Coherent Binary PSK Receiver.

Error Probability of Binary PSK using coherent Detection.

- with coherent detection, we partition the Signal Space into 2 regions

① The set of points closest to message point 1 at $\pm\sqrt{E_b}$ → Decision region Z_1.

② The set of points closest to message point 2 at $-\sqrt{E_b}$ → Decision region Z_2.

- The decision rule is ① to decide that signal $S_1(t)$ (Binary Symbol 1) was transmitted if the received signal point falls in region Z_1

② to decide that signal $S_2(t)$ (Binary Symbol was transmitted if the received signal point falls in region Z_2.

Two kinds of erroneous decisions may be made

i) Error of the first kind: Signal $S_2(t)$ is transmitted but the noise is such that the received signal point falls inside region Z_1, So the receiver decides in favour of signal $S_1(t)$

Error of the Second kind: Signal $s_1(t)$ is transmitted but the noise is such that the received signal point falls inside region Z2. So that the receiver decides in favor of signal $s_2(t)$.

Probability error of the first kind

- The decision region associated with Symbol 1 or signal $s_1(t)$ is described by

$$Z1 : 0 < x_1 < \infty$$

$x_1 \rightarrow$ observable element.

- x_1 is related to the recieved signal $x(t)$ by

$$x_1 = \int_0^{T_b} x(t)\,\phi_1(t)\,dt \longrightarrow (1)$$

- The conditional probability density function of random variable X_1, given that symbol 0 (i.e signal $S_2(t)$) was transmitted is defined by (with mean S_{ij} & variance $N_0/2$)

$$f_{X_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - S_{21})^2\right] \longrightarrow (2)$$

WKT $S_{21} = -\sqrt{E_b}$ $\longrightarrow (3)$

$$\therefore f_{X_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1+\sqrt{E_b})^2\right] \rightarrow (4)$$

- The conditional probability of the receiver deciding in favor of symbol 1, given that symbol 0 was transmitted, is therefore

$P_{10} = \int_0^\infty \text{CPdf}$

$$P_{10} = \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1+\sqrt{E_b})^2\right] dx_1 \rightarrow (5)$$

putting $z = \sqrt{\frac{2}{N_0}}(x_1+\sqrt{E_b}) \longrightarrow (6)$

$$P_{10} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2E_b/N_0}}^\infty \exp\left(\frac{-z^2}{2}\right) dz \rightarrow (7)$$

or

Q function

Standard Gaussian Random variable

Using $\boxed{Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-t^2/2\right) dt}$ in eqn (7)

$$\boxed{P_{10} = Q\sqrt{\frac{2E_b}{N_0}}} \rightarrow (8) \quad \text{where } x = \sqrt{\frac{2E_b}{N_0}}$$

Probability error of the second kind

- Since signal space is symmetric w.r.t origin

$$\therefore \boxed{P_{01} = P_{10} = Q\sqrt{\frac{2E_b}{N_0}}} \rightarrow (9)$$

- The average probability of symbol error or the BER for Binary PSK using coherent detection & assuming equiprobable symbols is given by

$$\boxed{P_e = Q\sqrt{\frac{2E_b}{N_0}}} \rightarrow (10) \quad \boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}}$$

- As we increase the transmitted signal energy per bit E_b for a specified noise spectral density $N_0/2$, the message points corresponding to symbol 1 & 0 move further apart and the average probability of error P_e is correspondingly reduced in accordance with eqn (10).

$\frac{N_0}{2} \rightarrow$ Noise power spectral density.

$$\boxed{Q(x) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{x}{\sqrt{2}}\right)}$$

$$\therefore P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{2E_b}{N_0} * \frac{1}{2}}\right)$$

$$\boxed{P_e = \frac{1}{2}\,\mathrm{erfc}\sqrt{\frac{E_b}{N_0}}}$$

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FREQUENCY SHIFT KEYING TECHNIQUES USING COHERENT DETECTION (BFSK)

- In Binary FSK, Symbols 1 & 0 are distinguished from each other by transmitting one of 2 sinusoidal waves that differ in frequency by a fixed amount.
- A typical pair of sinusoidal waves is described by

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases} \longrightarrow (1)$$

where $i = 1, 2$.

