Session 8 - 9

Probability

Prof. Jigar M. Shah

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Probability

- Theories of Probability
  - Introduction
  - Classical Theory
  - Relative Frequency Theory
- Axioms
  - Addition Rule
  - Multiplication Rule
  - Rule of At Least One Success
  - Expected Number of Trials
  - Bayes' Theorem

Probability

Theories of Probability

- Introduction
  - 2 types of outcomes of an experiment repeated under essentially homogeneous & similar conditions

Outcomes
- Unique or Certain — Predictable or Deterministic Phenomenon
- Not Unique, But One of the Several Possible Outcomes — Unpredictable or Probabilistic or Phenomena

Probability

Theories of Probability

- Introduction

Common Terms	
Trial	An experiment, which though repeated under essentially identical conditions, does not give unique results but may result in any one of the several possible outcomes
Events	Outcomes of the trial
Exhaustive Events	The total no. of possible outcomes in any trial
Favorable Events	Outcomes which entail the happening of the event
Mutually Exclusive Events	Set of events such that the happening of any one of them precludes the happening of all other events

Probability

Theories of Probability

- Introduction

Common Terms	
Equally Likely Events	Set of events for which, after considering all the relevant evidences, there is no reason to expect one in preference to the others
Independent Events	Set of events for which the happening (or non-happening) are not affected by the supplementary knowledge concerning the occurrence of any number of the remaining events
P(A)	Probability of event A happening
Marginal / Single / Unconditional Probability	Probability of a single event i.e. only one event taking place

Probability

Theories of Probability

- Introduction

Throwing a die

		1	2	3	4	5	6	Marginal Probability
Tossing a coin	Heads							½
	Tails							½
Marginal Probability		⅙	⅙	⅙	⅙	⅙	⅙	

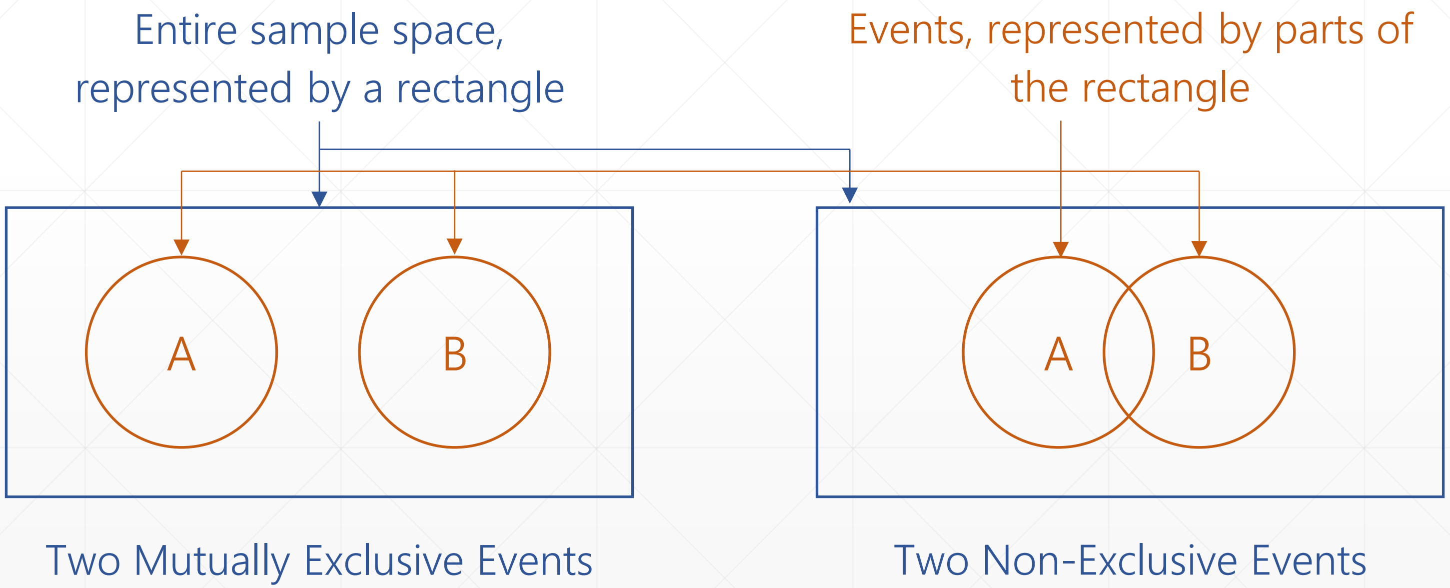
Probability of a single event is written in the margins, hence marginal probability

