

Assignment 2 (k-mean clustering)

Question:

Cluster the following eight points (with (x, y) representing locations) into three clusters:

$A_1 (2, 10)$,

$A_2 (2, 5)$,

$A_3 (8, 4)$,

$A_4 (5, 8)$,

$A_5 (7, 5)$,

$A_6 (6, 4)$,

$A_7 (1, 2)$,

$A_8 (4, 9)$.

Initial clusters are:

$A_1 (2, 10)$,

$A_4 (5, 8)$. and

$A_7 (1, 2)$.

The distance function between two points $a = (x_1, y_1)$ and $b = (x_2, y_2)$ is defined as

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Solution:

The initial cluster centers are:

$c_1 (2, 10)$

$c_2 (5, 8)$

$c_3 (1, 2)$

Distance calculations:

point	Co-ordinates	Distance to $c_1(2,10)$	Distance to $c_2(5,6)$	Distance to $c_3(1,2)$	Nearest cluster
A1	(2, 10)	0	3.61	8.06	c1
A2	(2, 5)	5	4.24	3.16	c3
A3	(8, 4)	8.49	5.00	7.28	c2
A4	(5, 8)	3.61	0	7.21	c2
A5	(7, 5)	7.07	3.61	6.71	c2
A6	(6, 4)	7.21	4.12	5.39	c2
A7	(1, 2)	8.06	7.21	0	c3
A8	(4, 9)	2.24	1.41	7.07	c2

$c_1 : (2, 10)$

$c_2 : A3, A4, A5, A6, A8$

$c_3 : A2, A7$

finding means

$\hookrightarrow c_1 (2, 10)$

$\hookrightarrow c_2 :$

mean of x-coordinates:

$$\frac{8 + 5 + 7 + 6 + 4}{5} = 6$$

mean of y-coordinates:

$$\frac{4 + 8 + 5 + 4 + 9}{5} = 6$$

$c_2 (6, 6)$

↳ c_3

mean of x-coordinates.

$$\frac{2+1}{2} = 1.5$$

mean of y-coordinates.

$$\frac{5+2}{2} = 3.5$$

$c_3(1.5, 3.5)$

Distance calculation.

point	coordinates	Distance to $c_1(2,10)$	Distance to $c_2(6,6)$	Distance to $c_3(1.5, 3.5)$	Nearest cluster
A1	(2,10)	0	5.66	6.40	c_1
A2	(2,8)	5	4.12	1.58	c_3
A3	(8,4)	8.49	2.83	6.80	c_2
A4	(5,8)	3.61	2.24	5.41	c_2
A5	(9,5)	7.07	1.41	5.70	c_2
A6	(6,4)	7.21	2.00	4.61	c_2
A7	(1,2)	8.06	6.40	1.58	c_3
A8	(4,9)	2.24	3.61	5.70	c_1

update cluster centers:

⇒ $c_1 : A1, A8$

mean of x-coordinates:

$$\frac{2+4}{2} = 3$$

mean of y-coordinates

$$\frac{10+9}{2} = 9.5$$

$$c_1(3, 9.5)$$

$$\Rightarrow c_2: A_3, A_4, A_5, A_6$$

mean of x-coordinates:

$$\frac{8+5+7+6}{4} = 6.5$$

mean of y-coordinates:

$$\frac{4+8+5+4}{4} = 5.25$$

$$c_2(6.5, 5.25)$$

$$\Rightarrow c_3: A_2, A_7$$

mean of x-coordinates

$$\frac{2+1}{2} = 1.5$$

mean of y-coordinates

$$\frac{5+2}{2} = 3.5$$

$$c_3(1.5, 3.5)$$

Distance calculation:

point	coordinates	Distance to $c_1 (3, 9.5)$	Distance to $c_2 (6.5, 5.25)$	Distance to $c_3 (1.5, 3.5)$	nearest cluster
A1	(2, 10)	1.72	6.02	6.80	c1
A2	(2, 5)	4.53	4.61	1.58	c3
A3	(8, 4)	7.43	1.68	6.50	c2
A4	(5, 8)	2.69	2.96	5.41	c1
A5	(7, 5)	5.41	0.79	5.70	c2
A6	(6, 4)	5.59	1.27	4.61	c2
A7	(1, 2)	7.59	6.10	1.58	c3
A8	(4, 9)	1.127	3.91	5.70	c1

update cluster centers:

$\Rightarrow c_1 : A1, A4, A8.$

mean x-coordinates:

$$\frac{2+5+4}{3} = 3.67,$$

mean of y-coordinates:

$$\frac{10+8+9}{3} = 9.$$

$c_1 (3.67, 9).$

$\Rightarrow c_2 : A3, A5, A6.$

mean of x-coordinates:

$$\frac{8+7+6}{3} = 7.$$

mean of y-coordinates:

$$\frac{4+5+4}{3} = 4.33$$

$$c_2(7, 4.33)$$

$$\Rightarrow c_3 : A_2, A_7$$

mean of x-coordinates:

$$\frac{2+1}{2} = 1.5$$

mean of y-coordinates:

$$\frac{5+2}{2} = 3.5$$

$$c_3(1.5, 3.5)$$

After calculating the distance for each point we get the same result as previous one, so we stop.

Final clusters:

$$\Rightarrow c_1 : A_1(2, 10), A_4(5, 8), A_8(4, 9)$$

$$\Rightarrow c_2 : A_3(8, 4), A_5(7, 5), A_6(6, 4)$$

$$\Rightarrow c_3 : A_2(2, 5), A_7(1, 2)$$

Final cluster centers:

$$\Rightarrow c_1 : (3.67, 9)$$

$$\Rightarrow c_2 : (7, 4.33)$$

$$\Rightarrow c_3 : (1.5, 3.5)$$