$E_b \rightarrow$ Transmitted signal Energy per bit.

$f_i = \frac{n_c + i}{T_b}$ for some fixed integer n_c & $i = 1, 2 \rightarrow (2)$

$f_i \rightarrow$ Transmitted frequency.

- Symbol 1 is represented by $s_1(t)$ & Symbol 0 by $s_2(t)$.
- The FSK signal described here is known as Sunde's FSK. It is continuous-phase signal, in the sense that phase continuity is always maintained, including the inter-bit switching times.

Using eqns (1) & (2), the signals $s_1(t)$ & $s_2(t)$ are orthogonal, but not normalized to have unit energy.

- The most useful form for the set of orthonormal basis functions is described by

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t) & 0 \le t \le T_b \\ 0 & \text{elsewhere} \end{cases} \rightarrow (3)$$

where $i = 1, 2$. correspondingly, the co-efficients S_{ij} where $i = 1, 2$ & $j = 1, 2$ is defined by

$$S_{ij} = \int_0^{T_b} S_i(t)\, \phi_j(t)\, dt$$

$$= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \cdot \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\, dt \rightarrow (4)$$

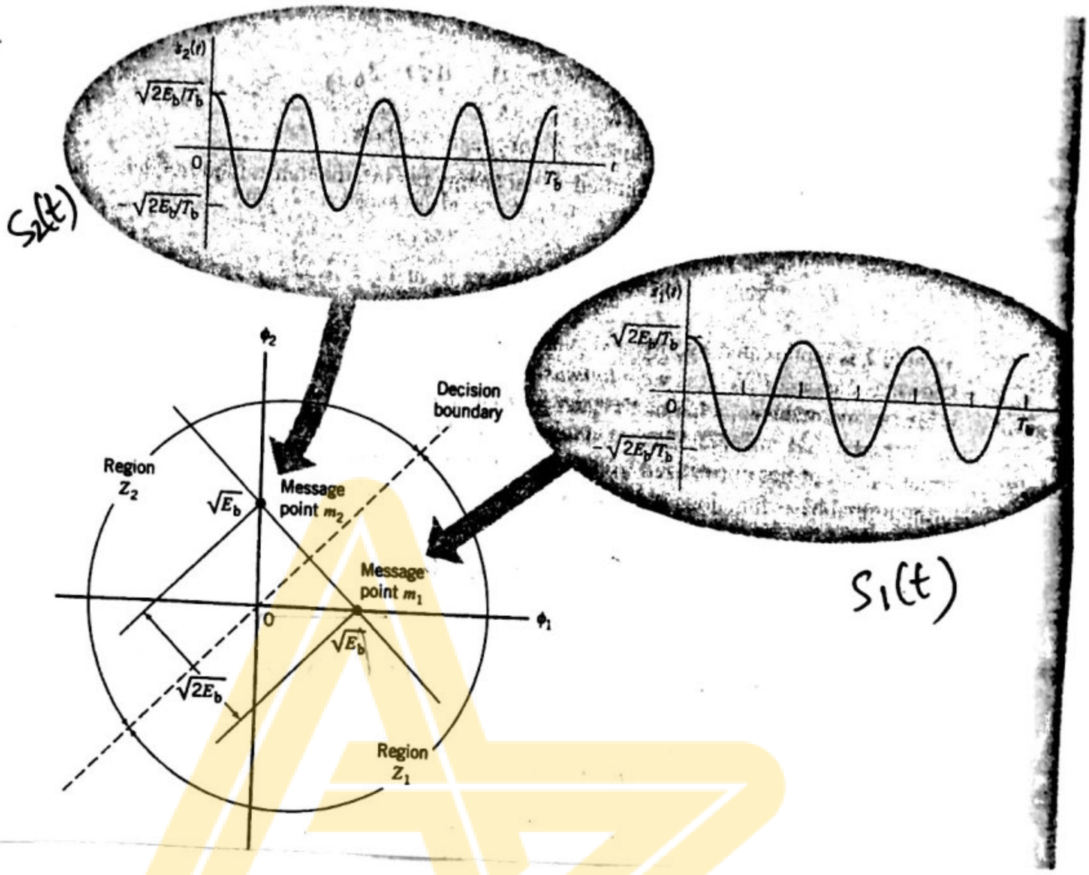
Carrying out the integration on equ (4)

$$S_{ij} = \begin{cases} \sqrt{E_b} & i = j \\ 0 & i \ne j \end{cases} \rightarrow (5)$$

