Chapter 1

Introduction

“One picture is worth more than ten thousand words”

The King of Jungle has two eyes……………
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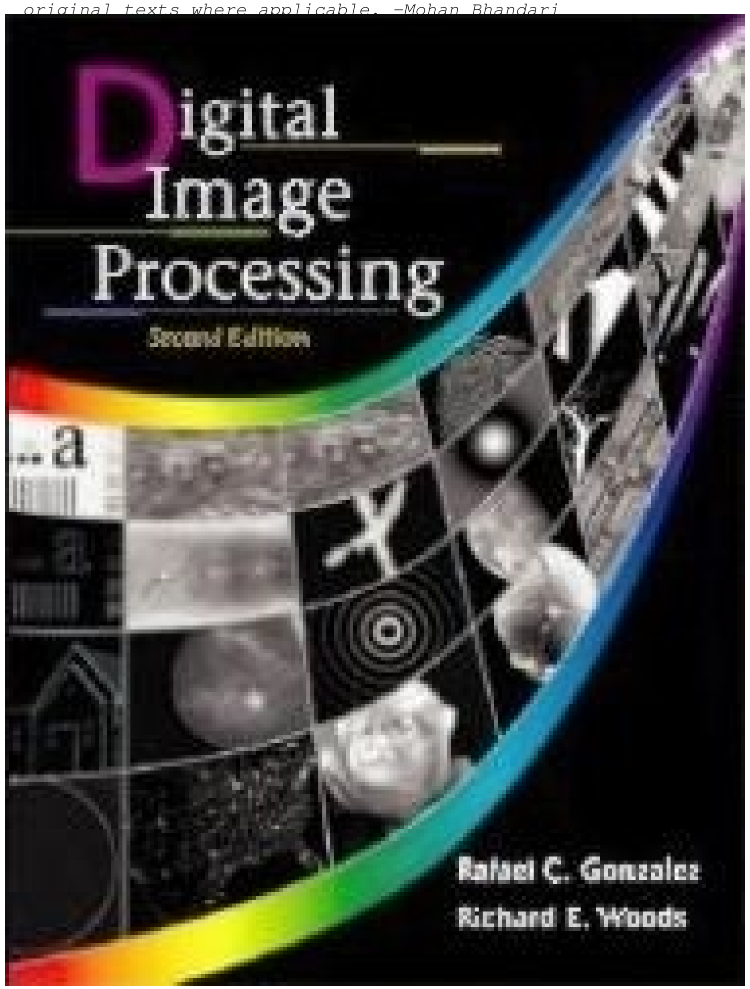
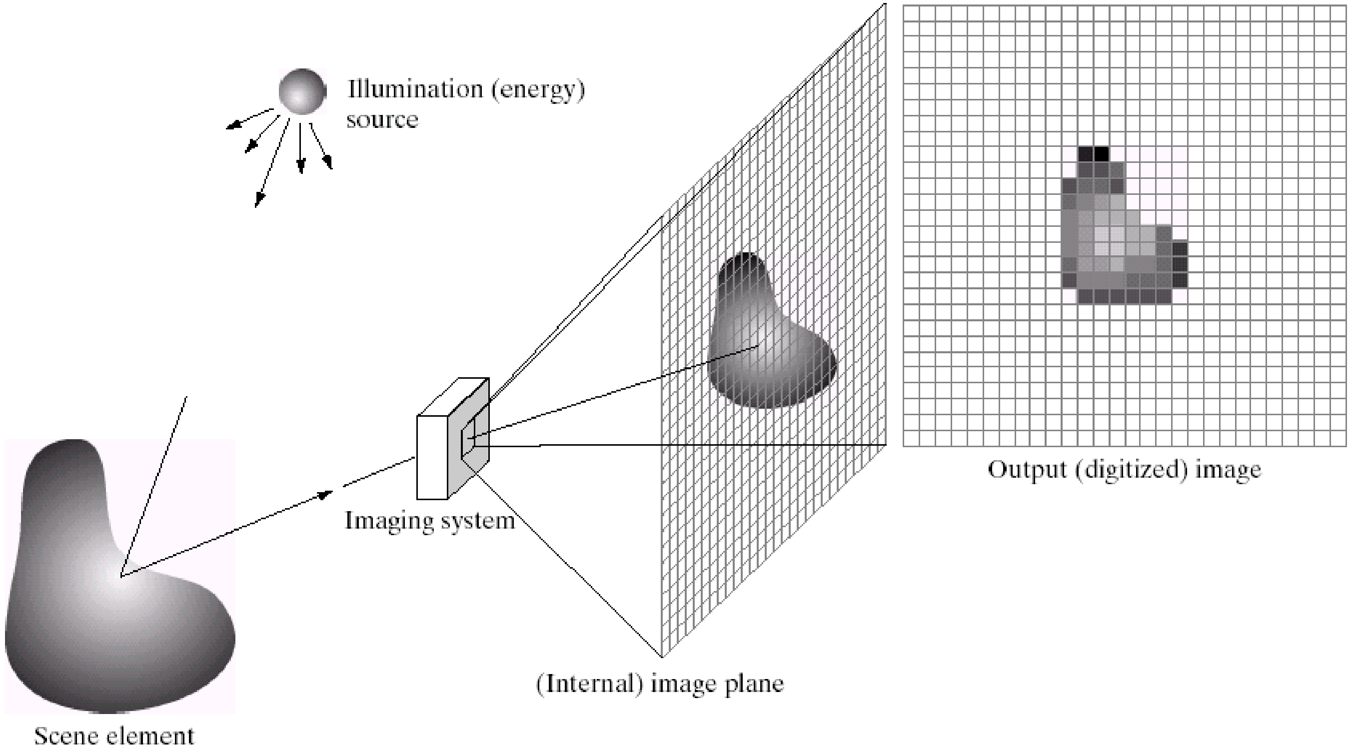
References



"Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002

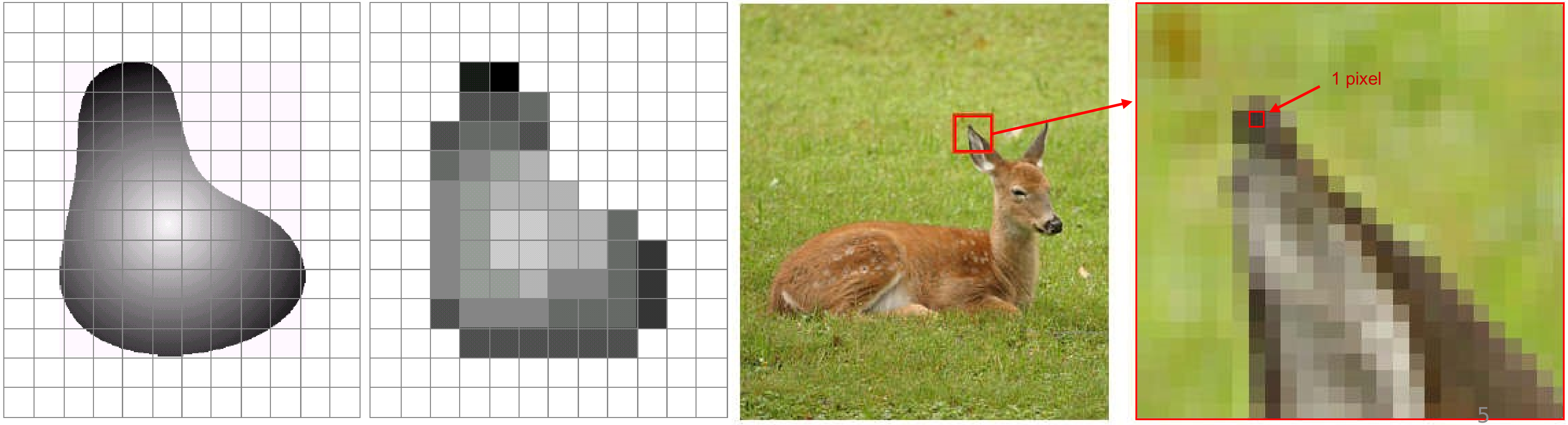
What is a Digital Image?

