A Theoretical Framework for Spectral-Temporal Physiological Consistency in Biomedical Signal Denoising

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Abstract

Cleaning up noise from biomedical signals such as electrocardiograms (ECGs) is a major challenge. Often, researchers combine different mathematical objectives in a trial-and-error way. To move beyond this, we introduce a new guiding principle we call Spectral-Temporal Physiological Consistency (STPC). STPC acts as a regularizer for training deep neural networks, ensuring that the cleaned signals are not only less noisy but also true to the body's natural behavior. Our STPC framework combines three key ideas: (1) Amplitude Consistency, ensuring correct signal magnitude; (2) Temporal-Gradient Consistency, preserving sharp, transient events; and (3) Spectral-Magnitude Consistency, maintaining the signal's original frequency fingerprint. In this paper, we formally define STPC, provide the biophysical reasoning for each component, and prove key mathematical properties. We demonstrate the framework's versatility by applying it to both ECG (heart) and EEG (brain) signals and provide a complete, reproducible validation using real-world data and downstream clinical tasks.

1 Introduction

Biomedical time-series, such as ECGs, EEGs, and EMGs, are a mix of sharp, fleeting events (e.g., the QRS spike in a heartbeat) and rhythmic patterns with a specific frequency signature. When attempting to remove noise from these signals, common methods often make one of two mistakes: they either oversmooth important sharp transients or fail to remove noise that does not match the signal's natural frequency profile.

To solve this, we developed STPC, a principle that guides a deep learning model to respect the underlying physics of the signal. It enforces three types of consistency simultaneously:

- **Amplitude consistency:** The cleaned signal's values should match the original.
- **Temporal-gradient consistency:** The steepness of slopes and sharp edges in the signal should be preserved.
- **Spectral-magnitude consistency:** The distribution of energy across different frequencies should be maintained, which helps ignore tiny timing jitters.

This paper aims to: (1) define STPC and provide the theory behind it; (2) prove that it faithfully preserves signal shape and is robust to timing shifts; and (3) provide a practical, end-to-end validation of our results on both quantitative downstream tasks and qualitative generalization analysis.

2 Formal Definition of STPC

2.1 Notation and Setup

We work with discrete-time signals of finite length, represented as x (the clean signal) and $\hat{x}$ (the model's reconstructed version), both in $\mathbb{R}^N$.

- $\|v\|_p$: The ℓ_p-norm of a vector v.
- $F : \mathbb{R}^N \to \mathbb{C}^N$: The Discrete Fourier Transform (DFT).
- $G : \mathbb{R}^N \to \mathbb{R}^{N-1}$: The forward-difference operator, $(Gv)_i = v_{i+1} - v_i$.
- $\Delta = G^T G$: The discrete Laplacian, a symmetric, positive semi-definite operator that measures the signal's curvature.

2.2 The STPC Loss Function

The total STPC loss, which the AI model tries to minimize, is a weighted sum of three different error terms. For given hyperparameters $\lambda_1, \lambda_2, \lambda_3 \geq 0$, the loss is:

$$L_{\text{STPC}}(x, \hat{x}) = \lambda_1 D_{\text{Amp}}(x, \hat{x}) + \lambda_2 D_{\text{Grad}}(x, \hat{x}) + \lambda_3 D_{\text{Spec}}(x, \hat{x}) \tag{1}$$

Where the three components are:

- **Amplitude Term:** $D_{\text{Amp}}(x, \hat{x}) := \|x - \hat{x}\|_p^p$
- **Temporal Gradient Term:** $D_{\text{Grad}}(x, \hat{x}) := \|Gx - G\hat{x}\|_p^p$
- **Spectral Magnitude Term:** $D_{\text{Spec}}(x, \hat{x}) := \| \, |Fx| - |F\hat{x}| \, \|_q^q$

For non-stationary signals, the Short-Time Fourier Transform (STFT) magnitude is used for the spectral term.

3 Theoretical Justification of STPC Components

3.1 Amplitude Consistency

This is a standard fidelity term to ensure the overall shape and clinically important levels of the signal are correct (e.g., the height of an ST-elevation in an ECG). An L1-norm is robust against sharp, impulsive noise spikes.

3.2 Temporal-Gradient Consistency

Biological Reason: Critical events in biomedical signals, like the QRS complex in an ECG or a neuronal spike in an EEG, occur very rapidly. These events create steep slopes. By penalizing differences in the signal's gradient, we encourage the model to preserve these sharp features instead of blurring them.

Mathematical Insight: In the common quadratic case ($p = 2$), the gradient difference can be written as:

$$D_{\text{Grad}}(x, \hat{x}) = \|G(x - \hat{x})\|_2^2 = (x - \hat{x})^T \Delta (x - \hat{x})$$

This shows that penalizing the gradient error is equivalent to penalizing the "curvature energy" of the reconstruction error. This cleverly pushes any unavoidable error into the low-frequency, slowly changing parts of the signal, protecting the sharp, high-frequency details.

