CHAPTER-9
TRIANGLES

1 Exercise 11.4

Question(4).Construct a triangle XYZ in which $\angle Y = 30^0$,$\angle Z = 90^0$ and $XY + YZ + ZX = 11cm$.

Solution:
Let **X**,**Y** and **Z** are the vertices of the triangle with coordinates. Given $XY + YZ + ZX = 8cm$.So the coordinate of the vertice **X** is:

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Also given $\angle Y = 30^0$ and $\angle Z = 90^0$ so by finding the length of sides we can form a required triangle.
The input parameters for this construction are

Symbol	Value	Description
$c + a + b$	11	$XY + YZ + ZX$
$\angle Y$	$30°$	$\angle Y$ in $\triangle ABC$
$\angle Z$	$90°$	$\angle Z$ in $\triangle XYZ$
$\mathbf{e_1}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	Basis vector

Table 1: Parameters

From the given information

$$a + b + c = k \tag{1}$$
$$b\cos Z + c\cos Y - a = 0 \tag{2}$$
$$b\sin Z - c\sin Y = 0 \tag{3}$$

Resulting in the matrix equations:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos Z & \cos Y \\ 0 & \sin Z & -\sin Y \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = ke1 \tag{4}$$

Substitute the values of k,$\mathbf{e_1}$,$\angle Y$ and $\angle Z$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos 90^\circ & \cos 30^\circ \\ 0 & \sin 90^\circ & -\sin 30^\circ \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 11 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & \sqrt{3}/2 \\ 0 & & -1/2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 11 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

From the above matrix we get the equations as:

$$a + b + c = 11 \tag{7}$$
$$-a + c\sqrt{3}/2 = 0 \tag{8}$$
$$b - c/2 = 0 \tag{9}$$

From the equations 8 and 9 we get:

$$a = c\sqrt{3}/2 \tag{10}$$
$$b = c/2 \tag{11}$$

Substitute the above equations in equation 6,we get the value of c:

$$c = \frac{22}{3 + \sqrt{3}} \tag{12}$$
$$c = 4.65 \tag{13}$$

Therefore we get the values of a and b :

$$a = 4.03 \tag{14}$$
$$b = 2.32 \tag{15}$$

Therefore the coodinates of the vertices are:

$$X = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{16}$$
$$Z = \begin{pmatrix} c/2 \\ 0 \end{pmatrix} = \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{pmatrix} 2.32 \\ 0 \end{pmatrix} \tag{17}$$
$$Y = \begin{pmatrix} c/2 \\ c\sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} 2.32 \\ 4.03 \end{pmatrix} \tag{18}$$

Construction:

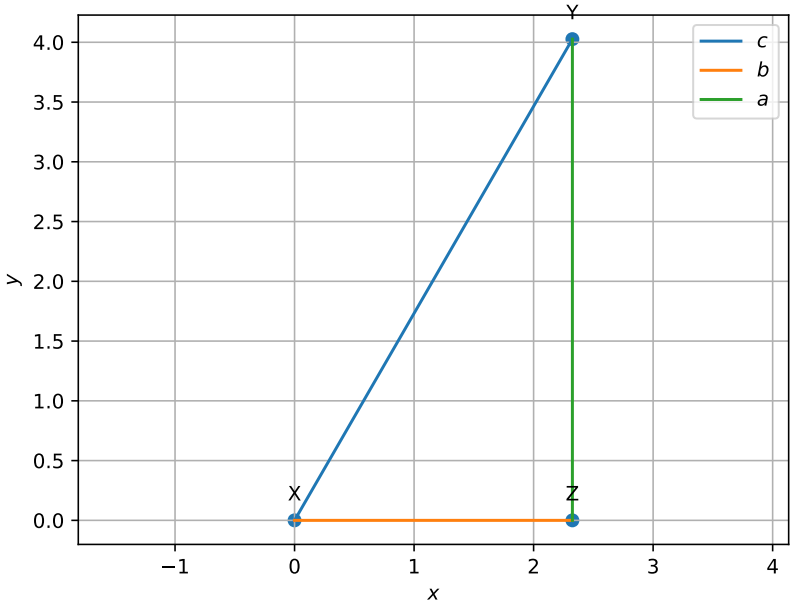

Figure 1: Triangle XYZ