MATHEMATICS

January 29, 2024

1. $y = (\sin^{-1} x) + (\cos^{-1} x)$,then find $\frac{dy}{dx}$
2. write the order and degree of the differential equation
   $(\frac{d^4y}{dx^4})^2 = (x + (\frac{dy}{dx})^2)^3$
3. if $*$ defined on the set $\mathbb{R}$ of all real numbers by $* : a * b = \sqrt{a^2 + b^2}$,find the identity element,if it exists in $\mathbb{R}$ with respect to $*$
4. if $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} KA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then find the values of k,a and b.
5. if $tan^{-1}x - cot^{-1}x = tan^{-1}\frac{1}{\sqrt{3}}$,$x > 0$,find the value of x and hence find the value of $sec^{-1}\left(()\frac{2}{x}\right)$
6. using the properties of determinants,prove that
   $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc$
7. if $y = \left(\sec^{-1} x\right)^2$,$x > 0$,show that $x^2\left(x^2 - 1\right)\frac{d^2y}{dx^2} + \left(2x^3 - x\right)\frac{dy}{dx} - 2 = 0$
8. find the equations of the tangent and the normal to the curve $y = \frac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.
9. Find $\int \frac{\sin 2x}{\left(\sin^2 x+1\left(\sin^2 x+3\right)\right)}$
10. Prove that

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

11. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}, dx$
12. let **a**,**b** and **c** be three vectors such that $\left|\overrightarrow{a}\right| = 1, \left|\overrightarrow{b}\right| = 2$ and $\left|\overrightarrow{c}\right| = 3$. if the projection of **b** along **a** is equal to the projection of **c** along **a**; and **b**,**c**,are perpendicular to each other, then find $\left|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}\right|$
13. find the values of λ for which the following lines are perpendicular to each other: $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$; $\frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$ hence,find whether the lines intersect or not
14. if $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, find A^{-1} hence solve the following system of equations: $x+y+z = 6, y+3z = 11 and x-2y+z = 0$
15. find the inverse of the following matrix,using elementary transformations: $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

16. show that the height of a cylinder,which is open at the top,having a given surface area and greatest volume ,is equal to the radius of its base

17. find the area of the traiagle whose vertices are $(-1, 1), (0, 5), (3, 2)$,using integration

18. find the area of the region bounded by the curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$,using integration

19. find the vector and cartesian equations of the plane passing through the points $(2, 5, -3), (-2, -3, 5)$ *and* $(5, 3, -3)$.also find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

20. find the equation of the plane passing through the intersection of the planes $\overrightarrow{r}.\left(\hat{i}+\hat{j}+\hat{k}\right) = 1$ and $\overrightarrow{r}.\left(2\hat{i}+3\hat{j}-\hat{k}\right) + 4 = 0$ and parallel to x-axis.Hence,find the distance of the plane from x-axis

21. There are two boxes I and II.Box I contains 3 red and 6 black balls.Box II contains 5 red and $'n'$ black balls.One of the two boxes,box I and box II is selected at random and a ball is drawn at random.The ball drawn is found to be red .If the probability that this red red ball comes out from box II is $\frac{3}{5}$,find the value of $'n'$.

22. A company manufactures two types of novelty souvenirs made of plywood.Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling.Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling.There are 3 hours and 20 minutes available for cutting and 4 hours available for assembling.The profit is₹50 each for type A and ₹60 each for type B souvenris.How many souvenris of each type should the company manufacture in order to maximize profit?Formuate the above LPP and solve it graphically and also find the maximum profit.