LAPLACE TRANSFORM

Little about Laplace Transform

MATH232 PROJECT

What is the Laplace Transform?

A function is said to be a piecewise continuous function if it has a finite number of breaks and it does not blow up to infinity anywhere.

Let us assume that the function f(t) is a piecewise continuous function, then f(t) is defined using the Laplace transform. The Laplace transform of a function is represented by L{f(t)} or F(s). Laplace transform helps to solve the differential equations, where it reduces the differential equation into an algebraic problem

Laplace form L {f (t)} = $\int_{0^-}^{\infty} f(t)e^{-st}\,dt$ (1)

The key thing to note is that Equation (1) is not a function of time, but rather a function of the
Laplace variable s = x + iy. Also, the Laplace transform only transforms functions defined over the
interval [0, ∞), so any part of the function which exists at negative values of t is lost! One of the most useful Laplace transformation theorems is the differentiation theorem where it reduces the differential equation into an algebraic problem.

Theorem 1 The Laplace transform of the first derivative of a function f is given by

$$L\{\frac{df}{dt}\}=sF(s) - f(0^-)$$

Here we will proof

$L\{\frac{df\ (t)}{dt}\}=\int_{0^-}^{\infty} \frac{df\ (t)}{dt}\ e^{-st}dt$

$= f(0^-)e^{-st}|_{0^-}^{\infty} + s\int_{0^-}^{\infty} f(t)e^{-st}dt$

$= sF(s) - f(0^-)$ #

By repeating the integration by parts, higher derivatives may be similarly transformed. Thus given
$A\ddot{x} + B\dot{x} + Cx = f(t)$

We have by taking the Laplace Transform of both sides

$A\,(s^2X(s) - sx(0^-) - \dot{x}(0^-)) + B\,(sX(s) - x(0^-)) + CX(s) = F(s)$

How can we calculate Laplace Transform?

1. Multiply the given function i.e. f(t) by e^{-St}, Where S is a complex number such that S = X + iY.
2. Integrate this product with respect to the time (t) by taking limits as 0 and ∞.
   This process results in Laplace transformation of f(t), and is denoted by F(t).

*T*able of basic functions after transformation:

No.	f(t)	L(f(t)) = F(s)	No.	f(t)	L(f(t)) = F(s)
1	1	$\frac{1}{s}$	11	$e^{\{at\}}$	$\frac{1}{(s-a)}$
2	t^n at n = 1,2, 3…	$\frac{n!}{s^{n+1}}$	12	t^p, at p>-1	$\frac{\Gamma(p+1)}{s^{p+1}}$
3	√(t)	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	13	$t^{\frac{n-1}{2}}$ at n = 1, 2...	$\frac{(1.3.5 \ldots (2n-1)\sqrt{\pi})}{2^n\, s^{\frac{n+1}{2}}}$
4	sin(at)	$\frac{a}{(s^2+a^2)}$	14	cos(at)	$\frac{s}{(s^2+a^2)}$
5	tsin(at)	$\frac{2as}{(s^2+a^2)^2}$	15	$tcos$(at)	$\frac{(s^2-a^2)}{(s^2+a^2)^2}$
6	sin(at + b)	$\frac{s\ \sin(b)+a\ \cos(b)}{s^2+a^2}$	16	cos(at+b)	$\frac{s\ \cos(b)+a\ \sin(b)}{s^2+a^2}$
7	sinh(at)	$\frac{a}{(s^2-a^2)}$	17	$cosh$(at)	$\frac{s}{(s^2-a^2)}$
8	$e^{\{at\}}$ sin(bt)	$\frac{b}{(s-a)^2-b^2}$	18	$e^{\{at\}}$ cos(bt)	$\frac{s-a}{(s-a)^2+b^2}$
9	$e^{\{ct\}}$f(t)	F(s-c)	19	t^nf(t) at n = 1,2, 3...	$(-1)^n F^n$ s
10	f'(t)	sF(s) – f(0)	20	f''(t)	s^2F(s) – sf(0) – f'(0)

[Example]

$$t^4+3t^3+5t-6$$

Solution

$$L\{t^4+3t^3+5t-6\}=\frac{4!}{s^5}+3\frac{3!}{s^4}+5.\frac{1}{s^2}-\frac{6}{s}$$

Properties of Laplace Transform

Some of the Laplace transformation properties are:

If $f_1(t) \leftrightarrow F_1(s)$ and [note: $\leftrightarrow$ implies Laplace Transform]

1-Linearity Property

If $L\{f_1(t)\} = F_1(s)$ and $f_2(t)\} = F_2(s)$

$\therefore L\{C_1 f_1(t) \pm C_2 f_2(t)\} = C_1 F_1(t) \pm C_2 F_2(s)$

Proof:

From Laplace Transform definition we deduce that

$$L\{C_1 f_1(t) \pm C_2 f_2(t)\} = \int_{0^-}^{\infty} (C_1 f_1(t) \pm C_2 f_2(t))e^{-st} dt$$

$$= C_1 \int_{0^-}^{\infty} f_1(t)e^{-st} dt \pm C_2 \int_{0^-}^{\infty} f_2(t)e^{-st} dt$$

$$\# = C_1 F_1(t) \pm C_2 F_2(s)$$

2-Division by t

If $L\{f(t)\} = F(s)$

But it is better to write it in the form $\therefore L\left\{\frac{f(t)}{t}\right\} = \int_S^{\infty} F(s)\, ds$

So, if The solution of the integration $\frac{\infty}{\infty}$ we can calculate it by lim $\lim_{R\to\infty} \int_S^R F(s)\, ds$

Proof:

$\because F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$

$\therefore \int_S^{\infty} F(s)\, ds = \int_S^{\infty} (\int_{0^-}^{\infty} f(t)e^{-st} dt)\, ds$

$= \int_0^{\infty} \int_S^{\infty} f(t)e^{-st} ds\, dt$

$= \int_0^{\infty} f(t)[-\frac{e^{-st}}{t}]_S^{\infty} dt$

$\# = \int_0^{\infty} \frac{f(t)}{t} e^{-st} dt = L\left\{\frac{f(t)}{t}\right\}$

[Example]

$L\left\{\frac{\sin t}{t}\right\}$

Solution

$L\left\{\frac{\sin t}{t}\right\} = F(s) = \frac{1}{s^2+1} = \lim_{R\to\infty} \int_S^R \frac{1}{s^2+1}\, ds$

$= \lim_{R\to\infty} \tan^{-1} s|_S^R = \lim_{R\to\infty} [\tan^{-1} R - \tan^{-1} S] = \frac{\pi}{2} - \tan^{-1} S$

3- Multiplication by Time

If $L\{f(t)\} = F(s)$

$\therefore L\{f(t)\ t\} = -\frac{dF(s)}{ds}$

Proof:

$\because F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\, dt$

$\therefore \frac{dF(s)}{ds} = \frac{d}{ds}\int_{0^-}^{\infty} f(t)e^{-st}\, dt$

by using diff. principle withim the integral sign with $= \int_0^\infty -t\ f(t)e^{-st}dt \rightarrow$
respect to "s"

$= -\int_0^\infty (f(t)\ t)\ e^{-st}\, dt$

$\# = -L\{f(t)\ t\}$

The previous relationship can be generalize to

$L\{f(t)\ t^n\} = (-1)^n \frac{d^n F(s)}{ds^n}$

[Example]

$L\{e^{-3t}\}$

Solution

$L\{e^{-3t}\} = \frac{1}{s+3}$

$L\{t^2 e^{-3t}\} = (-1)^2\ \frac{d^2}{ds^2}\left[\frac{1}{s+3}\right] = \frac{d}{ds}\left[\frac{-1}{(s+3)^2}\right] = \frac{2}{(s+3)^3}$

4-Transformation of derivatives

If $L\{f(t)\} = F(s)$

$$\therefore L\left\{\frac{df(t)}{dt}\right\} = SF(s) - f(0)$$

Proof:

$$L\{\frac{df\,(t)}{dt}\} = \int_0^\infty \frac{df\,(t)}{dt}\; e^{-st}dt$$

$$= f(t)e^{-st}|_0^\infty - \int_0^\infty f(t)(-se^{-st})dt$$

$$= f(0) + s\int_0^\infty f(t)e^{-st}dt$$

= SF(s) - $f(0)$ # ("Like Theorem 1 proof")

The previous relationship can be generalize to

$$L\left\{\frac{d^n f(t)}{dt^n}\right\} = S^n F(s) - S^{n-1} f(0) - f'(0)$$

5-Transformation of integrals

If $L\{f(t)\} = F(s)$

$$\therefore L\{\int_o^t f(x)dx\} = \frac{F(s)}{s}$$

Proof:

Let g(t)= $\int_o^t f(x)dx$

, g(0) =0$\therefore\ g'(t) = f(t)$

$$\because L\{g'(t)\} = sL\{g(t)\} - g(0)$$

$$\therefore L\{f(t)\} = sL\{\int_o^t f(x)dx\}$$

$$\therefore F(s) = sL\{\int_o^t f(x)dx\}$$

#$\therefore L\{\int_o^t f(x)dx\} = \frac{F(s)}{s}$

6-First shifting property

If $L\{f(t)\} = F(s)$

and $L\{e^{-kt}f(t)\} = F(s+k) \therefore L\{e^{kt}f(t)\} = F(s-k)$

Proof:

