

Jason Downing  
 Email: jason\_downing@student.uml.edu  
 Foundations of Computer Science  
 Homework #2 - Chapter 1: DFA, NFA  
 10/6/2016

### 1.3

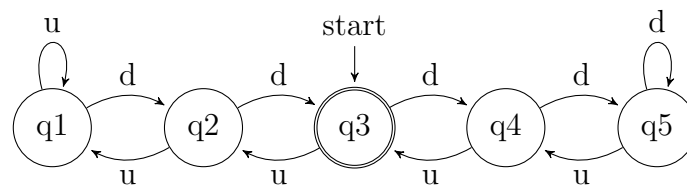
The formal description of a DFA  $M$  is  $\{q1, q2, q3, q4, q5\}$ ,  $\{u, d\}$ ,  $\delta$ ,  $q3$ ,  $\{q3\}$ , where  $\delta$  is given by the following table:

	u	d
q1	q1	q2
q2	q1	q3
q3	q2	q4
q4	q3	q5
q5	q4	q5

Initial state is  $q3$ , so that is where the machine will start.

We can use the table to create the nodes, and connect them as needed.

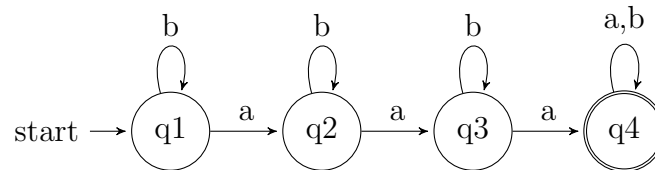
The accepted state is  $q3$  so we will mark that with a double circle to show that as the accepted state of the machine.



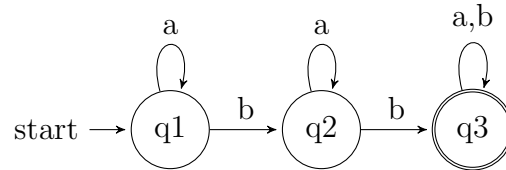
1.4: a, c, e, f, g

For all parts,  $\Sigma = \{a, b\}$

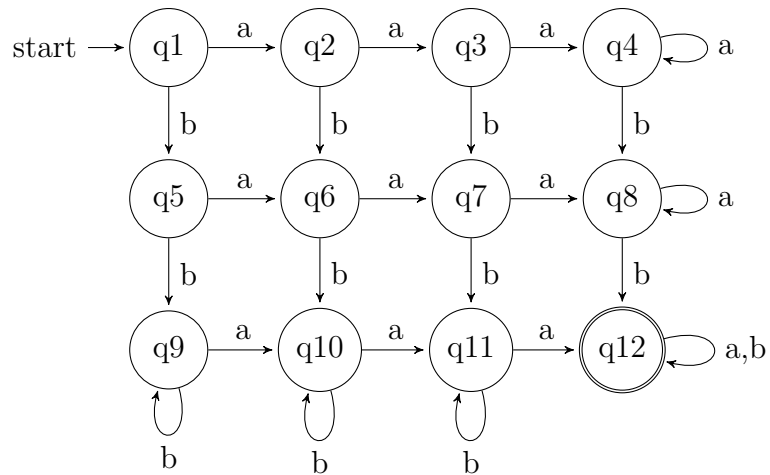
a.  $\{ w \mid w \text{ has at least three a's} \}$



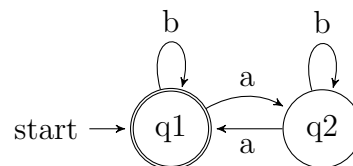
$\{ w \mid w \text{ has at least two b's} \}$



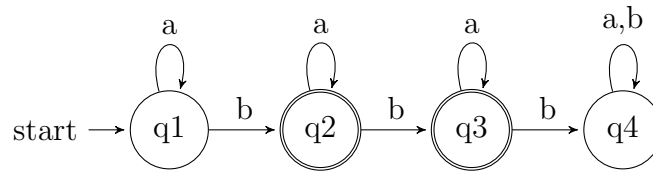
$\{ w \mid w \text{ has at least three a's and at least two b's} \}$



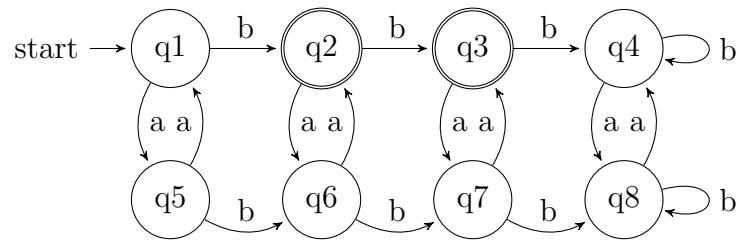
c.  $\{ w \mid w \text{ has an even number of a's} \}$



