Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

- F-Test
  - F Distribution
  - F-Test for Inferences About Two Population Variances
- Chi-Square Test
  - Chi-Square Statistic
  - Chi-Square Distribution
  - Chi-Square Tests
  - Precaution Using Chi-Square Test
- Analysis of Variance (ANOVA)
  - One-Way Classification Model
  - Two-Way Classification Model

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- F Distribution
  - A distribution used in comparing variances of two normal populations

F-Distribution - Sampling Distribution of $(s_1)^2 / (s_2)^2$ When $(\sigma_1)^2 = (\sigma_2)^2$

Whenever independent random samples of sizes n_1 & n_2 are selected from two normal populations with equal variances,

the sampling distribution of $\dfrac{(s_1)^2}{(s_2)^2}$ is an F Distribution

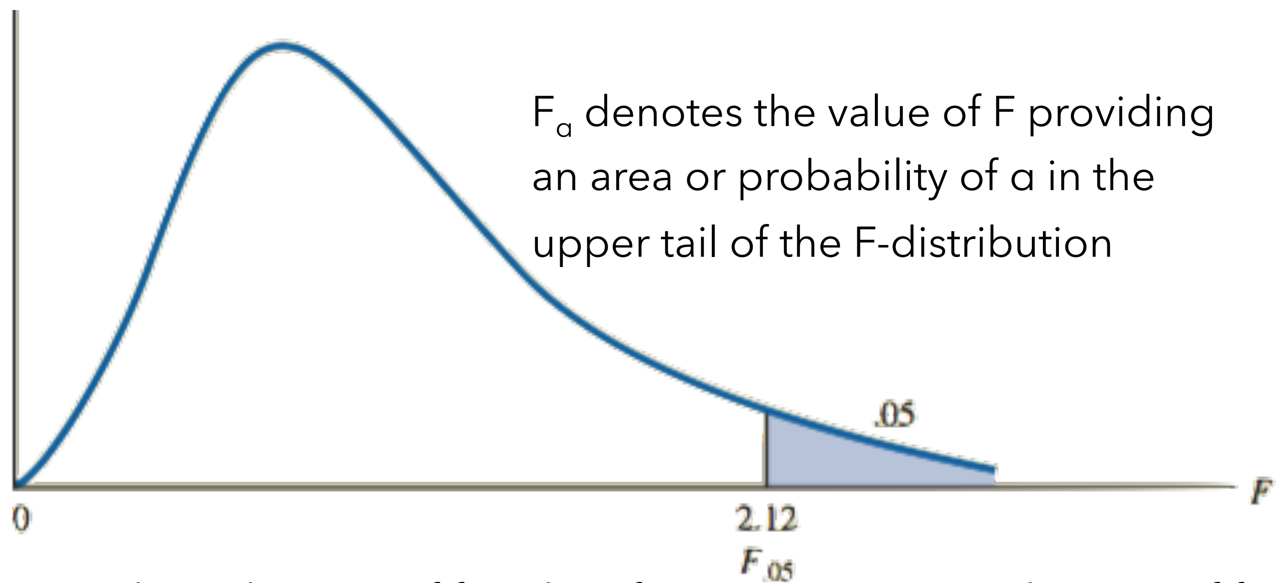
with $n_1 - 1$ degrees of freedom for numerator & $n_2 - 1$ degrees of freedom for denominator; $(s_1)^2$ is sample variance for random sample of n_1 items from population 1, $(s_2)^2$ is sample variance for random sample of n_2 items from population 2.

Session 25-30

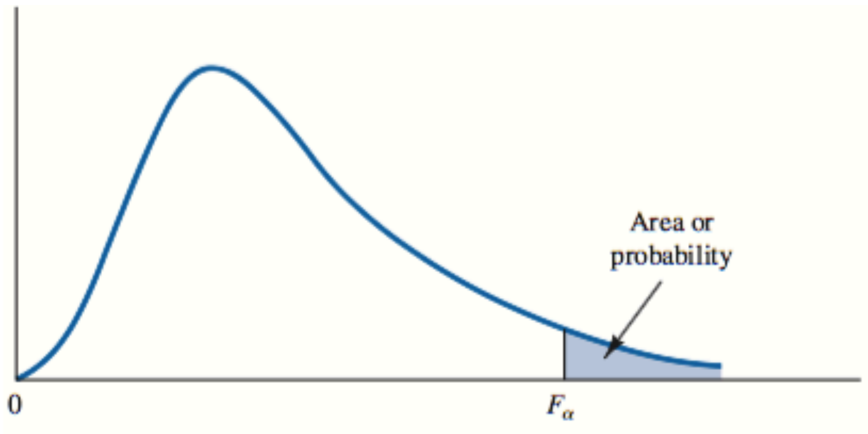
F-Test, Analysis of Variance & Chi-Square Test

F-Test

- F Distribution
  - F values can never be negative



F distribution with 20 degrees of freedom for numerator & 20 degrees of freedom for denominator



Entries in the table give F_α values, where α is the area or probability in the upper tail of the F distribution. For example, with 4 numerator degrees of freedom, 8 denominator degrees of freedom, and a .05 area in the upper tail, $F_{.05} = 3.84$.

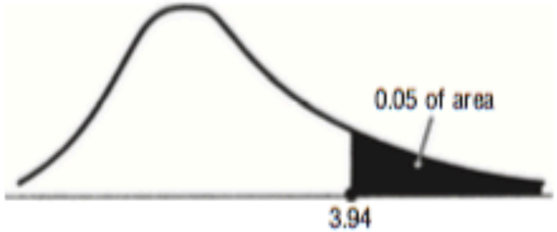
Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
1	.10	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	61.22	61.74	62.05	62.26	62.53	62.79	63.01	63.30
	.05	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	245.95	248.02	249.26	250.10	251.14	252.20	253.04	254.19
	.025	647.79	799.48	864.15	899.60	921.83	937.11	948.20	956.64	963.28	968.63	984.87	993.08	998.09	1001.40	1005.60	1009.79	1013.16	1017.76
	.01	4052.18	4999.34	5403.53	5624.26	5763.96	5858.95	5928.33	5980.95	6022.40	6055.93	6156.97	6208.66	6239.86	6260.35	6286.43	6312.97	6333.92	6362.80
2	.10	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
	.05	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.49
	.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
	.01	98.50	99.00	99.16	99.25	99.30	99.33	99.36	99.38	99.39	99.40	99.43	99.45	99.46	99.47	99.48	99.48	99.49	99.50
3	.10	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.20	5.18	5.17	5.17	5.16	5.15	5.14	5.13
	.05	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.63	8.62	8.59	8.57	8.55	8.53
	.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.25	14.17	14.12	14.08	14.04	13.99	13.96	13.91
	.01	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.34	27.23	26.87	26.69	26.58	26.50	26.41	26.32	26.24	26.14
4	.10	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
	.05	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
	.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.66	8.56	8.50	8.46	8.41	8.36	8.32	8.26
	.01	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.20	14.02	13.91	13.84	13.75	13.65	13.58	13.47
5	.10	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.324	3.21	3.19	3.17	3.16	3.14	3.13	3.11
	.05	6.61.	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.52	4.50	4.46	4.43	4.41	4.37
	.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.43	6.33	6.27	6.23	6.18	6.12	6.08	6.02
	.01	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.72	9.55	9.45	9.38	9.29	9.20	9.13	9.03

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
6	.10	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.87	2.84	2.81	2.80	2.78	2.76	2.75	2.72
	.05	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.83	3.81	3.77	3.74	3.71	3.67
	.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.27	5.17	5.11	5.07	5.01	4.96	4.92	4.86
	.01	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.56	7.40	7.30	7.23	7.14	7.06	6.99	6.89
7	.10	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.63	2.59	2.57	2.56	2.54	2.51	2.50	2.47
	.05	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.40	3.38	3.34	3.30	3.27	3.23
	.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.57	4.47	4.40	4.36	4.31	4.25	4.21	4.15
	.01	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.31	6.16	6.06	5.99	5.91	5.82	5.75	5.66
8	.10	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.30
	.05	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.11	3.08	3.04	3.01	2.97	2.93
	.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.10	4.00	3.94	3.89	3.84	3.78	3.74	3.68
	.01	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.52	5.36	5.26	5.20	5.12	5.03	4.96	4.87
9	.10	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.34	2.30	2.27	2.25	2.23	2.21	2.19	2.16
	.05	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.89	2.86	2.83	2.79	2.76	2.71
	.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.77	3.67	3.60	3.56	3.51	3.45	3.40	3.34
	.01	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	4.96	4.81	4.71	4.65	4.57	4.48	4.41	4.32
10	.10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.24	2.20	2.17	2.16	2.13	2.11	2.09	2.06
	.05	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.73	2.70	2.66	2.62	2.59	2.54
	.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.52	3.42	3.35	3.31	3.26	3.20	3.15	3.09
	.01	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.56	4.41	4.31	4.25	4.17	4.08	4.01	3.92
11	.10	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.17	2.12	2.10	2.08	2.05	2.03	2.01	1.98
	.05	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.60	2.57	2.53	2.49	2.46	2.41
	.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.33	3.23	3.16	3.12	3.06	3.00	2.96	2.89
	.01	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.25	4.10	4.01	3.94	3.86	3.78	3.71	3.61
12	.10	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.10	2.06	2.03	2.01	1.99	1.96	1.94	1.91
	.05	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.50	2.47	2.43	2.38	2.35	2.30
	.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.18	3.07	3.01	2.96	2.91	2.85	2.80	2.73
	.01	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.01	3.86	3.76	3.70	3.62	3.54	3.47	3.37
13	.10	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
	.05	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.41	2.38	2.34	2.30	2.26	2.21
	.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.05	2.95	2.88	2.84	2.78	2.72	2.67	2.60
	.01	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.82	3.66	3.57	3.51	3.43	3.34	3.27	3.18
14	.10	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.01	1.96	1.93	1.99	1.89	1.86	1.83	1.80
	.05	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.34	2.31	2.27	2.22	2.19	2.14
	.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	2.95	2.84	2.78	2.73	2.67	2.61	2.56	2.50
	.01	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.66	3.51	3.41	3.35	3.27	3.18	3.11	3.02
15	.10	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	1.97	1.92	1.89	1.87	1.85	1.82	1.79	1.76
	.05	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.28	2.25	2.20	2.16	2.12	2.07
	.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.86	2.76	2.69	2.64	2.59	2.52	2.47	2.40
	.01	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.52	3.37	3.28	3.21	3.13	3.05	2.98	2.88

Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
16	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.94	1.89	1.86	1.84	1.81	1.78	1.76	1.72
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.23	2.19	2.15	2.11	2.07	2.02
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.79	2.68	2.61	2.57	2.51	2.45	2.40	2.32
	.01	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.41	3.26	3.16	3.10	3.02	2.93	2.86	2.76
17	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.91	1.86	1.83	1.81	1.78	1.75	1.73	1.69
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.18	2.15	2.10	2.06	2.02	1.97
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.72	2.62	2.55	2.50	2.44	2.38	2.33	2.26
	.01	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.31	3.16	3.07	3.00	2.92	2.83	2.76	2.66
18	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.89	1.84	1.80	1.78	1.75	1.72	1.70	1.66
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.14	2.11	2.06	2.02	1.98	1.92
	.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.67	2.56	2.49	2.44	2.38	2.32	2.27	2.20
	.01	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.23	3.08	2.98	2.92	2.84	2.75	2.68	2.58
19	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.86	1.81	1.78	1.76	1.73	1.70	1.67	1.64
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.11	2.07	2.03	1.98	1.94	1.88
	.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.62	2.51	2.44	2.39	2.33	2.27	2.22	2.14
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.15	3.00	2.91	2.84	2.76	2.67	2.60	2.50
20	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.84	1.79	1.76	1.74	1.71	1.68	1.65	1.61
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.07	2.04	1.99	1.95	1.91	1.85
	.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.57	2.46	2.40	2.35	2.29	2.22	2.17	2.09
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.09	2.94	2.84	2.78	2.69	2.61	2.54	2.43
21	.10	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.83	1.78	1.74	1.72	1.69	1.66	1.63	1.59
	.05	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.18	2.10	2.05	2.01	1.96	1.92	1.88	1.82
	.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.53	2.42	2.36	2.31	2.25	2.18	2.13	2.05
	.01	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.03	2.88	2.79	2.72	2.64	2.55	2.48	2.37
22	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.81	1.76	1.73	1.70	1.67	1.64	1.61	1.57
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	2.02	1.98	1.94	1.89	1.85	1.79
	.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.50	2.39	2.32	2.27	2.21	2.14	2.09	2.01
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	2.98	2.83	2.73	2.67	2.58	2.50	2.42	2.32
23	.10	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.80	1.74	1.71	1.69	1.66	1.62	1.59	1.55
	.05	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.13	2.05	2.00	1.96	1.91	1.86	1.82	1.76
	.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.47	2.36	2.29	2.24	2.18	2.11	2.06	1.98
	.01	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	2.93	2.78	2.69	2.62	2.54	2.45	2.37	2.27
24	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.78	1.73	1.70	1.67	1.64	1.61	1.58	1.54
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.97	1.94	1.89	1.84	1.80	1.74
	.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.44	2.33	2.26	2.21	2.15	2.08	2.02	1.94
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	2.89	2.74	2.64	2.58	2.49	2.40	2.33	2.22

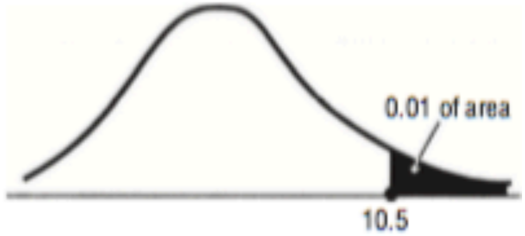
Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom																	
		1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	60	100	1000
25	.10	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.77	1.72	1.68	1.66	1.63	1.59	1.56	1.52
	.05	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.09	2.01	1.96	1.92	1.87	1.82	1.78	1.72
	.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.41	2.30	2.23	2.18	2.12	2.05	2.00	1.91
	.01	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.85	2.70	2.60	2.54	2.45	2.36	2.29	2.18
26	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.76	1.71	1.67	1.65	1.61	1.58	1.55	1.51
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.94	1.90	1.85	1.80	1.76	1.70
	.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.39	2.28	2.21	2.16	2.09	2.03	1.97	1.89
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.81	2.66	2.57	2.50	2.42	2.33	2.25	2.14
27	.10	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.75	1.70	1.66	1.64	1.60	1.57	1.54	1.50
	.05	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.06	1.97	1.92	1.88	1.84	1.79	1.74	1.68
	.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.36	2.25	2.18	2.13	2.07	2.00	1.94	1.86
	.01	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.78	2.63	2.54	2.47	2.38	2.29	2.22	2.11
28	.10	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.74	1.69	1.65	1.63	1.59	1.56	1.53	1.48
	.05	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.91	1.87	1.82	1.77	1.73	1.66
	.025	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.34	2.23	2.16	2.11	2.05	1.98	1.92	1.84
	.01	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.75	2.60	2.51	2.44	2.35	2.26	2.19	2.08
29	.10	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.73	1.68	1.64	1.62	1.58	1.55	1.52	1.47
	.05	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.03	1.94	1.89	1.85	1.81	1.75	1.71	1.65
	.025	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.32	2.21	2.14	2.09	2.03	1.96	1.90	1.82
	.01	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.73	2.57	2.48	2.41	2.33	2.23	2.16	2.05
30	.10	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.72	1.67	1.63	1.61	1.57	1.54	1.51	1.46
	.05	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.88	1.84	1.79	1.74	1.70	1.63
	.025	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.31	2.20	2.12	2.07	2.01	1.94	1.88	1.80
	.01	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.70	2.55	2.45	2.39	2.30	2.21	2.13	2.02
40	.10	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.66	1.61	1.57	1.54	1.51	1.47	1.43	1.38
	.05	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.78	1.74	1.69	1.64	1.59	1.52
	.025	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.18	2.07	1.99	1.94	1.88	1.80	1.74	1.65
	.01	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.52	2.37	2.27	2.20	2.11	2.02	1.94	1.82
60	.10	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.60	1.54	1.50	1.48	1.44	1.40	1.36	1.30
	.05	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.84	1.75	1.69	1.65	1.59	1.53	1.48	1.40
	.025	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.06	1.94	1.87	1.82	1.74	1.67	1.60	1.49
	.01	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.35	2.20	2.10	2.03	1.94	1.84	1.75	1.62
100	.10	2.76	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.69	1.66	1.56	1.49	1.45	1.42	1.38	1.34	1.29	1.22
	.05	3.94	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.97	1.93	1.77	1.68	1.62	1.57	1.52	1.45	1.39	1.30
	.025	5.18	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.24	2.18	1.97	1.85	1.77	1.71	1.64	1.56	1.48	1.36
	.01	6.90	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.59	2.50	2.22	2.07	1.97	1.89	1.80	1.69	1.60	1.45
1000	.10	2.71	2.31	2.09	1.95	1.85	1.78	1.72	1.68	1.64	1.61	1.49	1.43	1.38	1.35	1.30	1.25	1.20	1.08
	.05	3.85	3.00	2.61	2.38	2.22	2.11	2.02	1.95	1.89	1.84	1.68	1.58	1.52	1.47	1.41	1.33	1.26	1.11
	.025	5.04	3.70	3.13	2.80	2.58	2.42	2.30	2.20	2.13	2.06	1.85	1.72	1.64	1.58	1.50	1.41	1.32	1.13
	.01	6.66	4.63	3.80	3.34	3.04	2.82	2.66	2.53	2.43	2.34	2.06	1.90	1.79	1.72	1.61	1.50	1.38	1.16

EXAMPLE: IN AN *F* DISTRIBUTION WITH 15 DEGREES OF FREEDOM FOR THE NUMERATOR AND 6 DEGREES OF FREEDOM FOR THE DENOMINATOR, TO FIND THE *F* VALUE FOR 0.05 OF THE AREA UNDER THE CURVE LOOK UNDER THE 15 DEGREES OF FREEDOM COLUMN AND ACROSS THE 6 DEGREES OF FREEDOM ROW; THE APPROPRIATE *F* VALUE IS 3.94.



APPENDIX TABLE 6(a) VALUES OF *F* FOR *F* DISTRIBUTIONS WITH 0.05 OF THE AREA IN THE RIGHT TAIL

Degrees of Freedom for Denominator	Degrees of Freedom for Numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00



EXAMPLE: IN AN *F* DISTRIBUTION WITH 7 DEGREES OF FREEDOM FOR THE NUMERATOR AND 5 DEGREES OF FREEDOM FOR THE DENOMINATOR, TO FIND THE *F* VALUE FOR 0.01 OF THE AREA UNDER THE CURVE LOOK UNDER THE 7 DEGREES OF FREEDOM COLUMN AND ACROSS THE 5 DEGREES OF FREEDOM ROW; THE APPROPRIATE *F* VALUE IS 10.5.

APPENDIX TABLE 6(b) VALUES OF *F* FOR *F* DISTRIBUTIONS WITH 0.01 OF THE AREA IN THE RIGHT TAIL

Degrees of Freedom for Denominator	Degrees of Freedom for Numerator																		
	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,982	6,023	6,056	6,106	6,157	6,209	6,235	6,261	6,287	6,313	6,339	6,366
2	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	99.5
3	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	27.2	27.1	26.9	26.7	26.6	26.5	26.4	26.3	26.2	26.1
4	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.7	13.6	13.5
5	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	10.1	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.70	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.19	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.86	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.53	2.45	2.36	2.27	2.17
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- F-Test for Inferences About Two Population Variances
  - F distribution can be used to conduct a hypothesis test about the variances of two populations
  - Hypothesis testing about variances of two populations requires two independent random samples, one from each population
  - Population providing the larger sample variance is population 1

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- F-Test for Inferences About Two Population Variances

Forms for the Hypothesis Test About Two Population Variances	
$H_0 : (\sigma_1)^2 \leq (\sigma_2)^2$ $H_1 : (\sigma_1)^2 > (\sigma_2)^2$	$H_0 : (\sigma_1)^2 = (\sigma_2)^2$ $H_1 : (\sigma_1)^2 \neq (\sigma_2)^2$
Upper-tailed test	Two-tailed test
H_0 is Null Hypothesis H_1 is Alternate Hypothesis	$(\sigma_1)^2$ is variance of population with larger sample variance $(\sigma_2)^2$ is variance of population with lower sample variance
For a one-tailed test of two variances, the populations must be numbered such that the alternative hypothesis has the form $(\sigma_1)^2 > (\sigma_2)^2$	

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- F-Test for Inferences About Two Population Variances

Test Statistic for Hypothesis Tests about Two Population Variances		
$F = \dfrac{(s_1)^2}{(s_2)^2}$		The test statistic F has an F-Distribution with $n_1 - 1$ degrees of freedom for the numerator and $n_2 - 1$ degrees of freedom for the denominator
where	$(s_1)^2$ =	sample variance for sample of size n_1 from population 1 providing a larger sample variance
	$(s_2)^2$ =	sample variance for sample of size n_2 from population 2 providing a smaller sample variance

Because the F test statistic is constructed with the larger sample variance $(s_1)^2$ in numerator, the value of test statistic will be in the upper tail of F-Distribution

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example
  - A sociologist believing that incomes earned by college graduates show much greater variability than the earnings of those who did not attend colleges, takes a random sample of 21 college graduates & finds that their earnings have a sample standard deviation of Rs.17000, and also takes another random sample of 25 non-graduates & finds that their earnings have a sample standard deviation of Rs.7500. Verify the sociologist's belief at 0.01 level of significance.