$L\{e^{kt}f(t)\} = \int_{0^-}^{\infty} (e^{kt}f(t))e^{-st}dt$

$= \int_{0^-}^{\infty} (e^{-(s-k)t}f(t)dt$

#$= F(s-k)$

[Example]

$L\{e^{2t}t^{10}\}$

Solution

$As\ L\{t^{10}\} = \frac{10!}{s^{11}} \rightarrow L\{e^{2t}t^{10}\} = \frac{10!}{(s-2)^{11}}$

7-Second shifting property

If $L\{f(t)\} = F(s)$

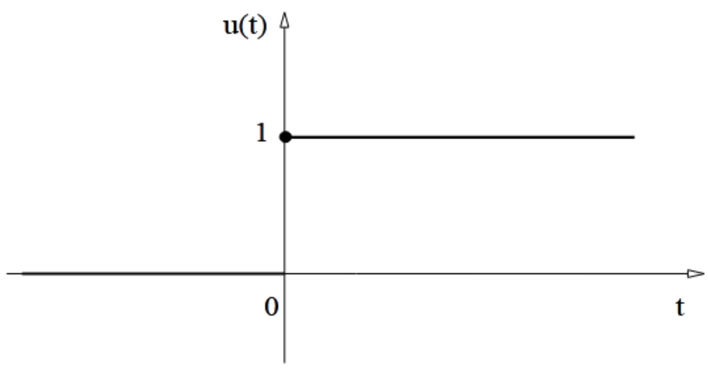
$$\boldsymbol{L\{u(t-a)f(t-a)\} = e^{-as}F(s)}$$

Before proving this property we must know

the definition of Unit step function

This function defined on two forms

1)$u(t) = \begin{cases} 0 , & t < 0 \\ 1 , & t \geq 0 \end{cases}$

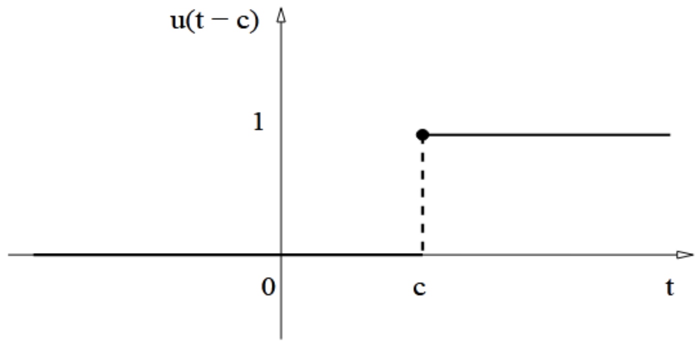


At $u(t) = 1$

$L\{u(t)\} = \int_0^\infty e^{-st}\,(1)\,dt = -\frac{1}{s}\,e^{-st}|_0^\infty = \frac{1}{s}$

$\therefore L\{u(t)\} = \frac{1}{s}$

2) $u(t-c) = \begin{cases} 0\ , & t < c \\ 1\ , & t \geq c \end{cases}$



At $u(t-c) = 1$

$$L\{u(t-c)\} = \int_c^\infty e^{-st}\,(1)\,dt = -\frac{1}{s}\,e^{-st}|_c^\infty = \frac{e^{-cs}}{s}$$

$\therefore L\{u(t-c)\} = \frac{e^{-cs}}{s}$

Proof for 7^{th} property:

