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Foundations of Computer Science
Homework #2 - Chapter 1: DFA, NFA
10/6/2016

1.3
The formal description of a DFA M is {q1, q2, q3, q4, q5 },
{u, d}, δ, q3 , {q3}, where δ is given by the following table:

	u	d
q1	q1	q2
q2	q1	q3
q3	q2	q4
q4	q3	q5
q5	q4	q5

Initial state is q3, so that is where the machine will start.
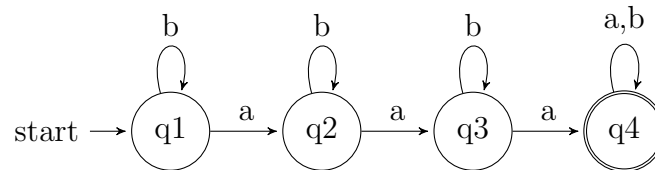We can use the table to create the nodes, and connect them as needed.
The accepted state is q3 so we will mark that with a double circle to
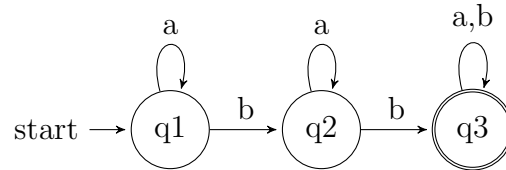show that as the accepted state of the machine.
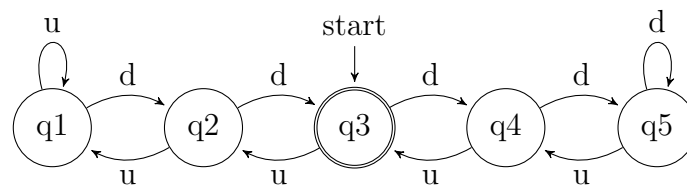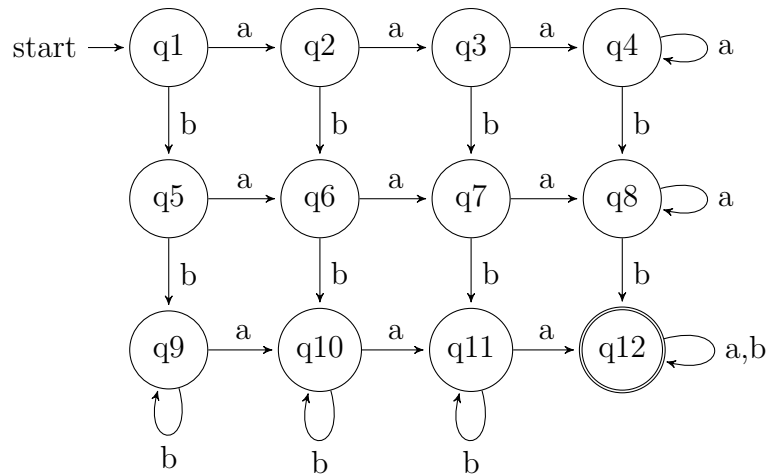
1.4: a, c, e, f, g
For all parts, $\sum = \{a, b\}$

a. { w| w has at least three a's }

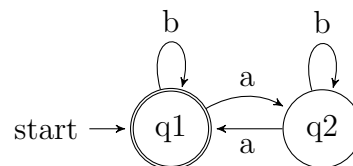{ w| w has at least two b's }

{w| w has at least three as and at least two bs}

c. { w| w has an even number of a's }

{ w| w has one or two b's }

```
        a           a           a          a,b
       ___         ___         ___         ___
      /   \       /   \       /   \       /   \
start → (q1) --b--> ((q2)) --b--> ((q3)) --b--> (q4)
```
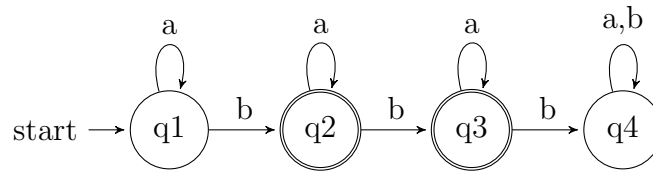
{ w| w has an even number of a's and one or two b's }

```
start → (q1) --b--> ((q2)) --b--> ((q3)) --b--> (q4) ⟲ b
          ⇅ a a        ⇅ a a        ⇅ a a        ⇅ a a
        (q5) --b--> (q6) --b--> (q7) --b--> ((q8)) ⟲ b
```

e. { w| w starts with an a }

```
   a,b          start          a,b
   ___            |            ___
  /   \           ↓           /   \
 (q1) ←--b-- (q2) --a--> ((q3))
```

{ w| w has at most one b }

```
        a           a          a,b
       ___         ___         ___
      /   \       /   \       /   \
start → ((q1)) --b--> ((q2)) --b--> (q3)
```