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example

$H_0: (\sigma_1)^2 \leq (\sigma_2)^2$ $\alpha = 0.01$ $n_1 = 21$ $s_1 = 17000$

$H_1: (\sigma_1)^2 > (\sigma_2)^2$ $n_2 = 25$ $s_2 = 7500$

$$F = \frac{(s_1)^2}{(s_2)^2} = 5.14 \qquad F_\alpha = 2.74$$

For right-tailed test, critical value is the smallest value of test statistic that will result in rejection of H_0, and the rejection rule is to reject H_0 if $F \geq F_\alpha$ For $\alpha = 0.01$, largest value of F resulting in rejection of H_0 is value of F for which area to right of curve is 0.01 for numerator degrees of freedom 20 (21 -1) and denominator degrees of freedom 24 (25 – 1)

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example

Since 5.14 > 2.74, $F > F_{\alpha}$, and hence H_0 is rejected

Thus the sociologist's belief that incomes earned by college graduates show much greater variability than the earnings of those who did not attend colleges is true

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example
  - A pharmaceutical firm has developed two new anesthetics, A & B using similar chemical structures. And hence it has predicted that they would exhibit the same variance in the effect delay time (time between injection and complete loss of sensation in the patient). Sample data from tests of the two anesthetics are as shown below. At 2 percent level of significance, test whether the two compounds have the same variance in effect delay time.

	Sample Size	Sample Variance
A	31	1296
B	41	784

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example

$H_0: (\sigma_1)^2 = (\sigma_2)^2$ $\alpha = 0.02$ $n_1 = 31$ $(s_1)^2 = 1296$

$H_1: (\sigma_1)^2 \neq (\sigma_2)^2$ $n_2 = 41$ $(s_2)^2 = 784$

$$F = \frac{(s_1)^2}{(s_2)^2} = 1.65 \qquad F_{\alpha/2} = 2.20$$

For two-tailed test, critical values are the boundary values of test statistic $F_{\alpha/2}$ that will result in rejection of $H_{0,}$ and the rejection rule is to reject H_0 if $F \leq F_{(1-(\alpha/2))}$ or if $F \geq F_{\alpha/2}$

For $\alpha = 0.02$, $\alpha/2 = 0.01$, & smallest value of F resulting in rejection of H_0 is value of F for which area to right of curve is 0.01 for numerator degrees of freedom 30 and denominator degrees of freedom 40

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example

For $\alpha = 0.02$, $\alpha/2 = 0.01$, & largest value of F resulting in rejection of H_0 is value of F for which area to left of curve is 0.01 for numerator degrees of freedom 30 and denominator degrees of freedom 40

i.e. the value of F for which area to right of curve is 0.99 for numerator degrees of freedom 30 and denominator degrees of freedom 40

If $F_{(n,\, d,\, \alpha)}$ represent the value of F for n numerator degrees of freedom, d denominator degrees of freedom for l level of significance, then

$$F_{(n,\, d,\, l)} = \frac{1}{F_{(d,\, n,\, 1-l)}}$$

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F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example

For $\alpha = 0.02$, $\alpha/2 = 0.01$, & largest value of F resulting in rejection of H_0 is value of F for which area to left of curve is 0.01 for numerator degrees of freedom 30 and denominator degrees of freedom 40

i.e. the value of F for which area to right of curve is 0.99 for numerator degrees of freedom 30 and denominator degrees of freedom 40

i.e. 1 ÷ (the value of F for which area to right of curve is 0.01 for numerator degrees of freedom 40 and denominator degrees of freedom 30)

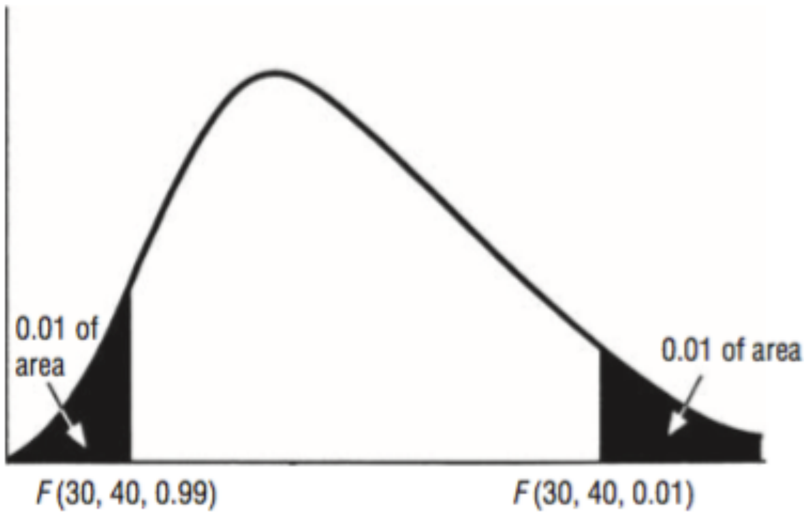
i.e. $1 \div 2.30 = 0.43$ $\quad F_{(1-(\alpha/2))} = 0.43$

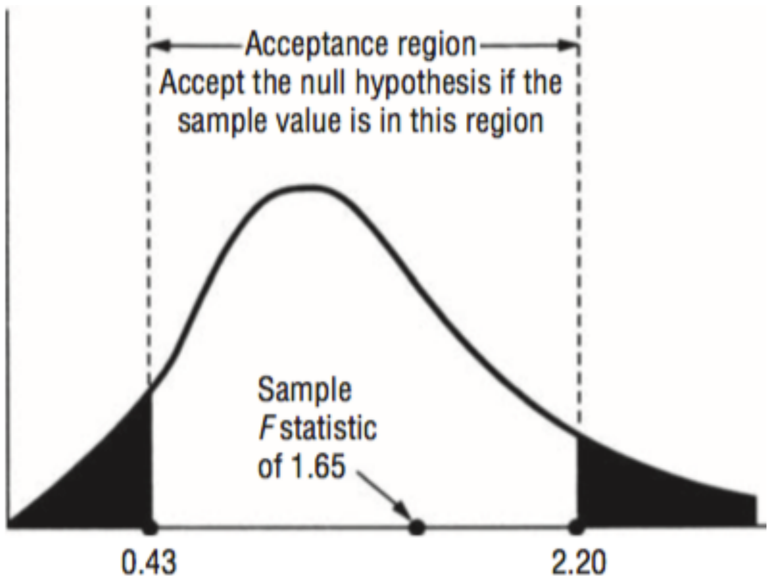
Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

F-Test

- Example





Since value of test statistic F = 1.65 which lies in the acceptance region, i.e. $F \geq F_{(1-(\alpha/2))}$ & $F \leq F_{(\alpha/2)}$, the null hypothesis is accepted.

Hence, it can be accepted that the two compounds have same variance in effect delay time.

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic
  - Used to compare proportions of more than two populations
  - When a population is classified into several categories with respect to these two attributes (e.g., age & occupation), used to determine whether the two attributes (e.g., age & occupation) are independent or not
  - Used to test the hypothesis about a population variance

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic
  - Comparing proportions of more than two populations

    Results of employee surveys in 4 regional offices of a firm about their preference of performance rating systems are summarized in the following 2×4 (no. of rows × no. of columns) contingency table

	North	East	West	South	Total
No. who prefer the current system	68	75	57	79	**279**
No. who prefer the new system	32	45	33	31	**141**
Total employees sampled	**100**	**120**	**90**	**110**	**420**

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic

Contingency Table

- A two-way table that represents categorical data with rows representing classification based on one categorical variable and columns representing classification based on another categorical variable

  - Chi-square test can be used to compare the proportion of employees preferring the current system in the 4 regional offices

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic

Let p_N be the proportion in North who prefer the current system

p_E be the proportion in East who prefer the current system

p_W be the proportion in West who prefer the current system

p_S be the proportion in South who prefer the current system

Let H_0: $p_N = p_E = p_W = p_S$

H_1: p_N, p_E, p_W, p_S are not all equal

If H_0 is true, then the proportion of total workforce who prefer the current system

$$= \frac{68 + 75 + 57 + 79}{100 + 120 + 90 + 110} = \frac{279}{420} = 0.6643$$

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic

Estimate of population proportion expected to prefer current system	=	0.6643
Estimate of population proportion expected to prefer new system	=	0.3357

	North	East	West	South
Number expected to prefer current system	100 × 0.6643 = 66.43	120 × 0.6643 = 79.72	90 × 0.6643 = 59.79	110 × 0.6643 = 73.07
Number expected to prefer new system	100 × 0.3357 = 33.57	120 × 0.3357 = 40.28	90 × 0.3357 = 30.21	110 × 0.3357 = 36.93

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic

	North	East	West	South
Frequency preferring current system				
Observed frequency	68	75	57	79
Expected frequency	66.43	79.72	59.79	73.07
Frequency preferring new system				
Observed frequency	32	45	33	31
Expected frequency	33.57	40.28	30.21	36.93

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic
  - Chi-square statistic to test hypothesis about proportions in 4 regional offices

Test Statistic for Hypothesis Tests about More than Two Population Proportions

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$$

where χ^2 = chi-square statistic

f_o = observed frequency

f_e = expected frequency

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Statistic
  - Higher values of χ^2 indicate greater difference between the observed & expected values
  - χ^2 value of 0 indicates that the observed & expected values are exactly the same

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Distribution

<table>
<tr><th colspan="2">Chi-Square Distribution</th></tr>
<tr><td colspan="2">If the null hypothesis is true, the sampling distribution of chi-square statistic, χ^2, can be closely approximated by a continuous curve known as chi-square distribution</td></tr>
<tr><td colspan="2">Different chi-square distributions for different degrees of freedom</td></tr>
<tr><td>For very low degrees of freedom, chi-square distribution is severely skewed towards the right</td><td>As no. of degrees of freedom increases, chi-square distribution rapidly becomes more symmetrical until it becomes large enough to be approximated by the normal distribution</td></tr>
<tr><td colspan="2">Being a probability distribution, the total area under the curve in each chi-square distribution is equal to 1</td></tr>
</table>