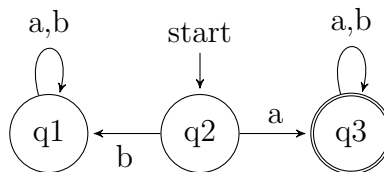
$\{ w \mid w \text{ has one or two b's} \}$



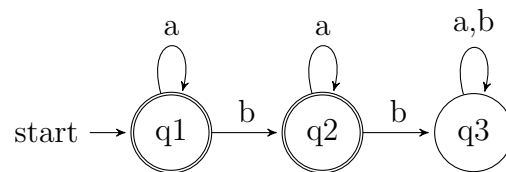
$\{ w \mid w \text{ has an even number of a's and one or two b's} \}$



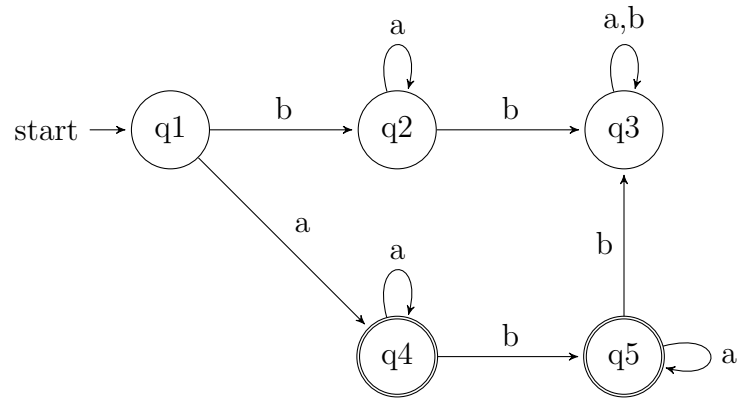
e.  $\{ w \mid w \text{ starts with an a} \}$



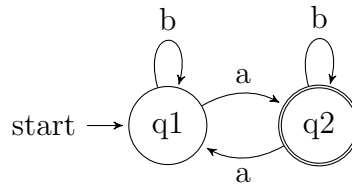
$\{ w \mid w \text{ has at most one b} \}$



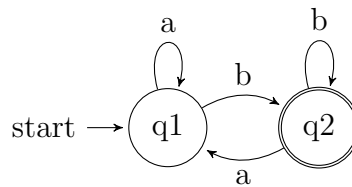
$\{ w \mid w \text{ starts with an } a \text{ and has at most one } b \}$



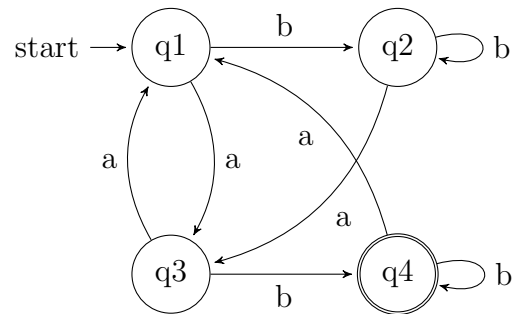
f.  $\{ w \mid w \text{ has an odd number of } a\text{'s} \}$



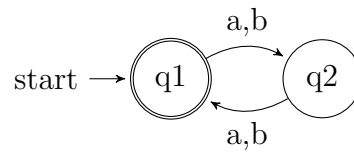
$\{ w \mid w \text{ ends with a } b \}$



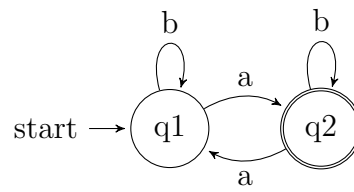
$\{ w \mid w \text{ has an odd number of } a\text{'s} \text{ and ends with a } b \}$



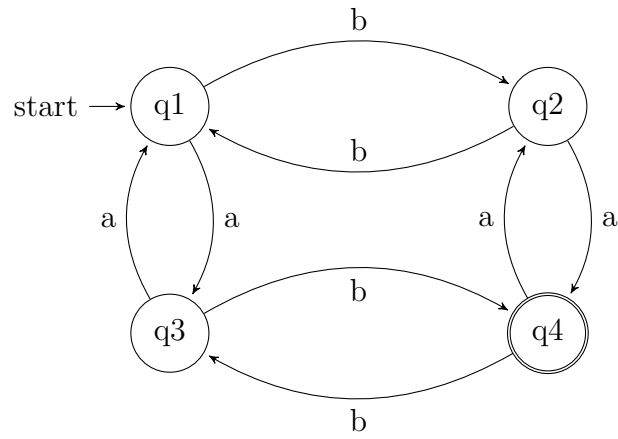
g.  $\{ w \mid w \text{ has even length} \}$



