

Laplace transform

Little about Laplace Transform



December 5, 2022

science

MATH232 PROJECT

**What is the Laplace Transform?**

A function is said to be a piecewise continuous function if it has a finite number of breaks and it does not blow up to infinity anywhere.

Let us assume that the function f(t) is a piecewise continuous function, then f(t) is defined using the Laplace transform. The Laplace transform of a function is represented by L{f(t)} or F(s). Laplace transform helps to solve the differential equations, where it reduces the differential equation into an algebraic problem

Laplace form L {f (t)} = (1)

The key thing to note is that Equation (1) is not a function of time, but rather a function of the  
Laplace variable s = x + iy. Also, the Laplace transform only transforms functions defined over the  
interval [0, ∞), so any part of the function which exists at negative values of t is lost! One of the most useful Laplace transformation theorems is the differentiation theorem where it reduces the differential equation into an algebraic problem.

**Theorem 1** The Laplace transform of the first derivative of a function f is given by

L{}=sF(s) -

Here we will proof

L{}=

=

# = sF(s) -

By repeating the integration by parts, higher derivatives may be similarly transformed. Thus given  
A ̈x + B ̇x + Cx = f (t)

We have by taking the Laplace Transform of both sides

A (X (s) − sx () − ̇x ()) + B (sX (s) − x ()) + CX (s) = F (s)

**Properties of Laplace Transform**

Some of the Laplace transformation properties are:

If f1(t) ⟷ F1(s) and [note: ⟷ implies Laplace Transform]

f2(t) ⟷ F2(s), then

|  |  |
| --- | --- |
| A f1(t) + B f2(t) ⟷ A F1(s) + B F2(s) | Linearity Property |
| es0t f(t)) ⟷ F(s – s0) | Frequency Shifting Property |
| t∫0 f(λ) dλ ⟷ 1⁄s F(s) | Integration |
| T f(t) ⟷ (−d F(s)⁄ds) | Multiplication by Time |
| f(t) e−at ⟷ F(s + a) | Complex Shift Property |
| f(-t) ⟷ F(-s) | Time Reversal Property |
| f(t⁄a) ⟷ a F(as) | Time Scaling Property |

**How can we calculate Laplace Transform?**

1. Multiply the given function i.e. ƒ(t) by , Where S is a complex number such that S = X + iY.
2. Integrate this product with respect to the time (t) by taking limits as 0 and ∞.

**This process results in Laplace transformation of ƒ(t), and is denoted by F(t).**

**𝑇able of basic functions after transformation:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***No.*** | ***f(t)*** | ***L(f(t)) = F(s)*** | ***No.*** | ***f(t)*** | ***L(f(t)) = F(s)*** |
| *1* | 1 |  | *11* |  |  |
| *2* | at n = 1,2, 3… |  | *12* | , at p>-1 |  |
| *3* | √(t) |  | *13* | at n = 1, 2... |  |
| *4* |  |  | *14* | (at) |  |
| *5* |  |  | *15* | (at) |  |
| *6* |  |  | *16* | (at+b) |  |
| *7* |  |  | *17* | (at) |  |
| *8* |  |  | *18* |  |  |
| *9* | f(t) | F(s-c) | *19* | f(t) at n = 1,2, 3... | (-1)n s |
| *10* | f'(t) | sF(s) – f(0) | *20* | f”(t) | s2F(s) − sf(0) − f'(0) |

**What about if the function is discontinuous forcing?**

**It's not a problem as we can solve it by**

**1- Heaviside (Step Function)**

Which it's defines by

i.e. The Heaviside step function is named Oliver Heaviside

Or Here if the function is impulsive forcing

**2- The Dirac delta is another important function (or distribution)**

which is often used to represent it Which it's defines by

"Hence, the Heaviside step function “turns on” at the right edge (t = ), and the Dirac delta function turns on and off at the same place. An additional property of the Dirac delta function is

The Area under the curve defined by the Dirac delta, the unit- step and the Dirac delta function are derivative and anti-derivative of one another

***i.e. Both the unit-step and Dirac delta belong to a class of functions called generalized function***

For regular functions, this fact not withstanding however we may define the following Relationships

**The Laplace transform of the Dirac delta function is revealed by taking the Laplace transform of the equation ():**

L{= L{} =

= H(t)

= 0 #

***In Dirac delta function we have additional property called ("Filtering property")***

***for a < b < c.***

***Here we can proof it as follows***

By integration by parts we will let we have

The last term in the brackets in previous equation vanishes because H(t) is zero in the interval of integration. Thus, we will have

Since the limit of H(t) as t approaches zero from left is zero, and the limit of H(t) as t approaches zero from right is 1, we will have

#

We have an additional useful theorem immediately follows from the filtering property; for all finite values of a:

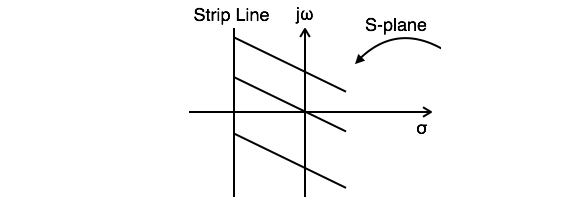
***Where the prime indicates differentiation with respect to x. In general, for the of the Dirac delta, we will have***

