CSC265 Assignment 3

Written by Kaiyang Wen, 1002123891, kai.wen@mail.utoronto.ca
Revised by Mohan Zhang, 1002748716, morgan.zhang@mail.utoronto.ca

1 Question 1.

a.
The hashing table tooks

b.
||| Let ADT = create (ADT, Δ) for some ADT and Δ. When the program executing line 4, we can surely create an ADT as we've previously specified in part a since there are only 2 elements (x_1 and x_2) and for that two elements, and $|x_1$ - $x_2|$ is greater than or equal to Δ by the definition of Δ.
Claim: after executed line 12, the elements in ADT are from $X_i = \{x_1, x_2, ..., x_i\}$, and every element in X_i are in ADT. The Δ for such ADT satisfied all the condition, And cp is the closet pair of points in the set X_i. Let cp = (m,n).
Prove by induction:
When i=3(the first loop):
If p==NIL, we insert x_3 into ADT, so the elements in ADT are from X_3, and every element in X_3 are in ADT. Since p==NIL, so $\forall$ y in X_2, $|x_3 - y| \geq \Delta$, so $\forall$ y_1, y_2 in X_3, $|y_1 - y_2| \geq \Delta$.
Else, since IsCloser(ADT, x_3) = p, then $\forall$ y_1, y_2 in X_2, $|y_1 - y_2| > |p - x_3|$. And, since $\Delta = |p - x_3|$, then $\forall$ y_1, y_2 in X_3, $|y_1 - y_2| \geq \Delta$. And, cp is the closet pair of points in the set X_3 since $\forall$ y_1, y_2 in X_3, $|y_1 - y_2| \geq |m - n|$.

Assume when i=k, the elements in ADT are from X_k, and every element in X_k are in ADT. $\forall$ y_1, y_2 in X_k, $|y_1 - y_2| \geq \Delta$. And for cp, $|m - n| = \Delta$. So, by the assumption, $\forall$ y_1, y_2 in X_k, $|y_1 - y_2| \geq |m - n|$, so cp is the closet pair of points in the set X_k. For i=k+1:
If p==NIL, we insert x_{k+1} into ADT, so from the previous assumption, the elements in ADT are from X_{k+1}, and every element in X_{k+1} are in ADT. Since p==NIL, so from the previous assumption, $\forall$ y in X_{k+1}, $|x_{k+1} - y| \geq \Delta$, so $\forall$ y_1, y_2 in X_{k+1}, $|y_1 - y_2| \geq \Delta$.
Else, since IsCloser(ADT, x_{k+1}) = p, then $\forall$ y_1, y_2 in X_{k+1}, $|y_1 - y_2| > |p - x_{k+1}|$. And, since $\Delta = |p - x_3|$, then $\forall$ y_1, y_2 in X_{k+1}, $|y_1 - y_2| \geq \Delta$. And, cp is the closet pair of points in the set X_{k+1} since $\forall$ y_1, y_2 in X_{k+1}, $|y_1 - y_2| \geq |m - n|$.
By induction, the claim is true. So, the algorithm works, and correctly outputs the closest pair of points in X = $\{x_1, x_2, ..., x_n\}$.

c.

d.
Among the program, all the operations except line 12 expected runs in constant time (from the definition of the creation, IsCloser and Insert on Y). so, the expected running time of ClosestPair only depends on how many times we execute line 12 for a certain i.

2 Question 2.

Lemma: Once Alice return 1 on a set T, Bob will finally discover an element of Alices set in O(log(size of T)) time, which is O(log(n)) time. Proof: When Alice return 1 on T, we can find that Using Faith

Ellen's algorithm in course lecture $https://www.youtube.com/watch?v = ILa6W3sCQJ4$, which takes O(log(n)) times.