$\boldsymbol{L\{u(t-a)f(t-a)\} = \int_a^\infty e^{-st}\,f(t-a)\,dt}$

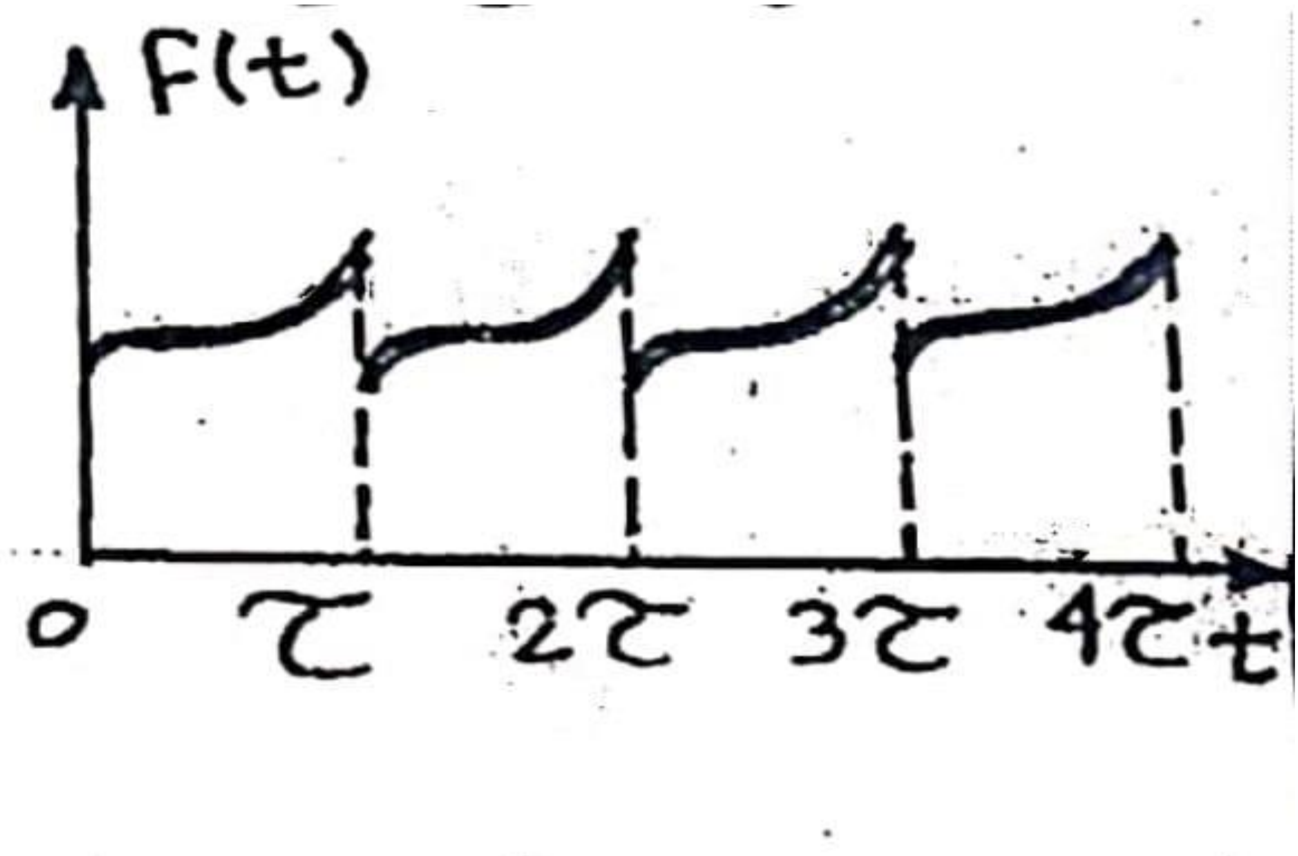
Put z= t-a → $\therefore dz = dt$

At t=a → z=0 , t= ∞ → z=∞

$\therefore \boldsymbol{L\{u(t-a)f(t-a)\} = \int_0^\infty e^{-s(z+a)}\,f(z)\,dz}$

$\boldsymbol{= e^{-as}\int_0^\infty e^{-sz}\,f(z)\,dz = e^{-as}\,F(s)}$ **#**

8-Laplace Transform of a periodic function



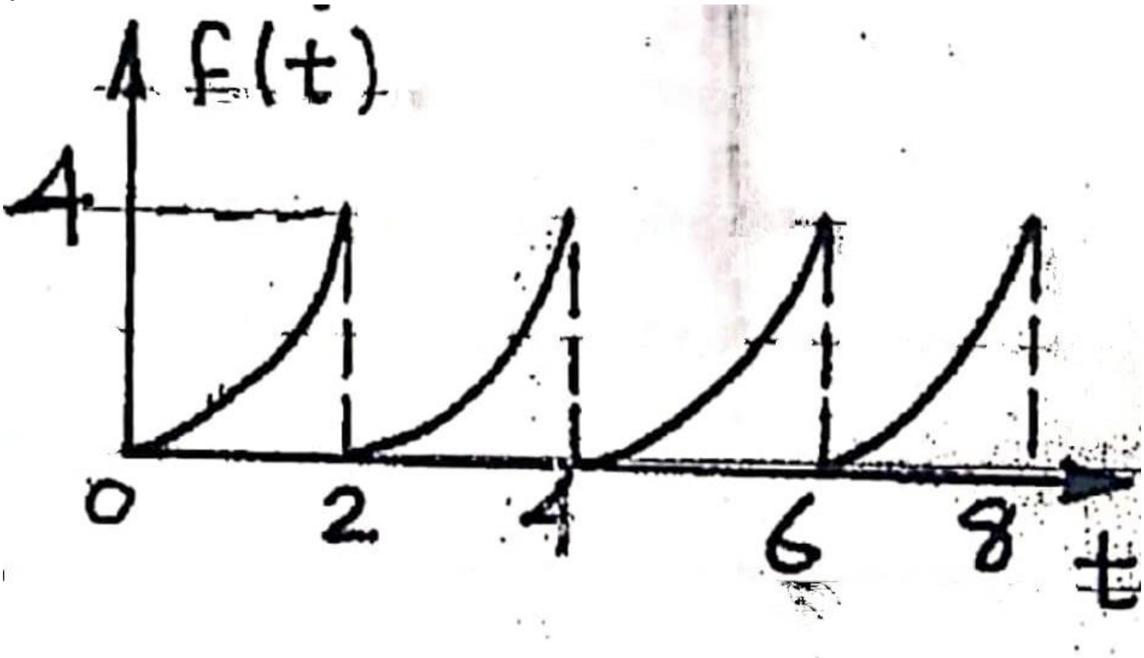
$$L\{f(t)\} = \frac{\int_0^{\tau} e^{-st}\, f(t)\, dt}{1 - e^{-\tau s}}$$