P(Heads) = ½ P(Tails) = ½ P(1) = ⅙ P(2) = ⅙ P(3) = ⅙ P(4) = ⅙ P(5) = ⅙ P(5) = ⅙

Probability

Theories of Probability

- Introduction
  - Venn Diagrams



Probability

Theories of Probability

- Classical Theory
  - For equally likely events, $$\text{Probability of an Event} = \frac{\text{No. of favorable outcomes}}{\text{Total no. of possible outcomes}}$$
  - Classical probability is also known as called a priori probability since probability can be known in advance (a priori) without actually conducting the experiment
  - Useful when dealing with card games, dice games, coin tosses, etc.
  - Not suitable for less orderly decision problems encountered in business management, as it assumes away situations that are very unlikely but that could conceivably happen

Probability

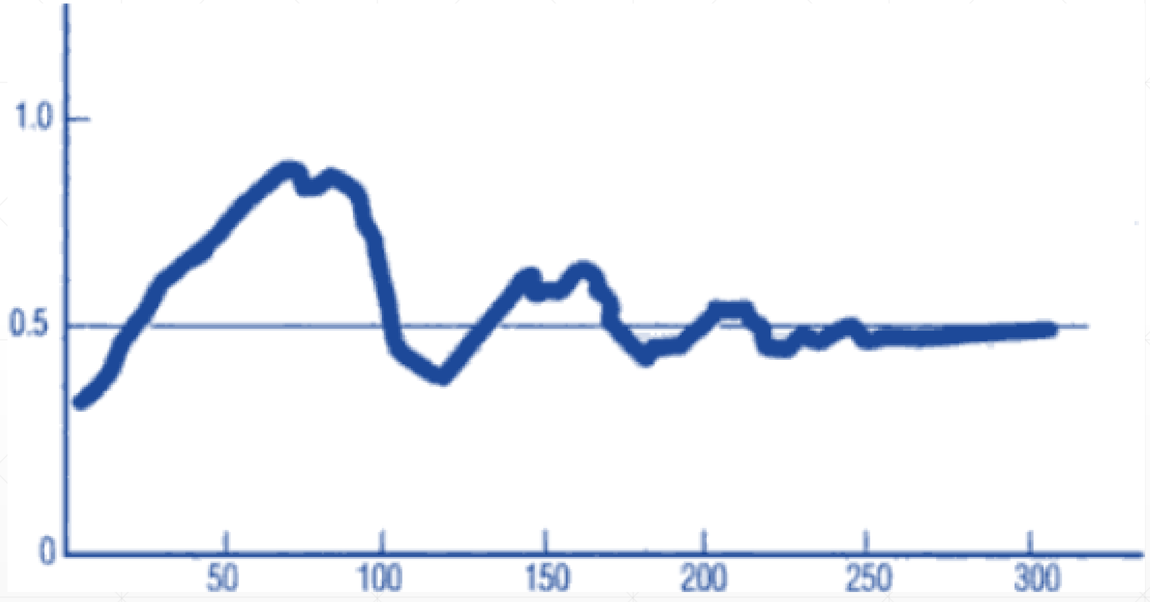
Theories of Probability

- Relative Theory
  - For cases where it may not be possible to state probabilities in advance, without actual experimentation
  - Defines probabilities from statistical data collected
  - Defines probabilities as either the observed relative frequency of an event in a very large number of trials or the proportion of times that an event occurs in the long run when conditions are stable
  - Uses relative frequencies of the past occurrences as probabilities
  - E.g. - Past actuarial data tells an insurance firm that of all males who are 40 years old, about 60 out of every 1,00,000 will die within a 1-year period. Based on this data, probability of death for the given age group can be computed as $\frac{60}{1,00,000} = 0.0006$