$\{ w \mid w \text{ has an odd number of a's} \}$



$\{ w \mid w \text{ has even length and an odd number of a's} \}$

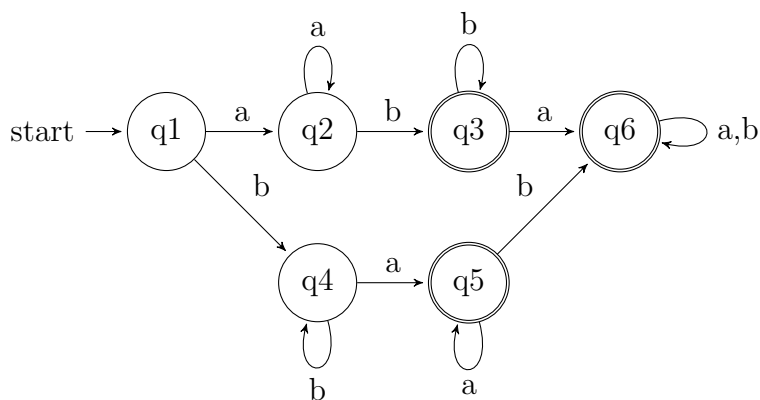


1.5: c, d, e, f, g, h

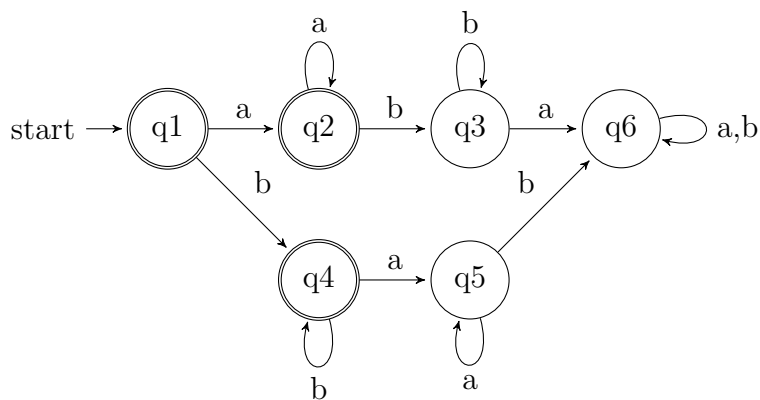
For all parts,  $\Sigma = \{a, b\}$

c.  $\{ w \mid w \text{ contains neither the substrings } ab \text{ nor } ba \}$

Simpler language

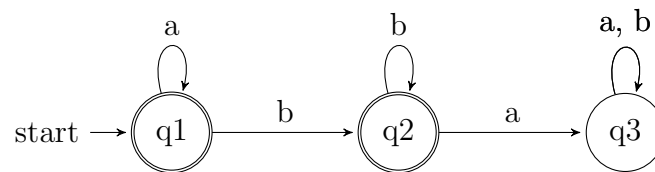


Language given.