3
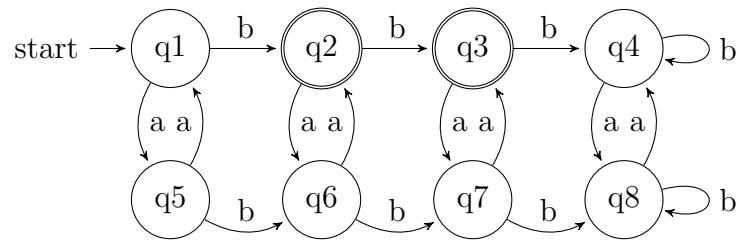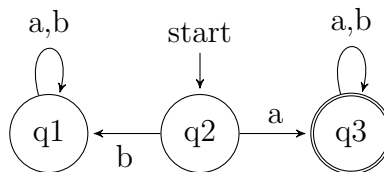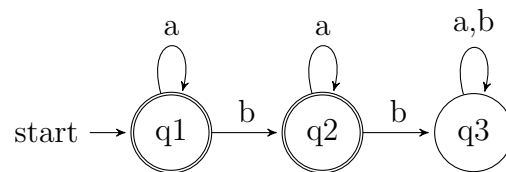
{ w| w starts with an a and has at most one b }

a

a,b

start → q1   b   q2   b   q3

a

a

b

a   b   a

q4   b   q5   a

f. { w| w has an odd number of a's }

b

b

a

start → q1   q2

a

{ w| w ends with a b }

a

b

b

start → q1   q2

a
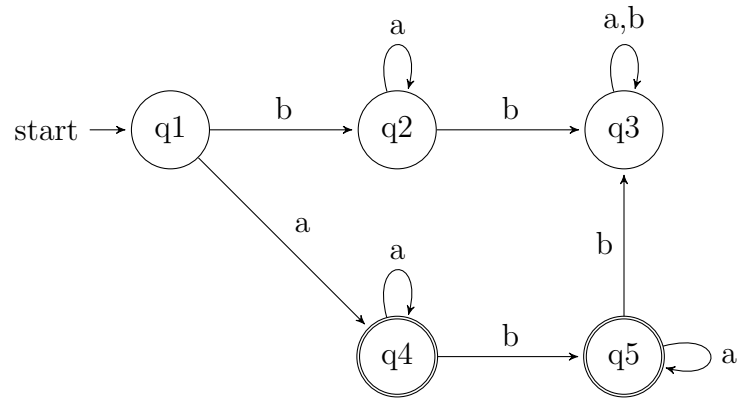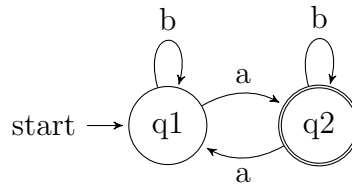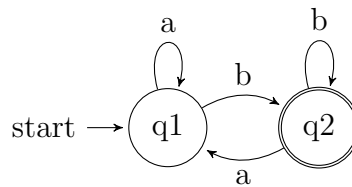
{ w| w has an odd number of a's and ends with a b }

start → q1   b   q2   b

a

a   a

a

a

q3   b   q4   b

4
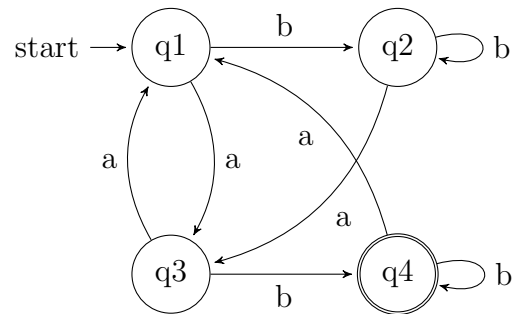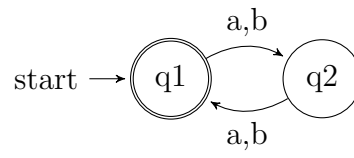
g. { w| w has even length }