A **digital image** is a representation of a two-dimensional image as a finite set of digital values, called picture elements or pixels



Pixel values typically represent gray levels, colors, heights, opacities etc.

Remember *digitization* implies that a digital image is an *approximation* of a real scene



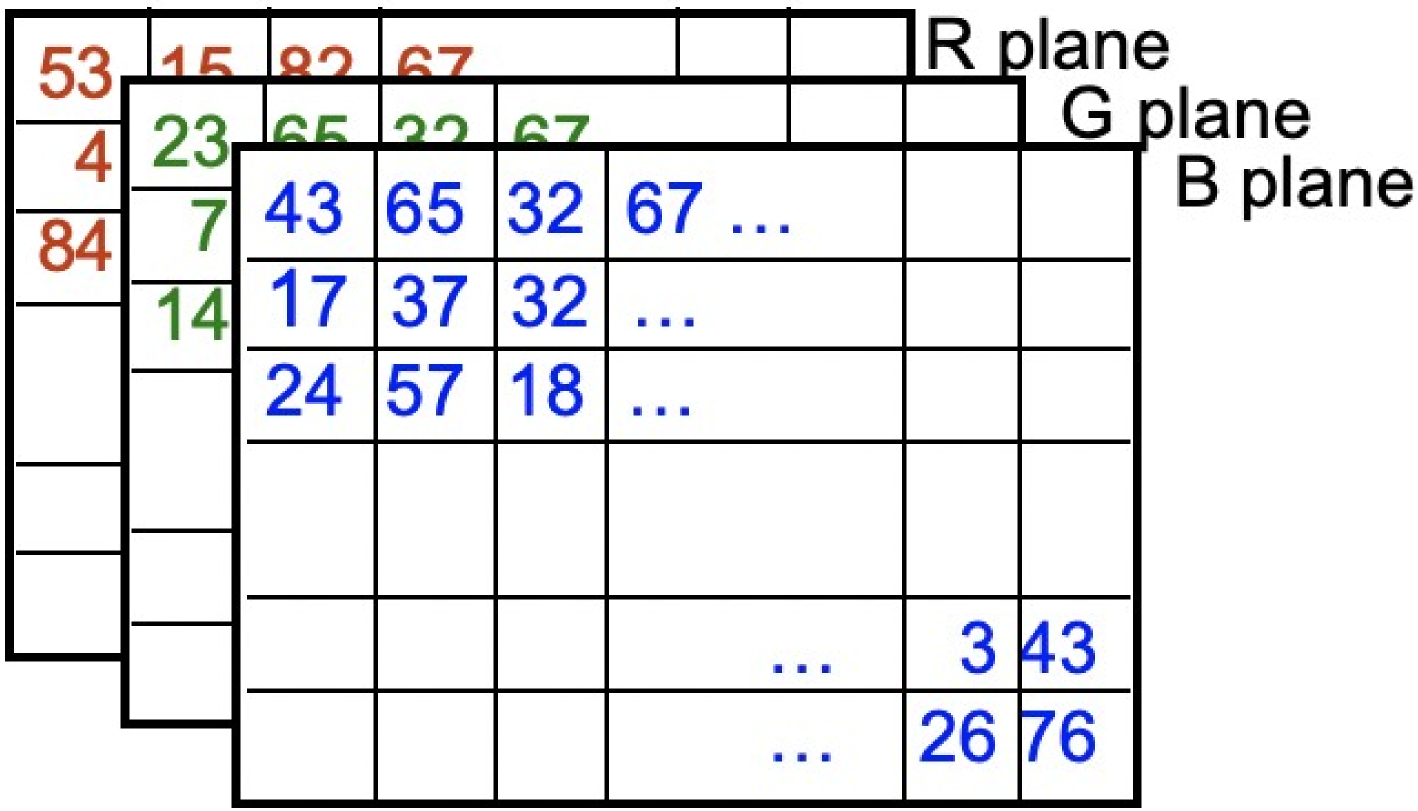
What is a Digital Image?

Common image formats include:

- 1 sample per point (B&W or Grayscale)
- 3 samples per point (Red, Green, and Blue)
- 4 samples per point (Red, Green, Blue, and "Alpha", (a.k.a. Opacity)

For most of this course we will focus on grey-scale images

What is a Digital Image?



What is Digital Image Processing?

- Processing of digital image for feature extraction is called digital image processing
- Digital image processing focuses on two major tasks
  - Improvement of pictorial information for human interpretation
  - Processing of image data for storage, transmission and representation for autonomous machine perception

What is DIP?

The continuum from image processing to computer vision can be broken up into low-, mid- and high-level processes

Low Level Process	Mid Level Process	High Level Process
Input: Image **Output:** Image	**Input:** Image **Output:** Attributes	**Input:** Attributes **Output:** Understanding
Examples: Noise removal, image sharpening	**Examples:** segmentation	**Examples:** Scene understanding, autonomous navigation

History of Digital Image Processing

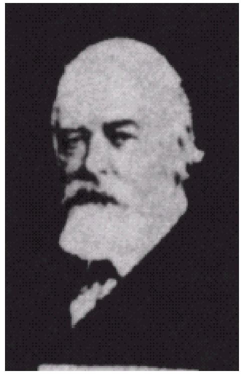
Early 1920s: One of the first applications of digital imaging was in the news-
paper industry

– The Bartlane cable picture transmission service
– Images were transferred by submarine cable between London and New York
– Pictures were coded for cable transfer and reconstructed at the receiving end on a telegraph printer

Early digital image

Mid to late 1920s: Improvements to the Bartlane system resulted in higher quality images

- New reproduction processes based on photographic techniques
- Increased number of tones in reproduced images

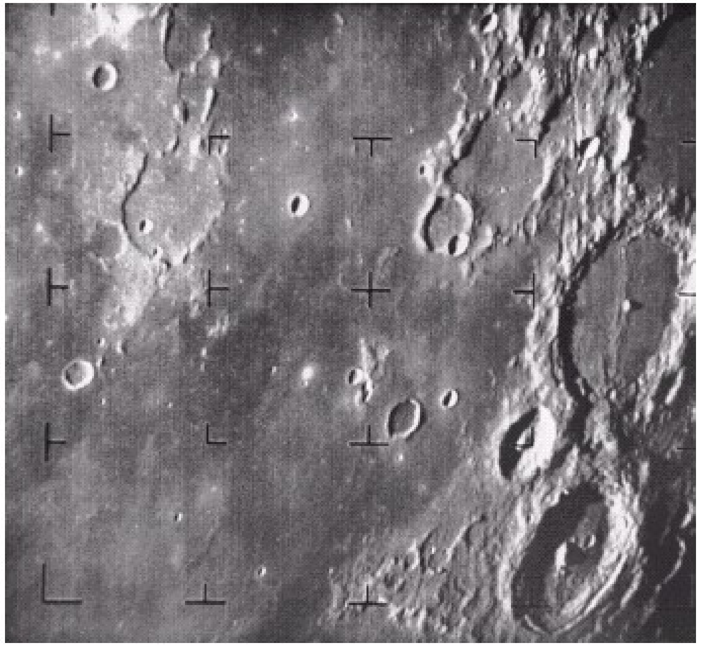


Improved digital image

Early 15 tone digital image

1960s: Improvements in computing technology and the onset of the space race led to a surge of work in digital image processing

- **1964:** Computers used to improve the quality of images of the moon taken by the *Ranger 7* probe
- Such techniques were used in other space missions including the Apollo landings



A picture of the moon taken by the Ranger 7 probe minutes before landing

History of DIP (cont…)

1970s: Digital image processing begins to be used in medical applications

- **1979:** Sir Godfrey N. Hounsfield & Prof. Allan M. Cormack share the Nobel Prize in medicine for the invention of tomography, the technology behind Computerised Axial Tomography (CAT) scans

Typical head slice CAT image

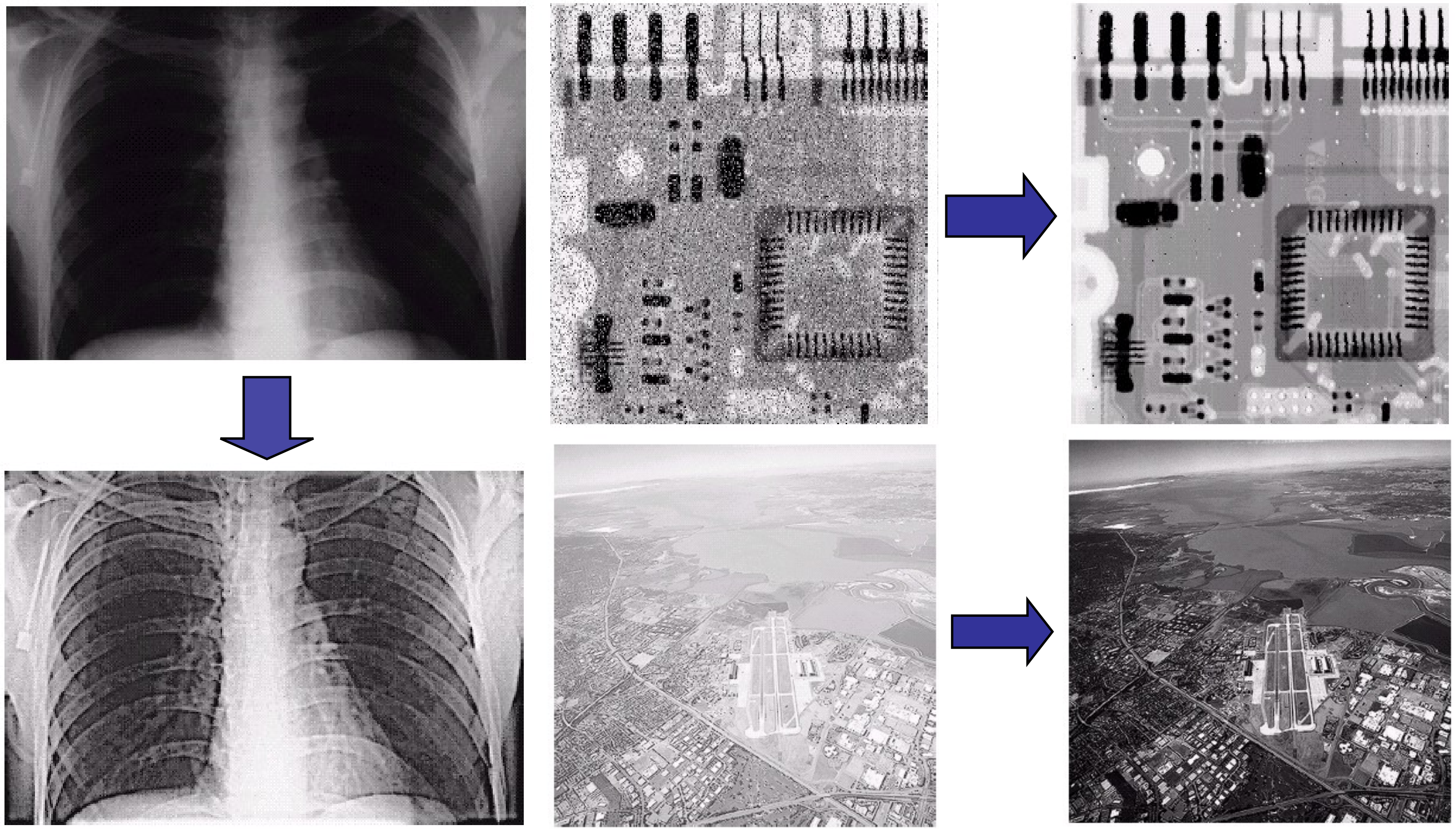
History of DIP (cont…)

1980s - Today: The use of digital image processing techniques has exploded and they are now used for all kinds of tasks in all kinds of areas

- Image enhancement/restoration
- Artistic effects
- Medical visualisation
- Industrial inspection
- Law enforcement
- Human computer interfaces

Examples: Image Enhancement

One of the most common uses of DIP techniques: improve quality, remove noise etc



Examples: The Hubble Telescope

Launched in 1990 the Hubble telescope can take images of very distant objects

However, an incorrect mirror made many of Hubble's images useless

Image processing techniques were used to fix this

Wide Field Planetary Camera 1

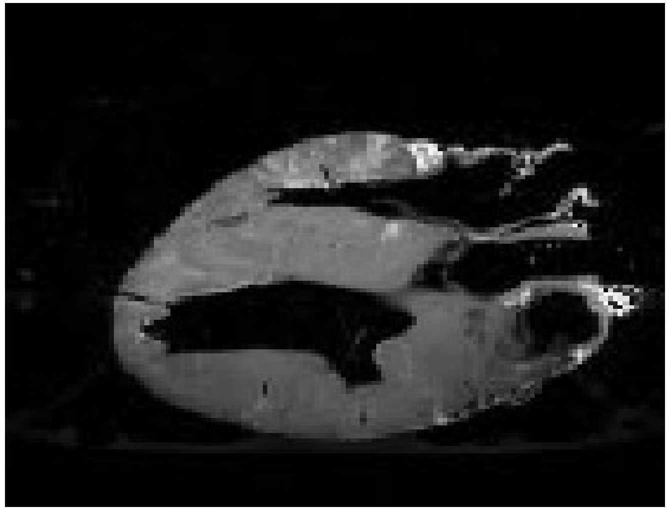
Wide Field Planetary Camera 2

Examples: Artistic Effects

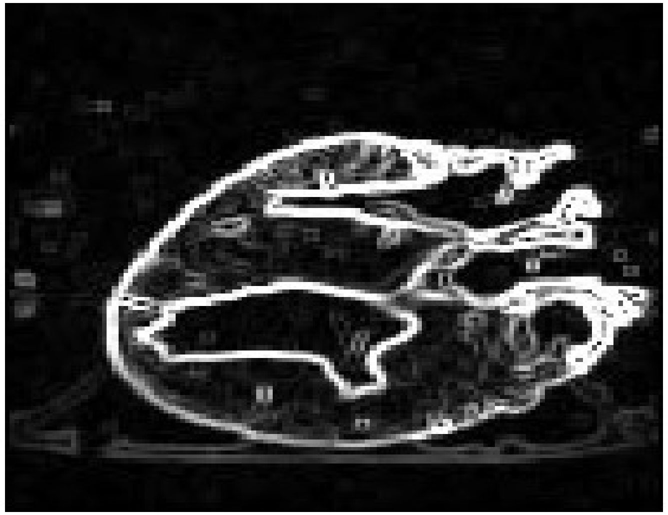
Artistic effects are used to make images more visually appealing, to add special effects and to make composite images