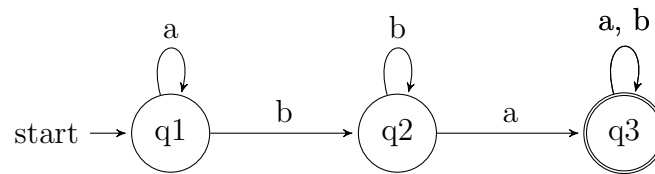


d.  $\{ w \mid w \text{ is any string not in } a^*b^* \}$

Simpler language

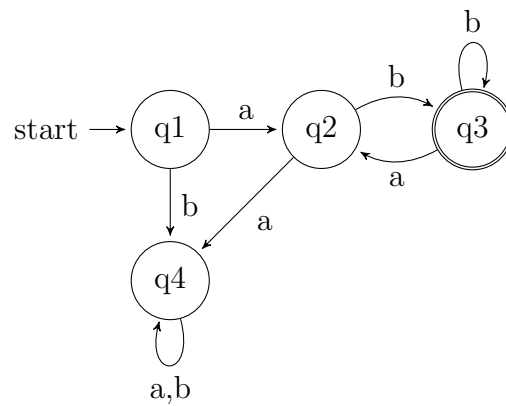


Language given

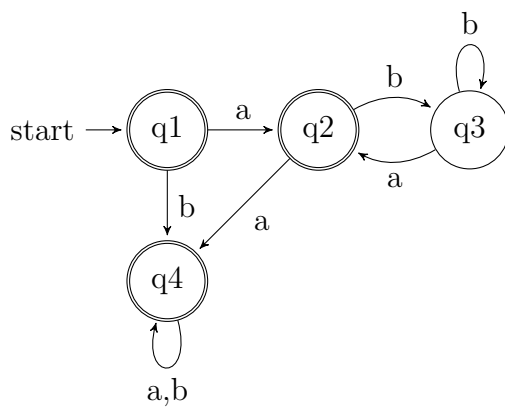


e.  $\{ w \mid w \text{ is any string not in } (ab^+)^* \}$

Simpler language

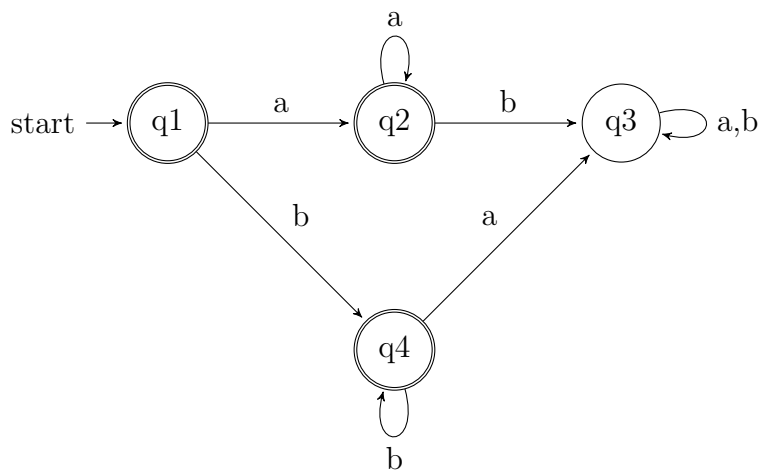


Language given



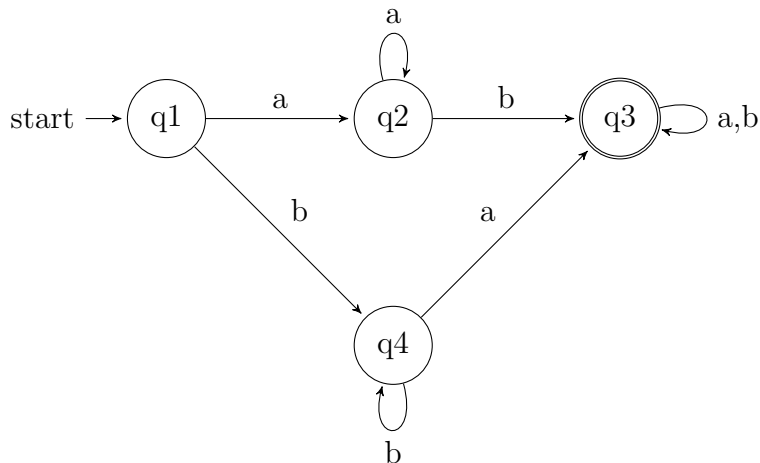
f.  $\{ w \mid w \text{ is any string not in } a^* \cup b^* \}$

