Session 17-24

Testing of Hypothesis

- Hypothesis Testing – One Sample Tests
  - Introduction
  - Null & Alternate Hypotheses
  - Significance Level
  - Type I & Type II Errors
  - Power of Hypothesis Test
  - Two-Tailed & One-Tailed Tests of Hypothesis
  - Procedure for Hypothesis Testing of Means
  - Hypothesis Testing of Means
  - Hypothesis Testing of Proportion of Success

Session 17-24

Testing of Hypothesis

- Hypothesis Testing – Two Sample Tests
  - Introduction
  - Hypothesis Testing for Difference between Means
    - When σ Is Known
    - When σ Is Not Known
  - Paired Difference Test
  - Hypothesis Testing for Difference between Proportions

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Introduction

Hypothesis

- An assumption made about a population parameter

Hypothesis Testing

- Testing the validity of hypothesis i.e. testing the validity of the assumption made about a population parameter

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Introduction
  - Hypothesis Testing
    - Making an assumption of the population parameter (hypothesized population parameter)
    - Gathering sample data & computing the sample statistic
    - Determining the difference between the hypothesized population parameter and the actual value of the sample statistic
    - Judging (objectively) whether difference is significant or not
    - The smaller the difference, the greater the likelihood that our hypothesized value for the mean is correct

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Introduction
  - E.g. - Specifications in the contract for a new sports complex, require the aluminum sheets used roofing to be 0.04-inch-thick. If sheets are appreciably thicker than 0.04 inch, the structure would not be able to support the additional weight. If sheets are appreciably thinner than 0.04 inch, roof strength would be inadequate. Of the 10,000 sheets required, a random sample of 100 sheets is drawn & its mean thickness is found to be 0.0408 inch. From past experience, it is believed that these sheets come from a thickness population with a standard deviation of 0.004 inch. On the basis of these data, the decision maker must decide whether the 10,000 sheets meet specifications.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Introduction
  - Assuming that aluminum sheets are 0.04 inch thick & that population standard deviation is 0.004 inch, decision maker must decide how likely is it that one would get a sample mean of 0.0408 inch or more from the population i.e. If true mean (of population) is 0.04 inch, and standard deviation is 0.004 inch, then what are the chances of getting a sample mean that differs from 0.04 inch by 0.0008 (0.0408 - 0.04) inch or more?
  - Need to compute the probability that a random sample with a mean thickness of 0.0408 inch will be selected from the population having a mean thickness of 0.04 inch & standard deviation 0.004 inch
  - If probability is too low, mean thickness of aluminum sheets ≠ 0.04 inch

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Introduction
  - Hypothesized population mean μ = 0.04 inch, σ = 0.004 inch, what is the probability of getting a sample with mean $\bar{x}$ = 0.0408 inch?
  - Standard error $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ = 0.004 / $\sqrt{100}$ = 0.0004 inch
  - $z = (\bar{x} - \mu) / \sigma_{\bar{x}}$ = (0.0408 – 0.04) / 0.0004 = 0.0008/0.0004 = 2
  - p(-2 ≤ z ≤ 2) = 0.045
  - There is a low probability that a population with true mean of 0.04 inch would produce a sample having sample mean as 0.0408 inch
  - Difference bet^{n}. sample mean & hypothesized population mean is large
  - How low or how high is the probability to accept or reject the assumption is the decision maker's choice to make

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - An automobile manufacturer claims that a particular model gets 28 kmpl mileage. The Environmental Protection Agency, using a sample of 49 automobiles of this model, finds the sample mean to be 26.8 kmpl. From previous studies, the population standard deviation is known to be 5 kmpl. Could we reasonably expect (within 2 standard errors) that we could select such a sample if indeed the population mean is actually 28 kmpl?
  - How many standard errors around the hypothesized value should be used to be 99.44 percent certain that hypothesis is accepted when it is true?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - A grocery store has specially packaged oranges and has claimed a bag of oranges will yield 2.5 liters of juice. After randomly selecting 42 bags, a stacker found the average juice production per bag to be 2.2 liters. Historically, we know the population standard deviation is 0.2 liters. Using this sample and a decision criterion of 2.5 standard errors, could we conclude the store's claims are correct?
  - If a hypothesized value is rejected because it differs from a sample statistic by more than 1.75 standard errors, what is the probability that the hypothesis rejected is in fact true?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - How many standard errors around the hypothesized value should be used to be 98 percent certain that hypothesis is accepted when it is true?
  - A magazine has asserted that the amount of time PC owners spend on their PCs averages 23.9 hours per week & has a std. dev. of 12.6 hours per week. A random sampling of 81 of its subscribers revealed a sample mean usage of 27.2 hours per week. On the basis of this sample, is it reasonable to conclude (using 2 standard errors as the decision criterion) that the magazine's subscribers are different from average PC owners?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Null & Alternate Hypotheses
  - Assumed value i.e. hypothesized value of the population parameter must be stated before beginning sampling procedure

Null Hypothesis (H_0)

- The assumption (about population parameter) to be tested
- Will be assumed to be true during the testing of the hypothesis
- Will be rejected only if sample data provide substantial contradictory evidence

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Null & Alternate Hypotheses

Alternate Hypothesis (H_A or H_a or H_1)

- Opposite of what is stated in the null hypothesis
- Includes all population values not included in the null hypothesis
- Will be selected only if there is strong enough sample evidence to support it
- Is deemed to be true if the null hypothesis is rejected

  - E.g. - to test that the population mean is 500,
    - H0: $\mu_0 = 500$ (Null hypothesis is that population mean is 500)
    - H1: $\mu_0 \neq 500$ (Alternate hypothesis is that population mean is not 500)

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Null & Alternate Hypotheses
  - E.g. - to test that the population mean is 500,
    - H0: $\mu_0 = 500$ (Null hypothesis is that population mean is 500)
    - H1: $\mu_0 > 500$ (Alternate hypothesis is that population mean is greater than 500)
  - E.g. - to test that the population mean is 500,
    - H0: $\mu_0 = 500$ (Null hypothesis is that population mean is 500)
    - H1: $\mu_0 < 500$ (Alternate hypothesis is that population mean is less than 500)

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Null & Alternate Hypotheses
  - Developing / Formulating Null & Alternate Hypotheses
    - Testing the status quo or challenging a claim
      - Used in process analysis or validating claims
      - Assuming the status quo or claim to be true (Null Hypothesis is that the status quo is true)
      - Null Hypothesis is rejected only if sample data provides strong enough evidence to accept the Alternate Hypothesis

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Null & Alternate Hypotheses
  - Developing / Formulating Null & Alternate Hypotheses
    - Testing a research hypothesis
      - Used in development of new products
      - Assuming (Null Hypothesis) that new product is no better than the original one
      - Null Hypothesis rejected only if sample data provides strong enough evidence to accept the Alternate Hypothesis (which is also known as Research Hypothesis in this case)

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Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Null & Alternate Hypotheses
  - Developing / Formulating Null & Alternate Hypotheses

Forms for the Hypothesis Test		
$H_0 : \mu \geq \mu_0$ $H_1 : \mu < \mu_0$	$H_0 : \mu \leq \mu_0$ $H_1 : \mu > \mu_0$	$H_0 : \mu = \mu_0$ $H_1 : \mu \neq \mu_0$
One-tailed tests		Two-tailed tests
H_0 is Null Hypothesis H_1 is Alternate Hypothesis	μ is population mean μ_0 is hypothesized value of population mean	

- Equality part (either $\geq$, $\leq$, or $=$) always appears in Null Hypothesis

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Significance Level

Goal of Hypothesis Testing

- The purpose of hypothesis testing is not to question the computed value of the sample statistic but to make a judgment about the difference between that sample statistic and a hypothesized population parameter

  - Significance level indicates the percentage of sample statistic that is outside certain limits if the hypothesis is assumed to be correct i.e. the probability of rejecting the null hypothesis when it is true as an equality

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Significance Level
  - For 5% level of significance, the null hypothesis will be rejected if the difference between the sample statistic & the hypothesized population parameter is so large that it or a larger difference would occur, on an average, only 5 or fewer times in every 100 samples when the hypothesized population parameter is correct
  - In interval estimation, the confidence level indicates the percentage of sample statistic that falls within the defined confidence interval