It can be written by easily form instead of integral forms and this by the definition of unit step function:

$$L\{f(t)\} = \frac{L\{f(t)[u(t) - u(t-\tau)]\}}{1 - e^{-\tau s}}$$

[Example] Find L.T of the periodic function

$f(t) = t^2 \;, 0 < t \le 2$



Solution

$As\ f(t)\ preiodic\ with\ \tau = 2$

$$L\{f(t)\} = \frac{\int_0^{\tau} e^{-st}\ f(t)\,dt}{1 - e^{-\tau s}} = \frac{\int_0^{2} e^{-st}\ t^2\ dt}{1 - e^{-2s}}$$

$$= \frac{L\{t^2\ [u(t) - u(t-2)]\}}{1 - e^{-2s}}$$

$$= \frac{1}{1 - e^{-2s}}\ [L\{t^2\} - L\{t^2\ u(t-2)\}]$$

$$= \frac{1}{1 - e^{-2s}} \left[\frac{2!}{s^3} - (-1)^2 \frac{d^2}{ds^2}\left[\frac{e^{-2s}}{s}\right]\right]$$

$$= \frac{1}{1 - e^{-2s}} \left[\frac{2!}{s^3} - \frac{2}{s^3} e^{-2s} - \frac{4}{s^2} e^{-2s} - 4\frac{e^{-2s}}{s}\right]$$

What about if the function is discontinuous forcing?

It's not a problem as we can solve it by

1- Heaviside (Step Function)

Which it's defines by $H(t)\begin{cases}1 & for\ t > 0\\ 0 & for\ t \leq 0\end{cases}$

i.e. The Heaviside step function is named Oliver Heaviside

Or Here if the function is impulsive forcing

2- The Dirac delta is another important function (or distribution)

which is often used to represent it Which it's defines by

$$\delta(t)\begin{cases}\infty & for\ t = 0^+\\ 0 & otherwise\end{cases}$$

"Hence, the Heaviside step function "turns on" at the right edge ($t = 0^+$), and the Dirac delta function turns on and off at the same place. An additional property of the Dirac delta function is

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

The Area under the curve defined by the Dirac delta, the unit- step and the Dirac delta function are derivative and anti-derivative of one another

$$\delta(t) = \frac{dH(t)}{dt}$$

i.e. Both the unit-step and Dirac delta belong to a class of functions called generalized function

For regular functions, this fact not withstanding however we may define the following Relationships

$$\frac{d\delta}{dt} = \frac{d^2H(t)}{dt^2}$$

$$\frac{d^n\delta}{dt^n} = \frac{d^{n+1}H(t)}{dt^{n+1}}$$

The Laplace transform of the Dirac delta function is revealed by taking the Laplace transform of the equation

$$\boldsymbol{(\delta(t) = \frac{dH(t)}{dt}):}$$

$$L\{\ \delta(t)\} = \int_0^\infty \delta(t)e^{-st}dt = L\{\frac{dH(t)}{dt}\} = \int_0^\infty \frac{dH(t)}{dt}\, e^{-st}dt$$

$$= H(t)\ e^{-st}|_0^\infty + s\int_0^\infty H(t)e^{-st}dt$$

$$= 0\ \ +s(\int_{0^-}^{0^+}(0)e^{-st} + \int_{0^+}^{\infty}(\ 1)e^{-st}) = 1 \qquad \#$$

In Dirac delta function we have additional property called ("Filtering property")

