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Foundations of Computer Science
Homework #1 - Basics + Using Latex
Problems: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
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0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

$a.$ $\{1, 3, 5, 7, ...\}$
This is a set of all real odd numbers.

$b.$ $\{..., -4, -2, 0, 2, 4, ...\}$
This is a set of all positive and negative even numbers.

$c.$ $\{n|\ n\ =\ 2m\ for\ some\ m\ in\ N\}$
This is a set of all even positive numbers.

$d.$ $\{n|\ n\ =\ 2m\ for\ some\ m\ in\ N,\ and\ n\ =\ 3k\ for\ some\ k\ in\ N\}$
This is a set of all positive numbers which are also a multiple of six.

$e.$ $\{w|\ w\ =\ is\ a\ string\ of\ 0s\ and\ 1s\ and\ w\ equals\ the\ reverse\ of\ w\}$
This is a set of all binary palindrome strings, which are strings that are the same when read forward or backwards.

$f.$ $\{n|\ n\ is\ an\ integer\ and\ n\ =\ n\ +\ 1\}$
This is an empty set: ø

0.2 Write formal descriptions of the following sets.

a. The set containing the numbers 1, 10, and 100
$\{1, 10, 100\}$

b. The set containing all integers that are greater than 5
$\{n \in \mathbb{Z} \mid n > 5\}$

c. The set containing all natural numbers that are less than 5
$\{n \in \mathbb{N} \mid n < 5\}$

d. The set containing the string aba
$\{aba\}$

e. The set containing the empty string
$\{\epsilon\}$

f. The set containing nothing at all
$\{\o\}$

0.3 Let A be the set {x, y, z} and B be the set {x, y}
a. Is A a subset of B?
No, A is not a subset of B because "z" is not in B, and to be a subset of another set all elements of A must be contained in B.

b. Is B a subset of A?
Yes, B is a subset of A because all elements in B are contained in A.

c. What is A ∪ B?
$A \cup B = \{x, y, z\}$

d. What is A ∩ B?
$A \cap B = \{x, y\}$

e. What is A × B?
$A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$

f. What is the power set of B?
$P(B) = \{\o, \{x\}, \{y\}, \{x, y\}\}$

0.4 If A has a elements and B has b elements, how many elements are in A × B? Explain your answer

For every element in A, there are B ordered pairs. This means that there will be A * B elements

0.5 If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

If $|C| = c$, then $P(C) = 2^C$. This is because if a set has n members, then the power set will have 2^n members.

As an example, consider set A: {1, 2, 3}:

subsets: {1}, {2} , {3}, {1,2}, {1,3}, {2,3} as well as {1,2,3} and the empty set of {}

When we combine all of these sets, we get the powerset:
$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

There are 3 elements in set A with 8, and $2^3 = 8$, which shows that $P(C) = 2^C$.

0.6

a. The value of $f(2)$ is 7.

b. The domain of f is X and the range is Y.

c. The value of $g(2, 10)$ is 6.

d. The domain of g is $X \times Y$ and the range is Y.

e. $g(4, f(4))$ is equal to $g(4, 7)$ which equals 8.

0.7 For each part, give a relation that satisfies the condition.

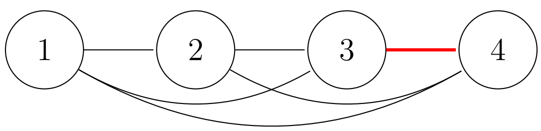
a. Reflexive and symmetric but not transitive
Set A contains $\{1, 2, 3\}$
$R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2)\}$

b. Reflexive and transitive but not symmetric
Set B contains $\{1, 2, 3\}$
$R = \{(1, 2), (2, 1), (2, 2), (1, 1)\}$

c. Symmetric and transitive but not reflexive
Set C contains 1, 2, 3
$R = \{(1,2),(2,1),(1,1),(2,2)\}$

0.8
Node 1 has a degree of 3
Node 2 has a degree of 3
Node 3 has a degree of 3
Node 4 has a degree of 3
Note: before we $3 \to 4$ was added, node 3
had a degree of 2 and node 4 has a degree of 2.

The graph looks like:



0.9 Write a formal description of the following graph.

G = (V, E) where V = {1, 2, 3, 4, 5, 6} and
E = {(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5),(3,6)}

Drawn in Latex this would be: