

CHAPTER-9
TRIANGLES

1 Exercise 11.2

Question(4).Construct a triangle XYZ in which $\angle Y = 30^\circ, \angle Z = 90^\circ$ and $XY + YZ + ZX = 11cm$.

Solution:

Let \mathbf{X}, \mathbf{Y} and \mathbf{Z} are the vertices of the triangle with coordinates. Given $XY + YZ + ZX = 8cm$.So the coordinate of the vertex \mathbf{X} is:

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (1)$$

Also given $\angle Y = 30^\circ$ and $\angle Z = 90^\circ$ so by finding the length of sides we can form a required triangle.

The input parameters for this construction are

Symbol	Value	Description
$c + a + b$	11	$XY + YZ + ZX$
$\angle Y$	30°	$\angle Y$ in $\triangle ABC$
$\angle Z$	90°	$\angle Z$ in $\triangle XYZ$
\mathbf{e}_1	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	Basis vector

Table 1: Parameters

From the given information

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{k} \quad (2)$$

$$\mathbf{b} \cos \mathbf{Z} + \mathbf{c} \cos \mathbf{Y} - \mathbf{a} = 0 \quad (3)$$

$$\mathbf{b} \sin \mathbf{Z} - \mathbf{c} \sin \mathbf{Y} = 0 \quad (4)$$

Resulting in the matrix equations:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos \mathbf{Z} & \cos \mathbf{Y} \\ 0 & \sin \mathbf{Z} & -\sin \mathbf{Y} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \mathbf{k} \mathbf{e}_1 \quad (5)$$

Substituting the values of $\mathbf{k}, \mathbf{e}_1, \angle Y, \angle Z$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos 90^\circ & \cos 30^\circ \\ 0 & \sin 90^\circ & -\sin 30^\circ \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = 11 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad (6)$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & \sqrt{3}/2 \\ 0 & 1 & -1/2 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \\ 0 \end{pmatrix} \quad (7)$$

Using row reduction methods to bring the values of $\mathbf{a}, \mathbf{b}, \mathbf{c}$ into row-reduced echelon form,

$$\xrightarrow{R_2 \rightarrow R_2 + R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{\sqrt{3}}{2} + 1 \\ 0 & 1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 11 \\ 11 \\ 0 \end{pmatrix} \quad (8)$$

$$\xrightarrow{R_1 \rightarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & \frac{\sqrt{3}}{2} + 1 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 0 \\ 11 \\ 0 \end{pmatrix} \quad (9)$$

$$\xrightarrow{R_3 \rightarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & \frac{\sqrt{3}}{2} + 1 \\ 0 & 0 & \frac{-\sqrt{3}-3}{2} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 0 \\ 11 \\ -11 \end{pmatrix} \quad (10)$$

$$\xrightarrow{R_3 \rightarrow \frac{2}{-\sqrt{3}-3} R_3} \begin{pmatrix} 1 & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & \frac{\sqrt{3}}{2} + 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 0 \\ 11 \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \quad (11)$$

$$\xrightarrow{R_1 \rightarrow R_1 + \frac{\sqrt{3}}{2} R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{\sqrt{3}}{2} + 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} \frac{11\sqrt{3}}{3+\sqrt{3}} \\ 11 \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \quad (12)$$

$$\xrightarrow{R_2 \rightarrow R_2 - (\frac{\sqrt{3}}{2} + 1) R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} \frac{11\sqrt{3}}{3+\sqrt{3}} \\ 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \quad (13)$$

After reduction the values of \mathbf{a} , \mathbf{b} , \mathbf{c} are:

$$\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} \frac{11\sqrt{3}}{3+\sqrt{3}} \\ 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \quad (14)$$

Therefore the coordinates of the vertices are:

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (15)$$

$$\mathbf{Z} = \begin{pmatrix} \mathbf{b} \\ 0 \end{pmatrix} = \begin{pmatrix} 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ 0 \end{pmatrix} \quad (16)$$

$$\mathbf{Y} = \begin{pmatrix} \mathbf{b} \\ \mathbf{a} \end{pmatrix} = \begin{pmatrix} 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ \frac{11\sqrt{3}}{3+\sqrt{3}} \end{pmatrix} \quad (17)$$

Construction:

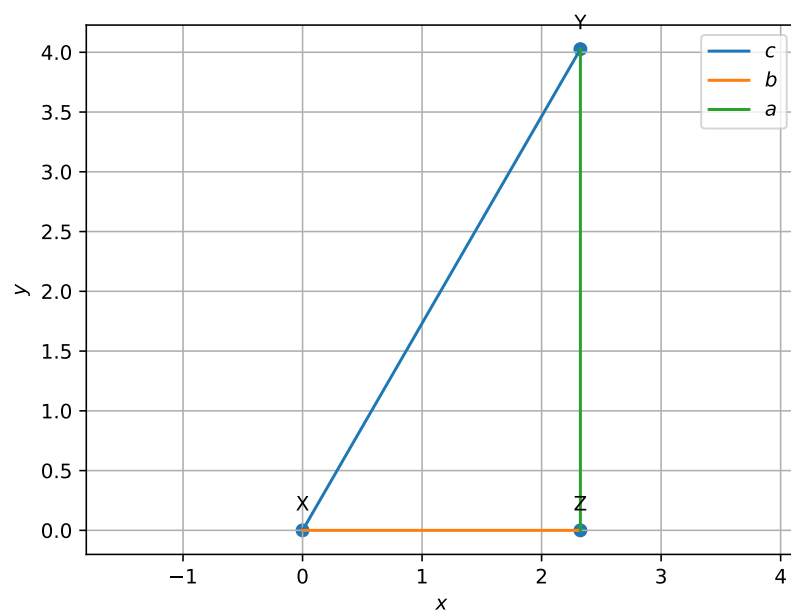


Figure 1: Triangle XYZ