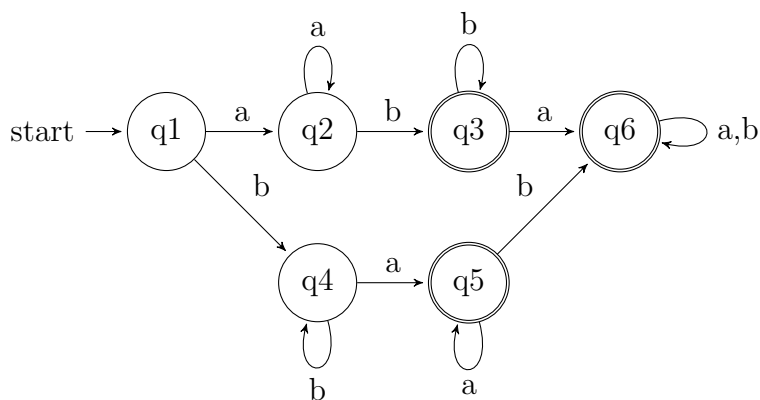{ w| w has an odd number of a's }
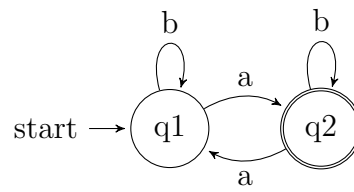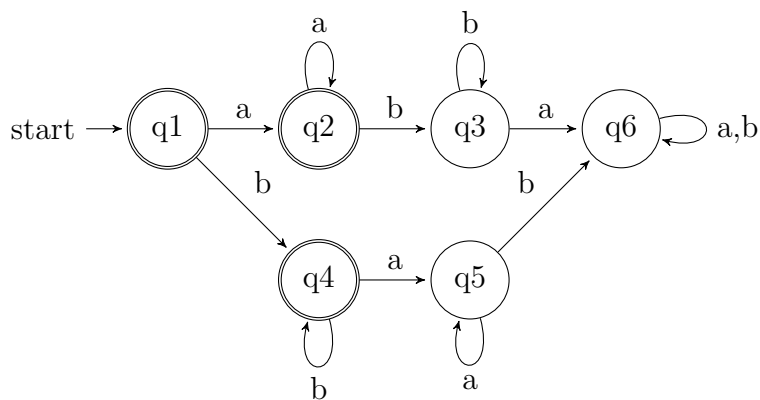

{ w| w has even length and an odd number of a's }
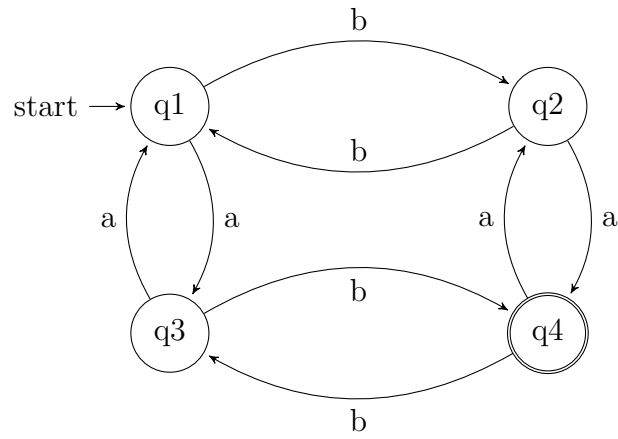
1.5: c, d, e, f, g, h

For all parts, $\sum = \{a, b\}$

c. { w| w contains neither the substrings ab nor ba }

Simpler language

Language given.

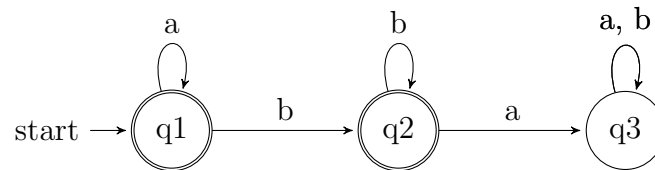d. { w| w is any string not in a*b* }
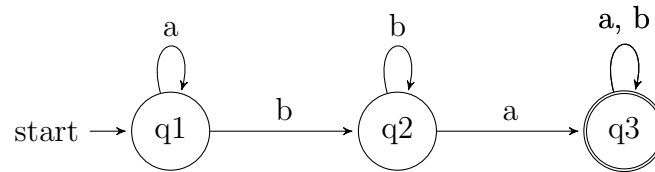
Simpler language

a   b   a, b

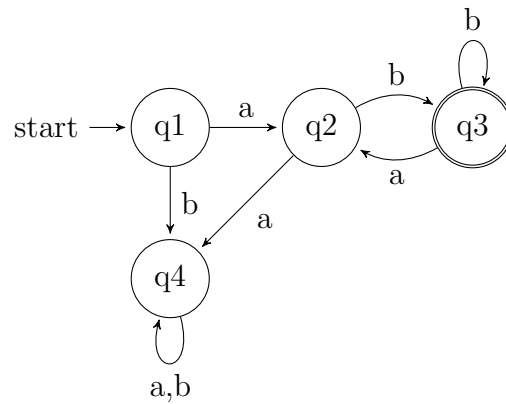start → ((q1)) —b→ ((q2)) —a→ (q3)