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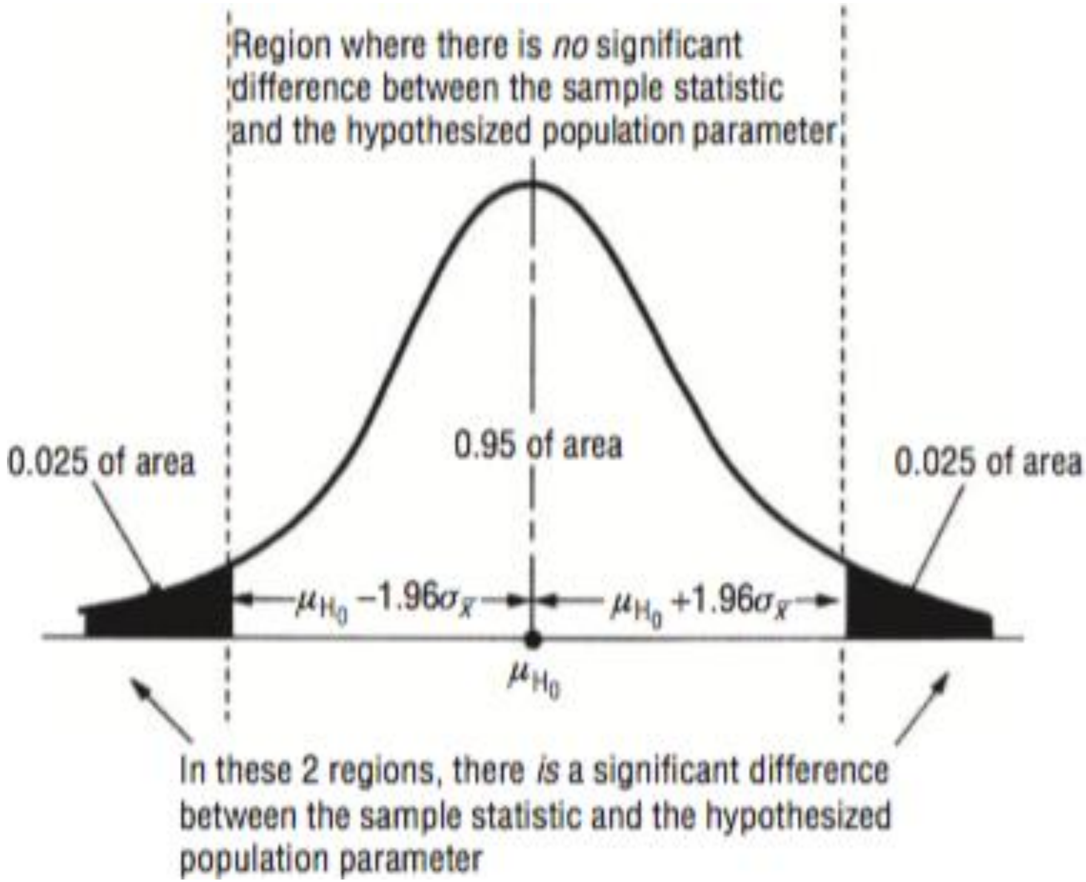
Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Significance Level

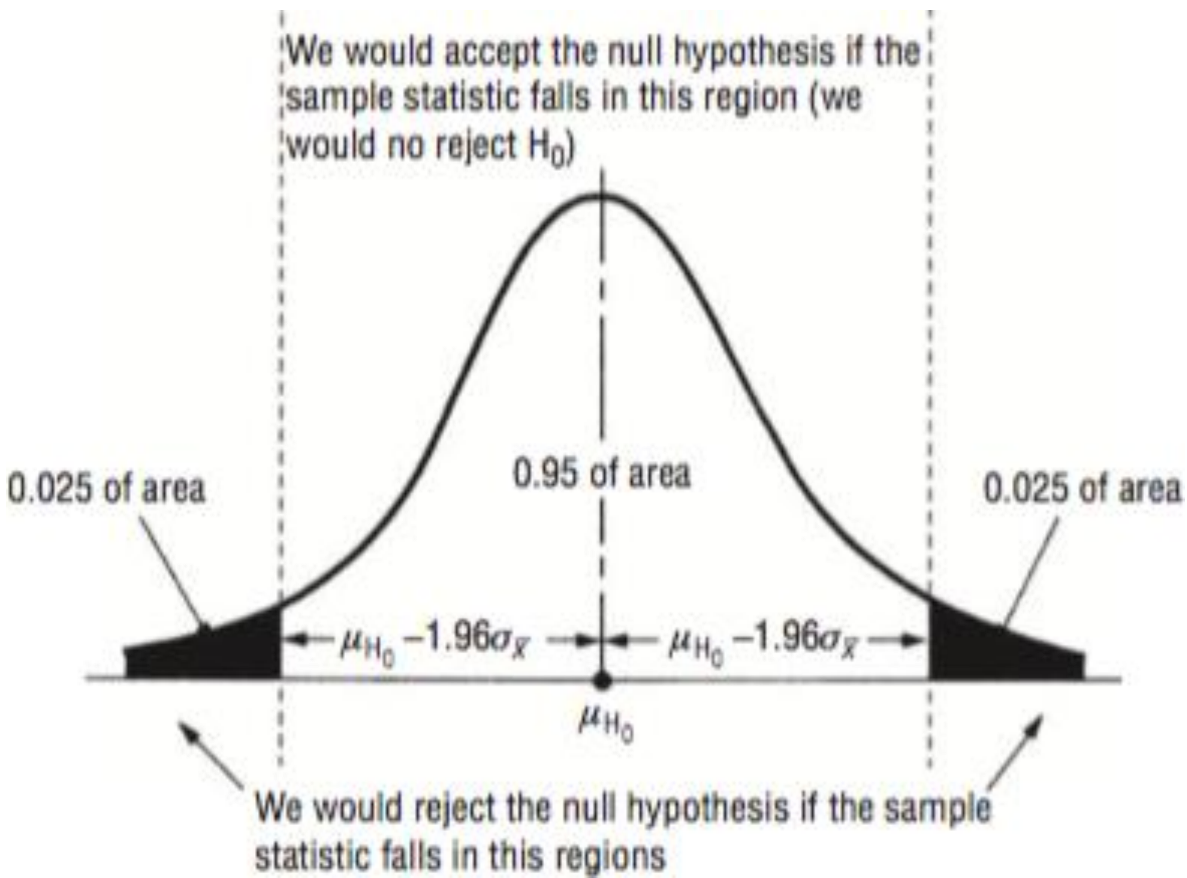


Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Significance Level



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Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Significance Level

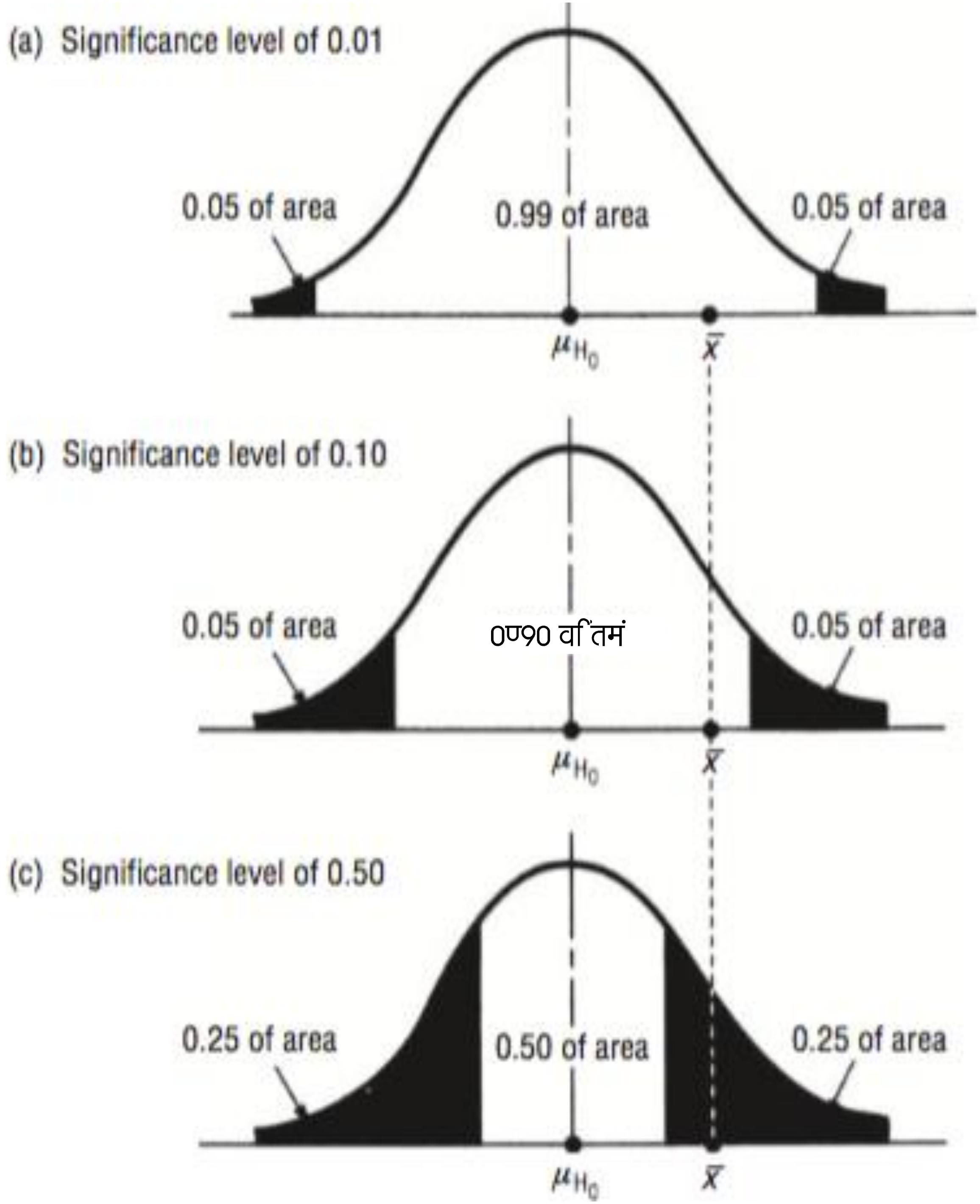
Even if the sample statistic falls in the acceptable region, this does not prove the the null hypothesis H_0 is true; it simply does not provide statistical evidence to reject it

The null hypothesis H_0 is accepted (not proven), when sample data do not cause it to be rejected

Higher the significance level, higher the probability of rejecting the null hypothesis when it is true as an equality

In parts *(a)* & *(b)*, the null hypothesis that the population mean is equal to the hypothesized value would be accepted

In part *(c)*, this same null hypothesis would be rejected because the significance level of 0.50 is so high that the null hypothesis would rarely be accepted when it is *not* true but, at the same time, it would often be rejected when it *is* true as an equality.