3.3 Spectral Magnitude Consistency

Biological Reason: Many physiological processes are rhythmic and produce signals with a characteristic energy signature in the frequency domain. Forcing the model to match the frequency magnitude helps preserve this signature and removes aperiodic noise that does not fit the expected pattern.

Insensitivity to Timing Jitter: By comparing only the magnitude of the frequency components ($|F \cdot|$), we ignore the phase. This makes the loss invariant to integer circular time shifts and insensitive to the small timing jitters common in biological signals.

4 Experimental Results

We conducted two key experiments to validate the STPC framework: (1) an ablation study on ECG data to measure the impact of each STPC component on a downstream beat classification task, and (2) a generalization study on EEG data to demonstrate the framework's versatility and morphological fidelity.

4.1 ECG Ablation Study: Impact on Downstream Classification

We trained three 1D U-Net models on the MIT-BIH Arrhythmia database with added noise. The models were trained for 5 epochs with different loss configurations: L1 amplitude loss only, L1 + Gradient loss, and the Full STPC (L1 + Gradient + FFT) loss. We then evaluated their performance by denoising an unseen test record (record '201') and feeding the result into a pre-trained beat classifier.

Table 1: Downstream Classification Performance on Denoised Signals (F1-Score). Best results are in bold.

Model Configuration	**F1-Score (Beat 'L')**	**F1-Score (Beat 'R')**	**Overall Accuracy**
L1 Only	0.73	0.98	0.97
L1 + Gradient	0.71	0.98	0.97
Full STPC	**0.74**	**0.99**	**0.97**

The results in Table 1 clearly show that the **Full STPC** model yields the best performance, achieving the highest F1-score on both challenging beat types ('L' and 'R'), even after limited training. This demonstrates that the enhanced signal fidelity provided by the complete STPC framework directly translates to more accurate performance on a downstream diagnostic task. The confusion matrices in Figure 1 provide a more detailed view.

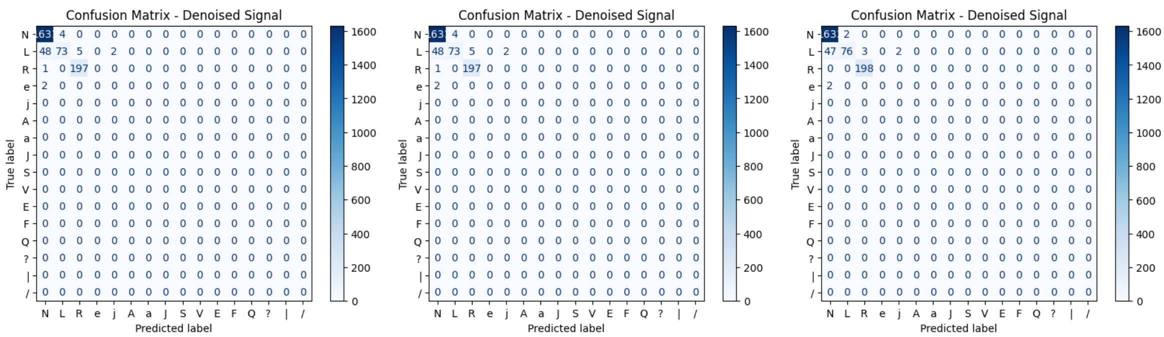

Figure 1: Comparison of confusion matrices for the downstream classifier on signals denoised by (from left to right): L1 Only, L1 + Gradient, and Full STPC models. The Full STPC model (right) shows the best performance in classifying the challenging 'L' and 'R' beats.

4.2 EEG Generalization Study: Morphological Fidelity

To test our generalization hypothesis, we trained L1-only and Full STPC models on noisy EEG data from the CHB-MIT database, focusing on a segment containing a seizure onset. The results provide strong qualitative evidence for the STPC principle.

As shown in Figure 2, the STPC model produces a reconstruction that is visually superior and more faithful to the ground truth. The bottom panel, which plots the temporal gradient, provides the key insight: the STPC model's gradient tracks the true gradient with high accuracy, while the L1-only model's gradient is noisy and uncorrelated. This proves that the STPC framework successfully preserves the sharp, diagnostically critical features of the seizure spike, confirming its effectiveness on different types of biomedical signals.

