Session 13 - 14

Sampling Distribution

Prof. Jigar M. Shah

Sampling Distribution

- Sampling Distribution
  - Introduction
  - Sampling Distribution for Arithmetic Mean
  - Sampling Distribution for Proportion of Success

Sampling Distribution

Sampling Distribution

- Introduction
  - Different samples drawn from a given population to compute a particular sample statistic may provide different values of the sample statistic
  - Thus, sample statistic is a random variable having a certain distribution known as sampling distribution

Sampling Distribution
Probability distribution of a sampling statistic i.e., Distribution of the various possible values of the sample statistic

Sampling Distribution

Sampling Distribution

- Sampling Distribution of Arithmetic Mean

Sampling Distribution of Arithmetic Mean

- Probability distribution of all the possible values of the sample arithmetic mean $\bar{x}$
- Random variable is $\bar{x}$ (the arithmetic mean of the sample, i.e., point estimator of the population mean, μ)

Mean of the Sampling Distribution of Arithmetic Mean i.e., Expected Value of $\bar{x}$

- Mean of all the sample means
- Denoted as $E(\bar{x})$

$$E(\bar{x}) = \mu$$

where $E(\bar{x})$ is the expected value of $\bar{x}$ i.e., mean of the sample means
μ is the mean of the population

When the expected value of a point estimator equals the population parameter, the point estimator is said to be unbiased

$\bar{x}$ is the unbiased estimator of population mean μ

Sampling Distribution

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- Sampling Distribution of Arithmetic Mean

Standard Deviation of the Sampling Distribution of Arithmetic Mean i.e., Standard Deviation of $\bar{x}$

$$\sigma_{\bar{x}} = \sqrt{\frac{N-n}{N-1}}\left(\frac{\sigma}{\sqrt{n}}\right) \qquad \sigma_{\bar{x}} = \left(\frac{\sigma}{\sqrt{n}}\right)$$

where
- $\sigma_{\bar{x}}$ is the standard deviation of $\bar{x}$
- σ is the standard deviation of the population
- n is the sample size
- N is the population size

whenever population is infinite

or

population is finite and $\frac{n}{N} \leq 0.05$

Standard deviation of a point estimator is known as the standard error

Standard error of the mean helps in determining how far a sample mean may be from the population mean

Sampling Distribution

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- Sampling Distribution of Arithmetic Mean

Shape of the Sampling Distribution of $\bar{x}$	
Normal Population i.e., Population having Normal Distribution	If population has normal distribution, sampling distribution of $\bar{x}$ is normally distributed
Non-Normal Population i.e., Population not having Normal Distribution	If population does not have normal distribution, sampling distribution of $\bar{x}$ can be approximated by a normal distribution as the sample size becomes large (generally more than 30) ….. by applying central limit theorem
The significance of the central limit theorem is that it permits using sample statistics to make inferences about population parameters without knowing anything about the shape of the frequency distribution of that population other that got from the sample	

Standardizing the Sampling Mean	$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$

Sampling Distribution

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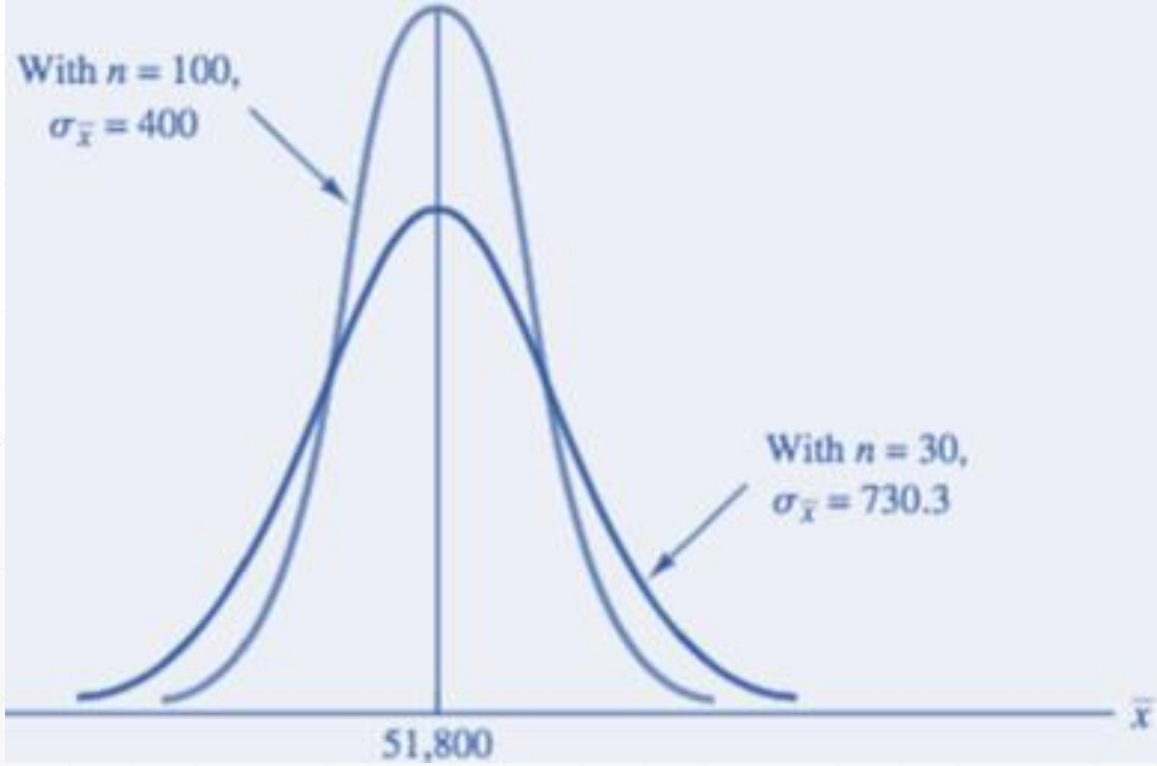
- Sampling Distribution of Arithmetic Mean
  - As the sample size increases, the standard error of the mean decreases
  - Larger sample size provides a higher probability that the sample mean will be within a specified distance from the population mean
  - Diminishing Returns in Sampling - as the sample size increases, the marginal improvement (reduction) in standard error decreases