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Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Type I & Type II Errors

Type I Error	Type II Error
Rejecting a null hypothesis when it is true is called a Type I error, and its probability (also known as the significance level) is symbolized as α (alpha)	Accepting a null hypothesis when it is false is called a Type II error, and its probability is symbolized as β (beta)

Trade-offs between the two Types of Error
Probability of making one type of error can be reduced only by increasing the probability of making the other type of error

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Type I & Type II Errors

		POPULATION CONDITION	
		H_0 True	H_1 True
CONDITION	Accept H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

Trade-offs between the two Types of Error
Probability of making one type of error can be reduced only by increasing the probability of making the other type of error

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Type I & Type II Errors

Preference for Type I Error

When the cost (consequence) of accepting a false hypothesis is high (severe) as compared to the cost (consequence) of rejecting a true hypothesis

E.g. – Testing a batch of chemicals to be ok or defective. Null Hypothesis: Batch is OK.

Type I Error: Rejecting the null hypothesis when it is true, i.e. rejecting a batch of chemicals when it is OK. It involves time & trouble of reworking a batch that should have been accepted i.e. unnecessary reworking for an OK batch.

Type II Error: Accepting the null hypothesis when it is false, i.e. accepting a batch which is poisonous, the consequences of which would be much sever.

Cost of Type II Error is much higher of Type I Error.

So a Type I Error would be preferred over a Type II Error

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Type I & Type II Errors

Preference for Type II Error

When the cost (consequence) of rejecting a true hypothesis is high (severe) as compared to the cost (consequence) of accepting a false hypothesis

E.g. – Testing an assembled engine to be ok or defective. Null Hypothesis: Engine is OK.

Type I Error: Rejecting the null hypothesis when it is true, i.e. rejecting an assembled engine when it is OK. It involves disassembling the entire engine at the factory, the cost of which is much higher.

Type II Error: Accepting the null hypothesis when it is false, i.e. accepting an assembled engine which is defective. It involves making necessary repairs as required.

Cost of Type I Error is much higher of Type II Error.

So a Type II Error would be preferred over a Type I Error

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Power of Hypothesis Test
  - Accepting a null hypothesis when it is false is called a Type II error, and its probability is symbolized as β (beta)
  - 1 - β is therefore, the probability of rejecting a null hypothesis when it is false
  - A good hypothesis test must reject a null hypothesis when it is false, and hence must provide as large a probability for rejecting a false null hypothesis as possible i.e. a large value of 1 - β

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Power of Hypothesis Test

A high value of 1 - β means the hypothesis test is working quite well (that it is rejecting the null hypothesis when it is false)

A low value of 1 - β means the hypothesis test is working very poorly (that it is not rejecting the null hypothesis when it is false)

Because the value of 1 - β is the measure of how well the hypothesis test is working, it is known as the power of the test

Session 17-24

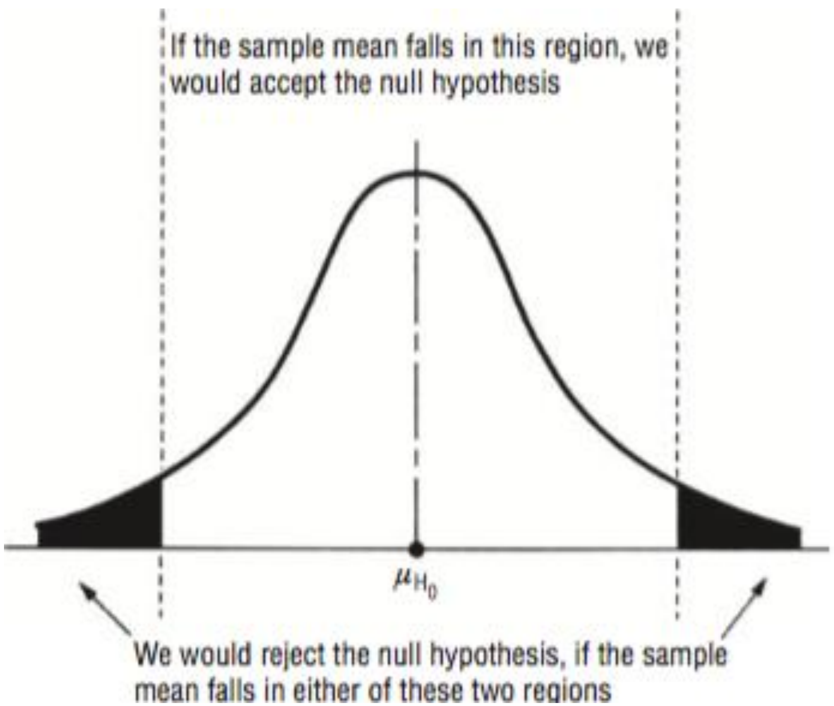
Testing of Hypothesis

Hypothesis Testing – One Sample Tests

•Two-Tailed & One-Tailed Tests of Hypothesis

Two-Tailed Test of Hypothesis

- Rejects the null hypothesis if the same mean is significantly higher / lower than the hypothesized population mean
- The two-tailed test thus has two rejection regions
- Appropriate when H_0: $\mu = \mu_0$ and H_1: $\mu \neq \mu_0$



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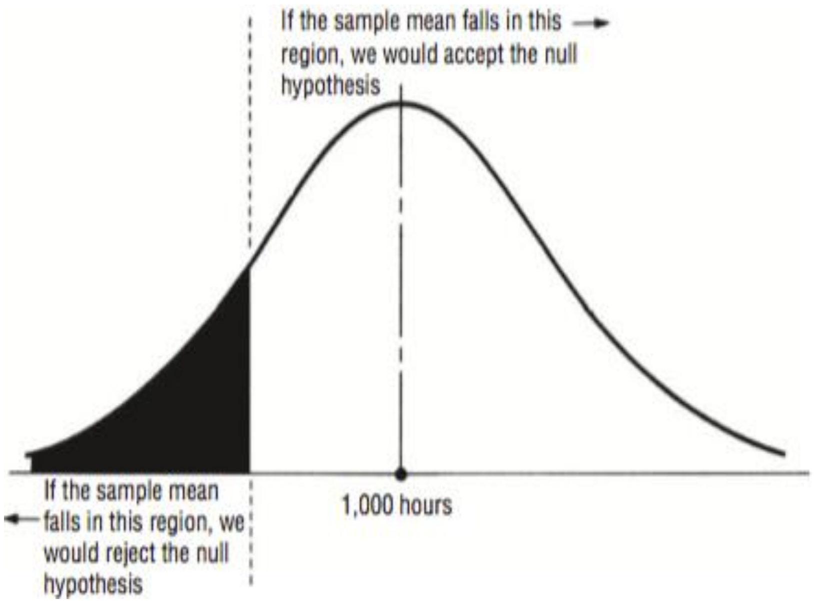
Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Two-Tailed & One-Tailed Tests of Hypothesis
  - Two types of One-Tailed Tests of Hypothesis

Left-Tailed Test of Hypothesis

- Left-tailed test (lower-tailed test) rejects the null hypothesis if the same mean is significantly lower than the hypothesized population mean
- H_0: $\mu \geq \mu_0$ and H_1: $\mu < \mu_0$