Examples: Medicine

- Take slice from MRI scan of canine heart, and find boundaries between types of tissue
  - Image with gray levels representing tissue density
  - Use a suitable filter to highlight edges



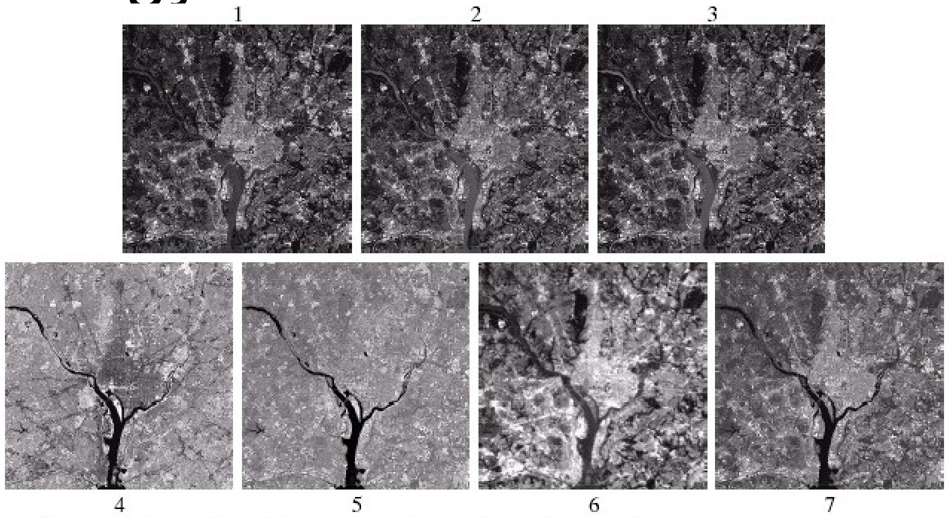
Original MRI Image of a Dog Heart

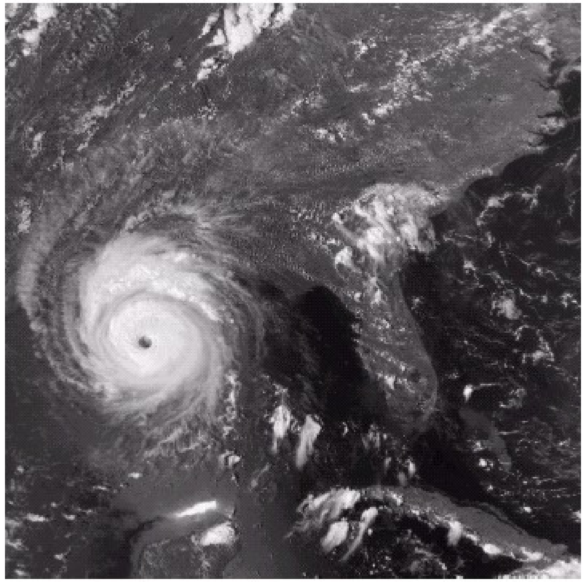


Edge Detection Image

Geographic Information Systems

- Digital image processing techniques are used extensively to manipulate satellite imagery
- Terrain classification
- Meteorology





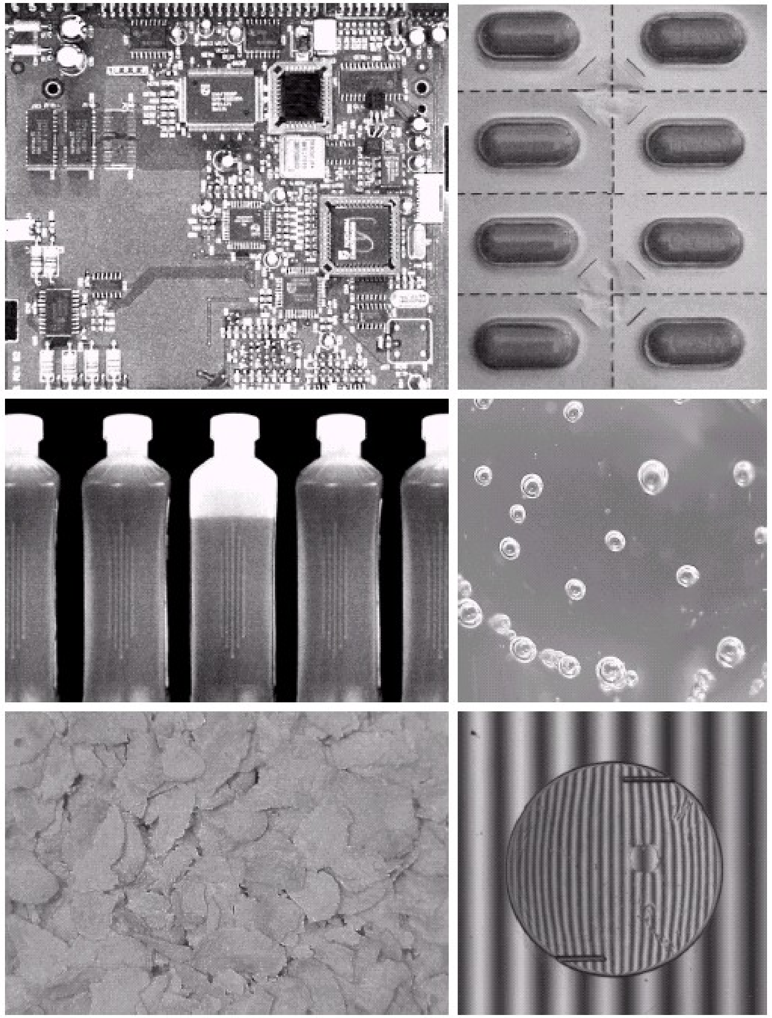
Examples: GIS (cont…)

Night-Time Lights of the World data set

- Global inventory of human settlement
- Not hard to imagine the kind of analysis that might be done using this data

Examples: Industrial Inspection

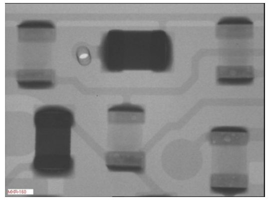
- Human operators are expensive, slow and unreliable
- Make machines do the job instead
- Industrial vision systems are used in all kinds of industries