Sampling Distribution

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- Sampling Distribution of Arithmetic Mean
  - A population has a mean of 200 & a standard deviation of 50. A sample of size 100 is taken & the sample mean is used to estimate the population mean.
    - What is the expected value of sample mean?
    - What is the standard deviation of sample mean?
    - Show the sampling distribution of sample mean.
    - What does the sampling distribution of sample mean show?
    - Find the probability that the sample mean will be within ±5 of the population mean.
    - What is the probability that the sample mean will be within ±10 of the population mean.

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- Sampling Distribution of Arithmetic Mean
  - Given the mean scores for the three parts of the Scholastic Aptitude Test (SAT) as shown in the adjacent table, & assuming the population std. dev. on each part of the test to be 100,
    - What is the probability a sample of 90 test takers will provide a sample mean test score within 10 points of the population mean of 502 on Critical Reading part of the test?
    - What is the probability a sample of 90 test takers will provide a sample mean test score within 10 points of the population mean of 515 on Mathematics part of the test?
    - What is the probability a sample of 100 test takers will provide a sample mean test score within 10 of the population mean of 494 on Writing part of the test?

SAT Section	Mean Score
Critical Reading	502
Mathematics	515
Writing	494

Sampling Distribution

Sampling Distribution

- Sampling Distribution for Proportion of Success

Sample Proportion $\bar{p}$

$$\bar{p} = \frac{x}{n}$$

where

x is the no. of elements in the sample that possess the characteristic of interest

n is the sample size

Sampling Distribution of $\bar{p}$

- Probability distribution of all the possible values of the sample proportion $\bar{p}$
- Random variable is $\bar{p}$ (the sample proportion, i.e., point estimator of the population proportion, p)

Mean of the Sampling Distribution of Sample Proportion $\bar{p}$ i.e., Expected Value of $\bar{p}$

- Mean of all the sample proportions
- Denoted as $E(\bar{p})$

$$E(\bar{p}) = p$$

where

$E(\bar{p})$ is the expected value of $\bar{p}$ i.e., mean of the sample proportions

p is the proportion of the population

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- Sampling Distribution for Proportion of Success

Standard Deviation of the Sampling Distribution of Arithmetic Mean i.e., Standard Deviation of $\bar{x}$

$$\sigma_{\bar{p}} = \sqrt{\frac{N-n}{N-1}}\sqrt{\frac{p(1-p)}{n}}$$

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$$

where $\sigma_{\bar{p}}$ is the standard deviation of $\bar{p}$

p is the proportion of the population

n is the sample size

N is the population size

whenever population is infinite

or

population is finite and $\frac{n}{N} \leq 0.05$

Standard deviation of a point estimator is known as the standard error

Standard error of the proportion helps in determining how far a sample proportion may be from population proportion

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- Sampling Distribution for Proportion of Success

Shape of the Sampling Distribution of $\bar{p}$

$\bar{p} = \frac{x}{n}$ where x is the binomial random variable indicating the number of elements in the sample with the characteristic of interest

Since n is constant, probability of $\frac{x}{n}$ is the same as the binomial probability of x. The sampling distribution of p is thus a discrete binomial probability distribution

Binomial distribution can be approximated by a normal distribution whenever the sample size is large enough to satisfy two conditions:

$np \geq 5$ $\quad n(1 - p) \geq 5$

Standardizing the Sampling Proportion $z = \frac{\bar{p} - p}{\sigma_{\bar{p}}}$

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- Sampling Distribution for Proportion of Success
  - A sample of size 100 is selected from a population with p = 0.4
    - What is the expected value of sampling proportion?
    - What is the standard error of sampling proportion?
  - A population proportion is 0.4. A sample of size 200 will be taken and the sample proportion will be used to estimate the population proportion.
    - What is the probability that the sample proportion will be within ±0.03 of the population proportion?
    - What is the probability that the sample proportion will be within ±0.05 of the population proportion?

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- Sampling Distribution for Proportion of Success
  - Assume that the population proportion is 0.55. Compute the standard error of the proportion, for sample sizes 100, 200, 500, & 1000.
  - The population proportion is 0.30. What is the probability that a sample proportion will be within ±0.04 of the population proportion for each of the following sample sizes?
    - 100
    - 200
    - 500
    - 1000
  - Forty-two percent of the primary care doctors think their patients receive unnecessary medical care. Suppose a sample of 300 primary care doctors were taken. What is the probability that the sample proportion will be within
    - ±0.03 of the population proportion?
    - ±0.05 of the population proportion?
    - What would be the effect of taking larger samples on the probabilities computed in the two cases above? Why?

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- Sampling Distribution for Proportion of Success
  - 17% of the households spend more than ₹2500 per week on groceries. A sample of 800 households will be selected from the population.
    - What is the probability that the sample proportion will be within ±0.02 of the population proportion?
    - What would be probability in the sample size were to be 1600 households?

Thank You

Prof. Jigar M. Shah