Session 17-24

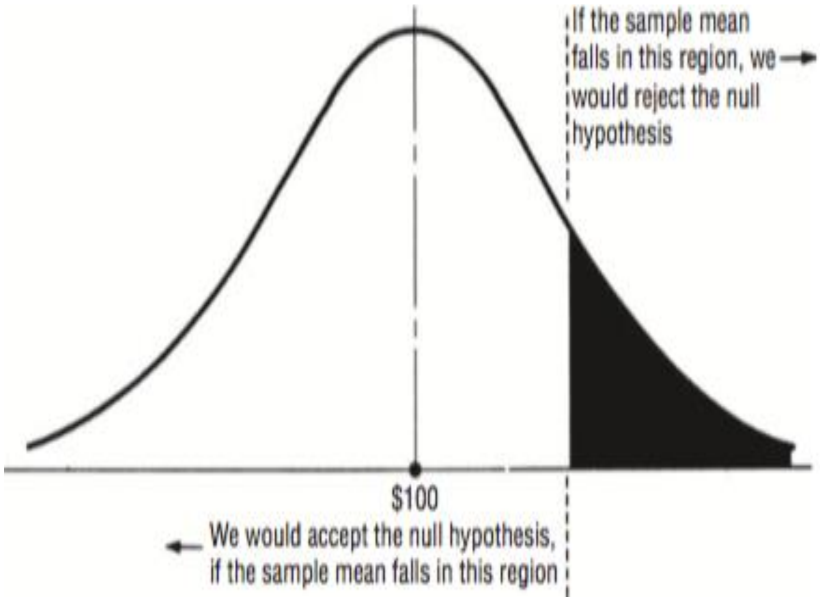
Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Two-Tailed & One-Tailed Tests of Hypothesis
  - Two types of One-Tailed Tests of Hypothesis

Right-Tailed Test of Hypothesis

- Right-tailed test (upper-tailed test) rejects the null hypothesis if the same mean is significantly higher than the hypothesized population mean
- H_0: $\mu \leq \mu_0$ and H_1: $\mu > \mu_0$



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Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - A highway safety engineer decides to test the load-bearing capacity of a bridge that is 20 years old. Considerable data are available from similar tests on the same type of bridge. Which is appropriate, a one-tailed or a two-tailed test? If the minimum load-bearing capacity of this bridge must be 10 tons, what are the null and alternative hypotheses?
  - Formulate null and alternative hypotheses to test whether the mean annual rainfall in Mumbai, exceeds 90 inches.

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Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - In a trial, the null hypothesis is that an individual is innocent of a certain crime. Would the legal system prefer to commit a Type I or a Type II error with this hypothesis?
  - The null hypothesis is that the battery for a heart pacemaker has an average life of less than or equal to 300 days, with the alternative hypothesis being that the battery life is more than 300 days. As the quality engineer for battery manufacturer:
    - Would you rather make a Type I or a Type II error?
    - For your preference of error, should you use a high or a low significance level?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - The statistics department installed energy-efficient lights, heaters, and air conditioners last year. Now they want to determine whether the average monthly energy usage has decreased. Should they perform a one-tailed or two-tailed test? If their previous average monthly energy usage was 3124 kilowatt hours, what are the null and alternative hypotheses?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Procedure for Hypothesis Testing of Means

1. Develop the null and alternative hypotheses
2. Specify the level of significance
3. Collect the sample data and compute the value of the test statistic

p-Value Approach	Critical Value Approach
4. Use the value of test statistic to compute the p-value 5. Reject H_0 if the p-value $\leq \alpha$	4. Use the level of significance to determine the critical value and the rejection rule 5. Use the value of test statistic & the rejection rule to determine whether to reject H_0

6. Interpret the statistical conclusion in the context of the application

p-Value Approach & Critical Value Approach will always provide the same conclusion

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Procedure for Hypothesis Testing of Means

3. Collect the sample data and compute the value of the test statistic

Test Statistic for Hypothesis Tests about a Population Mean: When σ Is Known
$z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

Test Statistic for Hypothesis Tests about a Population Mean: When σ Is Not Known
$t = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Procedure for Hypothesis Testing of Means

p-Value Approach
Uses value of test statistic (z or t) to compute a probability called a p-value p-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample Smaller p-values indicate more evidence against H_0 (Reject H_0 if p-value ≤ α)

Left-Tailed Test	Right-Tailed Test	Two-Tailed Test
p-value is the probability of obtaining a value for the test statistic as small as or smaller than that provided by the sample	p-value is the probability of obtaining a value for the test statistic as large as or larger than that provided by the sample	p-value is the probability of obtaining a value for the test statistic as unlikely as or more unlikely than that provided by the sample

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - The FDA is testing the claim on the label of a coffee brand that states the the coffee can contains 1000 gm. of coffee. It samples 36 coffee cans and finds a mean weight of 983 gm. of coffee. The FDA is willing to risk 1% chance of making an error of taking action against the coffee brand if it is meeting its weight specifications. Previous FDA tests suggest a population standard deviation of 38 gm. Can the FDA make a conclusion of under-filling and a charge of a label violation against the coffee brand?

$\mu_0 = 1000$ gm. $\bar{x} = 983$ gm. $\alpha = 0.01$ $n = 36$ $\sigma = 38$ gm.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

1. Develop the null and alternative hypotheses

   H_0: $\mu \geq 1000$ gm. H_1: $\mu < 1000$ gm.

2. Specify the level of significance

   $\alpha = 0.01$

3. Collect the sample data & compute the value of the test statistic

   $\bar{x} = 983$ gm. $n = 36$ $\sigma = 38$ gm.

   Since σ is known, test statistic is computed as

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = -2.68$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

p-Value Approach

4.Use the value of test statistic to compute the p-value

For left tail test, p-value is the probability of obtaining a value for the test statistic as small as or smaller than that provided by the sample

p-value = p $(z \leq -2.68) = 0.00368$

5.Reject H_0 if the p-value < α

p-value = 0.00368 $\alpha = 0.01$ p-value < α H_0 is rejected

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

  p-Value Approach

  6.Interpret statistical conclusion in the context of the application

  Probability of obtaining a value of $\bar{x} = 2.83$ or less when the null hypothesis is true as an equality is 0.00368. So null hypothesis is rejected, and FDA can make a conclusion of under-filling & a charge of a label violation against the coffee brand.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Procedure for Hypothesis Testing of Means

Critical Value Approach		
Determines critical value/s of the test statistic (z or t)		
Left-Tailed Test	**Right-Tailed Test**	**Two-Tailed Test**
Critical value is largest value of test statistic that will result in rejection of H_0 Reject H_0 if $z \leq -z_\alpha$ OR Reject H_0 if $t \leq -t_\alpha$ $-z_\alpha$ (or $-t_\alpha$) is critical value, i.e. z (or t) value providing area α in lower tail	Critical value is smallest value of test statistic that will result in rejection H_0 Reject H_0 if $z \geq z_\alpha$ OR Reject H_0 if $t \geq -t_\alpha$ z_α (or t_α) is critical value, i.e. z (or t) value providing area α in upper tail	Critical values are boundary values of test statistic that will result in rejection of H_0 Reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$ OR Reject H_0 if $t \leq -t_{\alpha/2}$ or if $t \geq t_{\alpha/2}$ $\pm z_{\alpha/2}$ (or $\pm t_{\alpha/2}$) are critical values i.e. z (or t) values providing area $\alpha/2$ in lower & upper tails

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - The FDA is testing the claim on the label of a coffee brand that states the the coffee can contains 1000 gm. of coffee. It samples 36 coffee cans and finds a mean weight of 983 gm. of coffee. The FDA is willing to risk 1% chance of making an error of taking action against the coffee brand if it is meeting its weight specifications. Previous FDA tests suggest a population standard deviation of 38 gm. Can the FDA make a conclusion of under-filling and a charge of a label violation against the coffee brand?

μ_0 = 1000 gm. $\bar{x}$ = 983 gm. α = 0.01 n = 36 σ = 38 gm.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

1. Develop the null and alternative hypotheses

   H_0: $\mu \geq 1000$ gm. H_1: $\mu < 1000$ gm.

2. Specify the level of significance

   $\alpha = 0.01$

3. Collect the sample data & compute the value of the test statistic

   $\bar{x} = 983$ gm. $n = 36$ $\sigma = 38$ gm.

