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 Foundations of Computer Science  
 Homework #2 - Chapter 1: DFA, NFA  
 10/6/2016

### 1.3

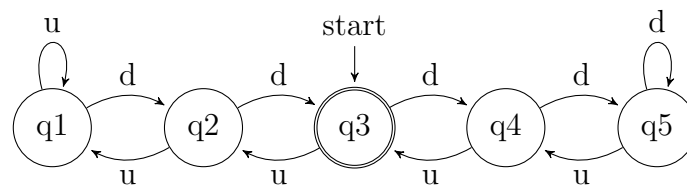
The formal description of a DFA  $M$  is  $\{q1, q2, q3, q4, q5\}$ ,  $\{u, d\}$ ,  $\delta$ ,  $q3$ ,  $\{q3\}$ , where  $\delta$  is given by the following table:

|    | u  | d  |
|----|----|----|
| q1 | q1 | q2 |
| q2 | q1 | q3 |
| q3 | q2 | q4 |
| q4 | q3 | q5 |
| q5 | q4 | q5 |

Initial state is  $q3$ , so that is where the machine will start.

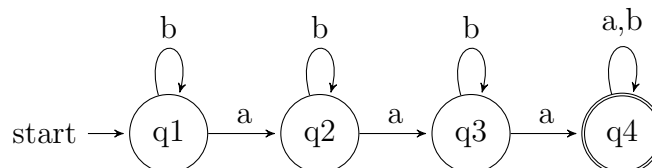
We can use the table to create the nodes, and connect them as needed.

The accepted state is  $q3$  so we will mark that with a double circle to show that as the accepted state of the machine.

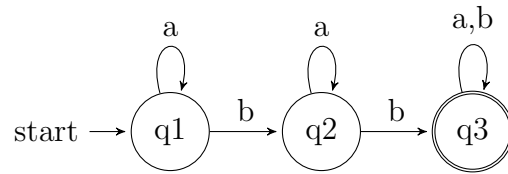


1.4: a, c, e, f, g. For all parts,  $\Sigma = \{a, b\}$

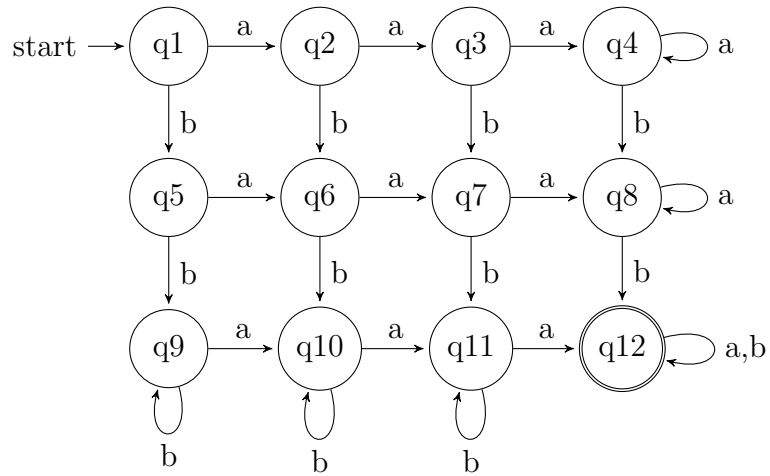
a.  $\{w \mid w \text{ has at least three a's}\}$



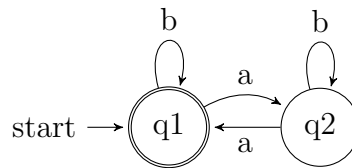
$\{ w \mid w \text{ has at least two b's} \}$



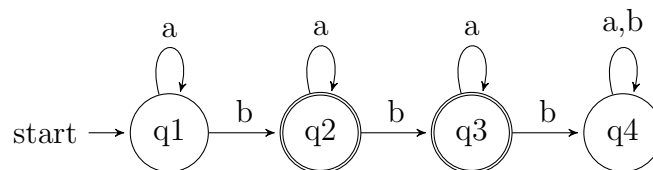
$\{ w \mid w \text{ has at least three a's and at least two b's} \}$



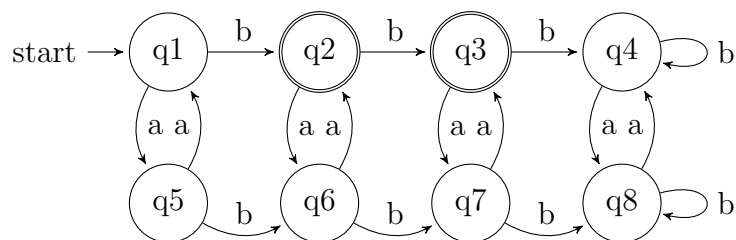
c.  $\{ w \mid w \text{ has an even number of a's} \}$



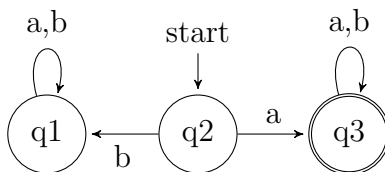
$\{ w \mid w \text{ has one or two b's} \}$



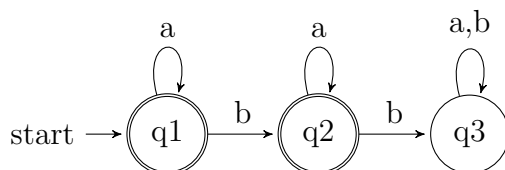
$\{ w \mid w \text{ has an even number of a's and one or two b's} \}$



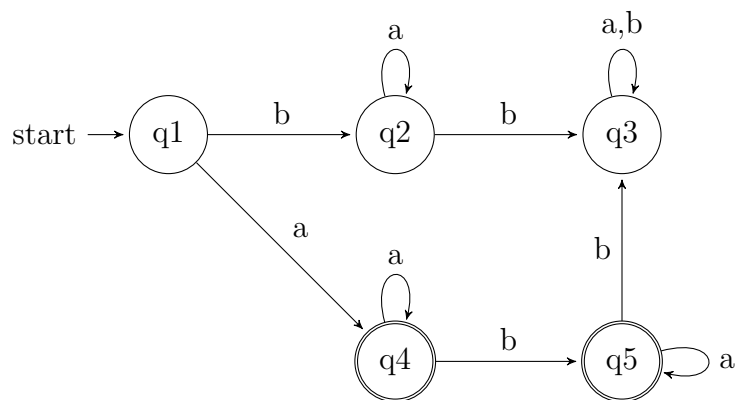
e.  $\{ w \mid w \text{ starts with an a} \}$



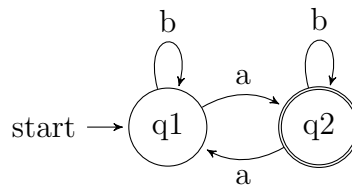
$\{ w \mid w \text{ has at most one b} \}$



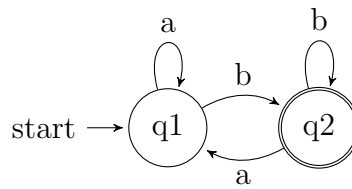
$\{ w \mid w \text{ starts with an a and has at most one b} \}$



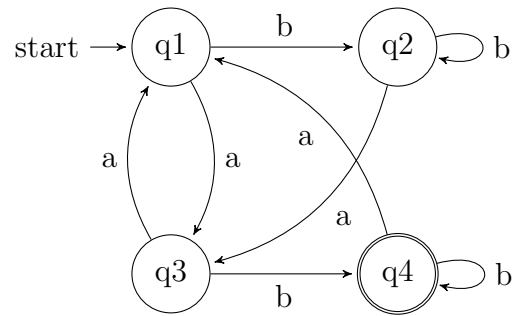
f.  $\{ w \mid w \text{ has an odd number of a's} \}$