Simpler language



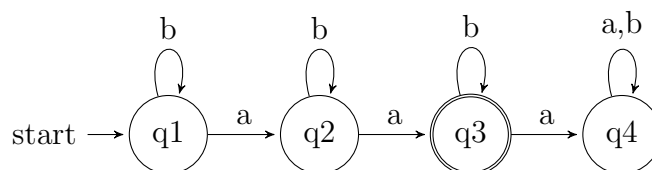


Language given

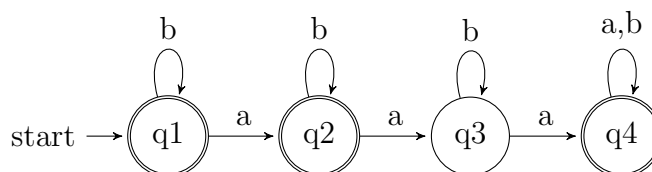


g.  $\{ w \mid w \text{ is any string that doesn't contain exactly two a's} \}$

Simpler language

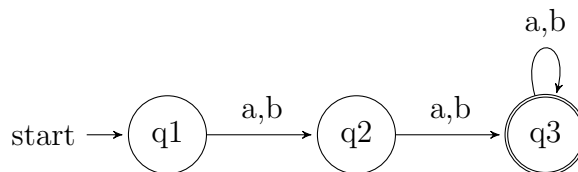


Language given

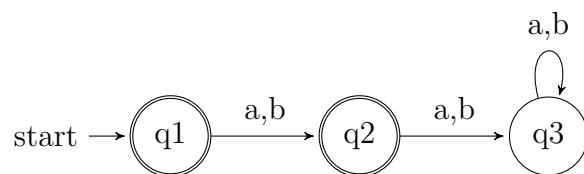


h.  $\{ w \mid w \text{ is any string except a and b} \}$

Simpler language

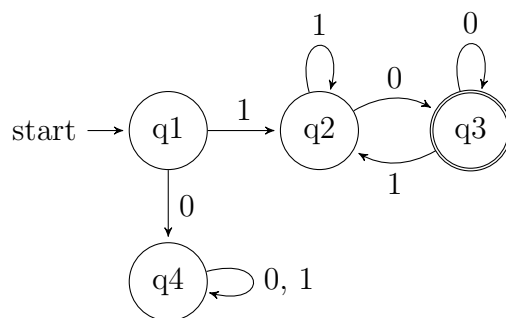


Language given

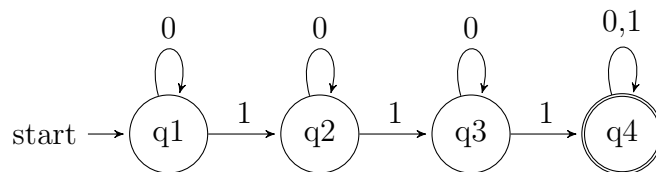


1.6: a, b, c, d, e, f, g, h, I, j, k, l, m, n

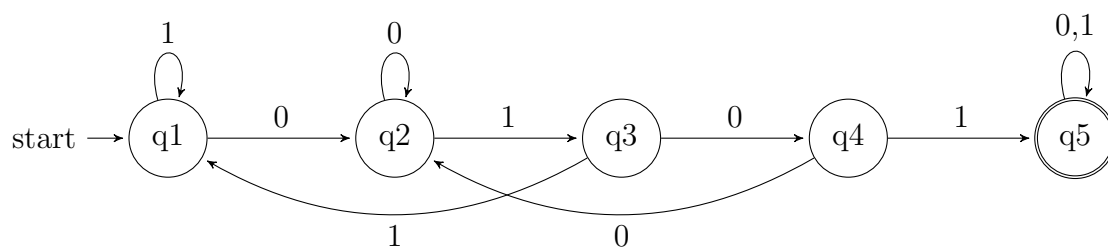
a.



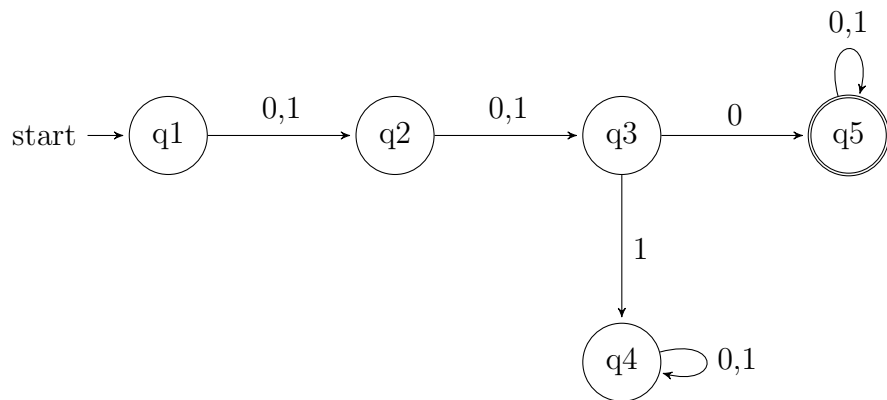
b.



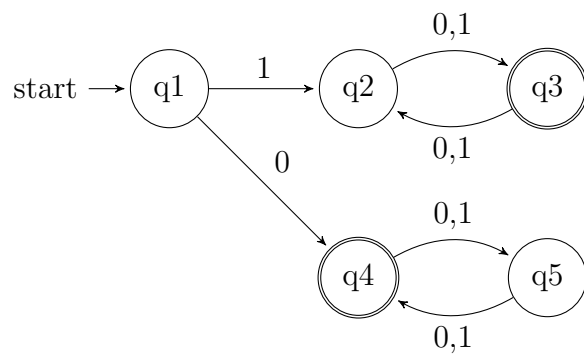
c.