$\int_a^c f(t)\delta(t-b)dt = f(b)$ ***for a < b < c.***

Here we can proof it as follows

$$\int_a^c f(t)\delta(t-b)dt$$

$$= \lim_{\varepsilon\to 0}[\int_a^{b-\varepsilon}(0)f(t)dt + \int_{b-\varepsilon}^{b+\varepsilon} f(t)\delta(t-b)dt + \int_{b+\varepsilon}^{c} f(t)(0)dt] = \int_{b-\varepsilon}^{b+\varepsilon} f(t)\delta(t-b)dt$$

By integration by parts we will let $\tau = t - b$ we have

$$\lim_{\varepsilon\to 0}\int_{b-\varepsilon}^{b+\varepsilon} f(t)\delta(t-b)dt\ = \lim_{\varepsilon\to 0}\int_{-\varepsilon}^{+\varepsilon} f(\tau+b)\delta(\tau)d\tau$$

$$= \lim_{\varepsilon\to 0}[\ H(\tau)f(\tau+b)|_{-\varepsilon}^{+\varepsilon} - \int_{-\varepsilon}^{+\varepsilon} H(\tau)f'(\tau+b)d\tau\ ]$$

The last term in the brackets in previous equation vanishes because H(t) is zero in the interval of integration. Thus, we will have

$$\lim_{\varepsilon\to 0}\int_{b-\varepsilon}^{b+\varepsilon} f(t)\delta(t-b)dt\ =$$

$$\lim_{\varepsilon\to 0}[H(+\varepsilon)f(b+\varepsilon) - H(-\varepsilon)f(b-\varepsilon)]$$

Since the limit of H(t) as t approaches zero from left is zero, and the limit of H(t) as t approaches zero from right is 1, we will have

$$\# \lim_{\varepsilon\to 0} \int_{b-\varepsilon}^{b+\varepsilon} f(t)\delta(t-b)dt \;=\; \lim_{\varepsilon\to 0} [H(+\varepsilon)f(b+\varepsilon) = H(0^{+})f(b^{+}) = f(b)$$

We have an additional useful theorem immediately follows from the filtering property; for all finite values of a: $\int_{-\infty}^{\infty} f(x)\delta'(x-a)dx = -f'(a)$

Where the prime indicates differentiation with respect to x. In general, for the n^{th} $derivative$ of the Dirac delta, we will have

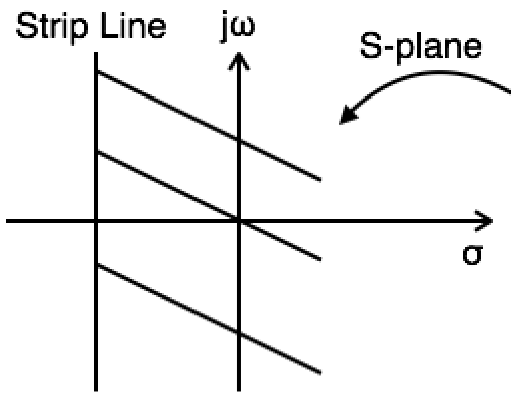
$$\int_{-\infty}^{\infty} f(x)\delta^{(n)}(x-a)dx = (-1)^{n} f^{(n)}(a)$$

What is Region of Convergence (ROC) for Laplace Transform?

The range of variation of σ for which the Laplace transform converges is called region of convergence

It's properties:

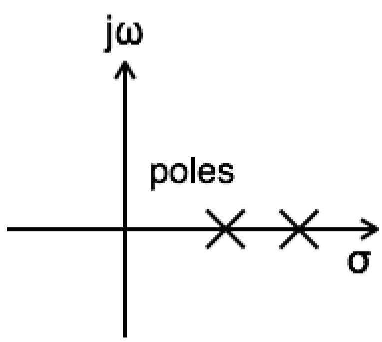
ROC contains strip lines parallel to jω axis in s-plane



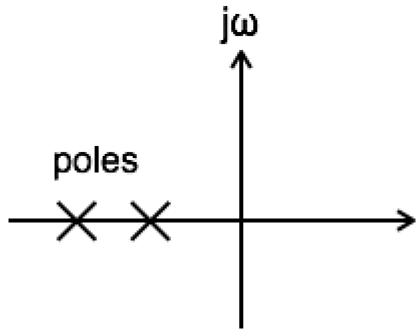
- If x(t) is absolutely integral and it is of finite duration, then ROC is entire s-plane.
- If x(t) is a right sided sequence then ROC: Re{s} > σ_0
- If x(t) is a left sided sequence then ROC: Re{s} < σ_0
- If x(t) is a two-sided sequence then ROC is the combination of two regions.

