

# ASSIGNMENT

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IITH - Future Wireless Communications (FWC)

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### 1 QUESTION

(GATE CS-2021)

Q.7. Let p and q be two propositions. Consider the following two formulae in propositional logic.

$$S1: (\neg p \wedge (p \vee q)) \longrightarrow q$$

$$S2: q \longrightarrow (\neg p \wedge (p \vee q))$$

S1 and S2 expressions in terms of boolean logic

$$S1: p'(p+q) \longrightarrow q = (p'(p+q))' + q$$

$$S2: q \longrightarrow p'(p+q) = q' + p'(p+q)$$

Which one of the following choices is correct?

- 1) Both S1 and S2 are tautologies.
- 2) S1 is a tautology but S2 is not a tautology.
- 3) S1 is not a tautology but S2 is a tautology.
- 4) Neither S1 nor S2 is a tautology.

### 2 COMPONENTS

| Component   | Values  | Quantity |
|-------------|---------|----------|
| ArduinoUNO  |         | 1        |
| JumperWires | M-M     | 10       |
| Breadboard  |         | 1        |
| LED         |         | 2        |
| Resistor    | 220ohms | 2        |

Figure.a

### 3 FORMULAE

- 1 A tautology is a compound statement in Maths which always results in Truth(True) value.

- 1 Tautology formula:

$$p \longrightarrow q = \neg p \vee q$$

- 1 where:

- 1  $\neg$  = Not operation

$\wedge$  = and operation

- 2  $\vee$  = or operation

- 2

a) Therefore the two propositional logics S1 and S2 formulae are:

$$S1: (\neg p \wedge (p \vee q)) \longrightarrow q = \neg((\neg p \wedge (p \vee q))) \vee q$$

$$S2: q \longrightarrow (\neg p \wedge (p \vee q)) = \neg q \vee (\neg p \wedge (p \vee q))$$

### 4 TRUTH TABLE

TABLE I: Truth table for expression S1

| p     | q     | $\neg((\neg p \wedge (p \vee q))) \vee q$ |
|-------|-------|---|
| false | false | true                                      |
| false | true  | true                                      |
| true  | false | true                                      |
| true  | true  | true                                      |

From the above truth table it is seen that all the outputs are true. Therefore the expression "S1 is a tautology".

TABLE II: Truth table for expression S2

| p     | q     | $\neg q \vee (\neg p \wedge (p \vee q))$ |
|-------|-------|--|
| false | false | true                                     |
| false | true  | true                                     |
| true  | false | true                                     |
| true  | true  | false                                    |

From the above truth table it is seen that one of the output is false, to meet the tautology condition all

the outputs must be true. Therefore the expression "S2 is not a tautology".

## 5 BOOLEAN EXPRESSIONS

Consider the propositional logic S1:

Assume the variables p and q as A and B (as there are two expressions with same variables we assume one of the expression with variables A and B).

Therefore the expression S1 becomes as

$$S1: (\neg A \wedge (A \vee B)) \longrightarrow B = \neg((\neg A \wedge (A \vee B))) \vee B$$

The boolean expression for the propositional expression can be written as:

$$S1: A'(A + B) \longrightarrow B = (A'(A + B))' + B$$

TABLE III: Logical Truth table for expression s1

| A | B | $(A'(A+B))' + B$ |
|---|---|------------------|
| 0 | 0 | 1                |
| 0 | 1 | 1                |
| 1 | 0 | 1                |
| 1 | 1 | 1                |

Consider the propositional logic S2:

S2:  $q \longrightarrow (\neg p \wedge (p \vee q)) = \neg q \vee (\neg p \wedge (p \vee q))$  The boolean expression for the propositional expression can be written as

$$S2: q \longrightarrow p'(p + q) = q' + p'(p + q)$$

TABLE IV: Logical truth table for expression S2

| p | q | $q' + p'(p+q)$ |
|---|---|----------------|
| 0 | 0 | 1              |
| 0 | 1 | 1              |
| 1 | 0 | 1              |
| 1 | 1 | 0              |

## 6 IMPLEMENTATION

TABLE V: connections

| Arduino pin | INPUT | OUTPUT |
|-------------|-------|--------|
| 2           | A     |        |
| 3           | B     |        |
| 4           | p     |        |
| 5           | q     |        |
| 6           |       | C      |
| 8           |       | R      |

### a) Procedure

1. Connect the circuit as per the above table.
2. Connect the Output pins C and R to the LED's.
3. Connect the other end of the LED's to the Ground terminal.
4. Connect inputs to Vcc for logic 1, ground for logic 0.
5. Execute the circuits using the below code.

<https://github.com/Mohan200305/ide/blob/main/code/cs.cpp>

6. Change the values of A,B,P,Q in the code and verify the Truth tables respectively.

Answer is : **Option2 : S1 is a tautology but S2 is not a tautology.**