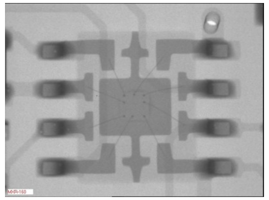


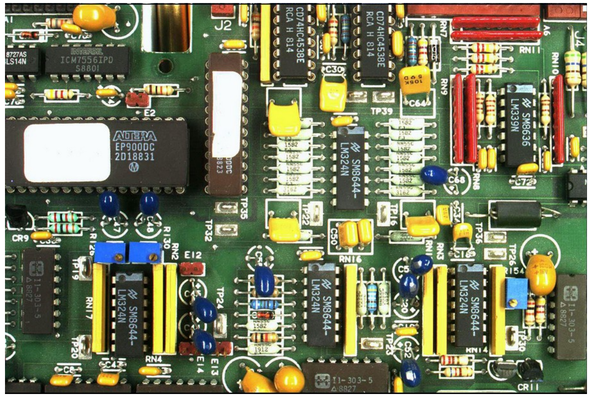
Examples: PCB Inspection

Printed Circuit Board (PCB) inspection

- Machine inspection is used to determine that all components are present and that all solder joints are acceptable
- Both conventional imaging and x-ray imaging are used



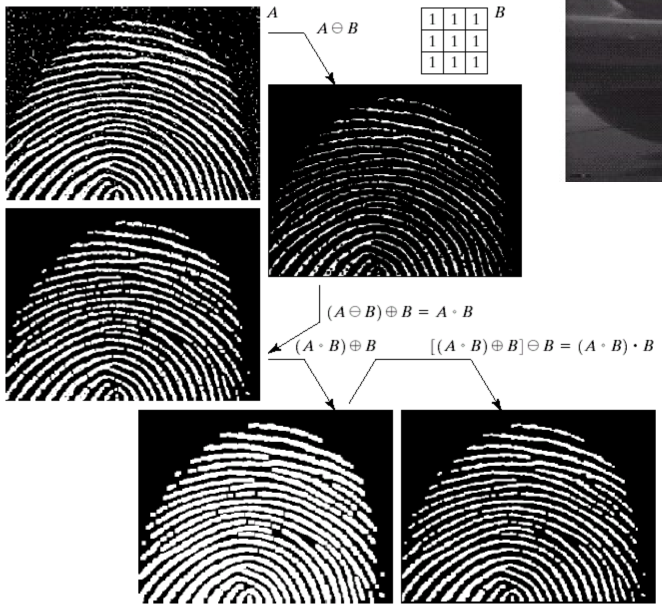




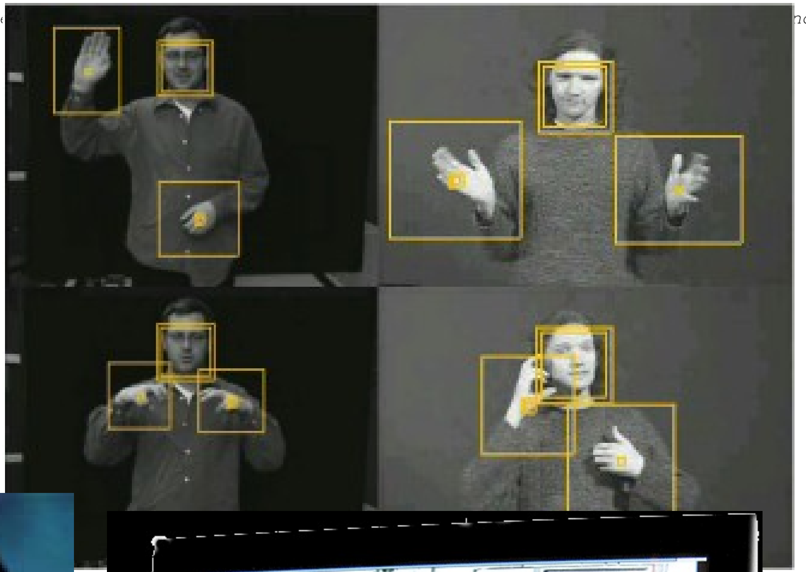
Examples: Law Enforcement

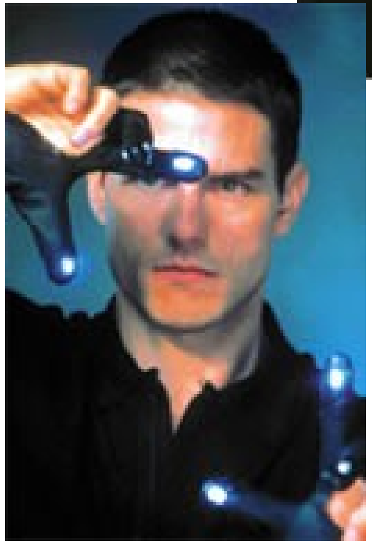
Image processing techniques are used extensively by law enforcers

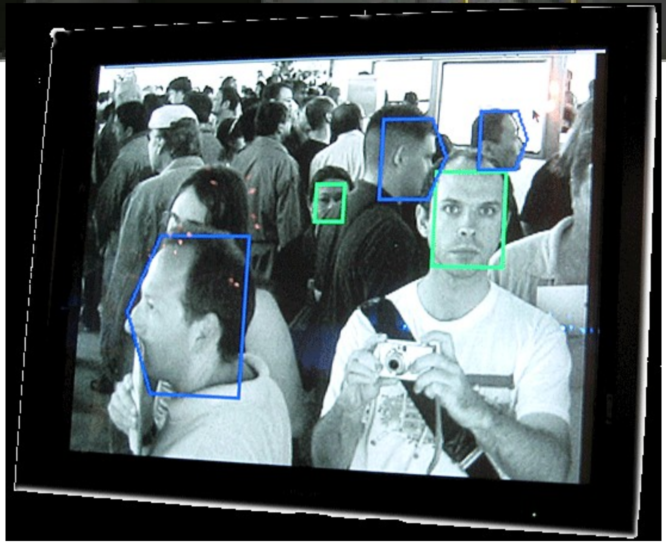
- Number plate recognition for speed cameras/automated toll systems
- Fingerprint recognition
- Enhancement of CCTV images



Examples: HCI







Try to make human computer interfaces more natural

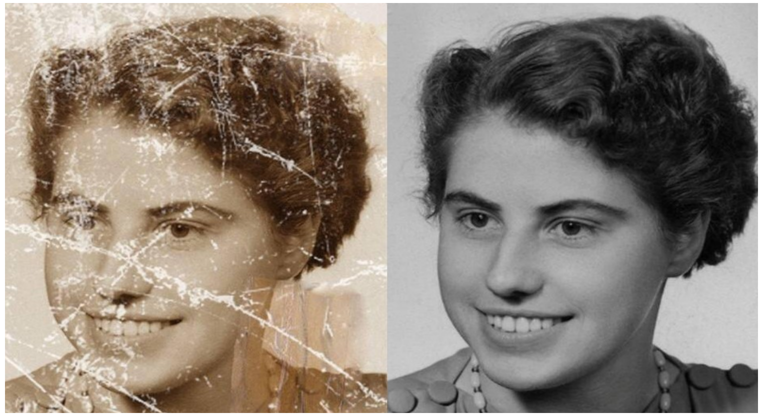
- Face recognition
- Gesture recognition

Does anyone remember the user interface from “Minority Report”?