Causality and Stability

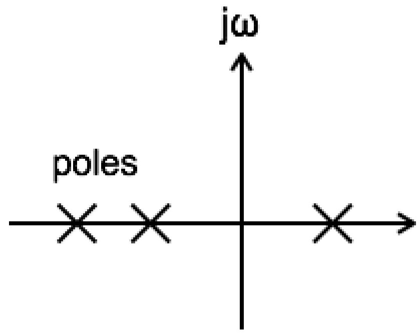
- For a system to be casual, all poles of its transfer function must be right half of s-plane.



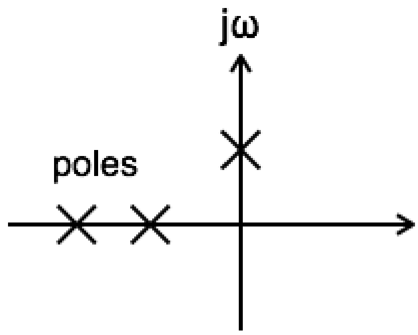
- For a system to be stable, all poles of its transfer function must be left half of s-plane.



- A system is said to be unstable when at least one pole of its transfer function is shifted to the right half of s-plane



- A system is said to be marginally stable when at least one pole of its transfer function lies on the jω axis of s-plane



ROC of basic functions

$f(t)$	$L(f(t)) = F(s)$	ROC
$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\} > 0$
$tu(t)$	$\frac{1}{s^2}$	$\mathrm{Re}\{s\} > 0$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\} > 0$
$e^{at}\, u(t)$	$\frac{1}{s-a}$	$\mathrm{Re}\{s\} > a$
$e^{-at}\, u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\} > -a$
$e^{at}\, u(-t)$	$-\frac{1}{s-a}$	$\mathrm{Re}\{s\} < a$
$e^{-at}\, u(-t)$	$-\frac{1}{s+a}$	$\mathrm{Re}\{s\} < -a$
$te^{at}\, u(t)$	$\frac{1}{(s-a)^2}$	$\mathrm{Re}\{s\} > a$
$t^n e^{at}\, u(t)$	$\frac{n!}{(s-a)^{n+1}}$	$\mathrm{Re}\{s\} > a$
$te^{-at}\, u(t)$	$\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} > -a$
$t^n e^{-at}\, u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\} > -a$
$te^{at}\, u(-t)$	$-\frac{1}{(s-a)^2}$	$\mathrm{Re}\{s\} < a$
$t^n e^{at}\, u(-t)$	$-\frac{n!}{(s-a)^{n+1}}$	$\mathrm{Re}\{s\} < a$
$te^{-at}\, u(-t)$	$-\frac{1}{(s+a)^2}$	$\mathrm{Re}\{s\} < -a$
$t^n e^{-at}\, u(-t)$	$-\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\} < -a$

Here we will talk about The Inverse of Laplace Transform

In the inverse Laplace transform, we are provided with the transform F(s) and asked to find what function we have initially. The inverse transform of the function F(s) is given by: $f(t) = L^{-1}\{F(s)\}$

To compute the inverse transform

1)By using the standard function table

$F(s)$	$f(t)$	s
$\frac{1}{s}$	1	s>0
$\frac{1}{s^2}$	t	s>0
$\frac{n!}{s^{n+1}}$	at n = 1,2, 3… t^n	s>0
$\frac{1}{(s-a)}$	$e^{\{at\}}$	s>a
$\frac{s}{(s^2+a^2)}$	(at)cos	s>0
$\frac{a}{(s^2+a^2)}$	sin(at)	s>0
$\frac{s-a}{(s-a)^2+b^2}$	$e^{\{at\}}$ cos(bt)	s>a
$\frac{b}{(s-a)^2-b^2}$	$e^{\{at\}}$ sin(bt)	s>a

2)By using the partial fraction

-This method used in case there's more than one bracket in the denominator of the function F(s) As it easily to find the equivalent partial fraction to the given function F(s) ,So we use this method to find each fraction separately

3)By using the 1^{st} shifting property

and $L^{-1}\{F(s+k)\} = e^{-kt}f(t)L^{-1}\{F(s-k)\} = e^{kt}f(t)$

-This method used in case every s written in form (s - k) OR (s + k) ,To get the inverse we forget The slope and get the inverse of the function without it then multiply this inverse by e^{kt} OR e^{-kt}

4)By using the 2^{nd} shifting property

$L^{-1}\{e^{-as}\,F(s)\} = f(t-a)u(t-a)$

-This method used in case every s written in form $e^{-as}\,F(s)$,To get the inverse we forget e^{-as} and get the inverse of $F(s) \to f(t)$ and put t-a instead of t in the function $f(t)$ then we multiply the result by $u(t-a)$