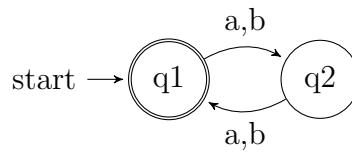
$\{ w \mid w \text{ ends with a b} \}$



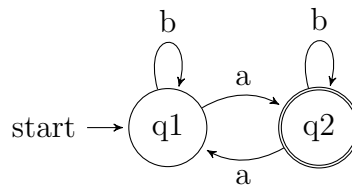
$\{ w \mid w \text{ has an odd number of a's and ends with a b} \}$



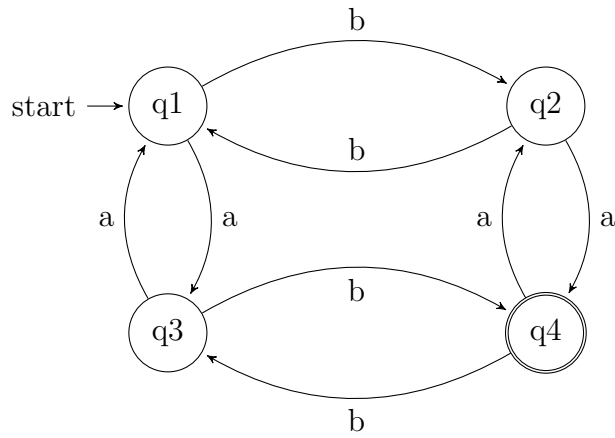
g.  $\{ w \mid w \text{ has even length} \}$



$\{ w \mid w \text{ has an odd number of a's} \}$



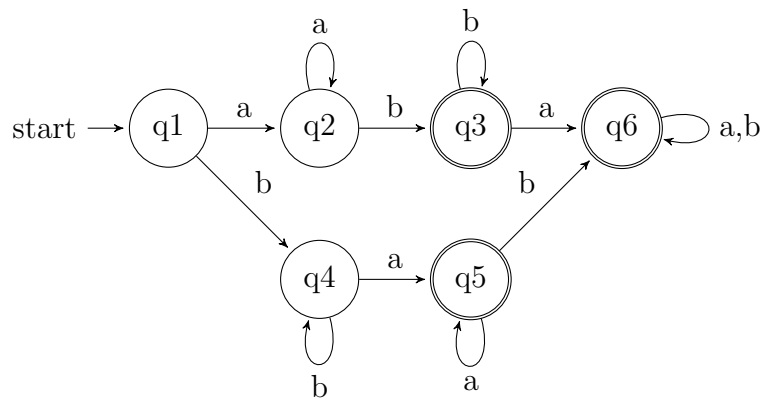
$\{ w \mid w \text{ has even length and an odd number of a's } \}$



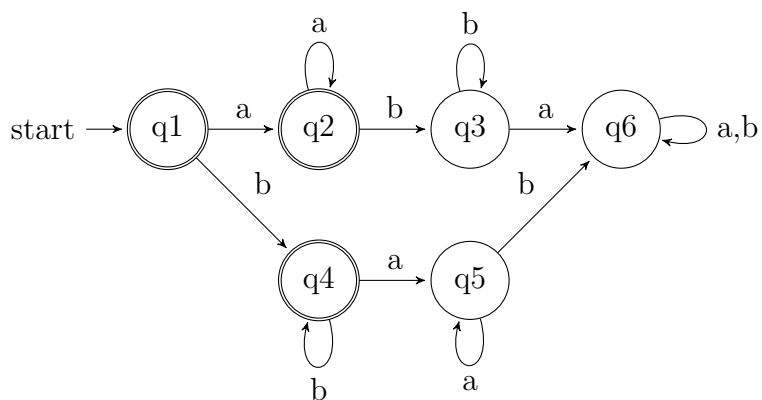
1.5: c, d, e, f, g, h. For all parts,  $\Sigma = \{a, b\}$

c.  $\{ w \mid w \text{ contains neither the substrings } ab \text{ nor } ba \}$

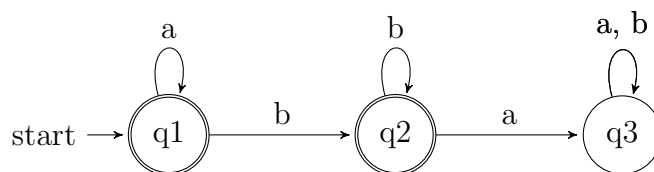
Simpler language



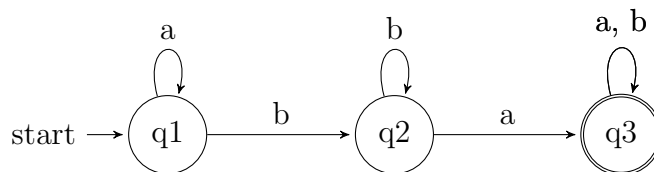
Language given.



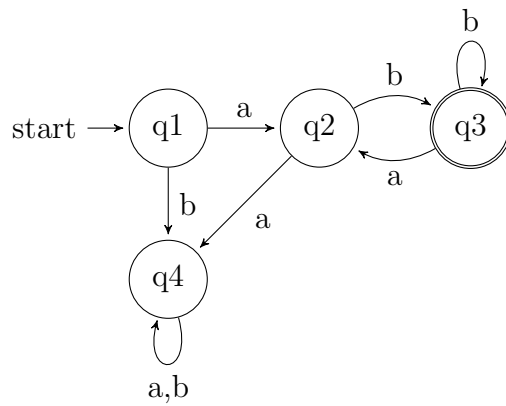
- d.  $\{ w \mid w \text{ is any string not in } a^*b^* \}$   
 Simpler language



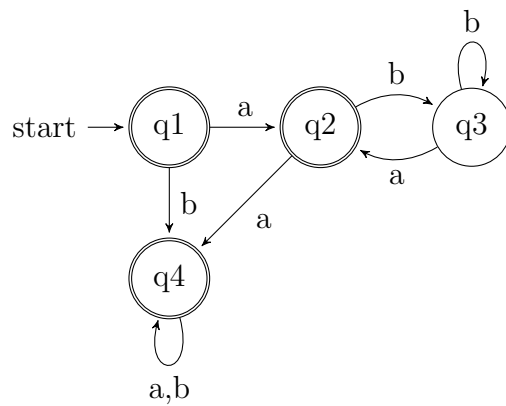
Language given



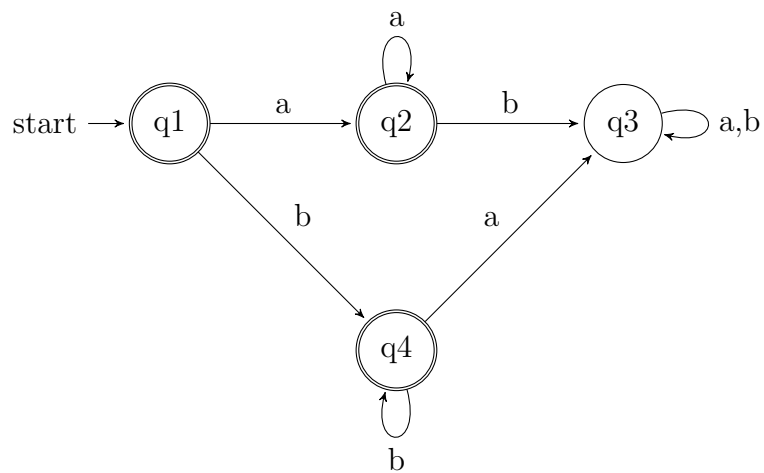
- e.  $\{ w \mid w \text{ is any string not in } (ab^+)^* \}$   
 Simpler language