Probability

Theories of Probability

- Relative Theory
  - Relative frequency becomes stable as the number of observations increases
  - Probability figure based on relative frequency approach will gain accuracy as the no. of observation increases



Probability

Theories of Probability

- Following are results of polling 30 machinists (M) & 30 supervisors (S) for their opinion of a revised wage package. What is the probability that:
  - Machinist randomly selected from the polled group mildly supports the package?
  - Supervisor randomly selected from the polled group is undecided about the package?
  - A worker (machinist/supervisor) randomly selected from the polled group strongly or mildly supports the package?
  - What types of probability estimates are these?

Opinion of Package	M	S
Strongly support	9	10
Mildly support	11	3
Undecided	2	2
Mildly oppose	4	8
Strongly oppose	4	7
	30	30

Probability

Theories of Probability

- Over the period of his 20-week tenure, a student intern at a firm has observed that his chances of getting between 50 & 74% of his weekly travel allowance claim approved are twice as good as those of getting between 75 & 99% approved, and 2.5 times as good as those of getting between 25 & 49% approved. Also, there is no chance of less than 25% of his weekly travel allowance claim being approved, and only once has 100% of it been approved. What are the probabilities of 0-24%, 25-49%, 50-74%, 75-99%, & 100% of the claim being approved?
- A manager at a bank has the following data on the functioning of copiers in his branch. What is the probability of a copier being out of service based on these data?

Copier No.	1	2	3	4	5
Days Functioning	209	217	258	229	247
Days Out of Service	51	43	2	31	13

Probability

Axioms

- Addition Rule
  - For Mutually Exclusive Events

Probability of Either A or B Happening				
Probability of Either A or B Happening	=	P(A or B)	=	P(A) + P(B)
Probability of Either A or B or C or D Happening	=	P(A or B or C or D)	=	P(A) + P(B) + P(C) + P(D)

Special Case - Complement Rule	
For any event A, either A happens or it doesn't, and hence, the events A and not A are exclusive & exhaustive	P(A) + P(not A) = 1
	P(A) = 1 - P(not A)

Probability

Axioms

- Addition Rule
  - For Events that are Not Mutually Exclusive

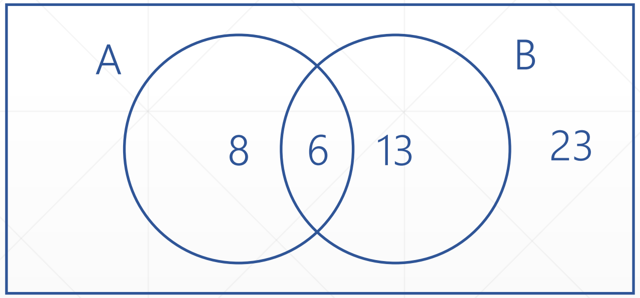
Probability of Either A or B Happening (when A & B are not mutually exclusive events)		
Probability of Either A or B Happening = P(A or B) = P(A) + P(B) - P(AB)		
Probability of A Happening	Probability of B Happening	Probability of Both A & B Happening