**What is Region of Convergence (ROC) for Laplace Transform?**

The range of variation of σ for which the Laplace transform converges is called region of convergence

It's properties:

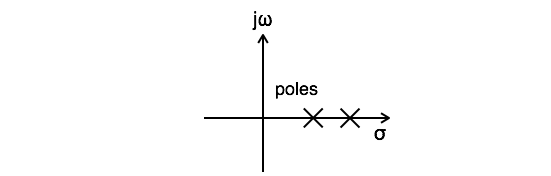
ROC contains strip lines parallel to jω axis in s-plane



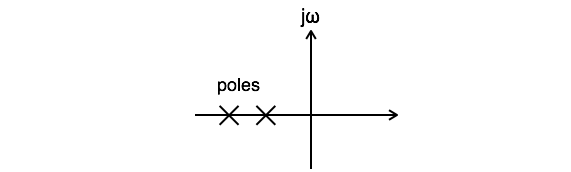
* If x(t) is absolutely integral and it is of finite duration, then ROC is entire s-plane.
* If x(t) is a right sided sequence then ROC: Re{s} >
* If x(t) is a left sided sequence then ROC: Re{s} <
* If x(t) is a two-sided sequence then ROC is the combination of two regions.

**Causality and Stability**

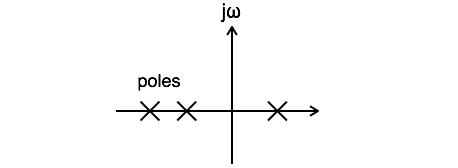
* For a system to be casual, all poles of its transfer function must be right half of s-plane.



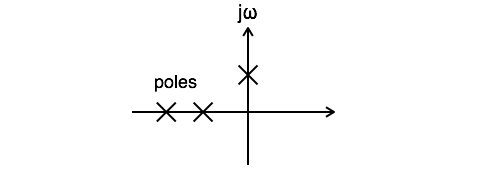
* For a system to be stable, all poles of its transfer function must be left half of s-plane.



* A system is said to be unstable when at least one pole of its transfer function is shifted to the right half of s-plane



* A system is said to be marginally stable when at least one pole of its transfer function lies on the jω axis of s-plane



**ROC of basic functions**

|  |  |  |
| --- | --- | --- |
|  |  | ROC |
|  |  | Re{s} > |
|  |  | Re{s} > |
|  |  | Re{s} > |
|  |  | Re{s} > |
|  |  | Re{s} > |
|  |  | Re{s} < |
|  |  | Re{s} < |
|  |  | Re{s} > |
|  |  | Re{s} > |
|  |  | Re{s} > |
|  |  | Re{s} > |
|  |  | Re{s} < |
|  |  | Re{s} < |
|  |  | Re{s} < |
|  |  | Re{s} < |

**Here we will talk about The Inverse of Laplace Transform**

In the inverse Laplace transform, we are provided with the transform F(s) and asked to find what function we have initially. The inverse transform of the function F(s) is given by:

To compute the inverse transform we will use this table

|  |  |  |
| --- | --- | --- |
|  |  |  |
| s>0 | 1 |  |
| s>0 | t |  |
| s>0 | at n = 1,2, 3… |  |
| s>a |  |  |
| s>0 | (at) |  |
| s>0 |  |  |
| s>a |  |  |
| s>a |  |  |

**Applications of Laplace Transform**

* Breaking down complex differential equations into simpler polynomial forms.
* Simplifies calculations in system modeling
* Gives information about steady as well as transient states
* In machine learning, used for making predictions and making analysis in data mining
* Analysis of electronic circuits (solve quickly differential equations occurring in the analysis of electronic circuits.)
* Solving digital signal processing problems
* Make easy to study analytic part of Nuclear physics possible to get the true form of radioactive
* Process Control (helps to analyze he variables which when altered, produce desired manipulations in the result)

**Here we will use programming languages to solve**

1. MATLAB

* Laplace Transform:

<https://github.com/Mohammedbih/Math_232-Project/blob/main/LT.M>

* Inverse of Laplace Transform:

https://github.com/Mohammedbih/Math\_232-Project/blob/main/ILT.M

1. Python

"Note: download and install **sympy** library On the PC to be able to run the code"

* Laplace Transform:

https://github.com/Mohammedbih/Math\_232-Project/blob/main/LT.py

**Sources**

* www. byjus.com
* A Brief Introduction To Laplace Transformation (Dr. Daniel S.Stutts)
* International Research Journal of Engineering and Technology

**Team Work**

1. *Radwa Rahoma Abdallah 2027212*
2. *Mohamed Sayed Fahim 2027228*
3. *Mohamed Sayed Mohamed 2027184*
4. *Omar Walid Omar 2027067*

At last thanks for reading the project, I hope it to be useful and cover all the most important points that help in understanding **The Laplace Transform.** For us, it was useful and enthusiastic to search about it to write the project.