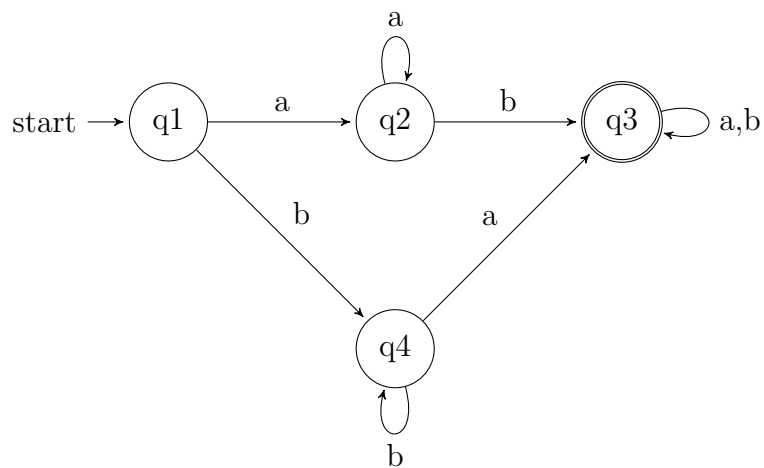
Language given



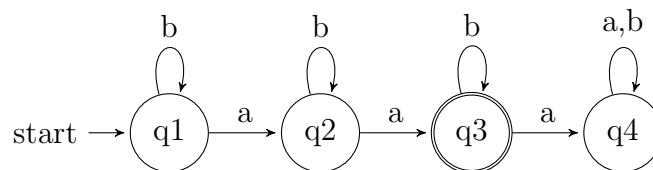
f.  $\{ w \mid w \text{ is any string not in } a^* \cup b^* \}$   
 Simpler language



Language given

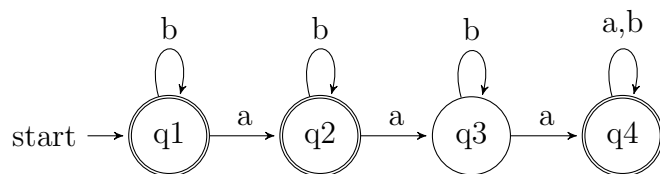


g.  $\{ w \mid w \text{ is any string that doesn't contain exactly two a's} \}$   
 Simpler language

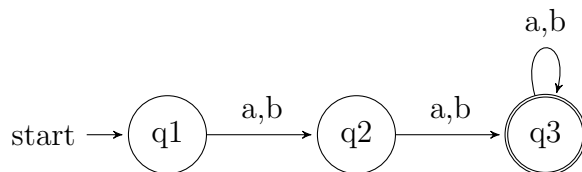




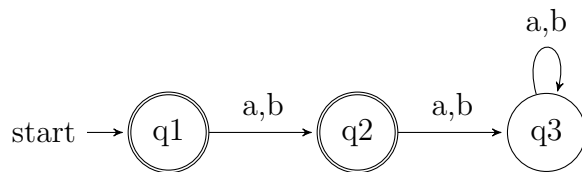
Language given



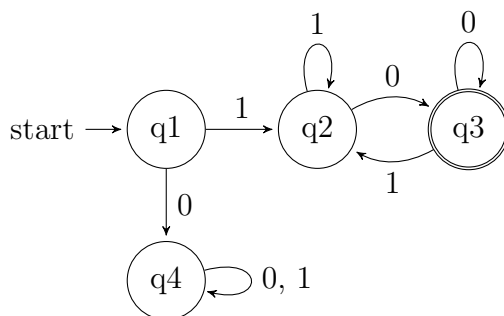
h.  $\{ w \mid w \text{ is any string except } a \text{ and } b \}$   
 Simpler language



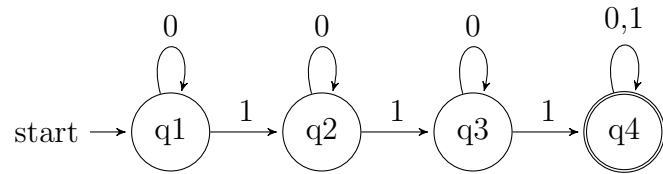
Language given



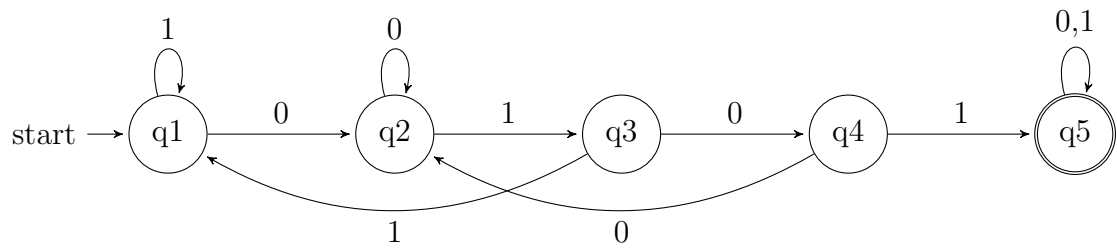
1.6: a, b, c, d, e, f, g, h, I, j, k, l, m, n  
 a.  $\{ w \mid w \text{ begins with a } 1 \text{ and ends with a } 0 \}$



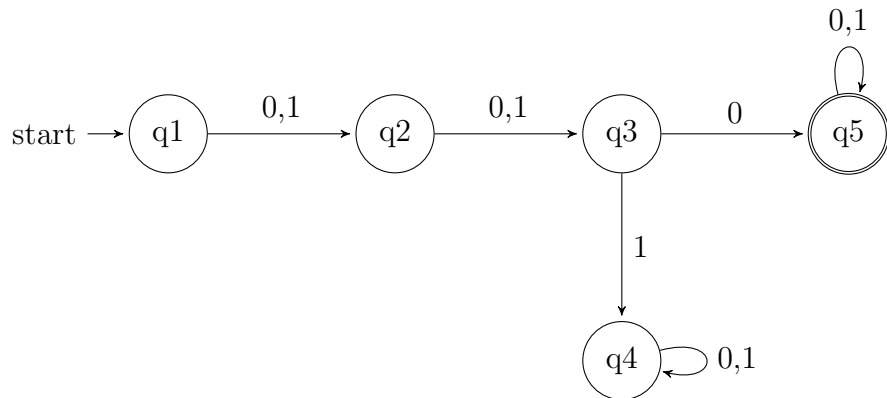
b.  $\{w \mid w \text{ contains at least three 1s}\}$



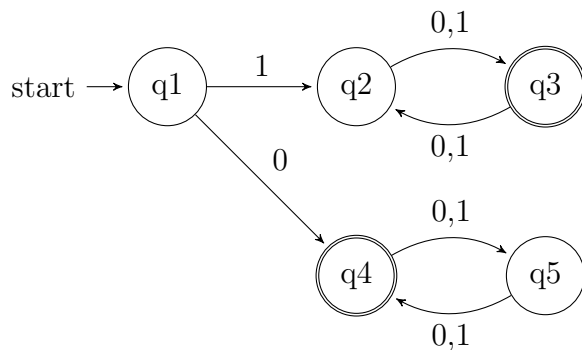
c.  $\{w \mid w \text{ contains the substring } 0101 \text{ (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$



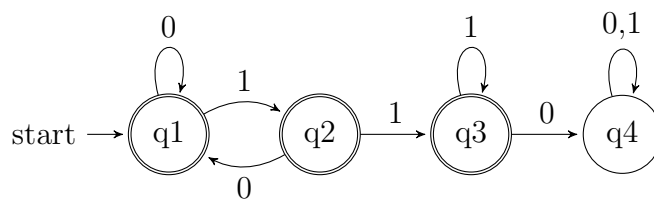
d.  $\{w \mid w \text{ has length at least 3 and its third symbol is a 0}\}$



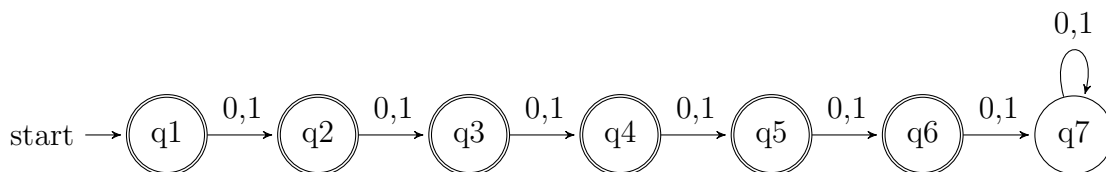
e.  $\{w \mid w \text{ starts with } 0 \text{ and has odd length, or starts with } 1 \text{ and has even length}\}$