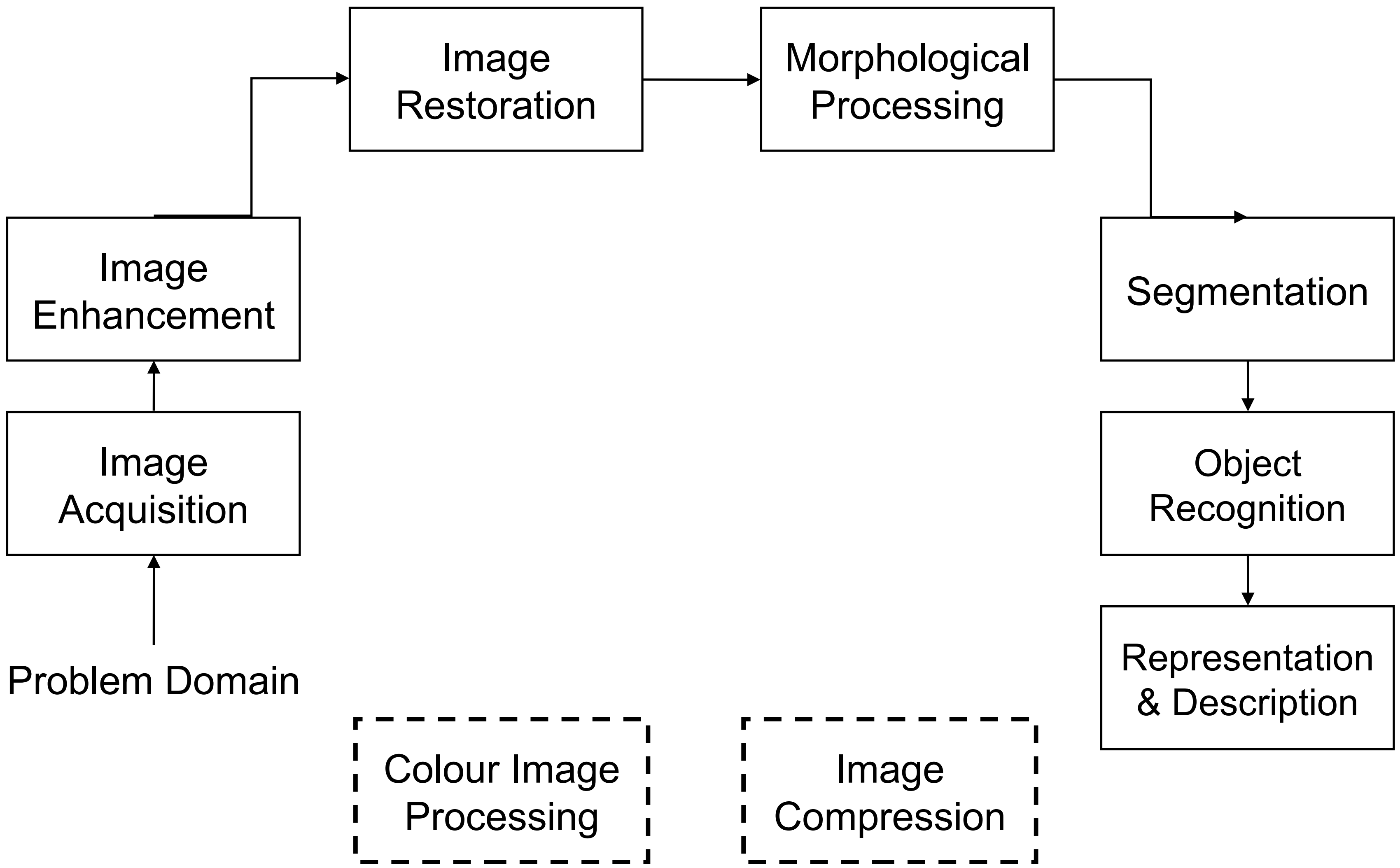
These tasks can be extremely difficult

Assignment

Explain the application areas of Image Processing



Key Stages in Digital Image Processing



Purpose of Digital Image Processing

- Visualization - Observe the objects that are not visible.
- Image sharpening and restoration - To create a better image.
- Image retrieval - Seek for the image of interest.
- Measurement of pattern – Measures various objects in an image.
- Image Recognition – Distinguish the objects in an image.