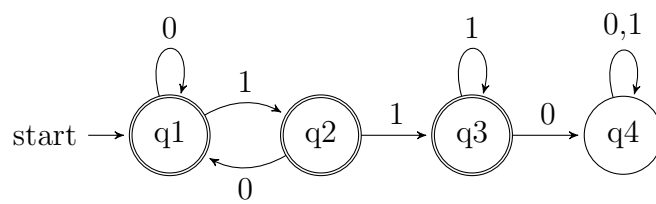
d.



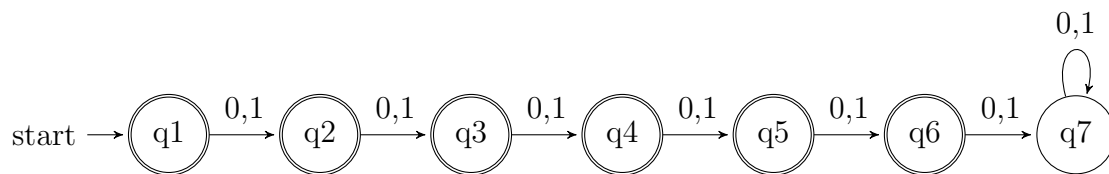
e.



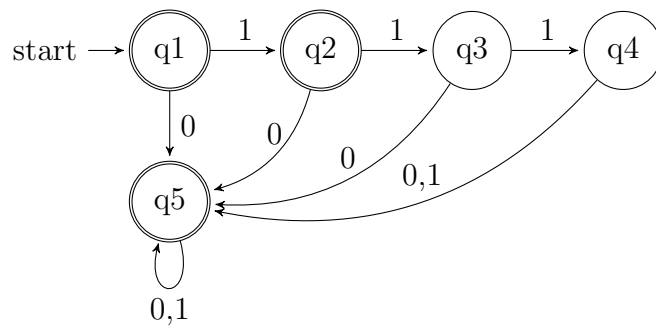
f.



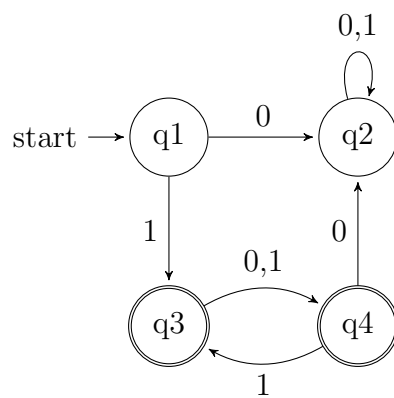
g.



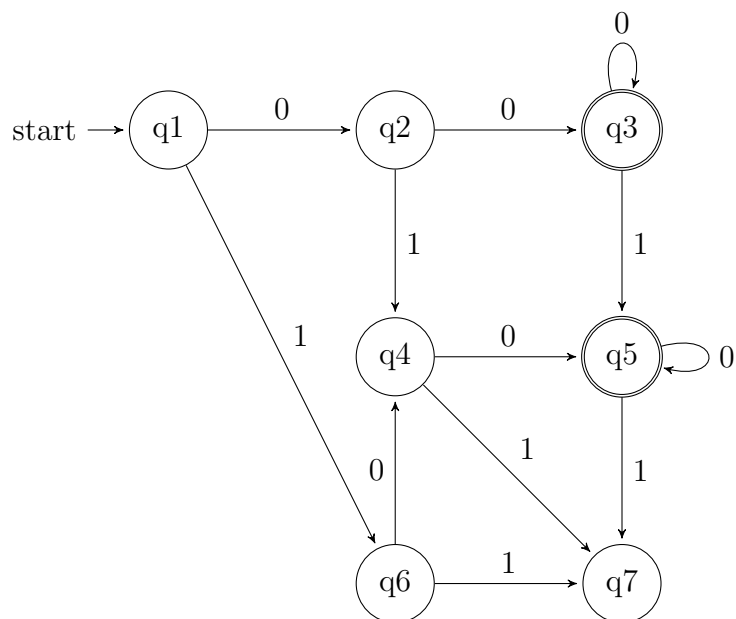
h.



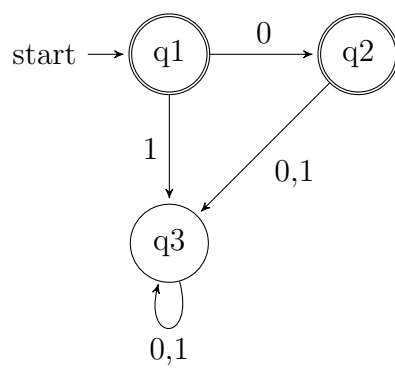
I.