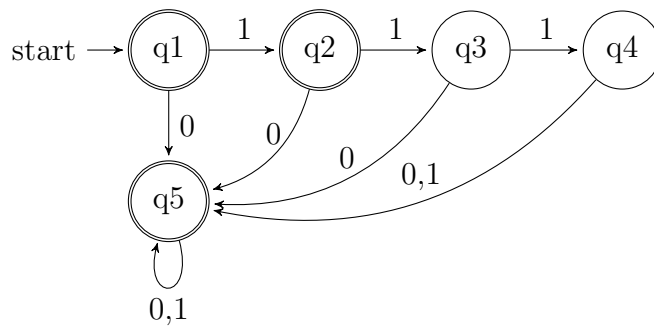
f.  $\{w \mid w \text{ doesn't contain the substring } 110\}$



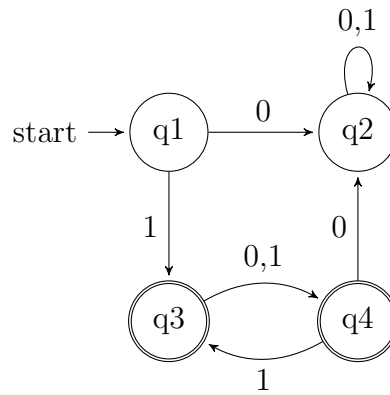
g.  $\{w \mid \text{the length of } w \text{ is at most } 5\}$



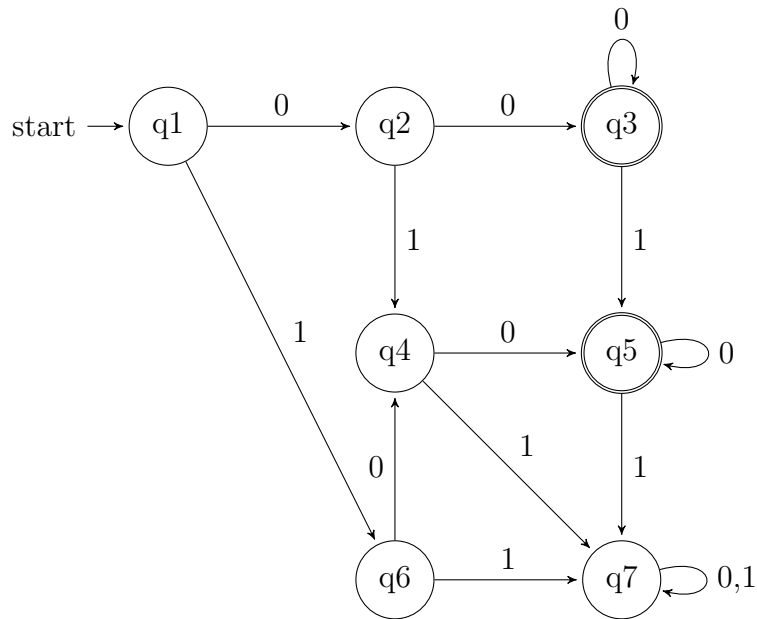
h.  $\{w \mid w \text{ is any string except } 11 \text{ and } 111\}$



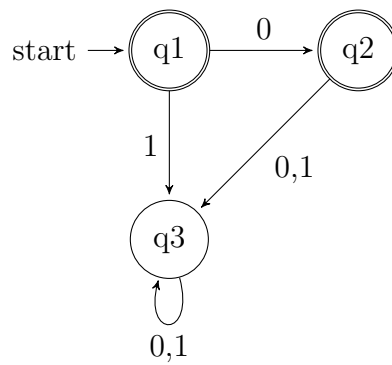
I.  $\{w \mid \text{every odd position of } w \text{ is a } 1\}$



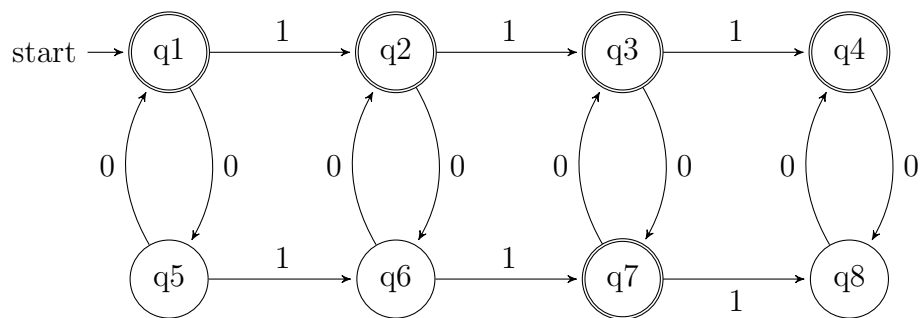
j.  $\{w \mid w \text{ contains at least two 0s and at most one 1}\}$



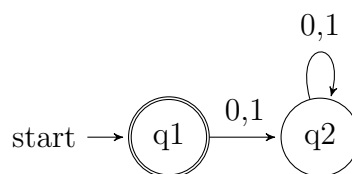
k.  $\{e, 0\}$



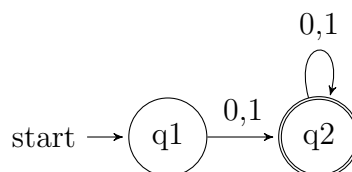
L.  $\{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s}\}$



m. The empty set



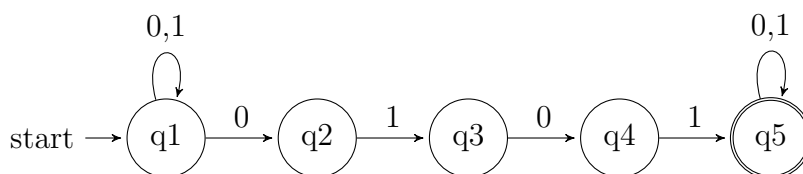
n. All strings except the empty string



1.7: b, c, d, e, g, h

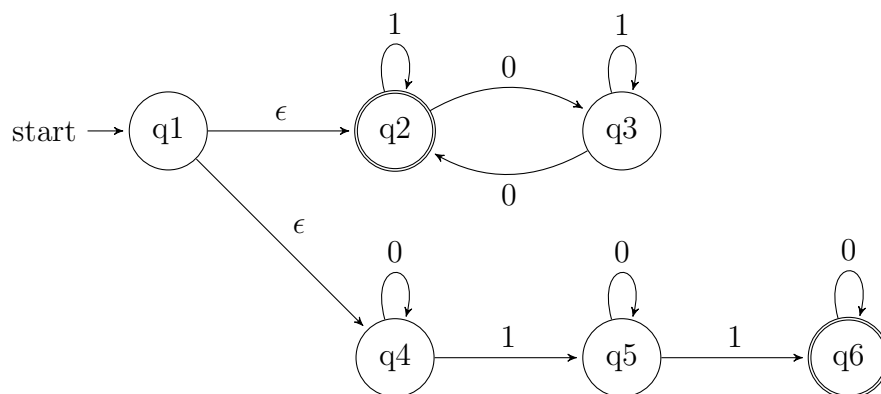
b. The language of Exercise 1.6l with six states

$\{w \mid w \text{ contains the substring } 0101 \text{ (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$

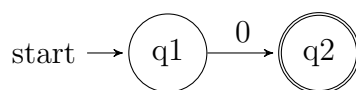


c. The language of Exercise 1.6l with six states

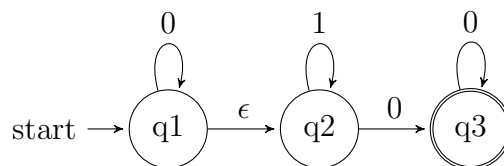
$\{w \mid w \text{ contains an even number of 0s, or contains exactly two 1s}\}$