Probability

Axioms

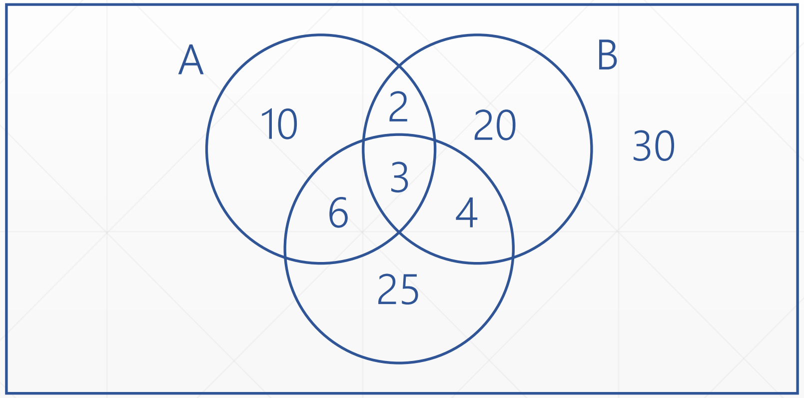
- Addition Rule
  - Compute P(A), P(B) and P(A or B).

Total Outcomes = 50



  - Compute P(A), P(B), P(C), P(A or B), P(A or C), P(B or C), P(A but not (B or C)), P(B but not (A or C)) , P(C but not (A or B)).

Total Outcomes = 100



Probability

Axioms

- Addition Rule
  - An urn has 75 marbles,: 35 are blue, and 25 of these blue marbles are swirled. The rest of them red, and 30 of the red ones are swirled. The marbles that are nor swirled are clear. What is the probability of drawing:
    - A blue marble from the urn?
    - A clear marble from the urn?
    - A blue, swirled marble from the urn?
    - A red, clear marble from the urn?
    - A swirled marbled from the urn?

Probability

Axioms

- Multiplication Rule
  - For Independent Events

Joint Probability of Two Independent Events

P(AB) = P(A) × P(B)

Marginal Probability of B occurring

Marginal Probability of A occurring

Probability of events A & B occurring together or in succession i.e., Joint Probability of A & B

Probability

Axioms

- Multiplication Rule
  - What is the probability of getting tails, heads, tails in that order on three successive tosses of a fair coin?
  - What is the probability of getting tails, tails, heads in that order on three successive tosses of a fair coin?
  - Out of 10 balls, 3 are colored & dotted, 1 is colored & striped, 2 are gray & dotted, 4 are gray & striped. If someone draws a colored ball, what is the probability that it is dotted? What is the probability that it is striped?

Probability

Axioms

- Multiplication Rule
  - For Dependent Events

Joint Probability of Two Dependent Events
$P(AB) = P(A) \times P(B\|A)$

$P(B|A)$: Probability of B occurring given that event A has occurred i.e., Conditional Probability of B given A

$P(A)$: Marginal Probability of A occurring

$P(AB)$: Probability of events A & B occurring together or in succession i.e., Joint Probability of A & B

Probability

Axioms

- Multiplication Rule
  - For Dependent Events

Joint Probability of Two Dependent Events
$P(AB) = P(B) \times P(A \mid B)$
Probability of A occurring given that event B has occurred i.e., Conditional Probability of A given B
Marginal Probability of B occurring
Probability of events A & B occurring together or in succession i.e., Joint Probability of A & B

Probability

Axioms

- Rule of At Least One
  - For Dependent Events

Probability of At Least One Event
P(at least one) = 1 - P(none)

Probability of none of the events occurring or Probability of an event not occurring

Probability of at least one of the events occurring or Probability of an event occurring at least once

Probability

Axioms

- Rule of At Least One
  - Find the probability of getting at least one defective product in a batch of 50 products, if the probability of a product being defective is 0.005.
  - Find the probability of getting at least two heads on three successive tosses of a fair coin.
  - Find the probability of getting at least 1 six in four throws of a die.
  - In a roll of two dice, find the probability of at least one of them rolling a six.

Probability

Axioms

- Two events A & B are statistically dependent. If P(A) = 0.39, P(B) = 0.21, and P(A or B) = 0.47, find the probability that:
    - Neither A nor B will occur
    - Both A and B will occur
    - B will occur given that A has occurred
    - A will occur given that B has occurred
- Assume that for two events A & B, P(A) = 0.65, and P(B) = 0.80, P(A|B) = P(A), and P(B|A) = 0.85. Is this consistent assignment of probabilities? Explain.