Language given

a   b   a, b

start → (q1) —b→ (q2) —a→ ((q3))

e. { w| w is any string not in (ab$^+$)* }

Simpler language

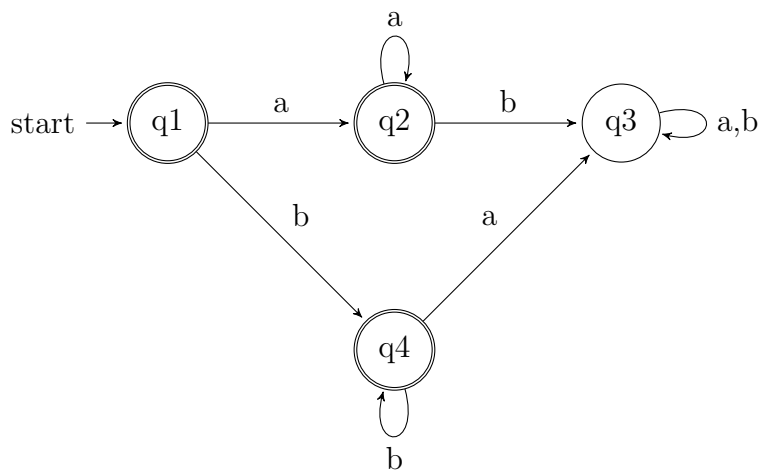start → (q1) —a→ (q2) —b→ ((q3)) with b self-loop on q3; q3 —a→ q2; q1 —b→ (q4); q2 —a→ q4; q4 has a,b self-loop

7
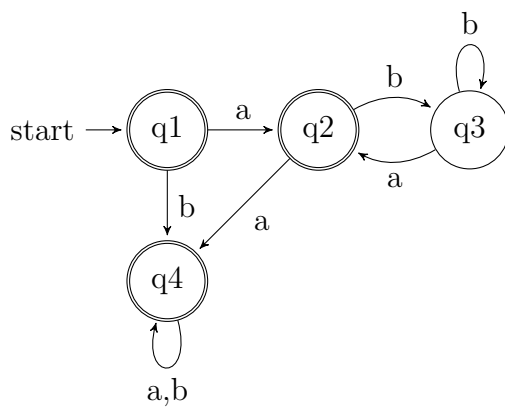
Language given

f. { w | w is any string not in a* $\bigcup$ b* }

Simpler language

8
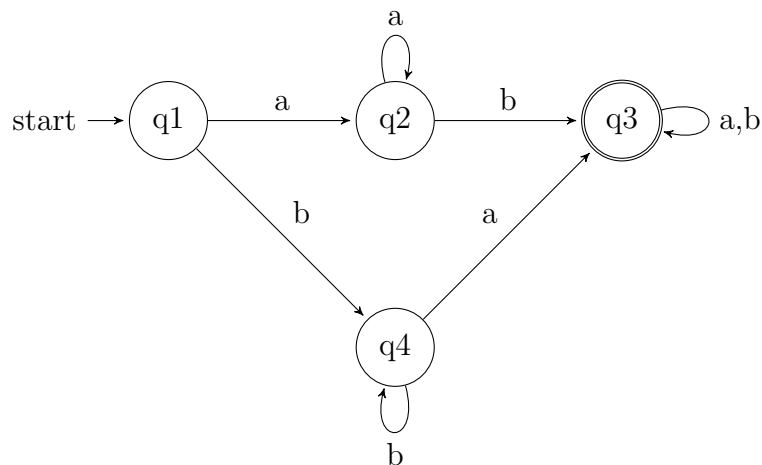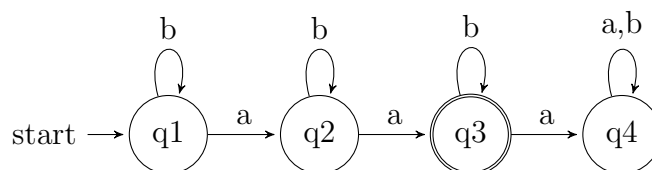
Language given
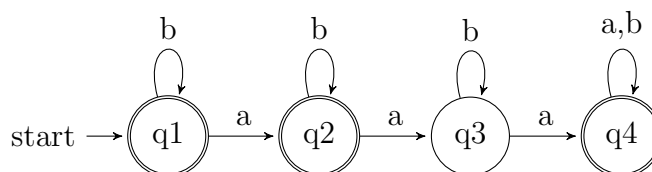

g. { w| w is any string that doesn't contain exactly two a's }

Simpler language
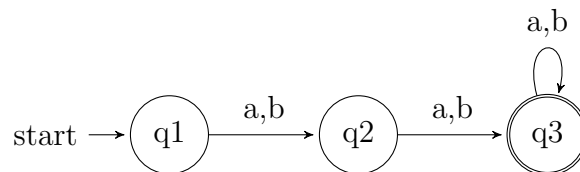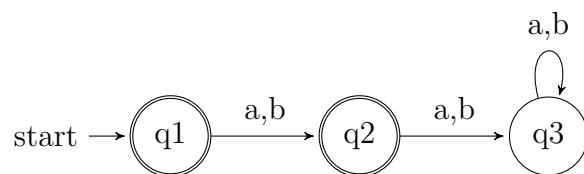