- Thus Binary FSK is characterised by having a single space diagram (2-D) with 2 message points as shown in fig (12).
- The 2 message points are defined by the vectors

$$S_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \quad \& \quad S_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

- The Euclidean distance $\|S_1 - S_2\| = \sqrt{2 \cdot E_b}$



fig(12) : Signal space diagram for BPSK.

GENERATION OF BFSK SIGNALS.

- The block diagram fig(13) describes a scheme for generating the BFSK signal.
- It consists of 2 components.

① on-off level encoder :- the o/p of which is a constant amplitude of $\sqrt{E_b}$ in response to input symbol 1 and zero in response to input symbol 0.

(2) Pair of oscillators :- whose frequencies f_1 & f_2 differ by an integer multiple of the bit rate $\frac{1}{T_b}$ in accordance with eqn (1).

- The lower oscillator with frequency f_2 is preceded by an inverter. When in a signaling interval, the input symbol is 1, the upper oscillator with frequency f_1 is switched on and signal $S_1(t)$ is transmitted, while the other lower oscillator is switched off.

- On the other hand, when the i/p symbol is 0, the upper oscillator is switched off, while the lower oscillator is switched on and signal $S_2(t)$ with frequency f_2 is transmitted. With phase continuity as a requirement, the 2 oscillators are synchronised with each other.

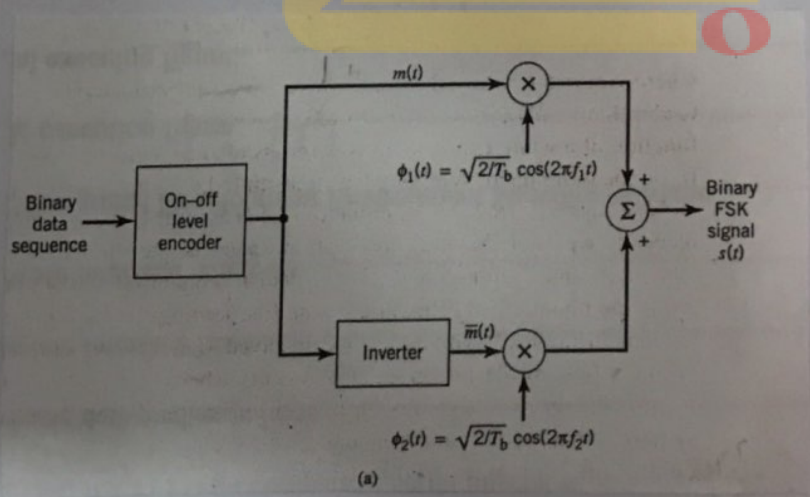
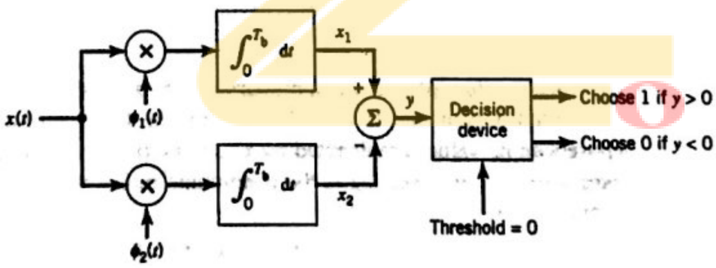


fig (13): BFSK Transmitter

DETECTOR OF BFSK.