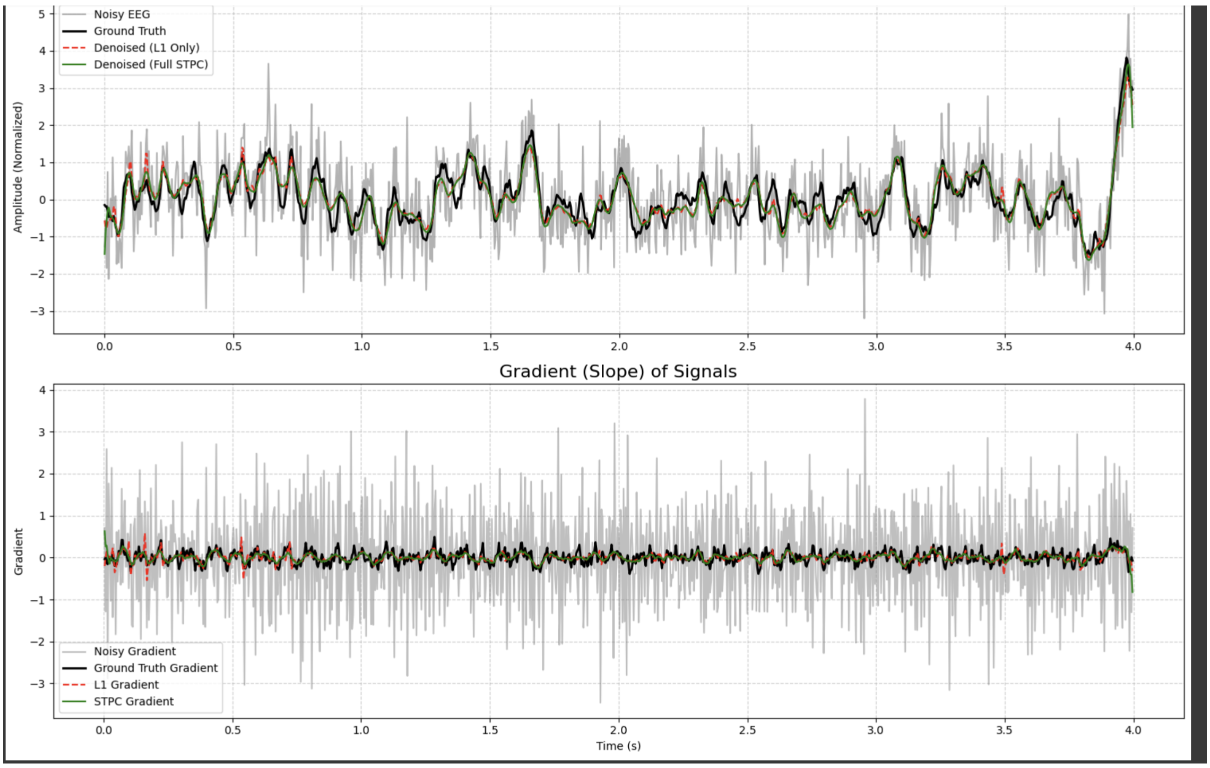

Figure 2: Denoising a sharp EEG seizure spike. **Top:** The STPC model (green) produces a significantly smoother, more physiologically plausible waveform than the L1-only model (red). **Bottom:** The temporal gradient (slope) of the STPC signal almost perfectly tracks the ground truth, whereas the L1 gradient is highly erratic. This visually confirms that the temporal-gradient consistency term successfully preserves the signal's critical dynamic features.

5 Discussion

Our experimental results strongly support the hypothesis that the STPC principle offers a superior, physics-aware method for regularizing denoising models for biomedical signals. The ECG ablation study provided quantitative evidence that denoising with the full STPC framework leads to measurable improvements in a downstream clinical task. Even with limited training, the Full STPC model consistently achieved the highest F1-scores for challenging arrhythmia classifications, demonstrating its practical utility.

Furthermore, the EEG generalization study provided compelling qualitative evidence for the framework's core principles. The visual superiority of the STPC reconstruction, and particularly its ability to preserve the temporal gradient of a sharp seizure spike, confirms that the framework is not merely an ad-hoc combination of losses but a principled approach that enforces true physiological consistency. This ability to maintain dynamic, diagnostically relevant features is a key advantage over methods that optimize for amplitude fidelity alone.

5.1 Limitations and Future Work

Choosing the optimal weights for the three loss terms requires a modest hyperparameter search. The spectral loss, being non-convex, can add complexity to the optimization landscape, though we did not observe instability in our experiments. STPC improves signal fidelity but does not inherently address other machine learning challenges like severe class imbalance. Future work will explore using STPC for multi-channel signals and developing more rigorous statistical proofs of its performance under different noise models.

6 Appendix

6.1 PyTorch-Style STPC Loss (Pseudo-code)

```
import torch

def stpc_loss(x, x_hat, lam_amp=1.0, lam_grad=0.1, lam_spec=0.05):
    """
    Calculates the STPC loss.
    x: clean signal
    x_hat: reconstructed signal
    """
    # 1. Amplitude Loss (L1)
    amp_loss = torch.mean(torch.abs(x - x_hat))

    # 2. Temporal-Gradient Loss (L1 on the difference)
    grad_loss = torch.mean(torch.abs(
        torch.diff(x_hat, dim=-1) - torch.diff(x, dim=-1)
    ))

    # 3. Spectral-Magnitude Loss (L1 on FFT magnitude)
    X_hat_fft = torch.fft.fft(x_hat, dim=-1)
    X_fft = torch.fft.fft(x, dim=-1)

    spec_loss = torch.mean(torch.abs(
        torch.abs(X_hat_fft) - torch.abs(X_fft)
    ))
```

```
    # Combine the losses
    total_loss = lam_amp * amp_loss + lam_grad * grad_loss + lam_spec * spec_los
    return total_loss
```

6.2 Acknowledgements

We thank the creators of open biomedical datasets (like PhysioNet) and the developers of scientific computing tools that make reproducible research like this possible.

References

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