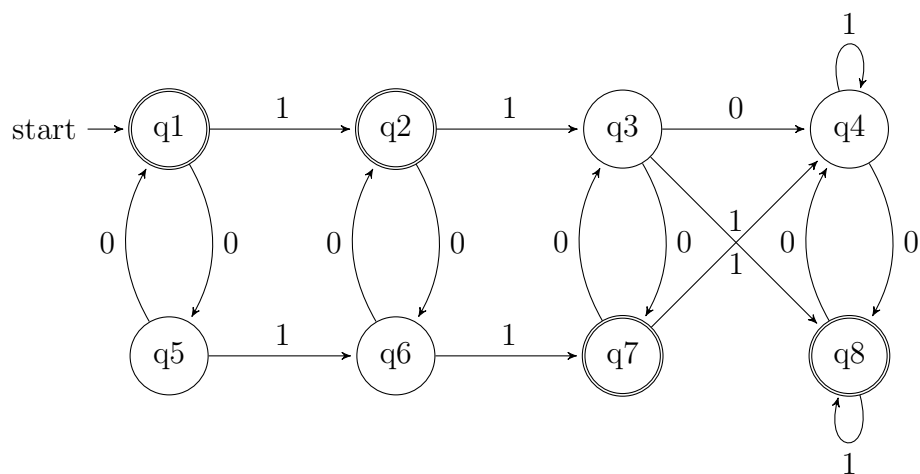
j.



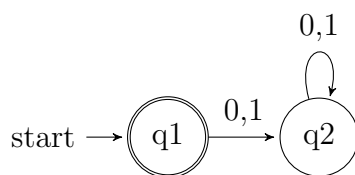
k.



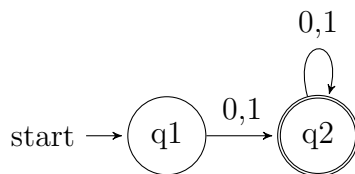
L.



m.

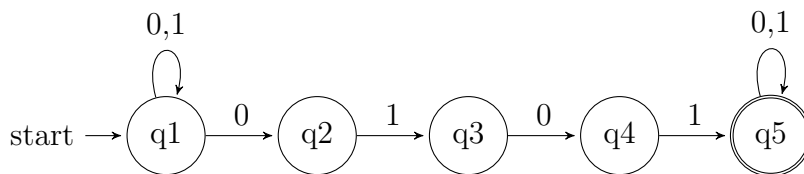


n.

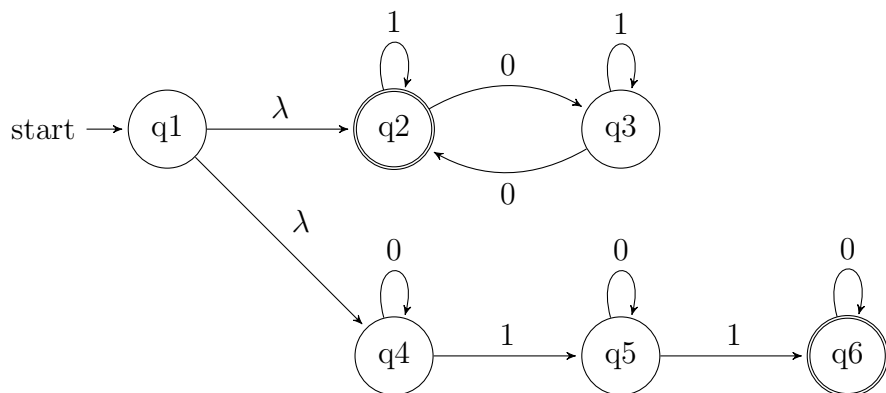


1.7: b, c, d, e, g, h

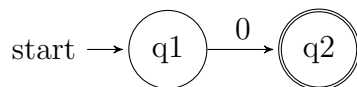
b. The language of Exercise 1.6l with six states



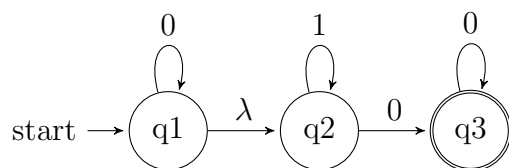
c. The language of Exercise 1.6l with six states



