CHAPTER-9
TRIANGLES

1 Exercise 11.4

Question(4).Construct a triangle XYZ in which $\angle Y = 30°$,$\angle Z = 90°$ and $XY + YZ + ZX = 11cm$.
Solution:
Let $\mathbf{X}$,$\mathbf{Y}$ and $\mathbf{Z}$ are the vertices of the triangle with coordinates. Given $XY + YZ + ZX = 8cm$.So the coordinate of the vertice $\mathbf{X}$ is:

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

Also given $\angle Y = 30°$ and $\angle Z = 90°$ so by finding the length of sides we can form a required triangle.
The input parameters for this construction are

Symbol	Value	Description
$c + a + b$	11	$XY + YZ + ZX$
$\angle Y$	30°	$\angle Y$ in $\triangle ABC$
$\angle Z$	90°	$\angle Z$ in $\triangle XYZ$
$\mathbf{e_1}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	Basis vector

Table 1: Parameters

From the given information

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{k} \tag{2}$$

$$\mathbf{b}\cos\mathbf{Z} + \mathbf{c}\cos\mathbf{Y} - \mathbf{a} = 0 \tag{3}$$

$$\mathbf{b}\sin\mathbf{Z} - \mathbf{c}\sin\mathbf{Y} = 0 \tag{4}$$

Resulting in the matrix equations:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos\mathbf{Z} & \cos\mathbf{Y} \\ 0 & \sin\mathbf{Z} & -\sin\mathbf{Y} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \mathbf{k}\mathbf{e_1} \tag{5}$$

Substituting the values of $\mathbf{k}$,$\mathbf{e_1}$,$\angle Y$,$\angle Z$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & \cos 90° & \cos 30° \\ 0 & \sin 90° & -\sin 30° \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = 11 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & \sqrt{3}/2 \\ 0 & 1 & -1/2 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

Using row reduction methods to bring the values of **a**,**b**,**c** into row-reduced echelon form,

$$\xrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{\sqrt{3}}{2}+1 \\ 0 & 1 & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 11 \\ 11 \\ 0 \end{pmatrix} \tag{8}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & \frac{\sqrt{3}}{2}+1 \\ 0 & 0 & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 0 \\ 11 \\ 0 \end{pmatrix} \tag{9}$$

$$\xrightarrow{R_3 \to R_3 - R_2} \begin{pmatrix} 1 & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & \frac{\sqrt{3}}{2}+1 \\ 0 & 0 & \frac{-\sqrt{3}-3}{2} \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 0 \\ 11 \\ -11 \end{pmatrix} \tag{10}$$

$$\xrightarrow{R_3 \to \frac{2}{-\sqrt{3}-3} R_3} \begin{pmatrix} 1 & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & \frac{\sqrt{3}}{2}+1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} 0 \\ 11 \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \tag{11}$$

$$\xrightarrow{R_1 \to R_1 + \frac{\sqrt{3}}{2} R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{\sqrt{3}}{2}+1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} \frac{11\sqrt{3}}{3+\sqrt{3}} \\ 11 \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \tag{12}$$

$$\xrightarrow{R_2 \to R_2 - (\frac{\sqrt{3}}{2}+1) R_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} \frac{11\sqrt{3}}{3+\sqrt{3}} \\ 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \tag{13}$$

After reduction the values of **a** , **b** , **c** are:

$$\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \\ \mathbf{c} \end{pmatrix} = \begin{pmatrix} \frac{11\sqrt{3}}{3+\sqrt{3}} \\ 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ \frac{22}{3+\sqrt{3}} \end{pmatrix} \tag{14}$$

Therefore the coordinates of the vertices are:

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{15}$$

$$\mathbf{Z} = \begin{pmatrix} \mathbf{b} \\ 0 \end{pmatrix} = \begin{pmatrix} 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ 0 \end{pmatrix} \tag{16}$$

$$\mathbf{Y} = \begin{pmatrix} \mathbf{b} \\ \mathbf{a} \end{pmatrix} = \begin{pmatrix} 11(1 - \frac{\sqrt{3}+2}{\sqrt{3}+3}) \\ \frac{11\sqrt{3}}{3+\sqrt{3}} \end{pmatrix} \tag{17}$$

Construction:

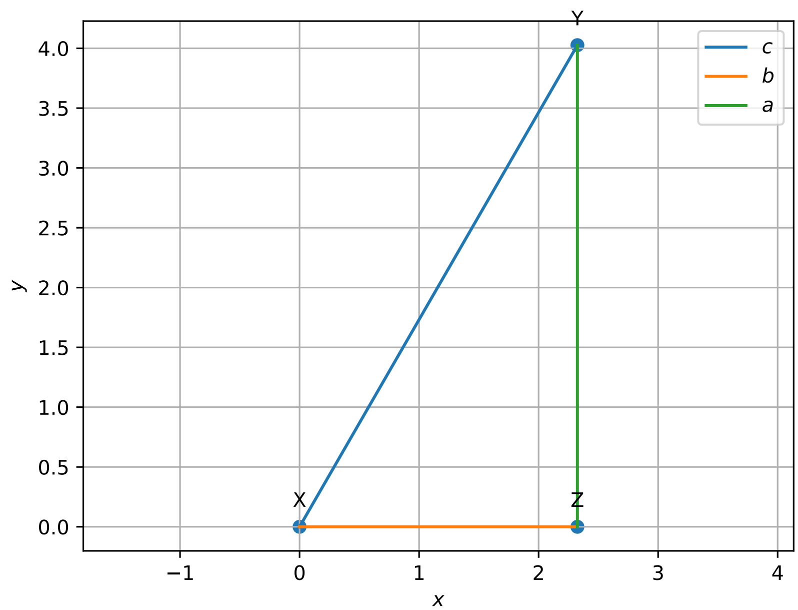

Figure 1: Triangle XYZ