   Since σ is known, test statistic is computed as

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = -2.68$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

Critical Value Approach

4.Use the level of significance to determine the critical value and the rejection rule

For left-tail test, critical value is the largest value of test statistic that will result in rejection of H_0, and the rejection rule is to reject H_0 if $z \leq z_\alpha$

For $\alpha = 0.01$, largest value of z that will result in rejection of H_0 is the value of z for which area to the left of the curve is 0.01

Critical Value is $z_\alpha = -2.325$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

Critical Value Approach

5.Use value of test statistic & rejection rule to determine whether to reject H_0

Test Statistic $z = -2.68$ Critical Value $z_\alpha = -2.325$

Rejection rule is to reject H_0 if $z \le z_\alpha$

Since $z \le z_\alpha$ ($-2.68 < -2.325$), H_0 is rejected

6.Interpret statistical conclusion in the context of the application

Since null hypothesis is rejected, FDA can make a conclusion of under-filling & a charge of a label violation against the coffee brand.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Means
  - When σ Is Known

	Lower Tail test	Upper Tail Test	Two-Tailed Test
Hypothesis	$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$
Test Statistic	$z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$		
Rejection Rule	Reject H_0 if		
p-Value Approach	p-value $\leq \alpha$		
Critical Value Approach	$z \leq -z_\alpha$	$z \geq z_\alpha$	$z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Means
  - When σ Is Not Known

	Lower Tail test	Upper Tail Test	Two-Tailed Test
Hypothesis	$H_0: \mu \geq \mu_0$ $H_1: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_1: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$
Test Statistic	$t = \dfrac{\overline{x} - \mu_0}{s / \sqrt{n}}$		
Rejection Rule	Reject H_0 if		
p-Value Approach	p-value $\leq \alpha$		
Critical Value Approach	$t \leq -t_\alpha$	$t \geq t_\alpha$	$t \leq -t_{\alpha/2}$ or $t \geq t_{\alpha/2}$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - Golf Association (GA) establishes rules that manufacturers of golf equipment must meet if their products are to be acceptable for use in GA events. ABC uses a high-technology manufacturing process to produce golf balls with a mean driving distance of 295 yards. Sometimes, however, the process gets out of adjustment and produces golf balls with a mean driving distance different from 295 yards. When the mean distance falls below 295 yards, the company worries about losing sales because the golf balls do not provide as much distance as advertised. When the mean distance passes 295 yards, ABC's golf balls may be rejected by the GA for exceeding the overall distance standard concerning carry and roll.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

ABC's quality control program involves taking periodic samples of 50 golf balls to monitor the manufacturing process. For each sample, a hypothesis test is conducted to determine whether the process has fallen out of adjustment. One such sample has resulted in a sample mean of 297.6 yards. The quality control team selected 0.05 as the level of significance for the test. Data from previous tests conducted tells that the population standard deviation can be assumed to be 12 yards when the process was known to be in adjustment. Is the value of the sample mean enough larger than 295 for ABC to worry?

μ_0 = 295 yards $\bar{x}$ = 297.6 yards α = 0.05 n = 50 σ = 12 yards

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

1. Develop the null and alternative hypotheses

   H_0: $\mu = 295$ yards $\quad H_1$: $\mu \neq 295$ yards

2. Specify the level of significance

   $\alpha = 0.05$

3. Collect the sample data & compute the value of the test statistic

   $\overline{x} = 297.6$ yards $\quad n = 50 \quad \sigma = 12$ yards

   Since σ is known, test statistic is computed as

$$z = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}} = 1.53$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

p-Value Approach

4.Use the value of test statistic to compute the p-values

For two-tailed test, p-value is the probability of obtaining a value for the test statistic as unlikely as or more unlikely than that provided by the sample

p-value = $p\ (z \le -1.53) + p\ (z \ge 1.53) = 0.06301 + 0.06301 = 0.12602$

5.Reject H_0 if the p-value < α

p-value = 0.12602 $\alpha = 0.05$ p-value > α H_0 is not rejected

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

p-Value Approach

6. Interpret statistical conclusion in the context of the application

Probability of obtaining a value of $\bar{x}$ = 297.6 or one which is at least as far away from the hypothesized mean as $\bar{x}$ when the null hypothesis is true as an equality is 0.12608. So null hypothesis is not rejected, implying that the value of the sample mean 297.6 is not enough larger than 295 for ABC to worry.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

1. Develop the null and alternative hypotheses

   H_0: μ = 295 yards H_1: $\mu \neq$ 295 yards

2. Specify the level of significance

   α = 0.05

3. Collect the sample data & compute the value of the test statistic

   $\bar{x}$ = 297.6 yards   n = 50 σ = 12 yards

   Since σ is known, test statistic is computed as

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = 1.53$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

Critical Value Approach

4.Use the level of significance to determine the critical value and the rejection rule

For two-tailed test, critical values are the boundary values of test statistic that will result in rejection of H_0, and the rejection rule is to reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq -z_{\alpha/2}$

For $\alpha = 0.05$, $\alpha/2 = 0.025$, largest value of z resulting in rejection of H_0 is value of z for which area to left of curve is 0.025, & smallest value of z resulting in rejection of H_0 is value of z for which area to right of curve is 0.025

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

Critical Value Approach

4.Use the level of significance to determine the critical value and the rejection rule

Critical Values are $z_{\alpha/2} = \pm 1.96$

5.Use value of test statistic & rejection rule to determine whether to reject H_0

Test Statistic $z = 1.53$ Critical Values $z_{\alpha/2} = \pm 1.96$

Rejection rule is to reject H_0 if $z \leq -z_{\alpha/2}$ or if $z \geq z_{\alpha/2}$

i.e. reject H_0 if $z \leq -1.96$ or if $z \geq 1.96$

Since $z \geq -z_{\alpha/2}$ & $z \leq z_{\alpha/2}$, there is no evidence to reject H_0

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise

  Critical Value Approach

  6. Interpret statistical conclusion in the context of the application

     Since there is no evidence to reject the null hypothesis, it means that the value of the sample mean 297.6 is not enough larger than 295 for ABC to worry.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - Hinton Press hypothesizes that the avg. life of its largest web press is 14500 hours. It knows that the std. dev. of press life is 2100 hours. From a sample of 25 presses, it finds a sample mean of 13000 hours. At a 0.01 significance level, should the company conclude that the avg. life of the presses is less than the hypothesized 14500 hours?
  - A certain hit movie ran an average of 84 days in each city, and the corresponding standard deviation was 10 days. The district manager in a region, interested in comparing the movie's popularity in his region with that in all other theaters in the country, randomly chose 75 theaters in his district and found that they ran the movie an average of 81.5 days.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
  - State the appropriate hypotheses for testing whether there was a significant difference in the length of the movie's run between the theaters chosen by the manager and all other theaters in the country.
  - At a 1 percent significance level, test these hypotheses.

- Given a sample mean of 83, a sample standard deviation of 12.5, and a sample size of 22, test the hypothesis that the value of the population mean is less than or equal to 70 against the alternative that it is more than 70. Use the 0.025 significance level.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Exercise
    - XYZ Ltd., a technology start-up, was planning the initial public offering of its stock in order to raise sufficient working capital to finance its expansion plans. With current earnings of Rs.161 a share, XYZ and its underwriters were contemplating an offering price of Rs.210, or about 13 times earnings. In order to check the appropriateness of this price, they randomly chose seven publicly traded technology start-ups and found that their average price/earnings ratio was 11.6, and the sample standard deviation was 1.3. At $\alpha = 0.02$, can XYZ conclude that the stocks of publicly traded technology start-ups have an average P/E ratio that is significantly different from 13?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Proportion of Success
  - 3 forms for a Hypothesis Test about Population Proportion

Forms for the Hypothesis Test about a Population Proportion		
$H_0 : p \geq p_0$ $H_1 : p < p_0$	$H_0 : p \leq p_0$ $H_1 : p > p_0$	$H_0 : p = p_0$ $H_1 : p \neq p_0$
Lower-tailed tests	Upper-tailed tests	Two-tailed tests
H_0 is Null Hypothesis H_1 is Alternate Hypothesis	p is population proportion p_0 is hypothesized value of population proportion	

- Equality part (either $\geq$, $\leq$, or $=$) always appears in Null Hypothesis

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Proportion of Success
  - Hypothesis tests about a population proportion are based on the difference between the sample proportion $\bar{p}$ and the hypothesized population proportion p_0
  - Procedure for hypothesis testing of population proportion is similar to that for the population means, expect for the computation of test statistic

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Proportion of Success

1. Develop the null and alternative hypotheses
2. Specify the level of significance
3. Collect the sample data and compute the value of the test statistic

p-Value Approach	Critical Value Approach
4. Use the value of test statistic to compute the p-value 5. Reject H_0 if the p-value $\leq \alpha$	4. Use the level of significance to determine the critical value and the rejection rule 5. Use the value of test statistic & the rejection rule to determine whether to reject H_0

6. Interpret the statistical conclusion in the context of the application

p-Value Approach & Critical Value Approach will always provide the same conclusion

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Proportion of Success

3. Collect the sample data and compute the value of the test statistic

Test Statistic for Hypothesis Tests about a Population Proportion

$$z = \frac{\bar{p} - p_0}{\sqrt{(p_0\,(1 - p_0) / n)}}$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Proportion of Success

p-Value Approach
Uses value of test statistic (z or t) to compute a probability called a p-value
p-value is a probability that provides a measure of the evidence against the null hypothesis provided by the sample
Smaller p-values indicate more evidence against H_0 (Reject H_0 if p-value $\leq \alpha$)

Left-Tailed Test	Right-Tailed Test	Two-Tailed Test
p-value is the probability of obtaining a value for the test statistic as small as or smaller than that provided by the sample	p-value is the probability of obtaining a value for the test statistic as large as or larger than that provided by the sample	p-value is the probability of obtaining a value for the test statistic as unlikely as or more unlikely than that provided by the sample

