Recursion

→ most important topic.

→ help us to solve complex problem in a very easy way.

→ will be a lot useful in future for lots of data structure and DP.

We might face some difficulties in starting but once it becomes clear, recursion is very easy.

Concept of function

Can a function call itself? Yes

2 things are more clear abt fn.

① Function stays in memory until resolved

② Only when a fn finishes the execution then only it comes out of program to the line it was called from & gets deleted from stack.

Recursion is a function calling itself.

fun1() → fun2()

HDSA() → HDSA()

"Recursion is when solution of a problem depends on the same smaller problem"

Factorial of no. :-

$$fact(n) = n! = n * (n-1) * (n-2) * \ldots 1$$

$$n! = n * \underbrace{(n-1) * (n-2) \ldots 1}$$

$$\Downarrow$$

$$n! = n * (n-1)!$$

$$\boxed{fact(n) = n * fact(n-1)}$$

$$fact(n-1) = (n-1) * fact(n-2)$$

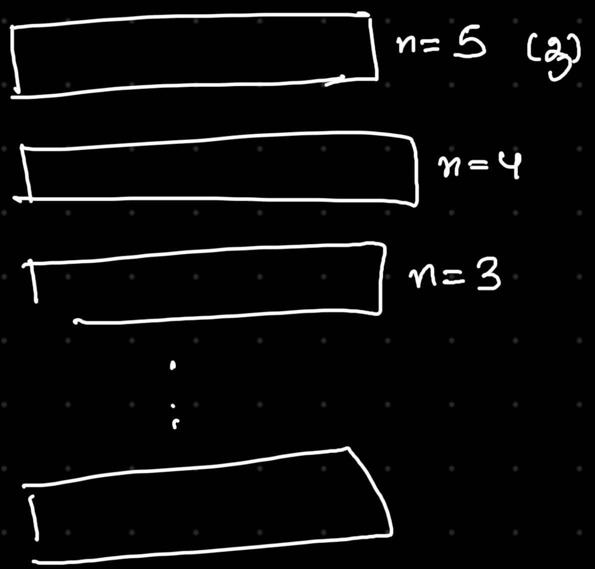
$$\Downarrow$$

$$(n-2) * (n-3) \ldots 1$$

$$=$$

Stack

fact(5)
↓
fact(4)
↓
fact(3)
↓
fact(2)
↓
fact(1) → fact(0) → fact(-1) → fact(-2)



Now we will not be making these boxes/f^n every time. and think of recursion this deep

Let us understand PMI (   ) which underlying principle of recursion.

PMI (Principle of mathematical Induction)

Initial few problem we will do can be done iteratively
but we have to use recursion to get mastery.
and as it is way to easy like this.

Recursion principle based on PMI, so understanding
this will further help us to master recursion.

PMI

It is used to prove some fact.

eg.

$$f(n) = \quad \forall n$$

using PMI, we can break this problem in 3 steps.

1. Prove $F(0)$ or $F(1)$ is true.
2. Assume $F(K)$ is true → Induction hypothesis
3. Using step 2, i.e. $F(K)$ is true*

   prove that $F(K+1)$ is true.

$$\Sigma n = \frac{n(n+1)}{2} = sum(n)$$

$$\Rightarrow \frac{(k+1)(k+2)}{2}$$

Let us try to prove using PMI

① $sum(1) = \Sigma 1$

$L.H.S = 1$

$R.H.S = \frac{1(1+1)}{2} = 1$

$L.H.S = R.H.S. \rightarrow$ ①

② Assume (don't question) that $f(k)$ is true

$$\Rightarrow \Sigma k = \frac{k(k+1)}{2} \Rightarrow \text{true} \rightarrow ②$$

③ We need to prove that $(k+1)$ true

$$\Rightarrow \Sigma k+1$$

$L.H.S.$ $\Sigma k+1 = k+1 + \Sigma k \Rightarrow$ ②

$$= k+1 + \frac{k(k+1)}{2}$$

$$= \frac{2(k+1)}{2} + \frac{k(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} \Rightarrow R.H.S$$

$\Sigma n = \frac{n(n+1)}{2}$ is true for all n

$f(3)$

$f(2)$

$f(1) =$

$k \to 0$ $\quad f(0) = 0$

$F(0) = 0$

so in every recursion problem we just have to apply the PMI and our work is done.

Just below 3 steps.

1. Base case
2. Assume for (n-1) is true ⇝ small problem
3. Using assumption in step(2), solve for bigger problem.

if all the above steps are properly followed.

P.M.I. says that we will get the answer.

ideally, no need to make f^n calls again and again.

Now, we will solve lots of question to make it clear.

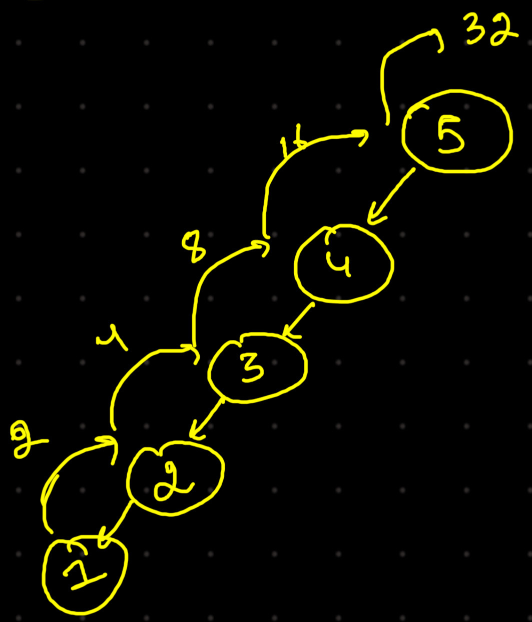
Q-1 Find 2 to the power of n for a given n, using recursion tree.

→ Also state recursive function and make recursion tree.

again it can be solved via iteration but we are going to use recursion.

$$2^n = 2 * 2^{n-1}$$

$$\underline{pow2(n)} = 2 * \underline{pow2(n-1)}$$



Q-2 Find no. of digits in a given number.

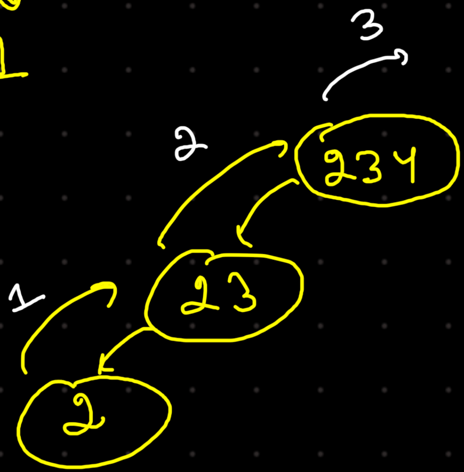
234 → 3

51234 → 5

draw recursion tree and recursion formula.

dig(234) = 1 + dig(23)

1



Fibonicci series