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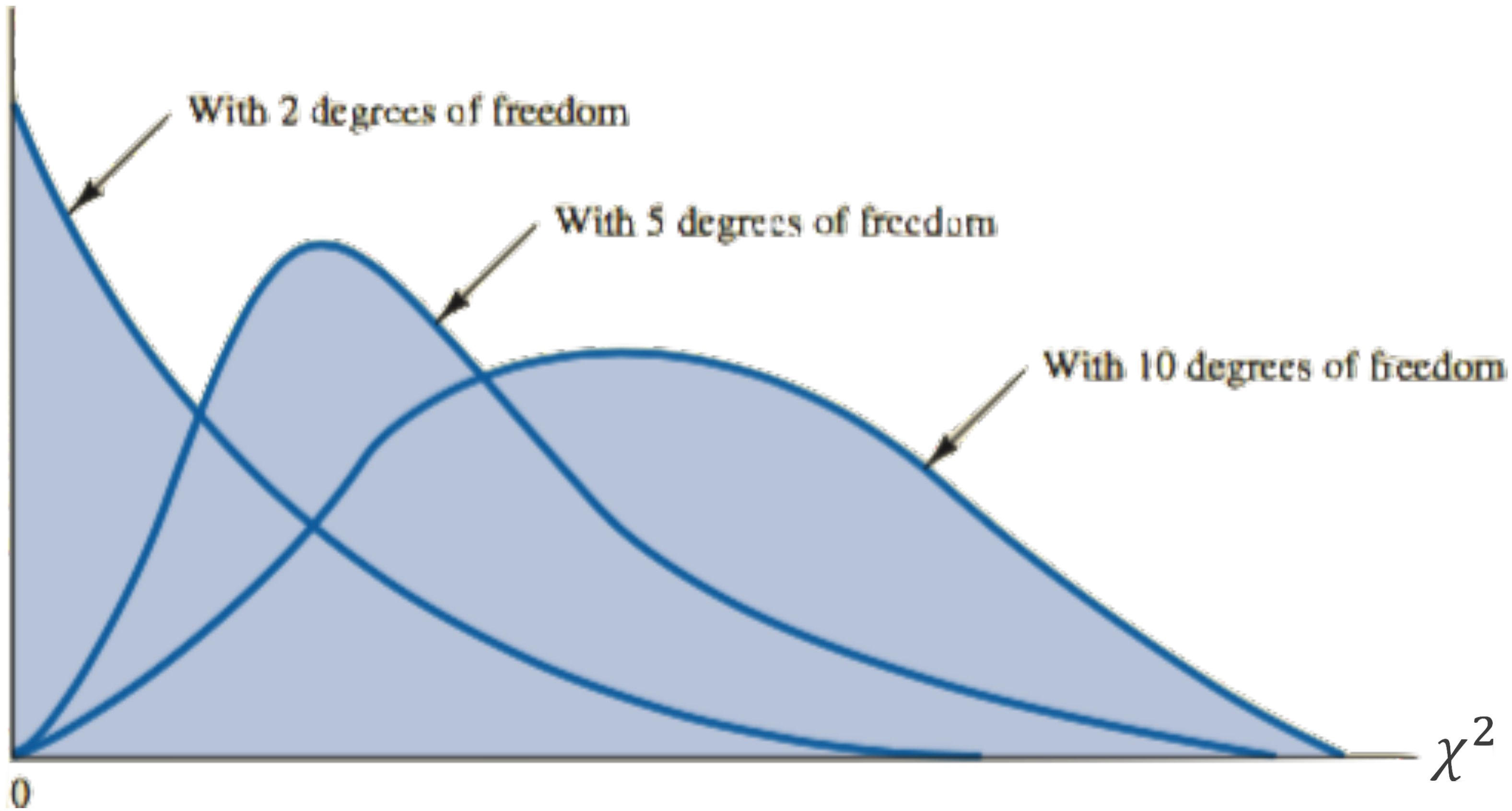
F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Distribution



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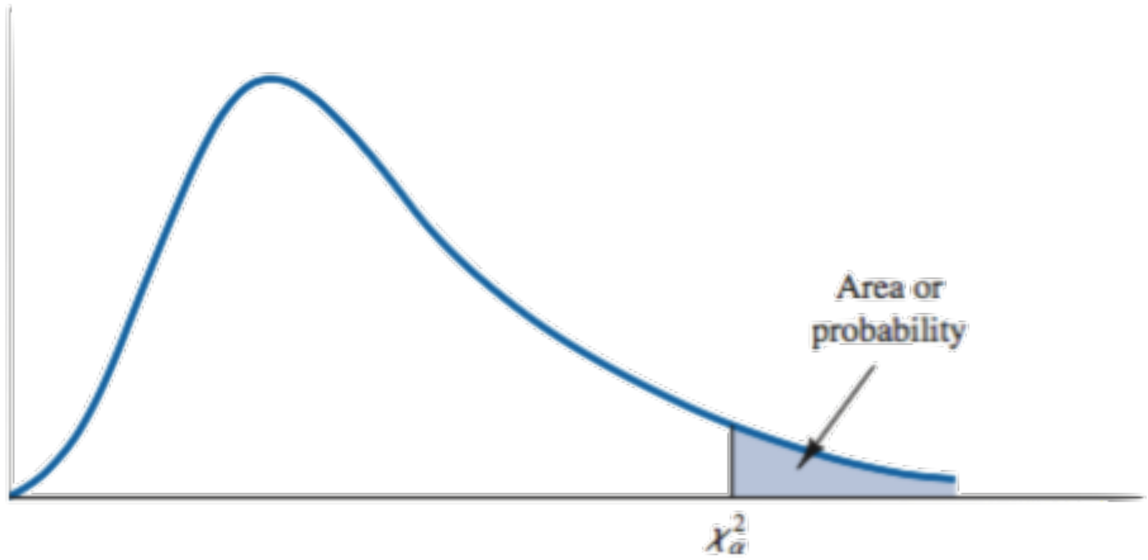
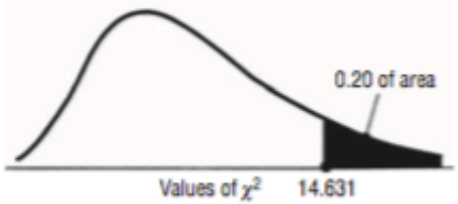
F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Distribution
  - χ^2_α denotes the value of chi-square distribution that provides an area or probability of a to the right of the χ^2_α value



EXAMPLE: IN A CHI-SQUARE DISTRIBUTION WITH 11 DEGREES OF FREEDOM, TO FIND THE CHI-SQUARE VALUE FOR 0.20 OF THE AREA UNDER THE CURVE (THE COLORED AREA IN THE RIGHT TAIL) LOOK UNDER THE 0.20 COLUMN IN THE TABLE AND THE 11 DEGREES OF FREEDOM ROW; THE APPROPRIATE CHI-SQUARE VALUE IS 14.631.

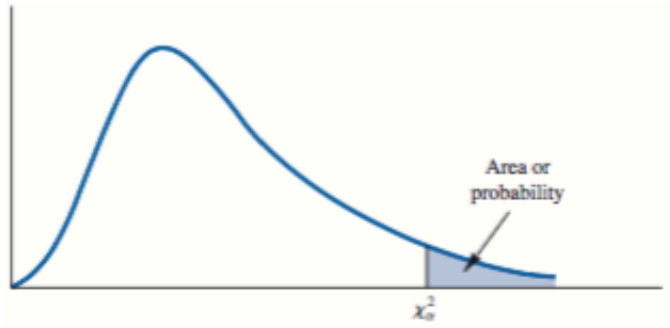


APPENDIX TABLE 5 AREA IN THE RIGHT TAIL OF A CHI-SQUARE (χ^2) DISTRIBUTION

Degrees of Freedom	Area in Right Tail 0.99	0.975	0.95	0.90	0.800
1	0.00016	0.00098	0.00398	0.0158	0.0642
2	0.0201	0.0506	0.103	0.211	0.446
3	0.115	0.216	0.352	0.584	1.005
4	0.297	0.484	0.711	1.064	1.649
5	0.554	0.831	1.145	1.610	2.343
6	0.872	1.237	1.685	2.204	3.070
7	1.239	1.690	2.167	2.833	3.822
8	1.646	2.180	2.733	3.490	4.594
9	2.088	2.700	3.325	4.168	5.380
10	2.558	3.247	3.940	4.865	6.179
11	3.053	3.816	4.575	5.578	6.989
12	3.571	4.404	5.226	6.304	7.807
13	4.107	5.009	5.892	7.042	8.634
14	4.660	5.629	6.571	7.790	9.467
15	5.229	6.262	7.261	8.547	10.307
16	5.812	6.908	7.962	9.312	11.152
17	6.408	7.564	8.672	10.085	12.002
18	7.015	8.231	9.390	10.865	12.857
19	7.633	8.907	10.117	11.651	13.716
20	8.260	9.591	10.851	12.443	14.578
21	8.897	10.283	11.591	13.240	15.445
22	9.542	10.982	12.338	14.041	16.314
23	10.196	11.689	13.091	14.848	17.187
24	10.856	12.401	13.848	15.658	18.062
25	11.524	13.120	14.611	16.473	18.940
26	12.198	13.844	15.379	17.292	19.820
27	12.879	14.573	16.151	18.114	20.703
28	13.565	15.308	16.928	18.939	21.588
29	14.256	16.047	17.708	19.768	22.475
30	14.953	16.791	18.493	20.599	23.364

Area in Right Tail					Degrees of Freedom
0.20	**0.10**	**0.05**	**0.025**	**0.01**	
1.642	2.706	3.841	5.024	6.635	1
3.219	4.605	5.991	7.378	9.210	2
4.642	6.251	7.815	9.348	11.345	3
5.989	7.779	9.488	11.143	13.277	4
7.289	9.236	11.070	12.833	15.086	5
8.558	10.645	12.592	14.449	16.812	6
9.803	12.017	14.067	16.013	18.475	7
11.030	13.362	15.507	17.535	20.090	8
12.242	14.684	16.919	19.023	21.666	9
13.442	15.987	18.307	20.483	23.209	10
14.631	17.275	19.675	21.920	24.725	11
15.812	18.549	21.026	23.337	26.217	12
16.985	19.812	22.362	24.736	27.688	13
18.151	21.064	23.685	26.119	29.141	14
19.311	22.307	24.996	27.488	30.578	15
20.465	23.542	26.296	28.845	32.000	16
21.615	24.769	27.587	30.191	33.409	17
22.760	25.989	28.869	31.526	34.805	18
23.900	27.204	30.144	32.852	36.191	19
25.038	28.412	31.410	34.170	37.566	20
26.171	29.615	32.671	35.479	38.932	21
27.301	30.813	33.924	36.781	40.289	22
28.429	32.007	35.172	38.076	41.638	23
29.553	33.196	36.415	39.364	42.980	24
30.675	34.382	37.652	40.647	44.314	25
31.795	35.563	38.885	41.923	45.642	26
32.912	36.741	40.113	43.194	46.963	27
34.027	37.916	41.337	44.461	48.278	28
35.139	39.087	42.557	45.722	49.588	29
36.250	40.256	43.773	46.979	50.892	30

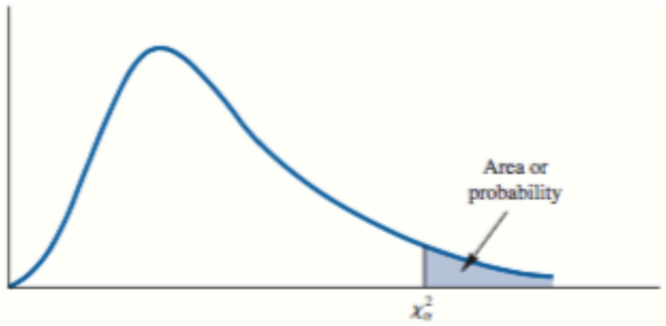
CHI-SQUARE DISTRIBUTION



Entries in the table give χ^2_α values, where α is the area or probability in the upper tail of the chi-square distribution. For example, with 10 degrees of freedom and a .01 area in the upper tail, $\chi^2_{.01} = 23.209$.