Language given

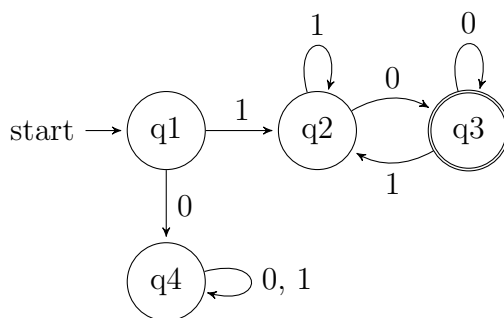
h. { w| w is any string except a and b }
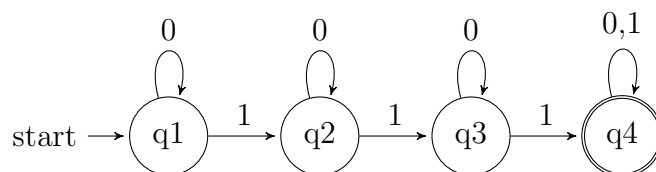
Simpler language

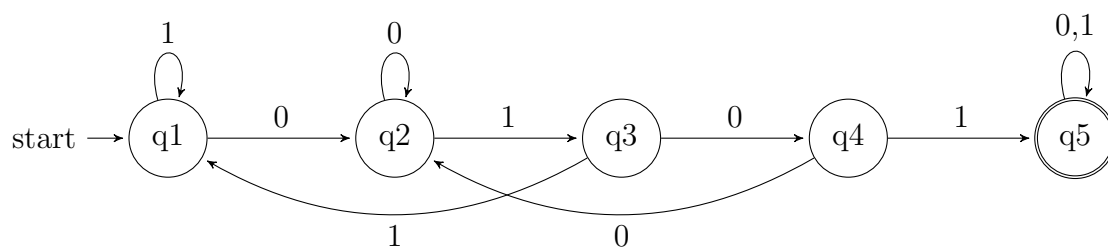Language given
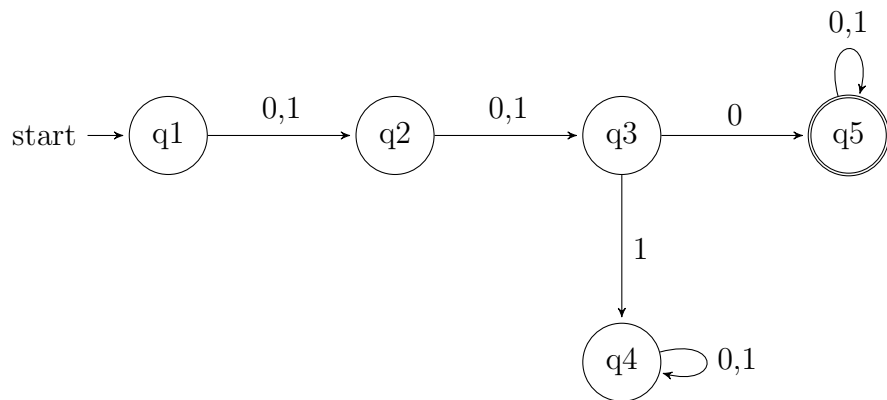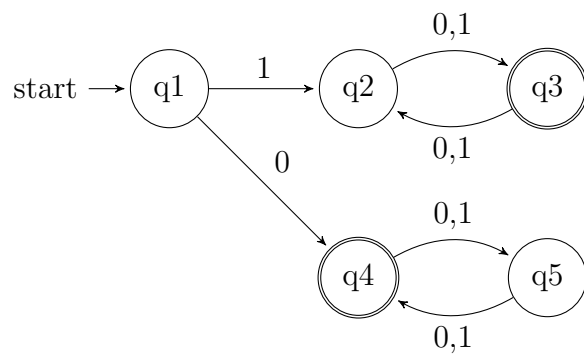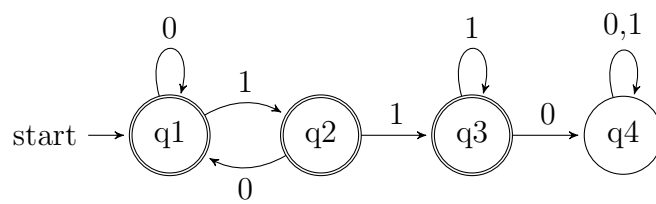

1.6: a, b, c, d, e, f, g, h, I, j, k, l, m, n

a.

b.

c.

d.

e.

f.

g.

h.

I.

j.

k.

L.

start → (q1) --1--> (q2) --1--> (q3) --1--> (q4)

q1 ⟷ q5 (0, 0), q2 ⟷ q6 (0, 0), q3 ⟷ q7 (0, 0), q4 ⟷ q8 (0, 0)

(q5) --1--> (q6) --1--> (q7) --1--> (q8)

m.