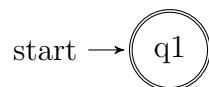
d. The language 0 with two states



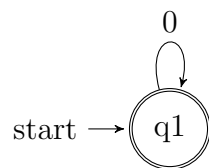
e. The language  $0^*1^*0^+$  with three states



g. The language  $\{\epsilon\}$  with one state



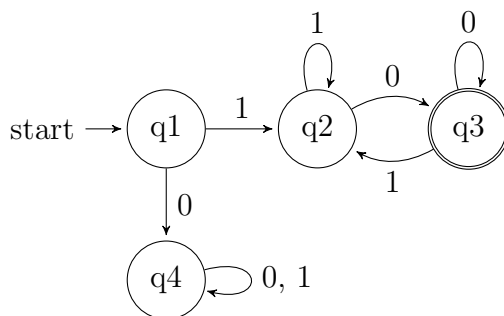
h. The language  $0^*$  with one state



1.8: a, b

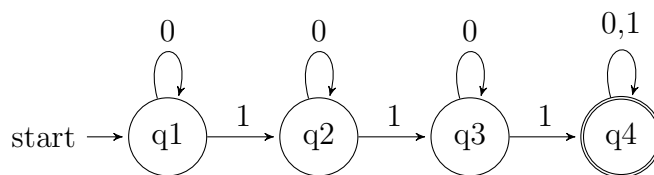
Use the construction in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages described in

1.8 a. Exercise 1.6a  $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$

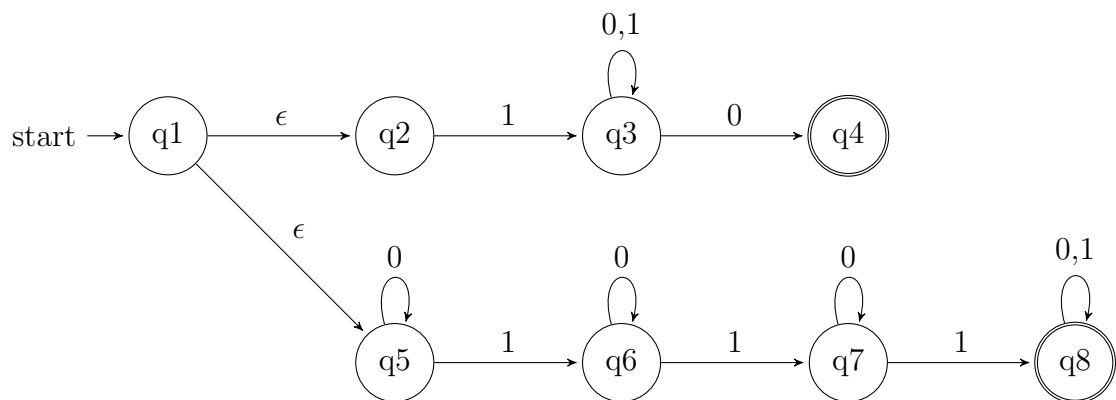


Exercise 1.6b

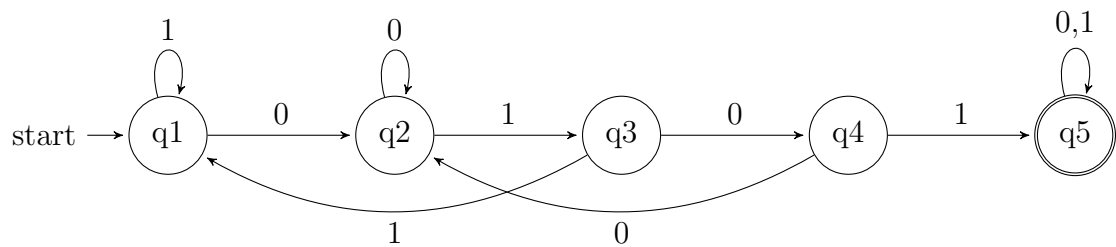
$\{w \mid w \text{ contains at least three 1s}\}$



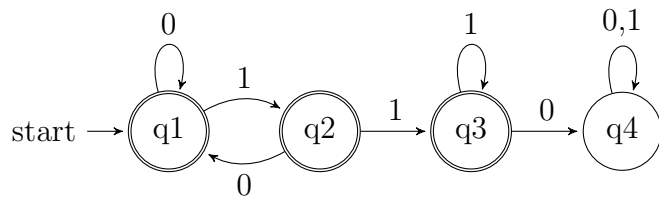
Exercise 1.6a and Exercise 1.b joined



1.8 b. Exercises 1.6c and 1.6f  
Exercises 1.6c

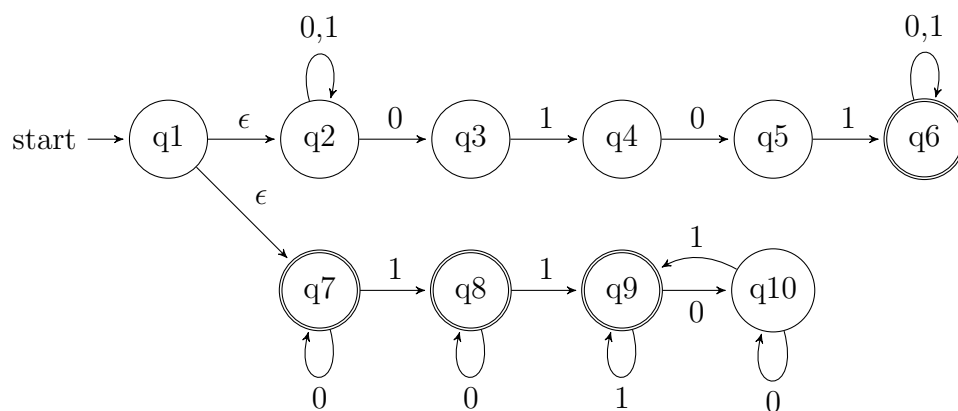


Exercises 1.6f





Exercises 1.6c and Exercises 1.6f joined

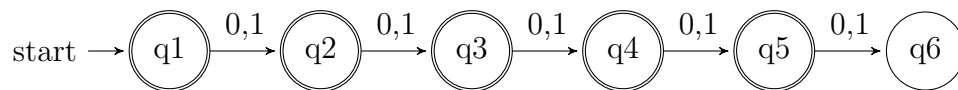


1.9: a, b

Use the construction in the proof of Theorem 1.47 to give the state diagrams of NFAs recognizing the concatenation of the languages described in