Degrees of Freedom	Area in Upper Tail									
	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	.000	.000	.001	.004	.016	2.706	3.841	5.024	6.635	7.879
2	.010	.020	.051	.103	.211	4.605	5.991	7.378	9.210	10.597
3	.072	.115	.216	.352	.584	6.251	7.815	9.348	11.345	12.838
4	.207	.297	.484	.711	1.064	7.779	9.488	11.143	13.277	14.860
5	.412	.554	.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	.676	.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.878	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.994
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.335

CHI-SQUARE DISTRIBUTION



Entries in the table give χ^2_α values, where α is the area or probability in the upper tail of the chi-square distribution. For example, with 10 degrees of freedom and a .01 area in the upper tail, $\chi^2_{.01} = 23.209$.

Degrees of Freedom	Area in Upper Tail									
	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
45	24.311	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957	73.166
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
55	31.735	33.571	36.398	38.958	42.060	68.796	73.311	77.380	82.292	85.749
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
65	39.383	41.444	44.603	47.450	50.883	79.973	84.821	89.177	94.422	98.105
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
75	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.285
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
85	55.170	57.634	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.324
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
95	63.250	65.898	69.925	73.520	77.818	113.038	118.752	123.858	129.973	134.247
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.170

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests

Chi-Square Test of Independence	Chi-Square Test of Goodness of Fit
To compare proportions of several populations To determine whether the two attributes / variable are independent or not (when a population is classified into several categories with respect to these two attributes / variables like age & occupation)	The test whether a particular probability distribution (Binomial, Poisson, or Normal) is the appropriate distribution

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Independence

Using the Chi-Square Test of Independence

1. Develop the null and alternative hypotheses
2. Specify the level of significance
3. Collect the sample data and compute the value of the test statistic, χ^2
   i. Draw the contingency table
   ii. Compute the observed & expected frequencies
   iii. Compute the value of the test statistic i.e. the chi-square statistic, χ^2
4. Determine the degrees of freedom & find out the critical chi-square value, χ^2_α
5. Reject H_0 if the value of the chi-square statistic ≥ critical chi-square value
6. Interpret the statistical conclusion in the context of the application

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Independence
    1. Develop the null and alternative hypotheses

       p_N is the proportion in North who prefer the current system

       p_E is the proportion in East who prefer the current system

       p_W is the proportion in West who prefer the current system

       p_S is the proportion in South who prefer the current system

       H_0: $p_N = p_E = p_W = p_S$ i.e. preferences about systems are independent of the geography

       H_1: p_N, p_E, p_W, p_S are not all equal
    2. Specify the level of significance

       $\alpha = 0.10$

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Independence
    3. Collect the sample data and compute the value of the test statistic, χ^2
       i. Draw the contingency table

	North	East	West	South	Total
No. who prefer the current system	68	75	57	79	**279**
No. who prefer the new system	32	45	33	31	**141**
Total employees sampled	**100**	**120**	**90**	**110**	**420**

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Independence
    3. Collect the sample data and compute the value of the test statistic, χ^2
       ii. Compute the observed & expected frequencies

	North	East	West	South
Frequency preferring current system				
Observed frequency (f_o)	68	75	57	79
Expected frequency (f_e)	66.43	79.72	59.79	73.07
Frequency preferring new system				
Observed frequency (f_o)	32	45	33	31
Expected frequency (f_e)	33.57	40.28	30.21	36.93

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Independence

    3. Collect the sample data and compute the value of the test statistic, χ^2

       iii. Compute value of test statistic i.e. chi-square statistic, χ^2

f_o	68	75	57	79	32	45	33	31	
f_e	66.43	79.72	59.79	73.07	33.57	40.28	30.21	36.93	
$f_0 - f_e$	1.57	-4.72	-2.79	5.93	-1.57	4.72	2.79	-5.93	
$(f_0 - f_e)^2$	2.46	22.28	7.78	35.16	2.46	22.28	7.78	35.16	
$(f_0 - f_e)^2 / f_e$	0.0370	0.2795	0.1301	0.4812	0.0733	0.5531	0.2575	0.9521	2.7638

$$\chi^2 = 2.7638$$

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
    - Chi-Square Test of Independence

4. Determine the degrees of freedom & find out the critical chi-square value, χ^2_α

Degrees of Freedom for Chi-Square Test of Independence				
No. of Degrees of Freedom	=	(No. of Rows* in Contingency Table – 1)	×	(No. of Columns* in Contingency Table – 1)

*Excluding the row & the column for total

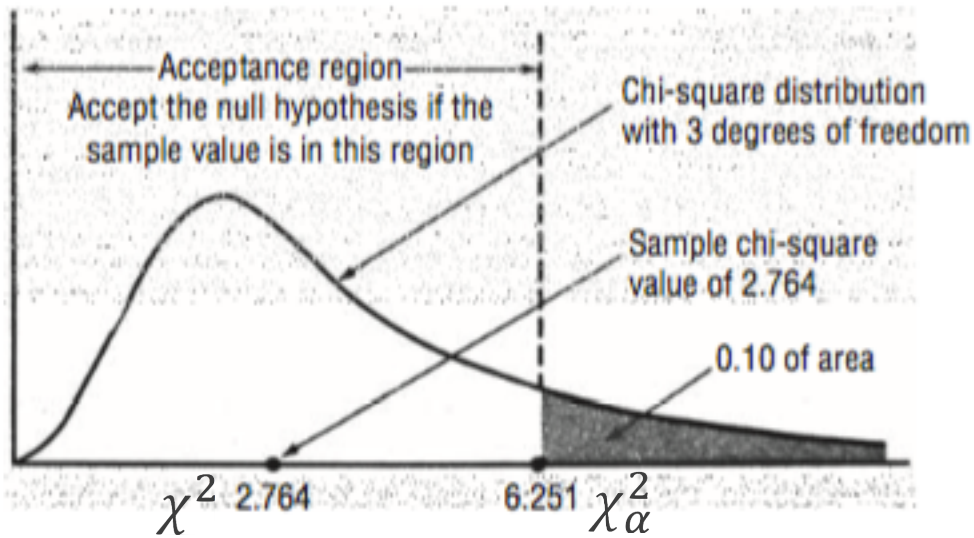
Degrees of Freedom = (2 - 1) × (4 – 1) = 3, and from table for 3 degrees of freedom, the chi-square value for which area to right of the curve is 0.10 = 6.251. Thus, critical chi-square value, χ^2_α= 6.251

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Independence
    5. Reject H_0 if the value of the chi-square statistic ≥ critical chi-square value



Since value of chi-square statistic χ^2 is 2.764 which is less than the critical value of chi-square χ^2_α of 6.251, H_0 is not rejected. Hence, H_0 is accepted.

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Independence
    6. Interpret the statistical conclusion in the context of the application

       Thus, it is accepted that there is no difference between the preferences about the systems in the four regions offices, and hence preference about the system is independent of the geography

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Example
  - Mr. George believes that lengths of stays in hospitals are dependent on types of health insurance people have. He collected data from a sample of 660 hospitals as follows. Test the hypothesis that length of stay & type of insurance are independent at $\alpha = 0.01$.

		Days in Hospital			
		< 5	5 – 10	> 10	**Total**
Fraction of Costs Covered by Insurance	< 25%	40	75	65	**180**
	25 – 50%	30	45	75	**150**
	> 50%	40	100	190	**330**
	Total	**110**	**220**	**330**	**660**

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

1. Develop the null and alternative hypotheses

   H_0: length of stay & type of insurance are independent

   H_1: length of stay depends on type of insurance

2. Specify the level of significance $\alpha = 0.10$

3. Collect the sample data & compute the value of the test statistic, χ^2

   i. Draw the contingency table

		Days in Hospital			
		< 5	5 – 10	> 10	**Total**
Fraction of Costs Covered by Insurance	< 25%	40	75	65	**180**
	25 – 50%	30	45	75	**150**
	> 50%	40	100	190	**330**
	Total	**110**	**220**	**330**	**660**

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

3. Collect the sample data and compute value of the test statistic, χ^2

   ii. Compute the observed & expected frequencies

   iii. Compute value of test statistic i.e. chi-square statistic, χ^2

Row	Column	f_o	f_e	=	(RT × CT) / n	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2 / f_e$
1	1	40	30	=	(180 × 110) / 660	10	100	3.33
1	2	75	60	=	(180 × 220) / 660	15	225	3.75
1	3	65	90	=	(180 × 330) / 660	-25	625	6.94
2	1	30	25	=	(150 × 110) / 660	5	25	1.00
2	2	45	50	=	(150 × 220) / 660	-5	25	0.50
2	3	75	75	=	(150 × 330) / 660	0	0	0.00
3	1	40	55	=	(330 × 110) / 660	-15	225	4.09
3	2	100	110	=	(330 × 220) / 660	-10	100	0.91
3	3	190	165	=	(330 × 330) / 660	25	225	3.79
							χ^2 =	**24.31**

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

4. Determine the degrees of freedom & find out the critical chi-square value, χ^2_α

   Degrees of Freedom = (3 - 1) × (3 – 1) = 4, and from table for 4 degrees of freedom, the chi-square value for which area to right of the curve is 0.01 = 13.277. Thus, critical chi-square value, χ^2_α = 13.277

5. Reject H_0 if the value of the chi-square statistic ≥ critical chi-square value

   Since value of chi-square statistic χ^2 is 24.31 which is greater than the critical value of chi-square χ^2_α of 13.277, H_0 is rejected.