- It consists of 2 correlators with a common input, which are supplied with locally generated coherent reference signals $\phi_1(t)$ & $\phi_2(t)$. as shown in fig (14)
- The correlators o/ps are then subtracted one from the other. The resulting difference y is then compared with a threshold of zero.
- If $y > 0$, the receiver decides in favour of 1. On the otherhand if $y < 0$, it decides in favour of 0.
- If $y = 0$, the receiver makes a random guess in favour of 1 or 0.



(b)

fig(14): Coherent BFSK receiver / Detector

ERROR PROBABILITY OF BINARY FSK

- The observation vector X has 2 elements x_1 and x_2 that are defined by

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\,dt \longrightarrow (1)$$

$$x_2 = \int_0^{T_b} x(t)\phi_2(t)\,dt \longrightarrow (2)$$

- where $x(t)$ is the received signal.

Given that symbol 1 was transmitted, $x(t) = s_1(t) + w(t)$

where $w(t) \rightarrow$ Sample function of a white Gaussian Noise process of zero mean and PSD $N_0/2$.

If symbol 0 was transmitted, then $x(t) = S_2(t) + w(t)$.

- Assuming the use of coherent detection at the receiver, the observation space is partitioned into 2 decision regions Z_1 & Z_2 as shown in fig(12).

- The decision boundary, separating region Z_1 from region Z_2, is the perpendicular bisector of the line joining the 2 message points.

- The receiver decides in favor of symbol 1 if the received signal point represented by the observation vector X falls inside region Z_1. This occurs when $x_1 > x_2$

- If we have, $x_1 < x_2$, the received signal point falls inside region Z_2 and the receiver decides in favor of symbol 0.

- If we have $x_1 = x_2$ in which the receiver makes a random guess in favor of symbol 1 0.

- We define a new Gaussian random variable Y whose sample value $y = x_1 - x_2$ $\longrightarrow$ (3)

- Given that symbol 1 was sent; the Gaussian random variables X_1 and X_2 whose sample values are x_1 & x_2 with mean $\sqrt{E_b}$ & 0 respectively.

Correspondingly the mean of the random variable Y given that symbol 1 was sent is

$$E[Y|1] = E[X_1|1] - E[X_2|1]$$
$$= +\sqrt{E_b} \longrightarrow (4)$$

$|||^{ly}$ the mean of random variable Y given that symbol 0 was sent is

$$E[Y|0] = E[X_1|0] - E[X_2|0]$$
$$= -\sqrt{E_b} \longrightarrow (5)$$

- The variance of random variable y is given by

$$Var[Y] = Var[x_1] + Var[x_2]$$
$$= \frac{N_0}{2} + \frac{N_0}{2} = N_0 \rightarrow (6)$$

- Suppose that symbol 0 was sent, The conditional probability density function of the random variable Y is given by

$$f_Y(y|0) = \frac{1}{\sqrt{2\pi N_0}} \exp\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right] \rightarrow (7)$$

Since $x_1 > x_2$ or $y > 0$, the receives making a decision in favor of symbol 1.

- We can represent conditional probability of error given that symbol 0 was sent is

$$P_{10} = P(y > 0 \mid \text{symbol 0 was sent})$$
$$= \int_0^\infty f_Y(y|0)\, dy$$
$$= \frac{1}{\sqrt{2\pi N_0}} \int_0^\infty \exp\left[-\frac{(y+\sqrt{E_b})^2}{2N_0}\right] dy \rightarrow (8)$$

Let $\frac{y+\sqrt{E_b}}{\sqrt{N_0}} = z$

(8) $\Rightarrow$ $$P_{10} = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp\left(-\frac{z^2}{2}\right) dz \rightarrow (9)$$

$$\boxed{P_{10} = \left(Q \sqrt{\frac{E_b}{N_0}} \right)}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^0 \exp\left(-\frac{t^2}{2}\right) dt$

Similarly $$\boxed{P_{01} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$

- Averaging P_{10} & P_{01} and assuming equiprobable symbols, we find that the average probability of bit error & BER for binary PSK using coherent detection is

$$\boxed{P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)} = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

*** For a binary FSK receiver to maintain the Same BER as in a Binary PSK receiver, the bit energy to noise density ratio E_b/N_0 has to be doubled