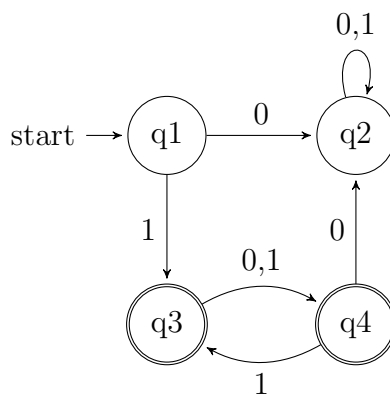
a. Exercises 1.6g and 1.6i

Exercise 1.6g

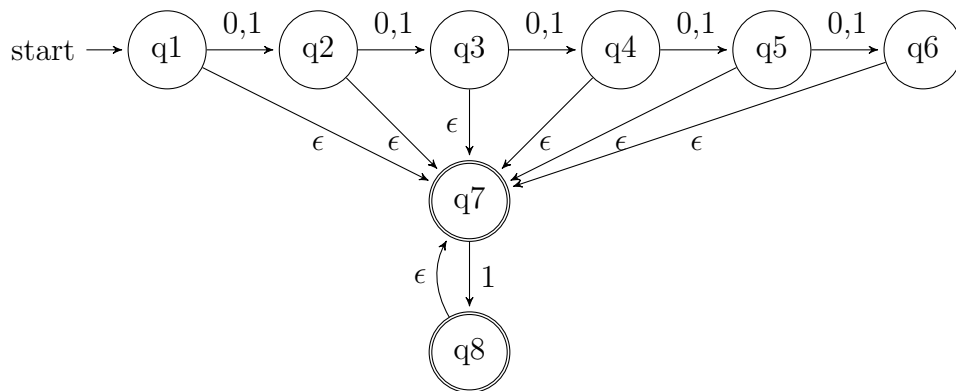


Exercise 1.6i

$\{w \mid \text{every odd position of } w \text{ is a } 1\}$



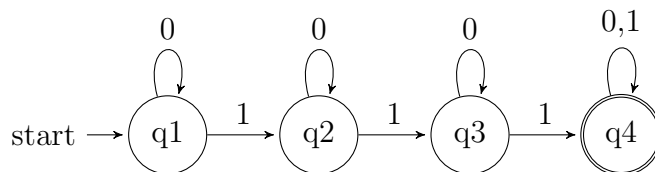
Exercise 1.6g and Exercise 1.6i joined



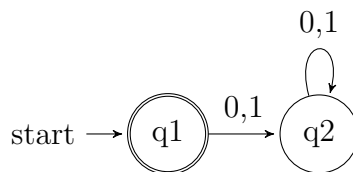
b. Exercises 1.6b and 1.6m

Exercise 1.6b

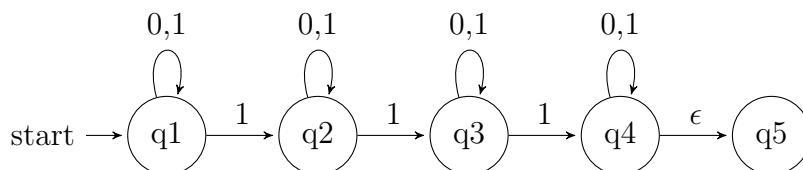
$\{w \mid w \text{ contains at least three 1s}\}$



Exercise 1.6m - The empty set



Exercises 1.6b and 1.6m joined

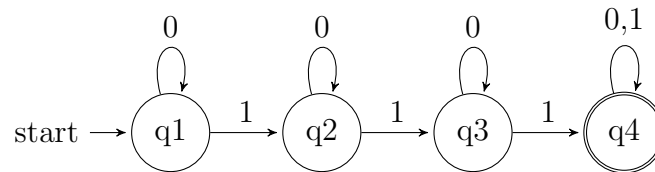


1.10: a, b, c

Use the construction in the proof of Theorem 1.49 to give the state diagrams of NFAs recognizing the star of the languages described in

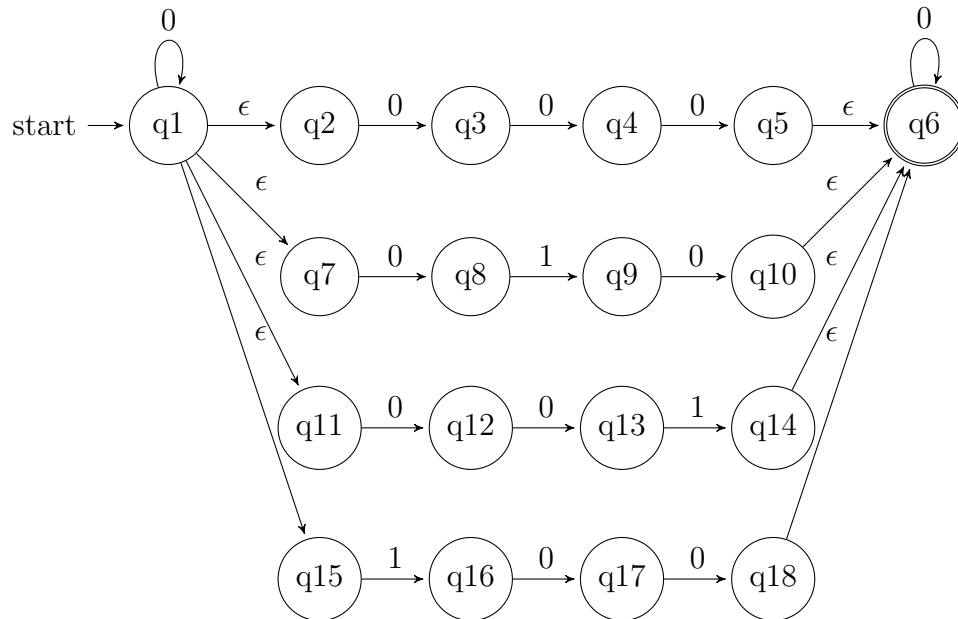
a. Exercise 1.6b

$\{w \mid w \text{ contains at least three 1s}\}$

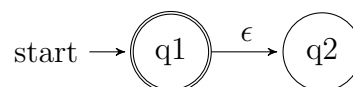


b. Exercise 1.6j

$\{w \mid w \text{ contains at least two 0s and at most one 1}\}$



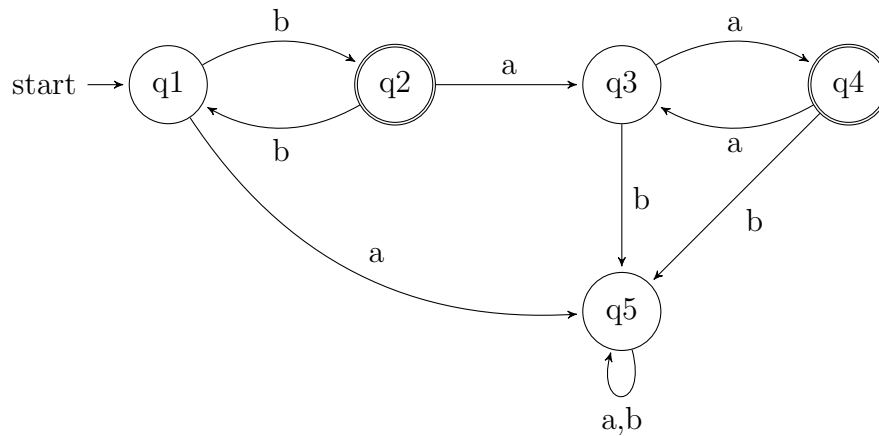
c. Exercise 1.6m



1.12

Let  $D = \{w \mid w \text{ contains an even number of a's and an odd number of b's and does not contain the substring } ab\}$ . Give a DFA with five states that recognizes  $D$  and a regular expression that generates  $D$ .

(Suggestion: Describe  $D$  more simply.)



$R1 = \{w \mid w \text{ contains an even number of a's}\}$

$R1 = (aa)^*$

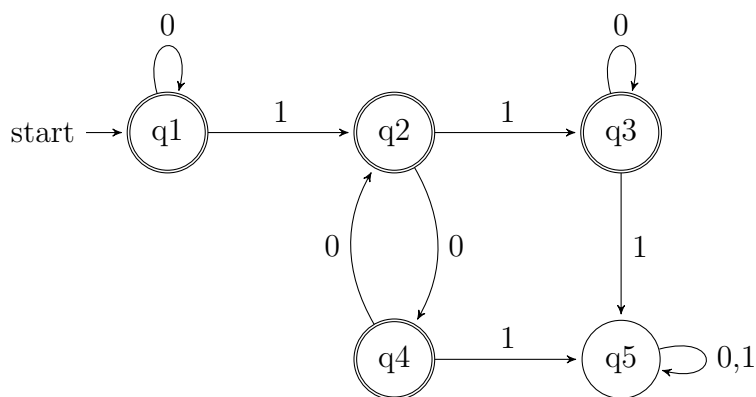
$R2 = \{w \mid w \text{ contains an odd number of b's and does not contain the substring } ab \}$

$R2 = b(bb)^*$

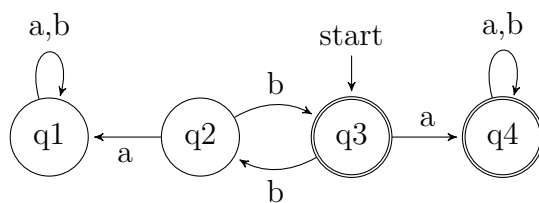
$R = b(bb)^*(aa)^*$

1.13

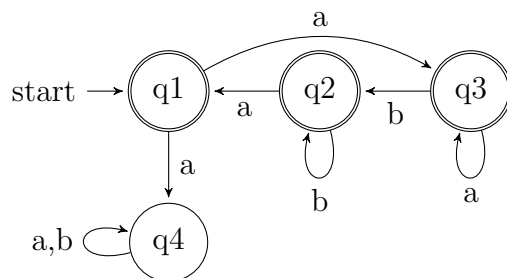
Let  $F$  be the language of all strings over  $0,1$  that do not contain a pair of 1s that are separated by an odd number of symbols. Give the state diagram of a DFA with five states that recognizes  $F$ .