Probability

Axioms

- In a study of highway accidents, NHAI found that 60% of all accidents occur at night, 52% are alcohol-related, and 37% occur at night & are alcohol-related. Find the probability that an accident:
  - Was alcohol-related, given that it occurred at night
  - Occurred at night, given that it was alcohol-related
- Given that $P(A) = 3/14$, $P(B) = 1/6$, $P(C) = 1/3$, $P(AC) = 1/7$, $P(B|C) = 5/21$, find $P(A|C)$, $P(C|A)$, $P(BC)$, $P(C|B)$.

Probability

Axioms

- Expected Number of Trials

Expected Number of Trials
$E = 1 \div P$

P: Probability of success in a trial

E: Expected number of trials

Probability

Axioms

- Bayes' Theorem
  - A priori probabilities are often revised based on the information about the event

New Information → Prior Probabilities → Application of Bayes' Theorem → Posterior Probabilities

  - Posterior Probabilities - probabilities revised based on additional information gained

Bayes' Theorem for Posterior Probabilities

$P(B|A) \quad = \quad P(BA) \quad \div \quad P(A)$

- $P(A)$: Probability of event A occurring
- $P(BA)$: Probability of events B & A occurring together or in succession i.e., Joint Probability of B & A
- $P(B|A)$: Probability of event B given that event A has occurred i.e., Conditional Probability of B given A

Probability

Axioms

- Bayes' Theorem
  - $P(A) = 0.35$, $P(B) = 0.45$, $P(C) = 0.2$. Assuming that A, B, or C has occurred, the probabilities of another event X occurring are $P(X|A) = 0.8$, $P(X|B) = 0.65$, $P(X|C) = 0.3$. Find $P(A|X)$, $P(B|X)$, $P(C|X)$.

Event	P(Event)	P(X|Event)	P(X & Event) = P(Event) × P(X|Event)	P(Event|X) = P(X & Event) ÷ P(X)
A	0.35	0.80	0.35 × 0.80 = 0.2800	0.2800 ÷ 0.6325 = 0.4427
B	0.45	0.65	0.45 × 0.65 = 0.2925	0.2925 ÷ 0.6325 = 0.4625
C	0.20	0.30	0.20 × 0.30 = 0. 0600	0.0600 ÷ 0.6325 = 0.0949
			0.6325	

Probability

Axioms

- Bayes' Theorem
  - Out of 200 patients, 50 are prescribed drug A, 50 drug B, and 100 both, by a doctor. The 200 patients were chosen such that each had an 80% chance of a heart attack if no drug was given. Drug A reduces the probability of heart attack by 25%, drug B by 20%, and the two drugs taken together work independently. If a randomly selected patient in the program has a heart attack, what is the probability that the patient was given both the drugs?

Let A represent the event that a patient was prescribed drug A	P(A) = 0.25
Let B represent the event that a patient was prescribed drug B	P(B) = 0.25
Let C represent the event that a patient was prescribed both the drugs A & B	P(C) = 0.50
Let H represent the event that a patient suffered a heart attack	P(H) = 0.80

Probability

Axioms

- Bayes' Theorem
  - P(A) = 0.35, P(B) = 0.45, P(C) = 0.2. Assuming that A, B, or C has occurred, the probabilities of another event X occurring are P(X|A) = 0.8, P(X|B) = 0.65, P(X|C) = 0.3. Find P (A|X), P(B|X), P(C|X).

Event	P(Event)	P(H|Event)	P(H & Event) = P(Event) × P(H|Event)	P(Event|H) = P(H & Event) ÷ P(H)
A	0.25	0.80 × 0.75 = 0.60	0.25 × 0.60 = 0.15	0.15 ÷ 0.55 = 0.2727
B	0.25	0.80 × 0.80 = 0.64	0.25 × 0.64 = 0.16	0.16 ÷ 0.55 = 0.2909
C	0.50	0.80 × 0.75 × 0.80 = 0.48	0.50 × 0.48 = 0. 24	0.24 ÷ 0.55 = 0.4364
			0.55	

Thank You

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