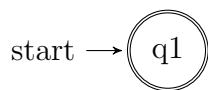
d. The language 0 with two states



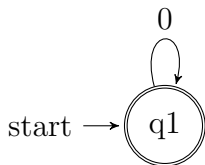
e. The language  $0^*1^*0^+$  with three states



g. The language  $\{\epsilon\}$  with one state



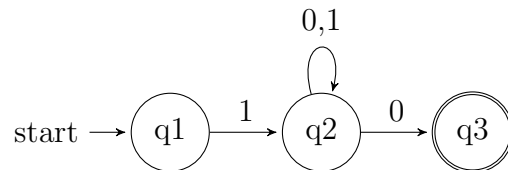
h. The language  $0^*$  with one state



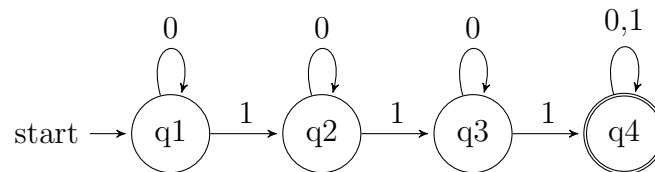
1.8: a, b

Use the construction in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages described in

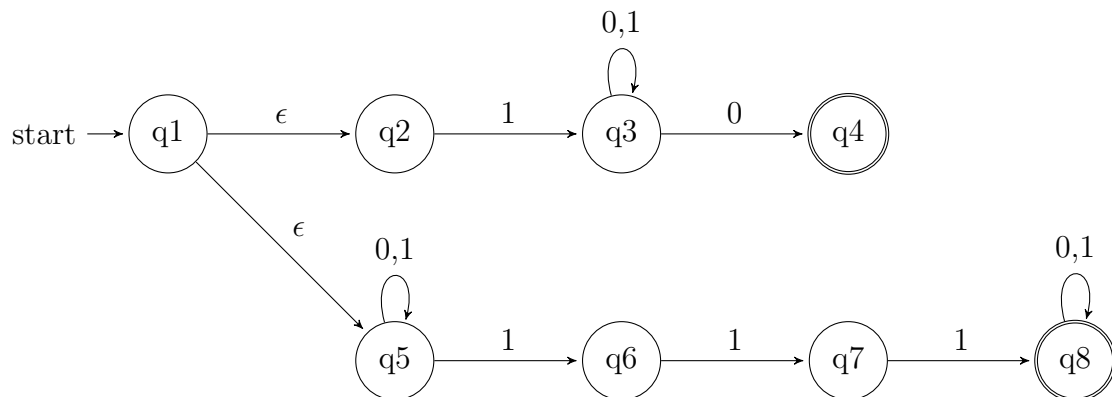
a. Exercise 1.6a



Exercise 1.6b



Exercise 1.6a and Exercise 1.6b joined



b. Exercises 1.6c and 1.6f

Exercises 1.6c

Exercises 1.6f

Exercises 1.6c and Exercises 1.6f joined

1.9: a, b

Use the construction in the proof of Theorem 1.47 to give the state



diagrams of NFAs recognizing the concatenation of the languages described in

- a. Exercises 1.6g and 1.6i

Exercise 1.6g

Exercise 1.6i

Exercise 1.6g and Exercise 1.6i joined

- b. Exercises 1.6b and 1.6m

1.10: a, b, c

Use the construction in the proof of Theorem 1.49 to give the state diagrams of NFAs recognizing the star of the languages described in

- a. Exercise 1.6b

- b. Exercise 1.6j

- c. Exercise 1.6m

1.12

1.13

1.16

1.17: a, b

- a.

- b.

1.18

- a.  $1 \Sigma^* 0$

- b.  $\Sigma^* 1 \Sigma^* 1 \Sigma^* 1 \Sigma^*$

c.  $\Sigma^*0101\Sigma^*$

d.  $\Sigma\Sigma^0\Sigma^*$

e.  $(0\cup 1\Sigma)(\Sigma\Sigma)^*$

f.  $0^*(10^+)^*1^*$

g.  $(\epsilon\cup\Sigma)^5$

h.  $\epsilon\cup\Sigma\cup 0\Sigma\cup 10\cup 0\Sigma\Sigma\cup 10\Sigma\cup 110\cup\Sigma^3\Sigma^+$

i.  $(1\Sigma)^*(\epsilon\cup 1)$

j.  $00^+\cup 100^+\cup 0^+10^+\cup 00^+1$

k.  $0\cup\epsilon$

l.  $1^*(01^*01^*)\cup 0^*10^*10^*$

m.  $\emptyset$

n.  $\Sigma^+$

1.20: a, b, c, d, e, f, g, h

a.

Members: ab, abb

Not members: ba, bba

b.

Members: ab, ababab

Not members: aba, bab

c.

Members: aaa, bbb

Not members: aabb, bbaa

d.

Members: aaa, aaaaaa

Not members: a, aaaa

e.

Members: aba, bbaaabaabb

Not members: a, b

f.

Members: aba, bab

Not members: ababab, ba

g.

Members: b, ab

Not members: a, ba

h.

Members: a, bbab

Not members: b,  $\epsilon$

1.21

1.22