1.16 a.

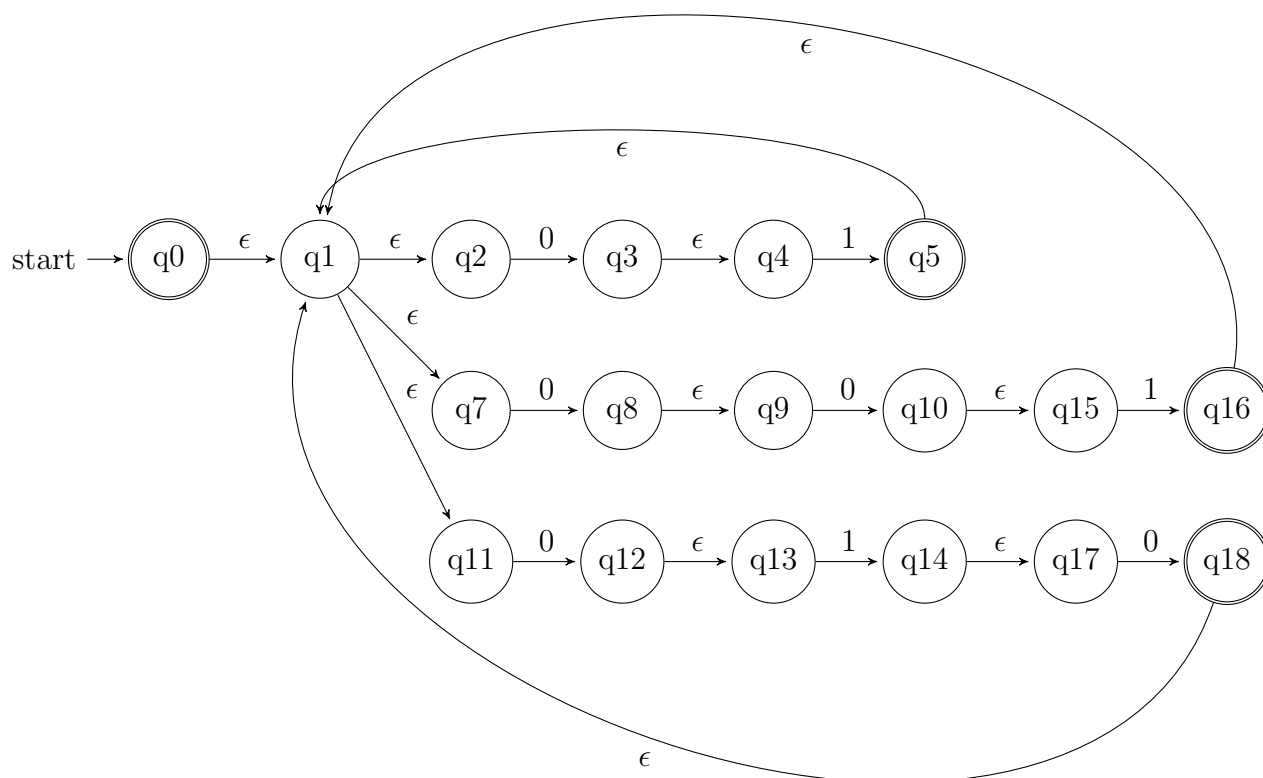


b.

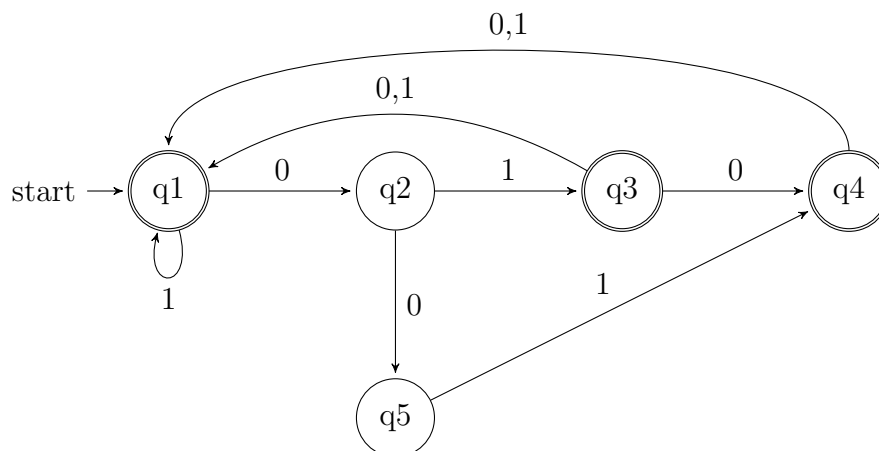


1.17: a, b

a. Give an NFA recognizing the language  $(01 \cup 001 \cup 010)^*$ .



- b. Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.



- 1.18 a.  $\{w \mid w \text{ begins with a 1 and ends with a 0}\}$

$$1 \Sigma^* 0$$

- b.  $\{w \mid w \text{ contains at least three 1s}\}$

$$\Sigma^* 1 \Sigma^* 1 \Sigma^* 1 \Sigma^*$$

- c.  $\{w \mid w \text{ contains the substring 0101 (i.e., } w = x0101y \text{ for some } x \text{ and } y)\}$

$$\Sigma^* 0101 \Sigma^*$$

- d.  $\{w \mid w \text{ has length at least 3 and its third symbol is a 0}\}$

$$\Sigma \Sigma 0 \Sigma^*$$

- e.  $\{w \mid w \text{ starts with 0 and has odd length, or starts with 1 and has even length}\}$

$$(0 \cup 1 \Sigma)(\Sigma \Sigma)^*$$

- f.  $\{w \mid w \text{ doesn't contain the substring 110}\}$

$$0^* (10^+)^* 1^*$$

g.  $\{w \mid \text{the length of } w \text{ is at most } 5\}$

$$(\epsilon \cup \Sigma)^5$$

h.  $\{w \mid w \text{ is any string except } 11 \text{ and } 111\}$

$$\epsilon \cup \Sigma \cup 0 \Sigma \cup 10 \cup 0 \Sigma \Sigma \cup 10 \Sigma \cup 110 \cup \Sigma^3 \Sigma^+$$

I.  $\{w \mid \text{every odd position of } w \text{ is a } 1\}$

$$(1 \Sigma)^*(\epsilon \cup 1)$$

j.  $\{w \mid w \text{ contains at least two } 0\text{s and at most one } 1\}$

$$00^+ \cup 100^+ \cup 0^+10^+ \cup 00^+1$$

k.  $\{", 0\}$

$$0 \cup \epsilon$$

L.  $\{w \mid w \text{ contains an even number of } 0\text{s, or contains exactly two } 1\text{s}\}$

$$1^*(01^*01^*) \cup 0^*10^*10^*$$

m. The empty set

$$\emptyset$$

n. All strings except the empty string

$$\Sigma^+$$

1.20: a, b, c, d, e, f, g, h

a. Members: ab, abb

Not members: ba, bba

b. Members: ab, ababab

Not members: aba, bab

c. Members: aaa, bbb

Not members: aabb, bbaa



d. Members: aaa, aaaaaa

Not members: a, aaaa

e. Members: aba, bbbaabaabb

Not members: a, b

f. Members: aba, bab

Not members: ababab, ba

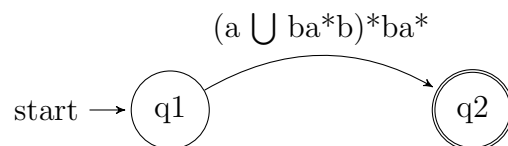
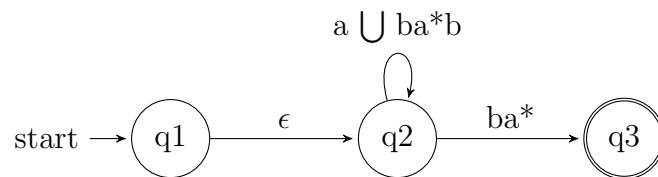
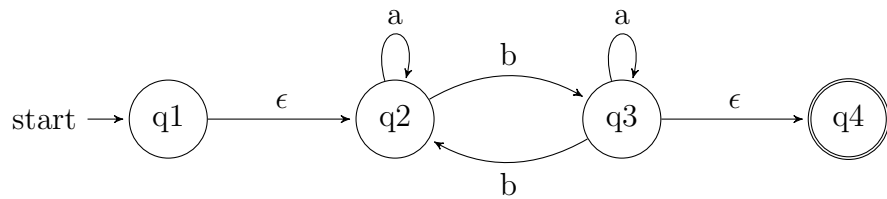
g. Members: b, ab