0,1

start → ((q1)) --0,1--> (q2) with self-loop 0,1

n.

0,1

start → (q1) --0,1--> ((q2)) with self-loop 0,1
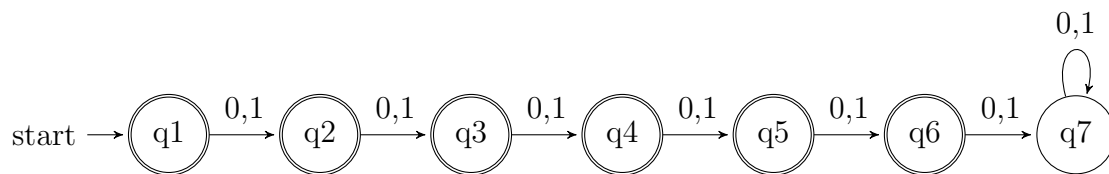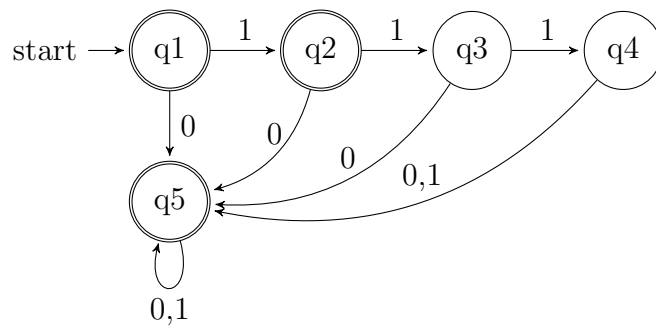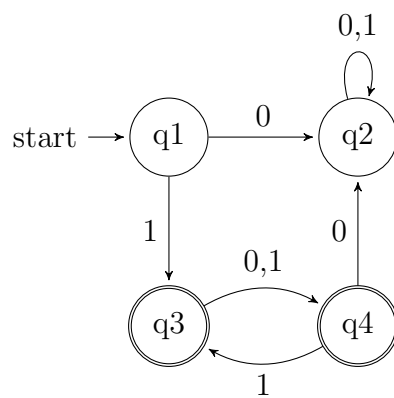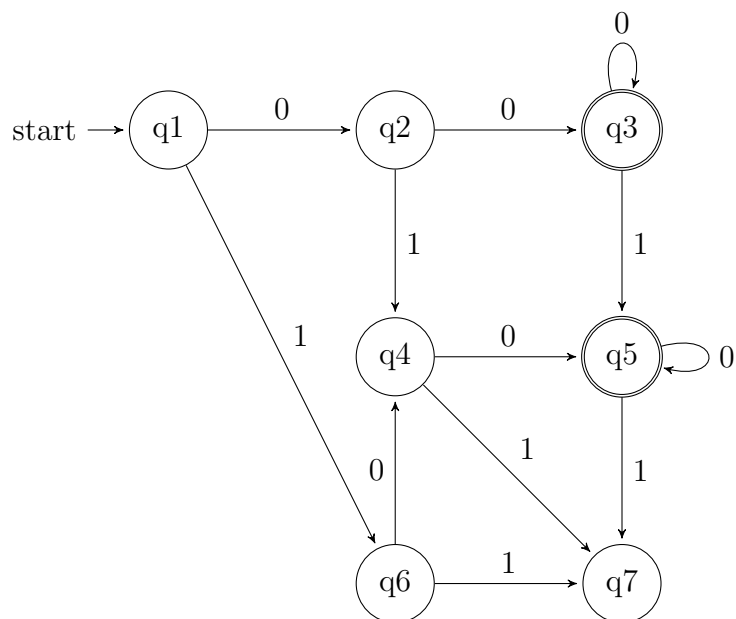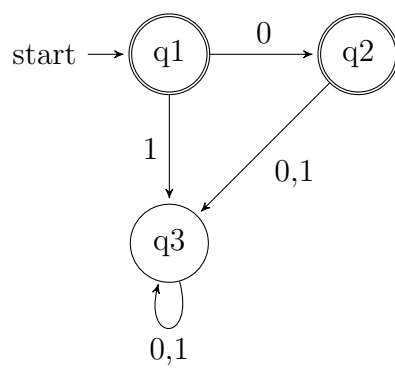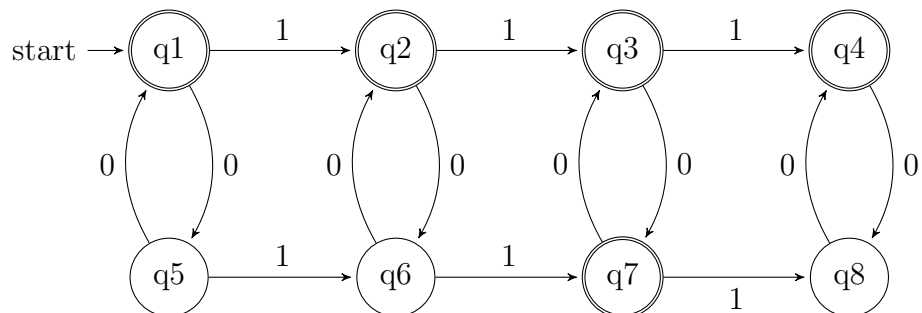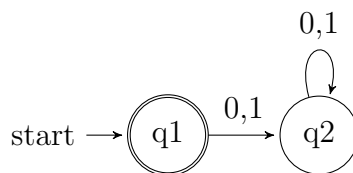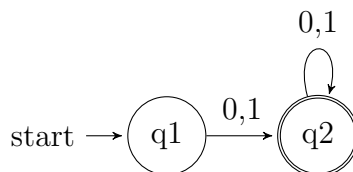
1.7: b, c, d, e, g, h