5)By using multiplication by t property

$L^{-1}\left\{-\frac{dF(s)}{ds}\right\} = f(t)t$

-This method used in case if we differentiation of the function F(s) with respect to s produce Function in s easily to get it's inverse

6)By using divison by t property

$$L^{-1}\left\{\int_{s}^{\infty} F(s)\ ds\right\} = \frac{f(t)}{t}$$

-This method used in case if we integrate the function F(s) with respect to s from (s to ∞) produce Function in s easily to get it's inverse

7)By using transformation of integrals property

$L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(t)dt$

-This method used in case the function given in form $\frac{F(s)}{s}$, Firstly we get the inverse of $F(s) \to f(t)$ then integrate the function F(s) with respect to t from (0 to t) "By increasing the number of power of s by n times we do the integration n times"

8)Evaluation of L^{-1} by using the convolution theorem

Definition of Convolution :

" The convolution of the two function f_1 and f_2 denoted by $f_1 * f_2$, is given by the following integral : $f_1 * f_2 = \int_0^t f_1(u) f_2(t-u) du$

Notice that: $f_1 * f_2 = f_2 * f_1$ So " $\int_0^t f_1(u) f_2(t-u) du = \int_0^t f_1(t-u) f_2(u) du$

The Convolution Theorem:

$$L\{f_1 * f_2\} = L\{ \int_0^t f_1(u) f_2(t-u) du\}$$

$$= L\{ \int_0^t f_1(t-u) f_2(u) du\}$$

$$L\{f_1(t)\} . L\{ f_2(t)\} = F_1(s).\ F_2(s)$$

The property:

$$L^{-1}\{ F(s)\} = L^{-1}\{F_1(s).\ F_2(s)\} = f_1 * f_2$$

$$= \int_0^t f_1(u) f_2(t-u) du \ \rightarrow 1$$

$$= \int_0^t f_1(t-u) f_2(u) du \rightarrow 2$$

This Theorem used to calculate L^{-1} For function F(s) In case The 3 conditions available :

1- We can put the given function as result of multiplying 2 functions

$$F(s) = F_1(s).\ F_2(s)$$

2- Calculation of L^{-1} for each function is easy

$$f_1(t) = L^{-1}\{ F_1(s)\}\ and\ f_2(t) = L^{-1}\{F_2(s)\}$$

3- by Finding $f_1(t)$, $f_2(t)$ We can integrate 1 OR 2 easily to find The required inverse

9)Evaluation of L^{-1} by using the binomial theorem

In this case the function is expressed as a binomial expansion with a form that allow us to find L^{-1} for each term in the expansion

"We use this method if the previous properties fail to find L^{-1} "

Applications of Laplace Transform

- Breaking down complex differential equations into simpler polynomial forms.
- Simplifies calculations in system modeling
- Gives information about steady as well as transient states
- In machine learning, used for making predictions and making analysis in data mining
- Analysis of electronic circuits (solve quickly differential equations occurring in the analysis of electronic circuits.)
- Solving digital signal processing problems
- Make easy to study analytic part of Nuclear physics possible to get the true form of radioactive
- Process Control (helps to analyze he variables which when altered, produce desired manipulations in the result)
- Solve I.V.P (initial value problem)
- Solution of integral and integrodifferential equations
- Solution for system of simultaneous equations
- In Mechanical engineering (used to solve differential equations occurring in mathematical modeling of mechanical system to find
- In Data mining ((which is the analysis step of Knowledge Discovery in Databases) focuses on the discovery of (previously) unknown properties on the data. Where the Laplace equation is used to determine the prediction and to analyse the step of knowledge in databases.)
- In Integrated circuit (Laplace Transformations helps to find out the current and some criteria for analyzing the circuits. It is used to build required ICs and chips for systems. So it plays a vital role in the field of computer science.)

Here we will use programming languages to solve

1- MATLAB

- Laplace Transform
- Inverse of Laplace Transform

2- Python

"Note: download and install **sympy** library On the PC to be able to run the code"

- Laplace Transform

Sources

- www. byjus.com
- A Brief Introduction To Laplace Transformation (Dr. Daniel S.Stutts)
- International Research Journal of Engineering and Technology
- Laplace Transforms: Theory, Problems, and Solutions (Marcel B.finan)
- https://en.wikipedia.org/wiki/Laplace_transform#Examples_and_applications

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At last thanks for reading the project, I hope it to be useful and cover all the most important points that help in understanding **The Laplace Transform.** For us, it was useful and enthusiastic to search about it to write this project.