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

6. Interpret the statistical conclusion in the context of the application

   Since H_0 is rejected, it can be concluded that the length of stays in hospitals is dependent on the type of insurance people have.

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Example
  - A brand manager is concerned that her brand's share may be unevenly distributed throughout the country. In a survey in which the the country was divided into 4 geographical regions, a random sampling of 100 consumers in each sample was surveyed with the following results. At $\alpha = 0.05$, test whether brand share is the same across the four regions.

	North	East	West	South	Total
Purchase the brand	40	55	45	50	**190**
Do not purchase the brand	60	45	55	50	**210**
Total	**100**	**100**	**100**	**100**	**400**

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Example
  - An advertising firm is trying to determine the demographics for a new product. They have randomly selected 75 people in each of 5 different age groups and introduced the product to them. The results of the survey are given in the following table. State the null & alternate hypotheses. Test null hypothesis at $\alpha = 0.01$.

Future Purchase Activity	**Age Group**				
	18-29	30-39	40-49	50-59	60-69
Purchase frequently	12	18	17	22	32
Seldom purchase	18	25	29	24	30
Never purchase	45	32	29	29	13

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Example
  - To see whether silicon chip sales are independent on the business cycle, data have been collected on the weekly chip sales, and business cycles as follows. State the null & alternate hypotheses, and test the null hypothesis at $\alpha = 0.10$.

Business Cycle	**Weekly Chip Sales**		
	High	Medium	Low
At Peak	20	7	3
At Trough	30	40	30
Rising	20	8	2
Falling	30	5	5

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit

Using the Chi-Square Test of Goodness of Fit

1. Develop the null and alternative hypotheses
2. Specify the level of significance
3. Collect the sample data and compute the value of the test statistic, χ^2
   1. Compute the observed & expected frequencies
   2. Compute the value of the test statistic i.e. the chi-square statistic, χ^2
4. Determine the degrees of freedom & find out the critical chi-square value, χ^2_α
5. Reject H_0 if the value of the chi-square statistic ≥ critical chi-square value
6. Interpret the statistical conclusion in the context of the application

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit

    Hiring process for senior managers at a company involves interview with three different executives each of whom provides a positive or a negative rating to each candidate interviewed. The HR manager at the company believes that the interview process can be approximated by a binomial distribution with $p = 0.40$ of a candidate receiving a positive rating on any one interview. The HR director wants to test this hypothesis at $\alpha = 0.20$ level of significance. The HR manager has collected following data on 100 candidates.

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit

Possible Positive Ratings from Three Interview	Number of Candidates Receiving Each of these Ratings
0	18
1	47
2	24
3	11
Total	**100**

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit
    1. Develop the null and alternative hypotheses

       H_0: Binomial distribution with p = 0.4 is a good description of the interview process

       H_1: Binomial distribution with p = 0.4 is not a good description of the interview process

    2. Specify the level of significance $\alpha = 0.20$
    3. Collect the sample data and compute the value of the test statistic, χ^2
       i. Compute value of observed & expected frequencies

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit

Possible Positive Ratings from Three Interview	Observed Frequency of Candidates Receiving These Ratings	Binomial Probability of Possible Outcomes	×	Number of Candidates Interviewed	=	Expected Frequency of Candidates Receiving These Ratings
0	18	0.2160	×	100	=	21.6
1	47	0.4320	×	100	=	43.2
2	24	0.2880	×	100	=	28.8
3	11	0.0640	×	100	=	6.4
Total	**100**	**1.0000**	**×**	**100**	**=**	**100**

Binomial Probability of Possible Outcomes = ${}^{n}C_{x}\, p^{x} (1 - p)^{n-x}$ ….. where $n = 3$, $x = \{0, 1, 2, 3\}$

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit
    3. Collect the sample data and compute the value of the test statistic, χ^2
       ii. Compute the value of the test statistic i.e. the chi-square statistic, χ^2

f_o	f_e	$f_o - f_e$	$(f_o - f_e)^2$	$(f_o - f_e)^2 / f_e$
18	21.6	-3.6	12.96	0.6000
47	43.2	3.8	14.44	0.3343
24	28.8	-4.8	23.04	0.8000
11	6.4	4.6	21.16	3.3063
			χ^2 =	5.0406

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F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit
    4. Determine the degrees of freedom & find out the critical chi-square value, χ^2_α

Degrees of Freedom for Chi-Square Test of Goodness of Fit
No. of Degrees of Freedom = (No. of Classes of Observed Frequencies – 1)

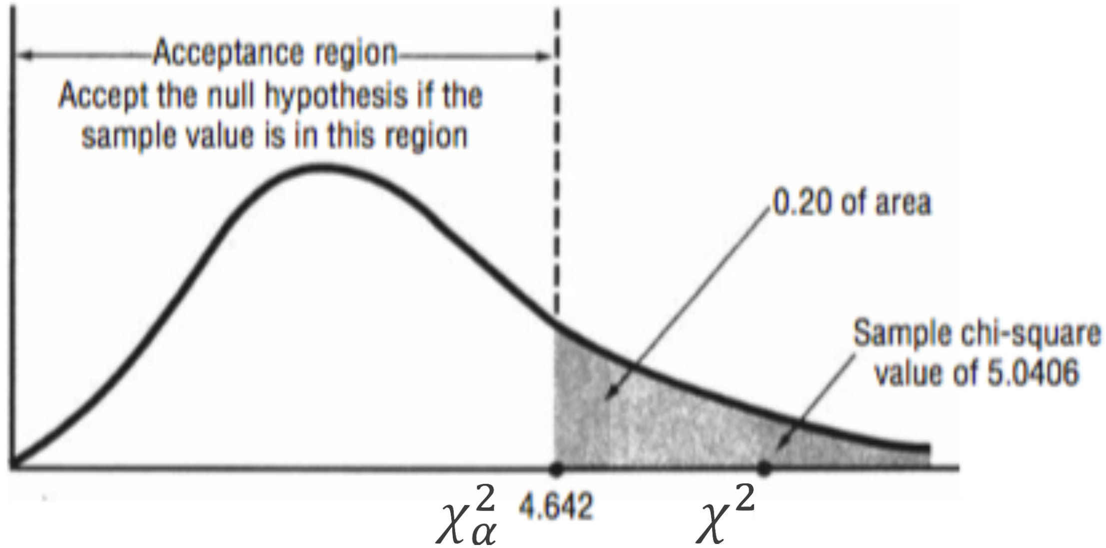
If any population parameters are to be estimated from the sample data, an additional degree of freedom is subtracted for each population parameter that has to be estimated from the sample data

Degrees of Freedom = (4 - 1) = 3, and from table for 3 degrees of freedom, the chi-square value for which area to right of the curve is 0.20 = 4.642. Thus, critical chi-square value, χ^2_α = 4.642

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit
    5. Reject H_0 if the value of the chi-square statistic ≥ critical chi-square value



Since value of chi-square statistic χ^2 is 5.041 which is greater than the critical value of chi-square χ^2_α of 4.642, H_0 is rejected.

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Chi-Square Tests
  - Chi-Square Test of Goodness of Fit
    6. Interpret the statistical conclusion in the context of the application

       Thus, the null hypothesis is rejected and it is concluded that binomial distribution with $p = 0.4$ fails to provide a good description of the interview process based on the observed frequencies

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Example
  - At 0.10 level of significance, can we conclude that following data about 400 observations follow a Poisson distribution with an average of 3 arrivals per hour?

No. of arrivals per hour	0	1	2	3	4	4+
Number of hours	20	57	98	85	78	62

1. Develop the null and alternative hypotheses

   H_0: Data follow Poisson distribution with avg. of 3 arrivals per hour

   H_1: Data does not follow Poisson distribution with avg. of 3 arrivals per hour
2. Specify the level of significance $\alpha = 0.10$

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

3. Collect the sample data & compute the value of the test statistic, χ^2
    i. Compute value of observed & expected frequencies

No. of Arrivals per Hour	Observed Frequency (No. of Hours)	Poisson Probability		Number of Observation		Expected Frequency (No. of Hours)
0	20	0.0498	×	400	=	19.92
1	57	0.1494	×	400	=	59.76
2	98	0.2240	×	400	=	89.60
3	85	0.2240	×	400	=	89.60
4	78	0.1680	×	400	=	67.20
4 +	62	0.1848	×	400	=	73.92

Poisson Probability = $\mu^x e^{-\mu} / x!$ ….. where $\mu = 3$, $x = \{0, 1, 2, 3, 4, 4+\}$

Poisson Probability of 4+ arrivals = 1 – Poisson probability of 4 or less arrivals

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

3. Collect the sample data & compute the value of the test statistic, χ^2
   ii. Compute the value of the statistic i.e. the chi-square statistic, χ^2

No. of Arrivals per Hour	f_o	f_e	$f_0 - f_e$	$(f_0 - f_e)^2$	$(f_0 - f_e)^2 / f_e$
0	20	19.92	0.08	0.0064	0.0003
1	57	59.76	-2.76	7.6176	0.1275
2	98	89.60	8.40	70.5600	0.7875
3	85	89.60	-4.60	21.1600	0.2362
4	78	67.20	10.80	116.6400	1.7357
4 +	62	73.92	-11.92	142.0864	1.9222
				$\chi^2 =$	4.8094

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

4. Determine the degrees of freedom & find out the critical chi-square value, χ^2_α

   Degrees of Freedom = (6 - 1) = 5, and from table for 5 degrees of freedom, the chi-square value for which area to right of the curve is 0.10 = 9.236. Thus, critical chi-square value, χ^2_α = 9.236

5. Reject H_0 if value of chi-square statistic ≥ critical chi-square value

   Since value of chi-square statistic χ^2 is 4.8094 which is less than the critical value of chi-square χ^2_α of 9.236, H_0 is not rejected.

6. Interpret the statistical conclusion in the context of the application

   Since H_0 is not rejected, it can be concluded that the data follow Poisson distribution with average 3 arrivals per hour.

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Chi-Square Test

- Precaution using Chi-Square Test

Precaution using Chi-Square Test
1. Sample size should be large enough to guarantee similarity between the theoretically correct distribution & the sampling distribution of χ^2, the chi-square statistic. 2. When the expected frequencies are too small, the value of χ^2 will be overestimated. So, for expected frequencies of less than 5, the frequencies can be appropriately combined to get expected frequencies of 5 or more. 3. If chi-square value is found to be zero, it must be verified whether absolutely no difference exists between observed and expected frequencies. If there are strong reasons to believe that some difference ought to exist, either the data collection method or the measurement technique, or both, must be examined to ascertain that existing differences were not obscured or missed in collecting sample data.