b. The language of Exercise 1.6l with six states

0,1

start → (q1) --0--> (q2) --1--> (q3) --0--> (q4) --1--> ((q5))

with self-loop 0,1 on q1 and self-loop 0,1 on q5
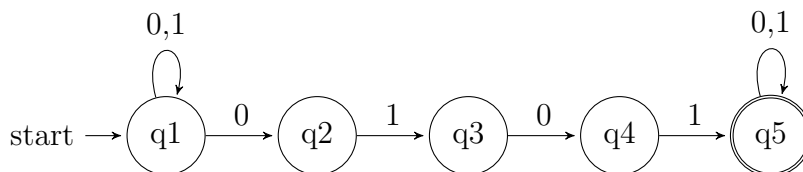
c. The language of Exercise 1.6l with six states

14
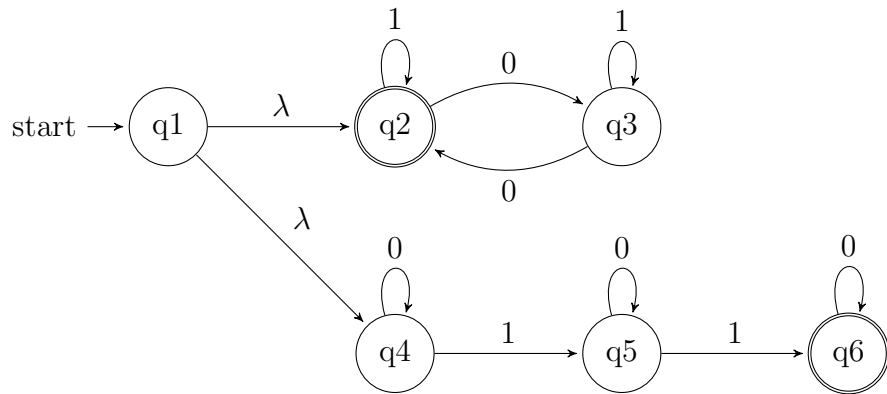
d. The language 0 with two states
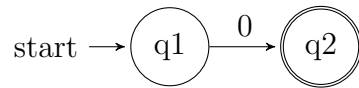
e. The language 0*1*0$^+$ with three states
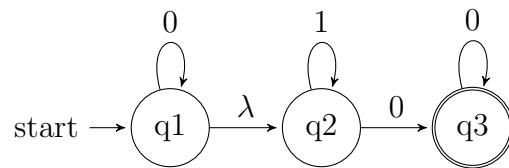
g. The language $\{\epsilon\}$ with one state
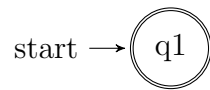
h. The language 0* with one state
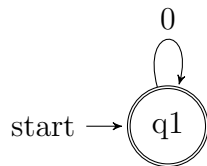
1.8: a, b
Use the construction in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages described in

a. Exercise 1.6a
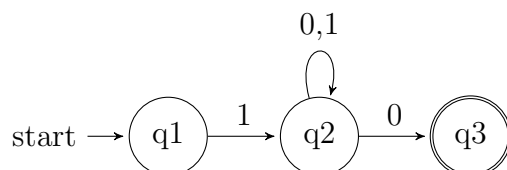
Exercise 1.6b

Exercise 1.6a and Exercise 1.b joined

b. Exercises 1.6c and 1.6f

Exercises 1.6c
Exercises 1.6f
Exercises 1.6c and Exercises 1.6f joined

1.9: a, b
Use the construction in the proof of Theorem 1.47 to give the state

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diagrams of NFAs recognizing the concatenation of the languages described in