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Proportion of Success

Critical Value Approach		
Determines critical value/s of the test statistic (z or t)		
Left-Tailed Test	**Right-Tailed Test**	**Two-Tailed Test**
Critical value is largest value of test statistic that will result in rejection of H_0 Reject H_0 if $z \le -z_\alpha$ OR Reject H_0 if $t \le -t_\alpha$ $-z_\alpha$ (or $-t_\alpha$) is critical value, i.e. z (or t) value providing area α in lower tail	Critical value is smallest value of test statistic that will result in rejection H_0 Reject H_0 if $z \ge z_\alpha$ OR Reject H_0 if $t \ge -t_\alpha$ z_α (or t_α) is critical value, i.e. z (or t) value providing area α in upper tail	Critical values are boundary values of test statistic that will result in rejection of H_0 Reject H_0 if $z \le -z_{\alpha/2}$ or if $z \ge z_{\alpha/2}$ OR Reject H_0 if $t \le -t_{\alpha/2}$ or if $t \ge t_{\alpha/2}$ $\pm z_{\alpha/2}$ (or $\pm t_{\alpha/2}$) are critical values i.e. z (or t) values providing area $\alpha/2$ in lower & upper tails

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Hypothesis Testing of Proportion of Success

	Lower Tail test	Upper Tail Test	Two-Tailed Test
Hypothesis	$H_0: p \geq p_0$ $H_1: p < p_0$	$H_0: p \leq p_0$ $H_1: p > p_0$	$H_0: p = p_0$ $H_1: p \neq p_0$
Test Statistic	$z = \dfrac{\overline{p} - p_0}{\sqrt{(p_0 (1 - p_0) / n)}}$		
Rejection Rule	Reject H_0 if		
p-Value Approach	p-value $\leq \alpha$		
Critical Value Approach	$z \leq -z_\alpha$	$z \geq z_\alpha$	$z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Example
  - Consider the following hypothesis test:

    H_0: p = 0.20 H_1: p ≠ 0.20

    A sample of 400 provided a sample proportion of 0.175.

    - Compute the value of the test statistic
    - What is the p-value?
    - At α = 0.05, what is your conclusion?
    - What is the rejection rule using the critical value? What is your conclusion?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

- Example
  - A ketchup manufacturer is in the process of deciding whether to produce a new extra-spicy brand. The company's marketing-research department used a national telephone survey of 6,000 households and found that the extra-spicy ketchup would be purchased by 335 of them. A much more extensive study made 2 years ago showed that 5 percent of the households would purchase the brand then. At a 2 percent significance level, should the company conclude that there is an increased interest in the extra-spicy flavor?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – One Sample Tests

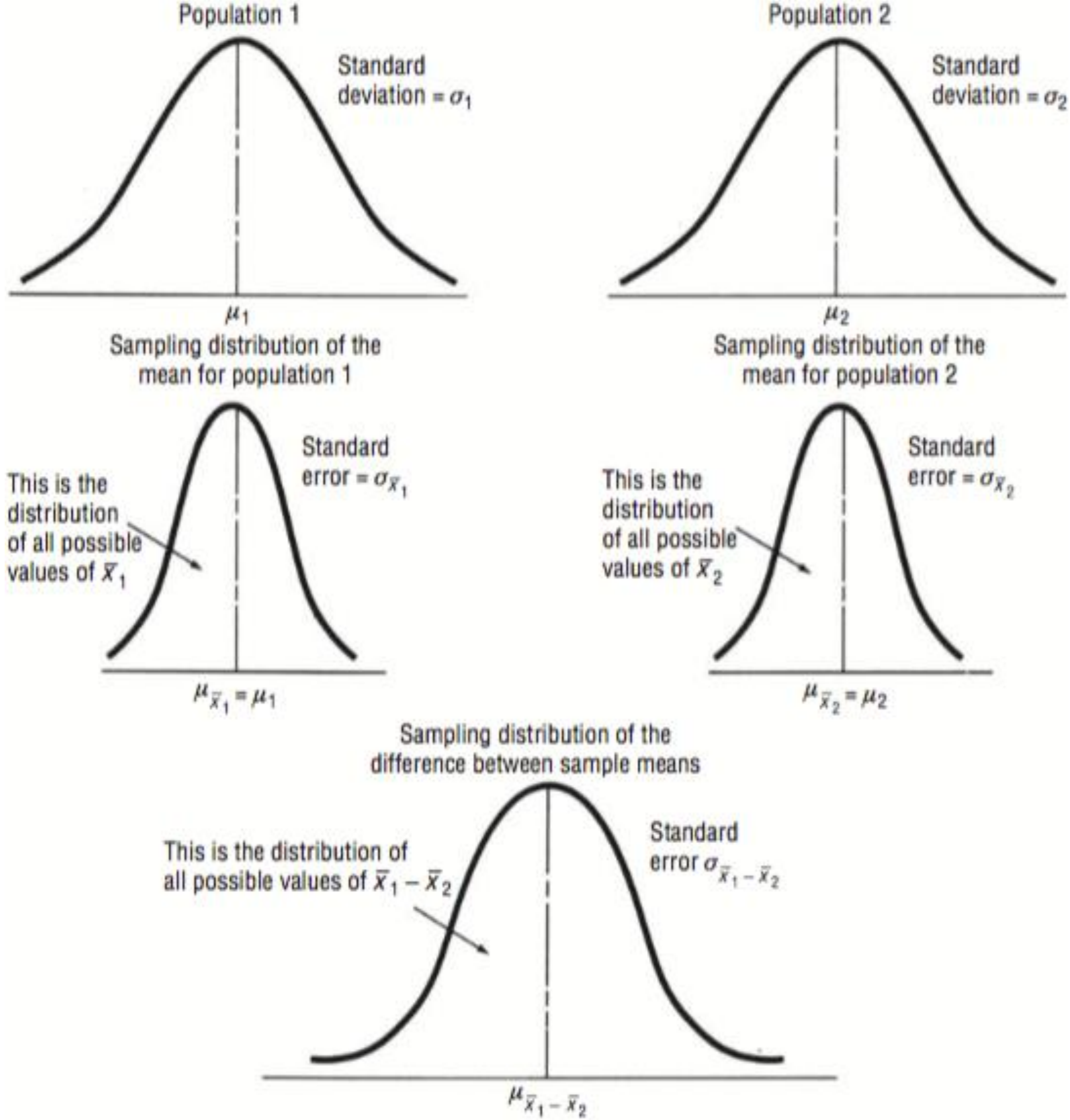
- Example
  - Steve Cutter sells Big Blade lawn mowers in his hardware store, and he is interested in comparing the reliability of the mowers he sells with the reliability of Big Blade mowers sold nationwide. Steve knows that only 15 percent of all Big Blade mowers sold nationwide require repairs during the first year of ownership. A sample of 120 of Steve's customers revealed that exactly 22 of them required mower repairs in the first year of ownership. At the 0.02 level of significance, is there evidence that Steve's Big Blade mowers differ in reliability from those sold nationwide?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Introduction
  - Used in comparing two populations when decision-makers are concerned with parameters of two populations
  - Actual value of the parameters of the two populations may not be as important as the relation between the values of those parameters, i.e. how they differ
  - Sampling distribution of interest is the sampling distribution of the diff. between the sample statistics



Population 1
Standard deviation = σ_1
μ_1
Population 2
Standard deviation = σ_2
μ_2
Sampling distribution of the mean for population 1
Standard error = $\sigma_{\bar{x}_1}$
This is the distribution of all possible values of $\bar{x}_1$
$\mu_{\bar{x}_1} = \mu_1$
Sampling distribution of the mean for population 2
Standard error = $\sigma_{\bar{x}_2}$
This is the distribution of all possible values of $\bar{x}_2$
$\mu_{\bar{x}_2} = \mu_2$
Sampling distribution of the difference between sample means
Standard error $\sigma_{\bar{x}_1 - \bar{x}_2}$
This is the distribution of all possible values of $\bar{x}_1 - \bar{x}_2$
$\mu_{\bar{x}_1 - \bar{x}_2}$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Hypothesis Testing for Difference between Means
    - 3 forms for a Hypothesis Test about Difference between Means

Forms for the Hypothesis Test about Difference Between Means		
$H_0 : \mu_1 - \mu_2 \geq D_0$ $H_1 : \mu_1 - \mu_2 < D_0$	$H_0 : \mu_1 - \mu_2 \leq D_0$ $H_1 : \mu_1 - \mu_2 > D_0$	$H_0 : \mu_1 - \mu_2 = D_0$ $H_1 : \mu_1 - \mu_2 \neq D_0$
Lower-tailed tests	Upper-tailed tests	Two-tailed tests

H_0 is Null Hypothesis

H_1 is Alternate Hypothesis

μ_1 is mean of population 1

μ_2 is mean of population 2

D_0 is hypothesized difference between μ_1 & μ_2

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Hypothesis Testing for Difference between Means

Test Statistic for Hypothesis Tests about Difference between Population Means: When σ Is Known
$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$
where $\bar{x}_1$ = sample mean for the simple random sample of size n_1 from population 1
$\bar{x}_2$ = sample mean for the simple random sample of size n_2 from population 2
σ_1 = standard deviation of population 1
σ_2 = standard deviation of population 2
$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ = standard error of $\bar{x}_1 - \bar{x}_2$ i.e. standard deviation of the sampling distribution of $\bar{x}_1 - \bar{x}_2$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Hypothesis Testing for Difference between Means

Test Statistic for Hypothesis Tests about Difference between Population Means: When σ Is Not Known

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

where $\bar{x}_1$ = sample mean for the simple random sample of size n_1 from population 1

$\bar{x}_2$ = sample mean for the simple random sample of size n_2 from population 2

s_1 = standard deviation of sample 1

s_2 = standard deviation of sample 2

and the degrees of freedom for t distribution is given by df

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Hypothesis Testing for Difference between Means

Test Statistic for Hypothesis Tests about Difference between Population Means: When σ Is Not Known (Assuming $\sigma_1 = \sigma_2 = \sigma$)

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where

$\bar{x}_1$ = sample mean for the simple random sample of size n_1 from population 1

$\bar{x}_2$ = sample mean for the simple random sample of size n_2 from population 2

df = degrees of freedom for t distribution = $n_1 + n_2 - 2$

s_p = pooled sample standard deviation

$(s_p)^2$ = pooled sample variance = $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example
  - Consider the following hypothesis test: $H_0 : \mu_1 - \mu_2 = 0$

    $H_1 : \mu_1 - \mu_2 \neq 0$

    The following results are for two independent samples taken from the two populations.