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction
  - A technique used in comparing means of more than two populations (by testing the significance of the differences among means of the samples taken from those populations) to make inferences about whether samples are drawn from populations having the same mean
  - E.g. – comparing mileage achieved from 5 brands of petrol, salaries of graduates of 8 schools, etc.

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction

Assumptions for using ANOVA	Logic behind ANOVA
1. Each population being considered is normally distributed i.e. each sample is drawn is from a normal population 2. The variance, σ^2, is same for all the populations 3. The observations are all independent	ANOVA is based on the development of two independent estimates of the common population variance, σ^2 One estimate of σ^2 is based on the **variability among the sample means** The other estimate of σ^2 is based on the **variability within samples themselves** By comparing these two estimates of σ^2, it can be determined whether the population means are equal

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction
  - Estimate based on variability among sample means

Variance Among the Sample Means

1. $(s_{\bar{x}})^2 = \dfrac{\Sigma(\bar{x}_j - \bar{\bar{x}})^2}{k - 1}$ .... as $s^2 = \dfrac{\Sigma(x_i - \bar{x})^2}{n - 1}$

where $(s_{\bar{x}})^2$ = variance among sample means

$\bar{x}_j$ = mean of sample j

$\bar{\bar{x}}$ = mean of the sample means

k = no. of samples

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction
    - Estimate based on variability among sample means

Population Variance

2. $$\sigma^2 = n \times (\sigma_{\bar{x}})^2 \quad \ldots \text{ as } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

where

σ = population standard deviation

$\sigma_{\bar{x}}$ = standard error of sampling distribution

n = sample size

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction
  - Estimate based on variability among sample means

Estimate of Population Variance based on Variance Among Sample Means

From 1. & 2. -> $(\hat{\sigma}_b)^2 = n_j \times (s_{\bar{x}})^2 = \dfrac{\Sigma n_j (\bar{x}_j - \bar{\bar{x}})^2}{k - 1}$

where $(\hat{\sigma}_b)^2$ = estimate of population variance based on variance among sample means **(between-sample variance)**

n_j = size of sample j

$\bar{x}_j$ = mean of sample j

k = no. of samples

$\bar{\bar{x}}$ = mean of sample means

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction
  - Estimate based on variability within the samples

Estimate of Population Variance based on Variance Within the Samples

$$(\hat{\sigma}_w)^2 = \sum \left[\frac{n_j - 1}{n_T - k} \right] (s_j)^2$$

where $(\hat{\sigma}_w)^2$ = estimate of population variance based on variance within the samples means **(within-sample variance)**

n_j = size of sample j

s_j = std. dev. of sample j

k = no. of samples

n_T = Σn_j = total sample size

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction

$(\hat{\sigma}_b)^2$ provides a good estimate of population variance only if the null hypothesis (population means are same) is true

over-estimates the population variance if null hypothesis (population means are same) is false

$(\hat{\sigma}_w)^2$ provides a good estimate of population variance in either case

Test Statistic for ANOVA

$$F = \frac{\text{Between Sample Variance}}{\text{Within Sample Variance}} = \frac{(\hat{\sigma}_b)^2}{(\hat{\sigma}_w)^2}$$

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction

If the null hypothesis (population means are same) is true, the two estimates of the population variance will be similar, and hence their ratio i.e. the test statistic F, will be close to 1

If the null hypothesis is false, the between-sample estimate will be larger than the within-sample estimate, and their ratio, the F statistic, will be large

Degrees of Freedom			
Numerator Degrees of Freedom	= $k - 1$	… where	k is no. of samples
Denominator Degrees of Freedom	= $n_T - k$		n_T is total sample size = Σn_j

Session 25-30

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction

Using ANOVA for comparing more than two population means

1. Develop the null and alternative hypotheses
2. Specify the level of significance
3. Collect the sample data and compute the value of the test statistic, F
   i. Compute one estimate of population variance from **variance among the sample means**
   ii. Compute another estimate from **variance within samples**
   iii. Compute the value of the test statistic, F
4. Determine degrees of freedom for numerator & denomination & find out the critical F value, F_α
5. Reject H_0 if the value of test statistic, $F \geq$ critical F value, F_α
6. Interpret the statistical conclusion in the context of the application

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example
  - The training director of a company is trying to evaluate three different methods of training new employees. Post-training daily production (in units) by 16 new employees assigned randomly to the three training methods is shown below. The director wonders whether there are differences in effectiveness among the training methods using a significance level of 0.05.

Method 1	15	18	19	22	11	
Method 2	22	27	18	21	17	
Method 3	18	24	19	16	22	15

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

1. Develop the null and alternative hypotheses

   H_0: $\mu_1 = \mu_2 = \mu_3$ i.e. effectiveness of training methods is same

   H_1: μ_1, μ_2, μ_3 are not all equal i.e. effectiveness of training methods is not the same

2. Specify the level of significance $\alpha = 0.05$
3. Collect the sample data and compute the value of the test statistic, F
   1. Compute one estimate of population variance from variance among the sample means
   2. Compute another estimate from variance within samples

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

  i. Compute one estimate of population variance from variance among the sample means

Sample	n	$\bar{x}$	$\bar{\bar{x}}$	$\bar{x} - \bar{\bar{x}}$	$(\bar{x} - \bar{\bar{x}})^2$	$n(\bar{x} - \bar{\bar{x}})^2$
1	5	17	19	-2	4	20
2	5	21	19	2	4	20
3	6	19	19	0	0	0
						40

$$(\hat{\sigma}_b)^2 = \frac{\Sigma n_j(\bar{x}_j - \bar{\bar{x}})^2}{k - 1} = \frac{40}{3 - 1} = 20$$

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

ii. Compute another estimate from variance within samples

$$(\hat{\sigma}_w)^2 = \sum\left[\frac{n_j - 1}{n_T - k}\right](s_j)^2$$

$$[(4/13) \times 17.5] + [(4/13) \times 15.5] + [(5/13) \times 12.0]$$

$$= 14.769$$

Sample 1			Sample 2			Sample 3		
x	$\bar{x}$	$(x - \bar{x})^2$	x	$\bar{x}$	$(x - \bar{x})^2$	x	$\bar{x}$	$(x - \bar{x})^2$
15	17	4	22	21	1	18	19	1
18	17	1	27	21	36	24	19	25
19	17	4	18	21	9	19	19	0
22	17	25	21	21	0	16	19	9
11	17	36	17	21	16	22	19	9
		70			62	15	19	16
								60
$(s_1)^2 =$ 70 / 4 = 17.5			$(s_2)^2 =$ 62 / 4 = 15.5			$(s_3)^2 =$ 60 / 5 = 12.0		

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

3. Collect the sample data and compute the value of the test statistic, F

   iii. Compute the value of the test statistic, F

$$F = \frac{(\hat{\sigma}_b)^2}{(\hat{\sigma}_w)^2} = \frac{20}{14.769} = 1.3542$$

4. Determine degrees of freedom for numerator & denomination & find out the critical F value, F_α

Numerator Degrees of Freedom	=	$k - 1$	=	$3 – 1$	=	2
Denominator Degrees of Freedom	=	$n_T - k$	=	$16 – 3$	=	13

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

4. Determine degrees of freedom for numerator & denomination & find out the critical F value, F_α

   The critical value of F, i.e. F_α is the value of F providing an area of 0.05 in the upper tail for F-distribution with 2 degrees of freedom for the numerator & 13 degrees of freedom for the denominator = 3.81

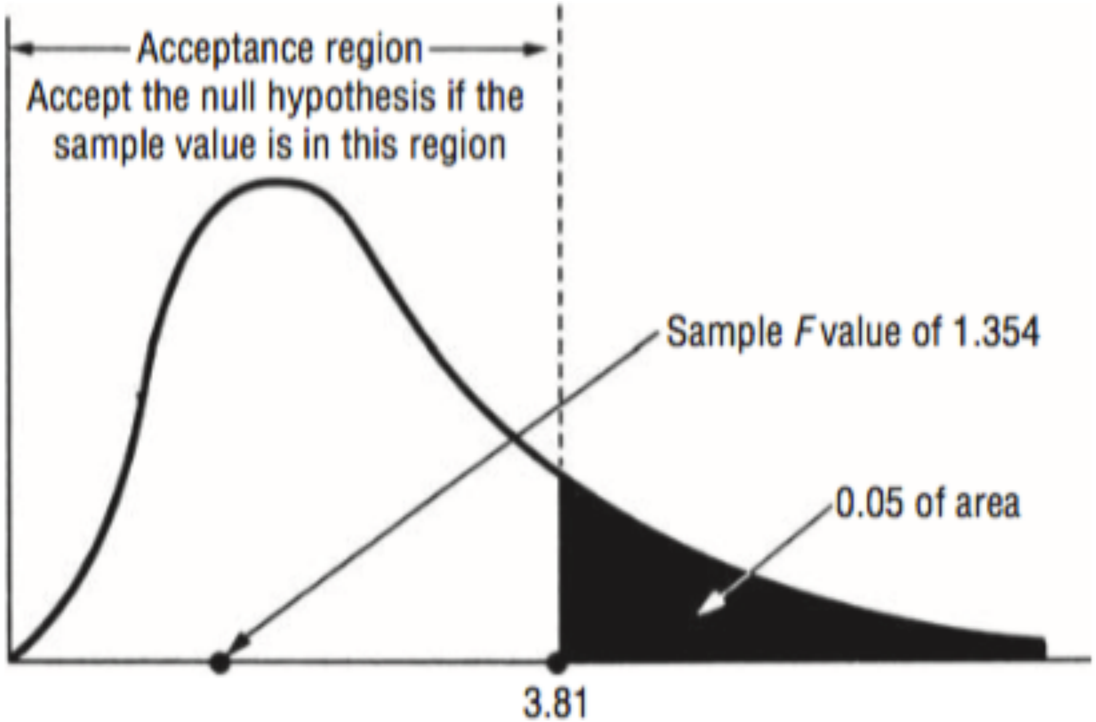
5. Reject H_0 if the value of test statistic, F ≥ critical F value, F_α

   Since F (1.3542) < F_α (3.81), we do not reject the null hypothesis i.e. we accept the null hypothesis

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example



6. Interpret the statistical conclusion in the context of the application

   Thus, there is no significant difference between the effectiveness of the three different training methods on employee productivity