    a. Exercises 1.6g and 1.6i

    Exercise 1.6g
    Exercise 1.6i
    Exercise 1.6g and Exercise 1.6i joined
    b. Exercises 1.6b and 1.6m
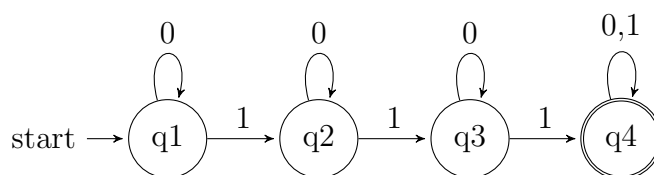
1.10: a, b, c
Use the construction in the proof of Theorem 1.49 to give the state
diagrams of NFAs recognizing the star of the languages described in
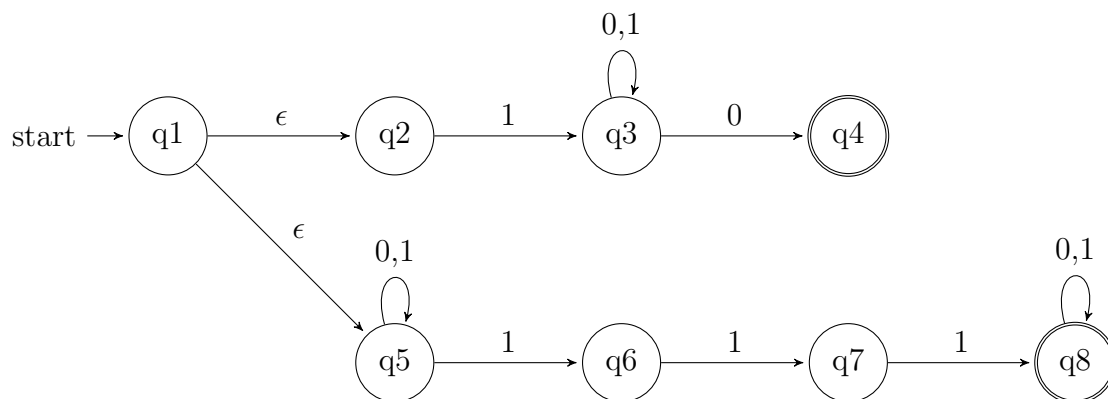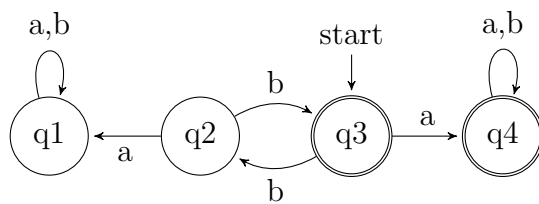
    a. Exercise 1.6b

    b. Exercise 1.6j

    c. Exercise 1.6m
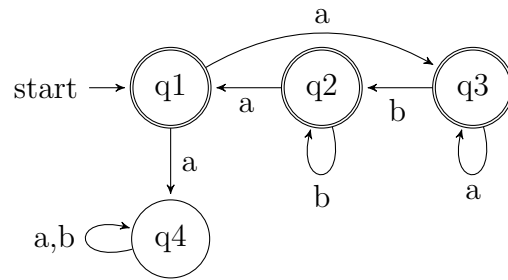
1.12

1.13

1.16
a.

    b.

1.17: a, b

    a.

    b.

1.18

    a. $1\sum{}^*0$

    b. $\sum{}^*1\sum{}^*1\sum{}^*1\sum{}^*$

    c. $\sum{}^*0101\sum{}^*$

    d. $\sum\sum 0\sum{}^*$

    e. $(0\bigcup 1\sum)(\sum\sum)^*$

    f. $0^*(10^+)^*1^*$

    g. $(\epsilon\bigcup\sum)^5$

    h. $\epsilon\bigcup\sum\bigcup 0\sum\bigcup 10\bigcup 0\sum\sum\bigcup 10\sum\bigcup 110\bigcup\sum{}^3\sum{}^+$

    I. $(1\sum)^*(\epsilon\bigcup 1)$

    j. $00^+\bigcup 100^+\bigcup 0^+10^+\bigcup 00^+1$

    k. $0\bigcup\epsilon$

L. $1^*(01^*01^*) \bigcup 0^*10^*10^*$

m. $\emptyset$

n. $\sum^+$

1.20: a, b, c, d, e, f, g, h

a.
Members: ab, abb
Not members: ba, bba

b.
Members: ab, ababab
Not members: aba, bab

c.
Members: aaa, bbb
Not members: aabb, bbaa

d.
Members: aaa, aaaaaa
Not members: a, aaaa

e.
Members: aba, bbaaabaabb
Not members: a, b

f.
Members: aba, bab
Not members: ababab, ba

g.
Members: b, ab
Not members: a, ba

h.
Members: a, bbab

Not members: b, ϵ

1.21

1.22