- What is the value of the test statistic?
- What is the p-value?
- With $\alpha = 0.05$, what is the hypothesis testing conclusion?

Sample 1	Sample 2
$n_1 = 80$	$n_2 = 70$
$\bar{x}_1 = 104$	$\bar{x}_2 = 106$
$\sigma_1 = 8.4$	$\sigma_2 = 7.6$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example
  - Following are details of customer satisfaction surveys conducted for 2 retailers:

    $\bar{x}_1$ & $\bar{x}_2$ are avg. customer satisfaction ratings (out of a max. rating of 100)

Retailer 1	Retailer 2
$n_1 = 25$	$n_2 = 30$
$\bar{x}_1 = 79$	$\bar{x}_2 = 71$

Results (out of a maximum customer satisfaction score of 100)

Assuming from past experience that satisfaction ratings have a population std. dev. of 12 for both retailers, test the hypothesis that retailer 1 has a significantly higher customer satisfaction rating that retailer 2 test at 0.05 level of significance.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example
  - A consumer-research organization routinely selects several car models each year and evaluates their fuel efficiency. In this year's study of two similar subcompact models from two different automakers, the average gas mileage for 12 cars of brand A was 27.2 kmpl, and the standard deviation was 3.8 kmpl. The nine brand B cars that were tested averaged 32.4 kmpl, and the standard deviation was 4.3 kmpl. At $\alpha = 0.01$, should it conclude that brand A cars have lower fuel efficiency than do brand B cars?

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Exercise

1. Develop the null and alternative hypotheses

   H_0: $\mu_1 - \mu_2 \geq 0$ H_1: $\mu_1 - \mu_2 < 0$

2. Specify the level of significance

   $\alpha = 0.01$

3. Collect the sample data & compute the value of the test statistic

Sample 1			Sample 2		
n_1	=	12	n_2	=	9
$\bar{x}_1$	=	27.2	$\bar{x}_2$	=	32.4
s_1	=	3.8	s_2	=	4.3

Since σ_1 & σ_1 are no known, test statistic is computed as

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = -2.88$$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Exercise

Critical Value Approach

4.Use the level of significance to determine the critical value and the rejection rule

For left-tailed test, critical value is the largest value of test statistic that will result in rejection of H_0, and the rejection rule is to reject H_0 if $t \leq t_\alpha$

For $\alpha = 0.01$, largest value of t resulting in rejection of H_0 is value of t for which area to left of curve is 0.01 for df degrees of freedom

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Exercise

Critical Value Approach

4.Use the level of significance to determine the critical value and the rejection rule

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2} = 16.09 \approx 17$$

Critical Value is $t_\alpha = -2.567$

5.Use value of test statistic & rejection rule to determine whether to reject H_0

Test Statistic $t = -2.88$ Critical Value $t_\alpha = -2.567$

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Exercise

Critical Value Approach

Rejection rule is to reject H_0 if $t \le t_\alpha$

Since -2.88 < 2.567, $t \le t_\alpha$, there is evidence to reject H_0

6.Interpret statistical conclusion in the context of the application

Since there no evidence to reject the null hypothesis, it can be concluded that brand A cars have lower fuel efficiency than that of brand B cars

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Exercise
  - As reported by a leading newspaper, salary data show staff journalists in city 1 earn less than staff journalists in city 2. In a follow-up study of 40 staff journalists in city 1and 50 staff journalists in city the following results were obtained.

City 1		City 2	
n_1	= 40	n_2	= 50
$\bar{x}_1$	= 56100	$\bar{x}_2$	= 59400
s_1	= 6000	s_2	= 7000

Formulate & test the hypothesis at α = 0.05 so that, if the null hypothesis is rejected, it can be concluded that salaries for staff journalists in city 1 are significantly lower than for those in city 2.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Paired Difference Test

Paired Samples (Matched Samples, Dependent Samples, Matched Pairs)

- Samples paired such that elements of samples share every characteristic except for the one under investigation
- Provide more precise analysis as they allow control for extraneous factors leading to smaller sampling errors
- Use same basic procedure for hypothesis testing as that for independent samples, only difference being the use of a different formula for the estimated standard error of the sample differences

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Paired Difference Test
  - 3 forms for a Hypothesis Test about Difference between Means for Paired Samples

Forms for the Hypothesis Test about Difference Between Means – Paired Samples		
$H_0 : \mu_d \geq \mu_{d0}$ $H_1 : \mu_d < \mu_{d0}$	$H_0 : \mu_d \leq \mu_{d0}$ $H_1 : \mu_d > \mu_{d0}$	$H_0 : \mu_d = \mu_{d0}$ $H_1 : \mu_d \neq \mu_{d0}$
One-tailed tests		Two-tailed tests
H_0 is Null Hypothesis H_1 is Alternate Hypothesis	μ_d is mean of diff. bet. matched pairs μ_{d0} is hypothesized value of mean of diff. bet. matched pairs	

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Paired Difference Test

Test Statistic for Hypothesis Tests about Difference between Population Means: Paired Samples

$$t = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}}$$

where	$\bar{d}$	=	mean of the diff. bet. the matched pairs
	s_d	=	std. deviation of the diff. bet. the matched pairs
	n	=	no. of matched pairs (size of samples of the two populations)

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example
  - Pre & post records of 10 participants in a weight-reducing program at a health spa are shown below. The health spa has claimed that the average participant in the program loses more than 17 kg. Test at 5 percent level of significance the claimed average weight loss of more than 17 kg.

		Participants									
		1	2	3	4	5	6	7	8	9	10
Weights (kg)	Before	95	101	110	104	97	89	97	101	104	117
	After	76	78	93	89	75	73	78	86	82	88

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

$H_0 \quad : \quad \mu_d \quad \leq \quad 17$

$H_1 \quad : \quad \mu_d \quad > \quad 17$

$\alpha = 0.05$

$n = 10$

$\bar{d} = \Sigma d / n$

$= 197 / 10$

$= 19.7$

$$(s_d)^2 = \frac{\Sigma (d - \bar{d})^2}{(n - 1)}$$

$= 174.1 / 9$

$s_d = 4.40$

Participants	Before Wt. (kg.)	After Wt. (kg.)	d (kg.)	$d - \bar{d}$	$(d - \bar{d})^2$
1	95	76	19	-0.7	0.49
2	101	78	23	3.3	10.89
3	110	93	17	-2.7	7.29
4	104	89	15	-4.7	22.09
5	97	75	22	2.3	5.29
6	89	73	16	-3.7	13.69
7	97	78	19	-0.7	0.49
8	101	86	15	-4.7	22.09
9	104	82	22	2.3	5.29
10	117	88	29	9.3	86.49
Total			197		174.1

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example

$$t = \frac{\bar{d} - \mu_{d0}}{s_d / \sqrt{n}}$$ --> Standard Error of Mean Difference

$$t = \frac{19.7 - 17}{4.40 / \sqrt{10}}$$

$$t = 1.94$$

For a right-tailed test with $\alpha = 0.05$, & 9 degrees of freedom, critical value

$$t_\alpha = 1.833$$

Rejection rule for right-tailed test is to reject H_0 if $t \geq t_\alpha$. Since, $t > t_\alpha$, null hypothesis is rejected. Hence, alternate hypothesis is accepted i.e. the health spa's claim of average weight loss of more than 17 kg is accepted.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Hypothesis Testing for Difference between Proportions
  - 3 forms for a Hypothesis Test about Difference between Proportions