Key Stages in Digital Image Processing: Image Aquisition

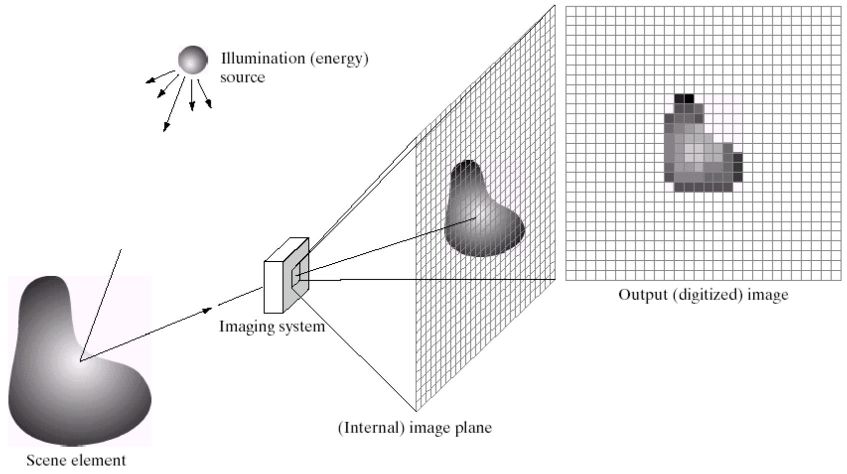
Image Restoration

Morphological Processing

Image Enhancement

Image Acquisition

Problem Domain



Segmentation

Object Recognition

Representation & Description

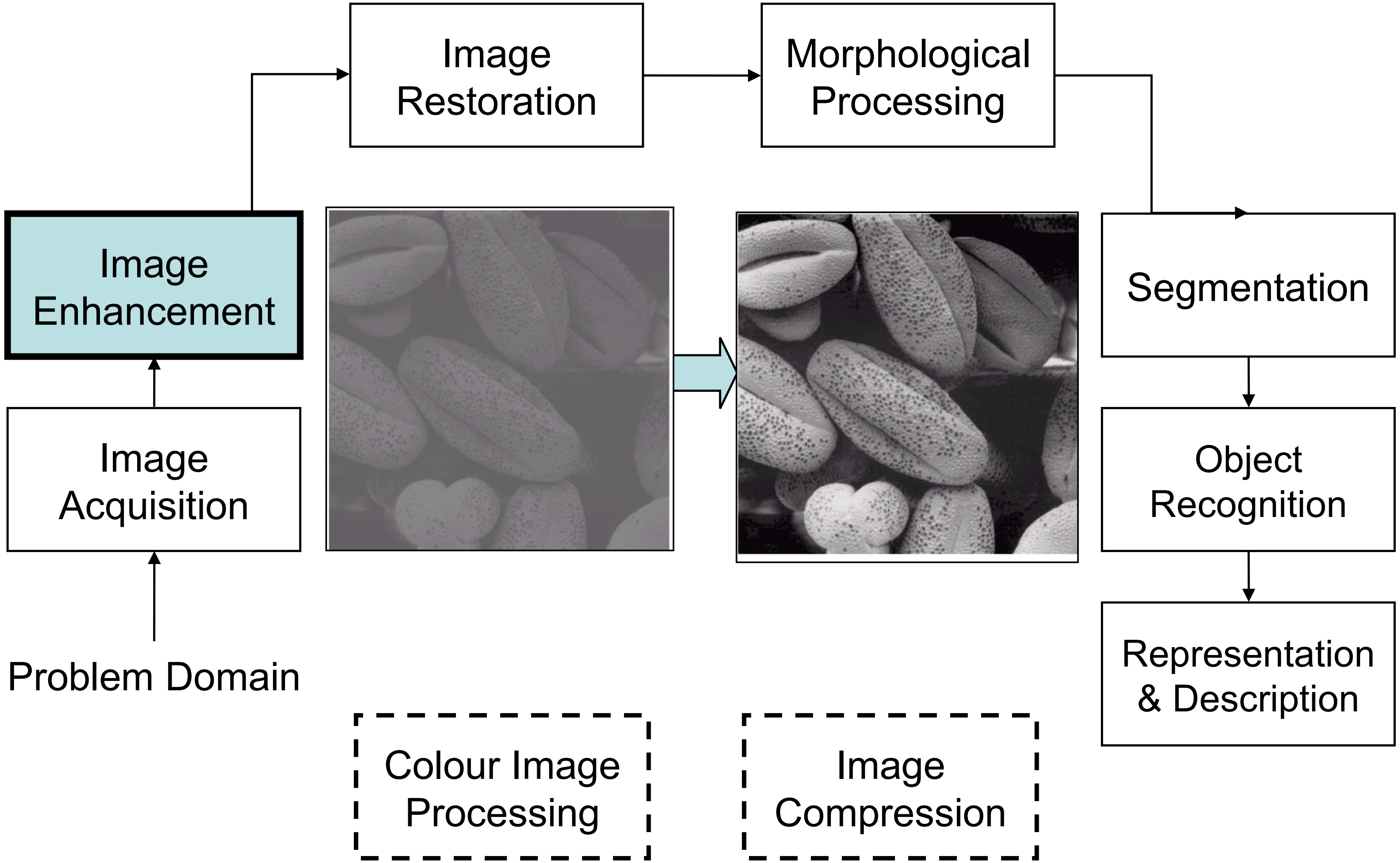
Colour Image Processing

Image Compression

Key Stages in Digital Image Processing: Image Aquisition

- Acquire a digital image using an image sensor
  - a monochrome or color TV camera: produces an entire image of the problem domain every 1/30 second
  - a line-scan camera: produces a single image line at a time, motion past the camera produces a 2-dimensional image
- If not digital, an analog-to-digital conversion process is required
- The nature of the image sensor (and the produced image) are determined by the application
  - Mail reading applications rely greatly on line-scan cameras
  - CCD and CMOS imaging sensors are very common in many applications

Key Stages in Digital Image Processing: Image Enhancement

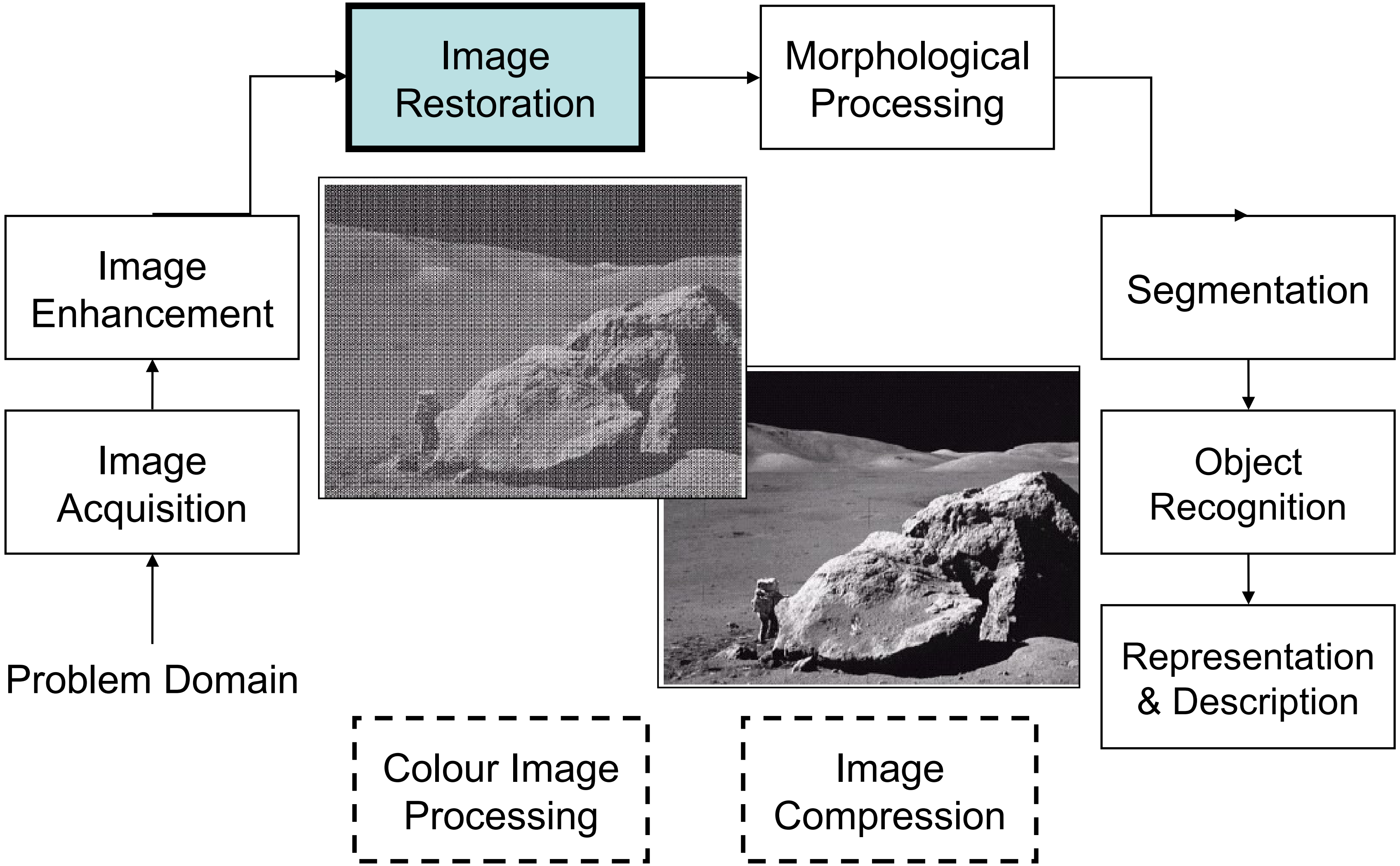


Key Stages in Digital Image Processing: Image Enhancement

- Image enhancement is among the simplest and most appealing areas of digital image processing.

- Basically, the idea behind enhancement techniques is to bring out detail that is obscured, or simply to highlight certain features of interest in an image. Such as, changing brightness & contrast etc.