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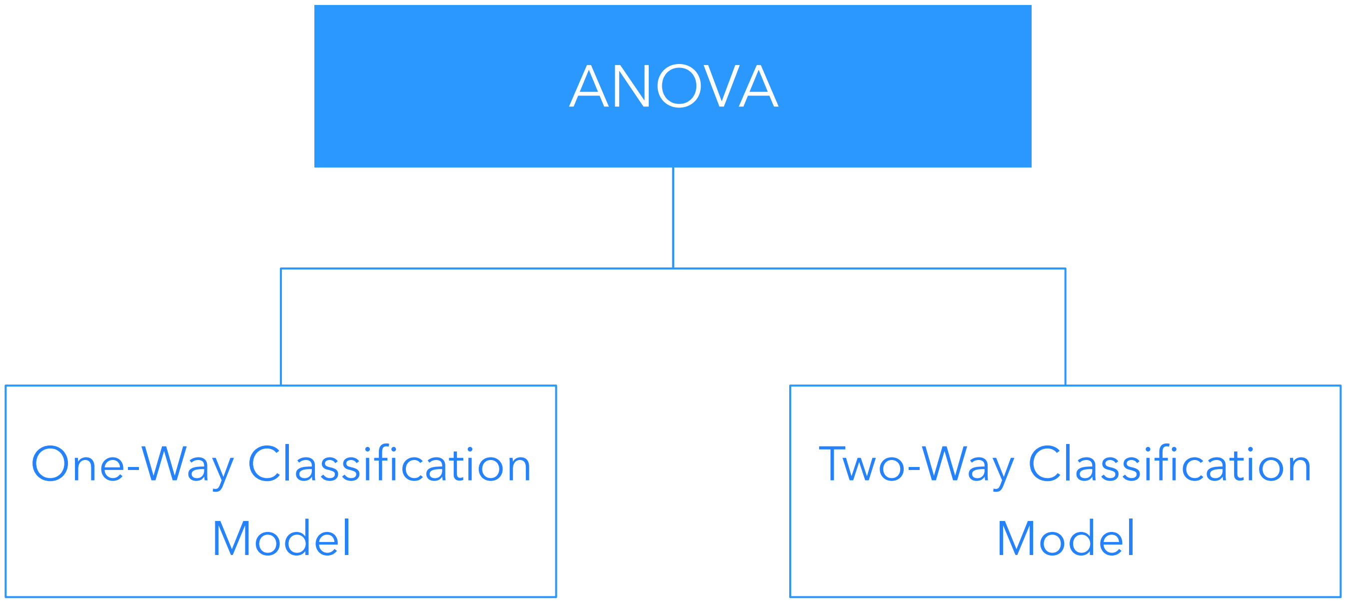
F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Introduction
    - Two ANOVA models, depending on the factors used in ANOVA



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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- One-Way Classification Model

One-Way Classification Model

Effect of one independent variable (factor) examined on dependent variable

Independent variable categorized into various levels / groups / classes

E.g.: Effect of training method on employee productivity

Independent Variable	:	Training method
Dependent Variable	:	Employee productivity
Classes of Independent Variable	:	Method 1, Method 2, Method 3, etc.

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- One-Way Classification Model

H_0 : $\mu_1 = \mu_2 = \mu_3 \quad \dots \quad = \mu_i \quad \dots \quad \mu_m$

H_1 : At least two μ_is are different

Factor A							Total	Mean
A_1	x_{11}	x_{12}	$\dots$	x_{1j}	$\dots$	x_{1n1}	T_1	$\bar{x}_1$
A_2	x_{21}	x_{22}	$\dots$	x_{2j}	$\dots$	x_{2n2}	T_2	$\bar{x}_2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_i	x_{i1}	x_{i2}	$\dots$	x_{ij}	$\dots$	x_{ini}	T_i	$\bar{x}_i$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	x_{m1}	x_{m2}	$\dots$	x_{mj}	$\dots$	x_{mnm}	T_m	$\bar{x}_m$
							G	$\bar{\bar{x}}$

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Two-Way Classification Model

Two-Way Classification Model

Effect of two independent variables (factors) examined on dependent variable

Independent variables categorized into various levels / groups / classes

E.g.: Effect of training method & age of employee on productivity

Independent Variables	:	Training Method
	:	Age of Employee
Dependent Variable	:	Employee Productivity
Classes of Independent Variables	:	Method 1, Method 2, Method 3, etc.
	:	Age Group 1, Age Group 2, etc.

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Two-Way Classification Model
  - Since two factors are involved, there are at least two sets of hypothesis

For Factor A H_{0A} : $\alpha_1 = \alpha_2 = \alpha_3 \; \dots = \alpha_i \; \dots \; \alpha_m$ like one-way ANOVA for the row factor

H_{1A} : At least two α_is are different

For Factor B H_{0B} : $\beta_1 = \beta_2 = \beta_3 \; \dots = \beta_i \; \dots \; \beta_m$ like one-way ANOVA for the column factor

H_{1B} : At least two β_is are different

H_{0AB} : There is no interaction between factors A & B

H_{1AB} : There is interaction between factors A & B

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Two-Way Classification Model

Factor A	**Factor B**						**Total**	**Mean**
	B_1	B_2	$\dots$	B_j	$\dots$	B_n		
A_1	x_{11}	x_{12}	$\dots$	x_{1j}	$\dots$	x_{1n1}	$T_{1\cdot}$	$\bar{x}_{1\cdot}$
A_2	x_{21}	x_{22}	$\dots$	x_{2j}	$\dots$	x_{2n2}	$T_{2\cdot}$	$\bar{x}_{2\cdot}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_i	x_{i1}	x_{i2}	$\dots$	x_{ij}	$\dots$	x_{ini}	$T_{i\cdot}$	$\bar{x}_{i\cdot}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
A_m	x_{m1}	x_{m2}	$\dots$	x_{mj}	$\dots$	x_{mnm}	$T_{m\cdot}$	$\bar{x}_{m\cdot}$
Total	$T_{\cdot 1}$	$T_{\cdot 2}$		$T_{\cdot j}$		$T_{\cdot m}$	$\mathbf{G}$	
Mean	$\bar{x}_{\cdot 1}$	$\bar{x}_{\cdot 2}$		$\bar{x}_{\cdot J}$		$\bar{x}_{\cdot m}$		$\bar{\bar{x}}_{\cdot\cdot}$

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example
  - A study compared the effects of four 1-month point-of-purchase promotions on sales. The unit sales for five stores using all four promotions in different months are as follows.

Free gift	78	87	81	89	85
Extra 25%	94	91	87	90	88
Price Discount	73	78	69	83	76
Cashback	79	83	78	69	81

a. mean unit sales for each promotion & then the grand

b. population variance using the between-sample variance, and using the within-sample variance.

c. Calculate the F- ratio. At 0.01 level of significance, do the promotions produce different effect on sales?

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

1. Develop the null and alternative hypotheses

   H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ i.e. mean sales for all promotions are same, i.e. all promotions produce same effect on sales

   H_1: $\mu_1, \mu_2, \mu_3, \mu_4$ for all promotions are not same, i.e. all promotions do not produce same effect on sales

2. Specify the level of significance $\alpha = 0.01$

3. Collect the sample data and compute the value of the test statistic, F

   i. Compute one estimate of population variance from variance among the sample means

   ii. Compute another estimate from variance within samples

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example
  i. Compute one estimate of population variance from variance among the sample means

Promotion	n	$\bar{x}$	$\bar{\bar{x}}$	$\bar{x} - \bar{\bar{x}}$	$(\bar{x} - \bar{\bar{x}})^2$	$n(\bar{x} - \bar{\bar{x}})^2$
Free gift	5	84	81.95	2.05	4.2025	21.0125
Extra 25%	5	90	81.95	8.05	64.8025	324.0125
Price Discount	5	75.8	81.95	-6.15	37.8225	189.1125
Cashback	5	78	81.95	-3.95	15.6025	78.0125
						612.15

$$(\hat{\sigma}_b)^2 = \frac{\Sigma n_j (\bar{x}_j - \bar{\bar{x}})^2}{k - 1} = \frac{612.15}{4 - 1} = 204.05$$

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

  ii. Compute another estimate from variance within samples

Free Gift			Extra 25%			Price Discount			Cashback		
x	$\bar{x}$	$(x - \bar{x})^2$	**x**	$\bar{x}$	$(x - \bar{x})^2$	**x**	$\bar{x}$	$(x - \bar{x})^2$	**x**	$\bar{x}$	$(x - \bar{x})^2$
78	84	36	94	90	16	73	75.8	07.84	79	78	01
87	84	09	91	90	01	78	75.8	04.84	83	78	25
81	84	09	87	90	09	69	75.8	46.24	78	78	00
89	84	25	90	90	00	83	75.8	51.84	69	78	81
85	84	01	88	90	04	76	75.8	0.04	81	78	09
		80			30			110.80			116
$(s_1)^2 =$		80 / 4	$(s_2)^2 =$		30 / 4	$(s_3)^2 =$		110.8 0 / 4	$(s_1)^2 =$		116 / 4
	=	20		=	7.5		=	27.7		=	29

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

ii. Compute another estimate from variance within samples

$$(\hat{\sigma}_w)^2 = \sum \left[\frac{n_j - 1}{n_T - k} \right] (s_j)^2$$

$$= \begin{array}{l} [(4/16) \times 20.0] + \\ [(4/16) \times 07.5] + \\ [(4/16) \times 27.7] + \\ [(4/16) \times 29.0] \end{array}$$

$$= 21.05$$

F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

3. Collect the sample data and compute the value of the test statistic, F

   iii. Compute the value of the test statistic, F

$$F = \frac{(\hat{\sigma}_b)^2}{(\hat{\sigma}_w)^2} = \frac{204.05}{21.05} = 9.6936$$

4. Determine degrees of freedom for numerator & denomination & find out the critical F value, F_α

Numerator Degrees of Freedom	=	k - 1	=	4 – 1	=	3
Denominator Degrees of Freedom	=	n_T - k	=	20 – 4	=	16

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F-Test, Analysis of Variance & Chi-Square Test

Analysis of Variance (ANOVA)

- Example

4. Determine degrees of freedom for numerator & denomination & find out the critical F value, F_α

   The critical value of F, i.e. F_α is the value of F providing an area of 0.01 in the upper tail for F-distribution with 3 degrees of freedom for the numerator & 16 degrees of freedom for the denominator = 5.29

5. Reject H_0 if the value of test statistic, F ≥ critical F value, F_α

   Since F (9.6936) > F_α (5.29), we reject the null hypothesis

6. Interpret the statistical conclusion in the context of application

   All promotions do not produce same effect on sales i.e. promotions have significantly different effects on sales