Forms for the Hypothesis Test about Difference Between Proportions		
$H_0 : p_1 - p_2 \geq D_0$ $H_1 : p_1 - p_2 < D_0$	$H_0 : p_1 - p_2 \leq D_0$ $H_1 : p_1 - p_2 > D_0$	$H_0 : p_1 - p_2 = D_0$ $H_1 : p_1 - p_2 \neq D_0$
Lower-tailed tests	Upper-tailed tests	Two-tailed tests
H_0 is Null Hypothesis H_1 is Alternate Hypothesis	p_1 is proportion of population 1 p_2 is proportion of population 2 D_0 is hypothesized difference between p_1 & p_2	

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Hypothesis Testing for Difference between Proportions

Test Statistic[*] for Hypothesis Tests about Difference between Population Proportions

$$z = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

--> Estimated Standard Error of $\bar{p}_1 - \bar{p}_2$ when $p_1 = p_2 = p$

where $\bar{p}_1$ = sample proportion for a simple random sample from population 1

$\bar{p}_2$ = sample proportion for a simple random sample from population 2

$\bar{p}$ = pooled estimator of overall proportion p in two populations when $p_1 = p_2 = p$

$$= \frac{n_1 \bar{p}_1 + n_2 \bar{p}_2}{n_1 + n_2}$$ = weighted average of $\bar{p}_1$ & $\bar{p}_2$

[*]For large sample sample sizes when n_1p_1, $n_1(1-p_1)$, n_2p_2, $n_2(1-p_2)$ are all ≥ 5

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example
  - A taxation firm wants to compare the quality of work at two of its regional offices. It randomly selects samples of tax returns prepared at each office & verifies the sample returns' accuracy to estimate the proportion of erroneous returns prepared at each office. Sample details are as follows. For 0.10 level of significance test the hypothesis whether the error proportions differ between the two offices.

	Office 1	Office 2
Sample Size	250	300
Erroneous Returns	35	27

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example

$H_0: p_1 - p_2 = 0$ $\alpha = 0.10$ $n_1 = 250$ $\bar{p}_1 = 35 / 250 = 0.14$

$H_1: p_1 - p_2 \neq 0$ $n_2 = 300$ $\bar{p}_2 = 27 / 300 = 0.09$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = 0.1127 \qquad z = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 1.85$$

p-value = $p(z \leq -1.85) + p(z \geq 1.85) = 0.03216 + 0.03216 = 0.06432$

Rejection rule is to reject H_0 if p-value $\leq \alpha$

Since p-value (0.06432) < α (0.10), H_0 is rejected, and it is concluded that the error proportions differ significantly between the two offices at a significance level of 0.10

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example
    - A large hotel chain is trying to decide whether to convert more of its rooms to nonsmoking rooms. In a random sample of 400 guests last year, 166 had requested nonsmoking rooms. This year, 205 guests in a sample of 380 preferred the nonsmoking rooms. Would you recommend that the hotel chain convert more rooms to nonsmoking? Support your recommendation by testing the appropriate hypotheses at a 0.01 level of significance.

Session 17-24

Testing of Hypothesis

Hypothesis Testing – Two Sample Tests

- Example

$H_0: p_1 - p_2 \geq 0$ $\alpha = 0.01$ $n_1 = 400$ $\bar{p}_1 = 166 / 400 = 0.4150$

$H_1: p_1 - p_2 < 0$ $n_2 = 380$ $\bar{p}_2 = 205 / 380 = 0.5395$

$$\bar{p} = \frac{n_1\bar{p}_1 + n_2\bar{p}_2}{n_1 + n_2} = 0.4756 \qquad z = \frac{(\bar{p}_1 - \bar{p}_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -3.48$$

p-value = $p(z \leq -3.48) = 0.00025$ Rejection rule is to reject H_0 if p-value $\leq \alpha$

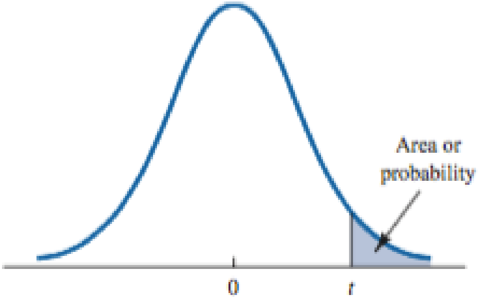
Since p-value (0.00025) < α (0.01), H_0 is rejected, & it is concluded that a greater proportions of guests preferred nonsmoking rooms this year than in the previous year & hence the hotel chain convert more rooms to nonsmoking

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756
30	.854	1.310	1.697	2.042	2.457	2.750
31	.853	1.309	1.696	2.040	2.453	2.744
32	.853	1.309	1.694	2.037	2.449	2.738
33	.853	1.308	1.692	2.035	2.445	2.733
34	.852	1.307	1.691	2.032	2.441	2.728

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
35	.852	1.306	1.690	2.030	2.438	2.724
36	.852	1.306	1.688	2.028	2.434	2.719
37	.851	1.305	1.687	2.026	2.431	2.715
38	.851	1.304	1.686	2.024	2.429	2.712
39	.851	1.304	1.685	2.023	2.426	2.708
40	.851	1.303	1.684	2.021	2.423	2.704
41	.850	1.303	1.683	2.020	2.421	2.701
42	.850	1.302	1.682	2.018	2.418	2.698
43	.850	1.302	1.681	2.017	2.416	2.695
44	.850	1.301	1.680	2.015	2.414	2.692
45	.850	1.301	1.679	2.014	2.412	2.690
46	.850	1.300	1.679	2.013	2.410	2.687
47	.849	1.300	1.678	2.012	2.408	2.685
48	.849	1.299	1.677	2.011	2.407	2.682
49	.849	1.299	1.677	2.010	2.405	2.680
50	.849	1.299	1.676	2.009	2.403	2.678
51	.849	1.298	1.675	2.008	2.402	2.676
52	.849	1.298	1.675	2.007	2.400	2.674
53	.848	1.298	1.674	2.006	2.399	2.672
54	.848	1.297	1.674	2.005	2.397	2.670
55	.848	1.297	1.673	2.004	2.396	2.668
56	.848	1.297	1.673	2.003	2.395	2.667
57	.848	1.297	1.672	2.002	2.394	2.665
58	.848	1.296	1.672	2.002	2.392	2.663
59	.848	1.296	1.671	2.001	2.391	2.662
60	.848	1.296	1.671	2.000	2.390	2.660
61	.848	1.296	1.670	2.000	2.389	2.659
62	.847	1.295	1.670	1.999	2.388	2.657
63	.847	1.295	1.669	1.998	2.387	2.656
64	.847	1.295	1.669	1.998	2.386	2.655
65	.847	1.295	1.669	1.997	2.385	2.654
66	.847	1.295	1.668	1.997	2.384	2.652
67	.847	1.294	1.668	1.996	2.383	2.651
68	.847	1.294	1.668	1.995	2.382	2.650
69	.847	1.294	1.667	1.995	2.382	2.649
70	.847	1.294	1.667	1.994	2.381	2.648
71	.847	1.294	1.667	1.994	2.380	2.647
72	.847	1.293	1.666	1.993	2.379	2.646
73	.847	1.293	1.666	1.993	2.379	2.645
74	.847	1.293	1.666	1.993	2.378	2.644
75	.846	1.293	1.665	1.992	2.377	2.643
76	.846	1.293	1.665	1.992	2.376	2.642
77	.846	1.293	1.665	1.991	2.376	2.641
78	.846	1.292	1.665	1.991	2.375	2.640
79	.846	1.292	1.664	1.990	2.374	2.639

Degrees of Freedom	Area in Upper Tail .20	.10	.05	.025	.01	.005
80	.846	1.292	1.664	1.990	2.374	2.639
81	.846	1.292	1.664	1.990	2.373	2.638
82	.846	1.292	1.664	1.989	2.373	2.637
83	.846	1.292	1.663	1.989	2.372	2.636
84	.846	1.292	1.663	1.989	2.372	2.636
85	.846	1.292	1.663	1.988	2.371	2.635
86	.846	1.291	1.663	1.988	2.370	2.634
87	.846	1.291	1.663	1.988	2.370	2.634
88	.846	1.291	1.662	1.987	2.369	2.633
89	.846	1.291	1.662	1.987	2.369	2.632
90	.846	1.291	1.662	1.987	2.368	2.632
91	.846	1.291	1.662	1.986	2.368	2.631
92	.846	1.291	1.662	1.986	2.368	2.630
93	.846	1.291	1.661	1.986	2.367	2.630
94	.845	1.291	1.661	1.986	2.367	2.629
95	.845	1.291	1.661	1.985	2.366	2.629
96	.845	1.290	1.661	1.985	2.366	2.628
97	.845	1.290	1.661	1.985	2.365	2.627
98	.845	1.290	1.661	1.984	2.365	2.627
99	.845	1.290	1.660	1.984	2.364	2.626
100	.845	1.290	1.660	1.984	2.364	2.626
∞	.842	1.282	1.645	1.960	2.326	2.576

If the degrees of freedom exceed 100, the infinite degrees of freedom row can be used to approximate the actual *t* value; in other words, for more than 100 degrees of freedom, the standard normal *z* value provides a good approximation to the *t* value.