Not members: a, ba

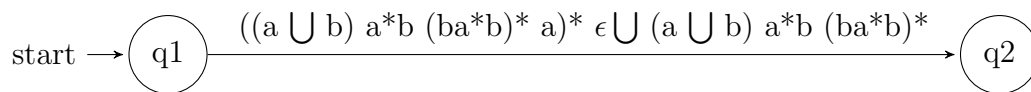
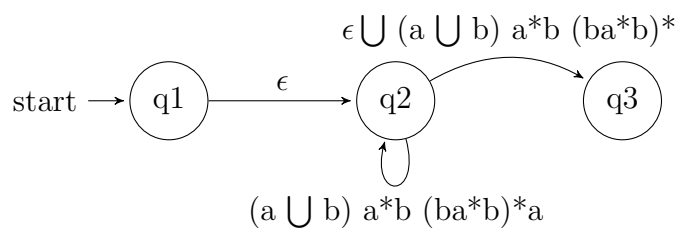
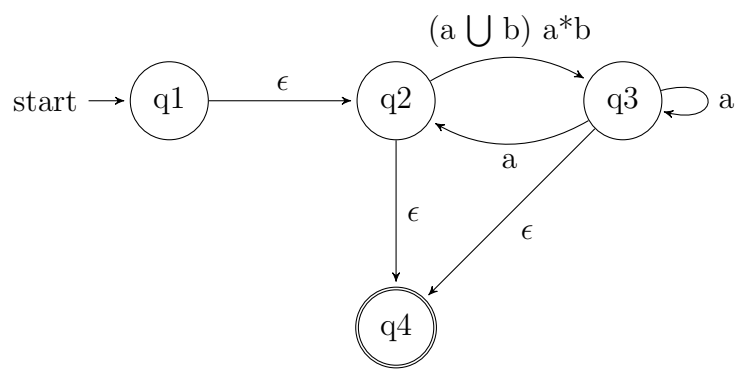
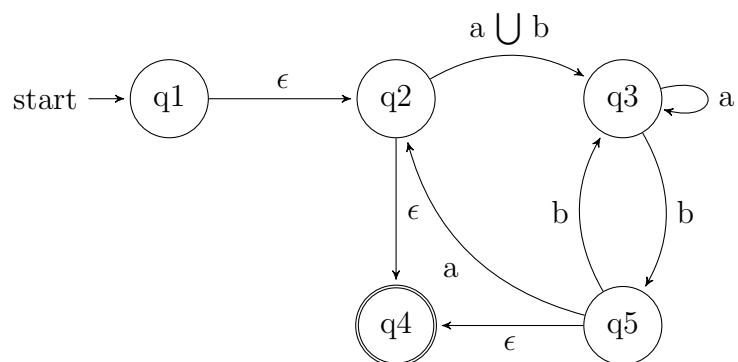
h. Members: a, bbab

Not members: b,  $\epsilon$

1.21 a.

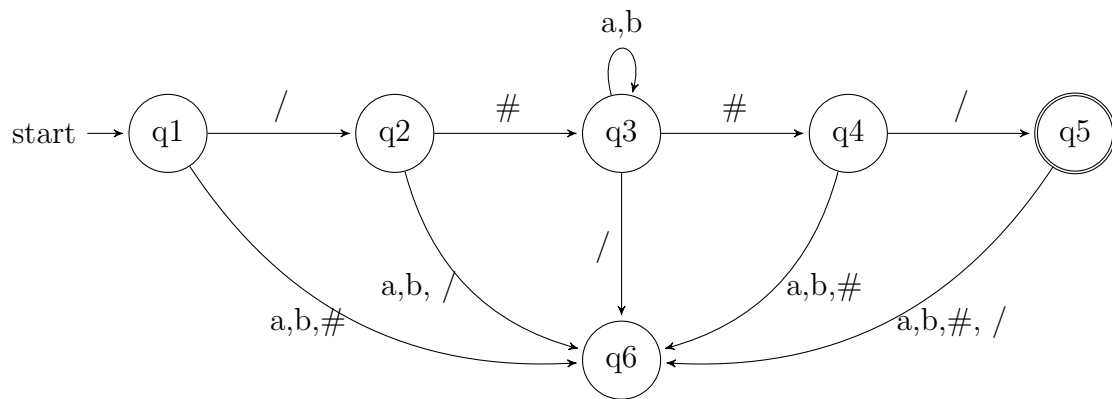


b.

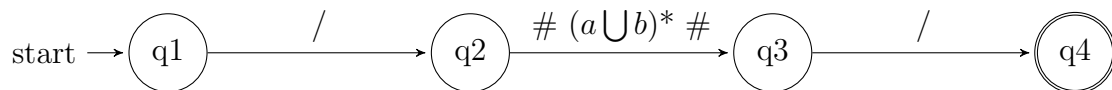
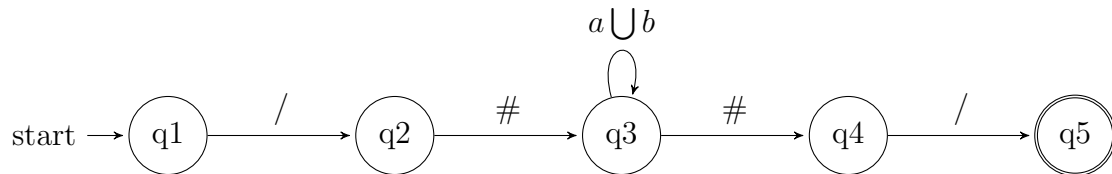
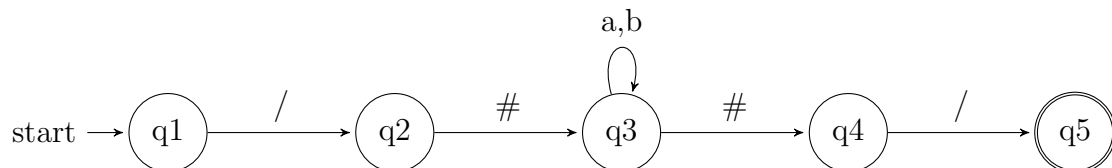


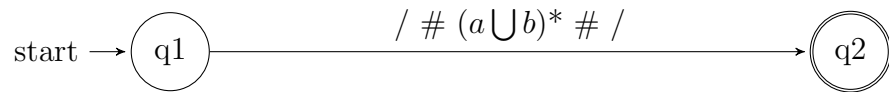
1.22 In certain programming languages, comments appear between delimiters such as  $/\#$  and  $\#/\#$ . Let  $C$  be the language of all valid delimited comment strings. A member of  $C$  must begin with  $/\#$  and end with  $\#/\#$  but have no intervening  $\#/\#$ . For simplicity, assume that the alphabet for  $C$  is  $\Sigma = \{a, b, /, \#\}$ .

a. Give a DFA that recognizes  $C$ .



b. Give a regular expression that generates  $C$ .





The regular expression is:  $/ \# (a \cup b)^* \# /$