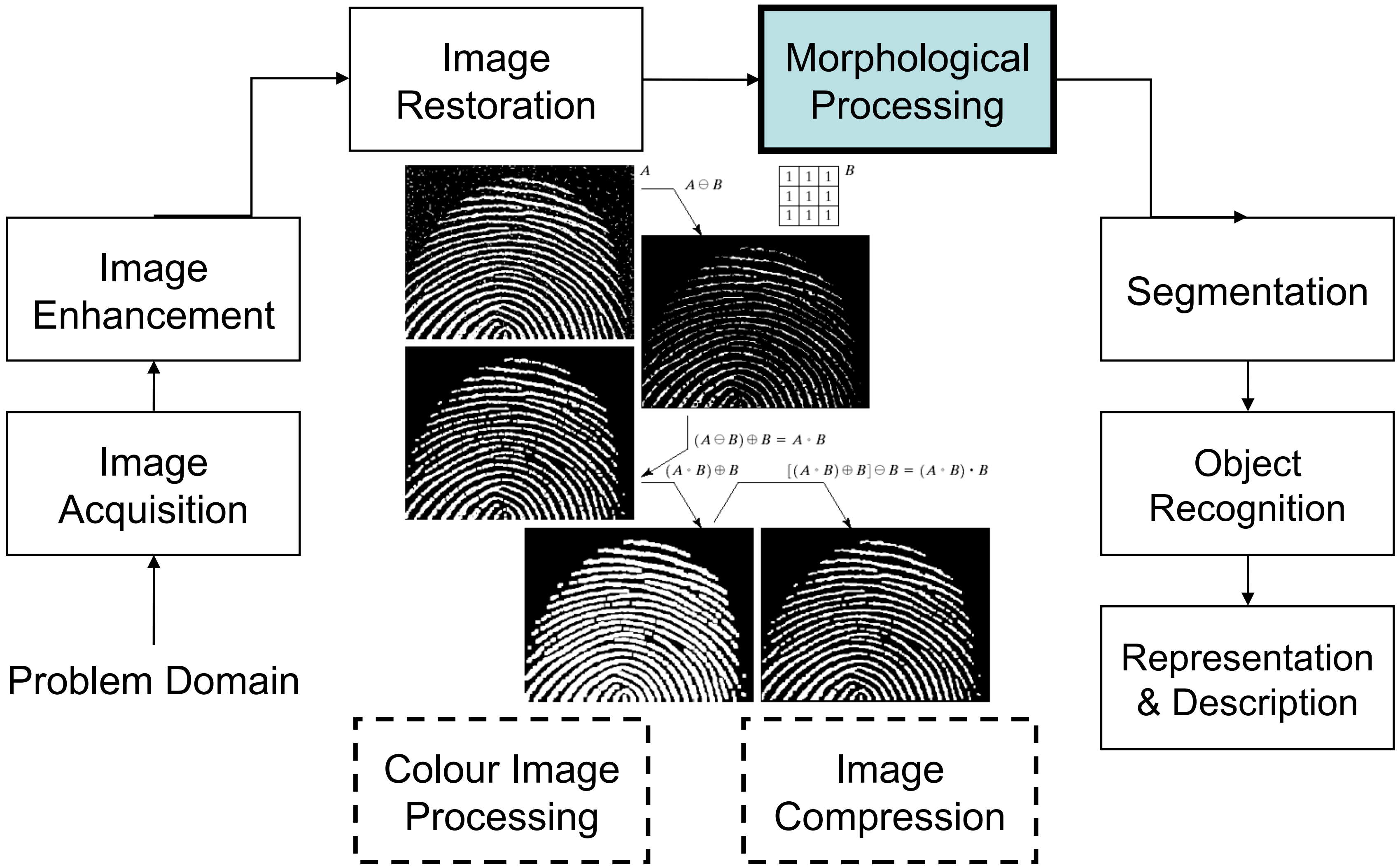
Key Stages in Digital Image Processing: Image Restoration



Key Stages in Digital Image Processing: Image Restoration

- Image restoration is an area that also deals with improving the appearance of an image.
- However, unlike enhancement, which is subjective, image restoration is objective, in the sense that restoration techniques tend to be based on mathematical or probabilistic models of image degradation.

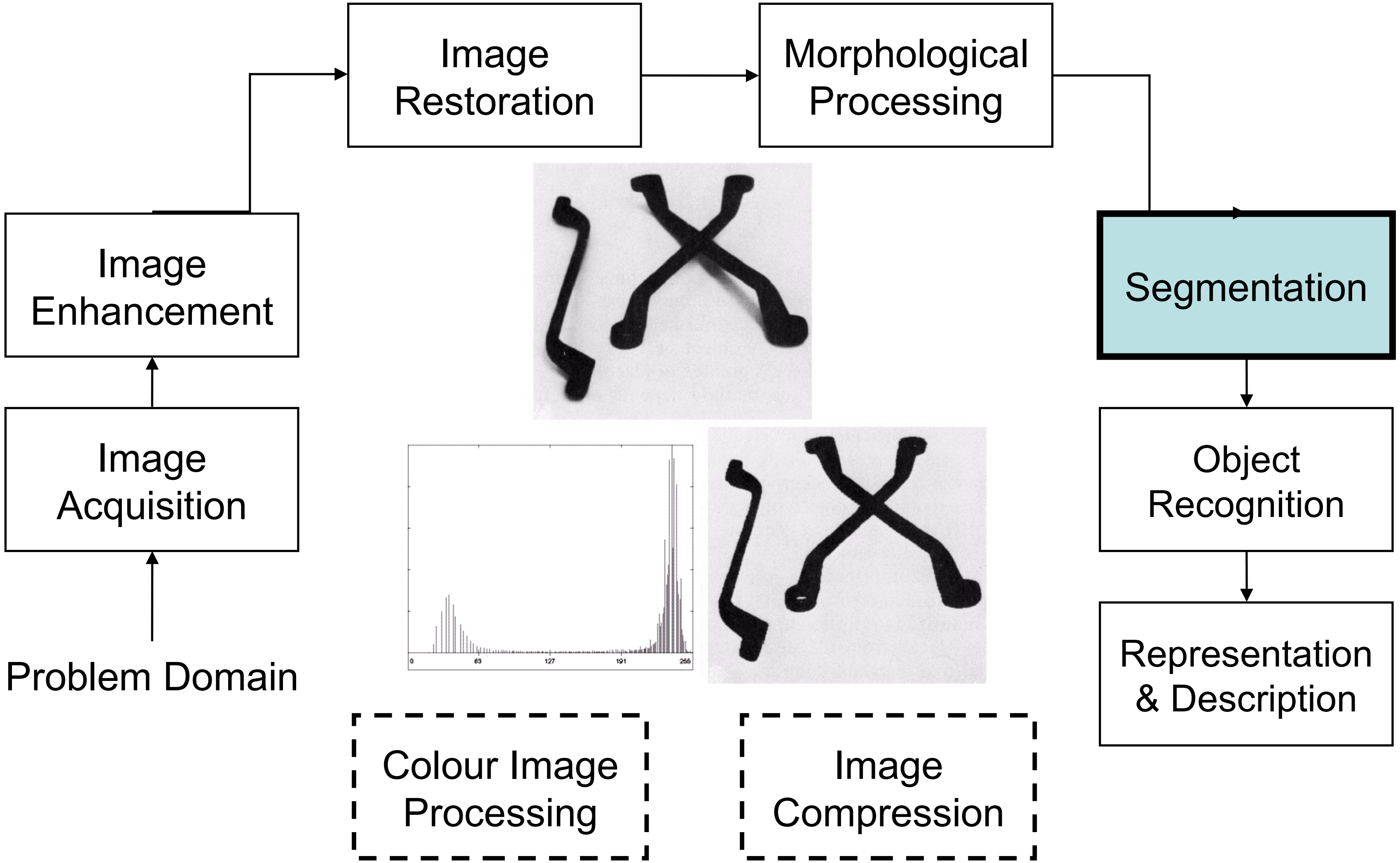
Key Stages in Digital Image Processing: Morphological Processing



Key Stages in Digital Image Processing: Morphological Processing

Morphological processing deals with tools for extracting image components that are useful in the representation and description of shape.

Key Stages in Digital Image Processing: Segmentation

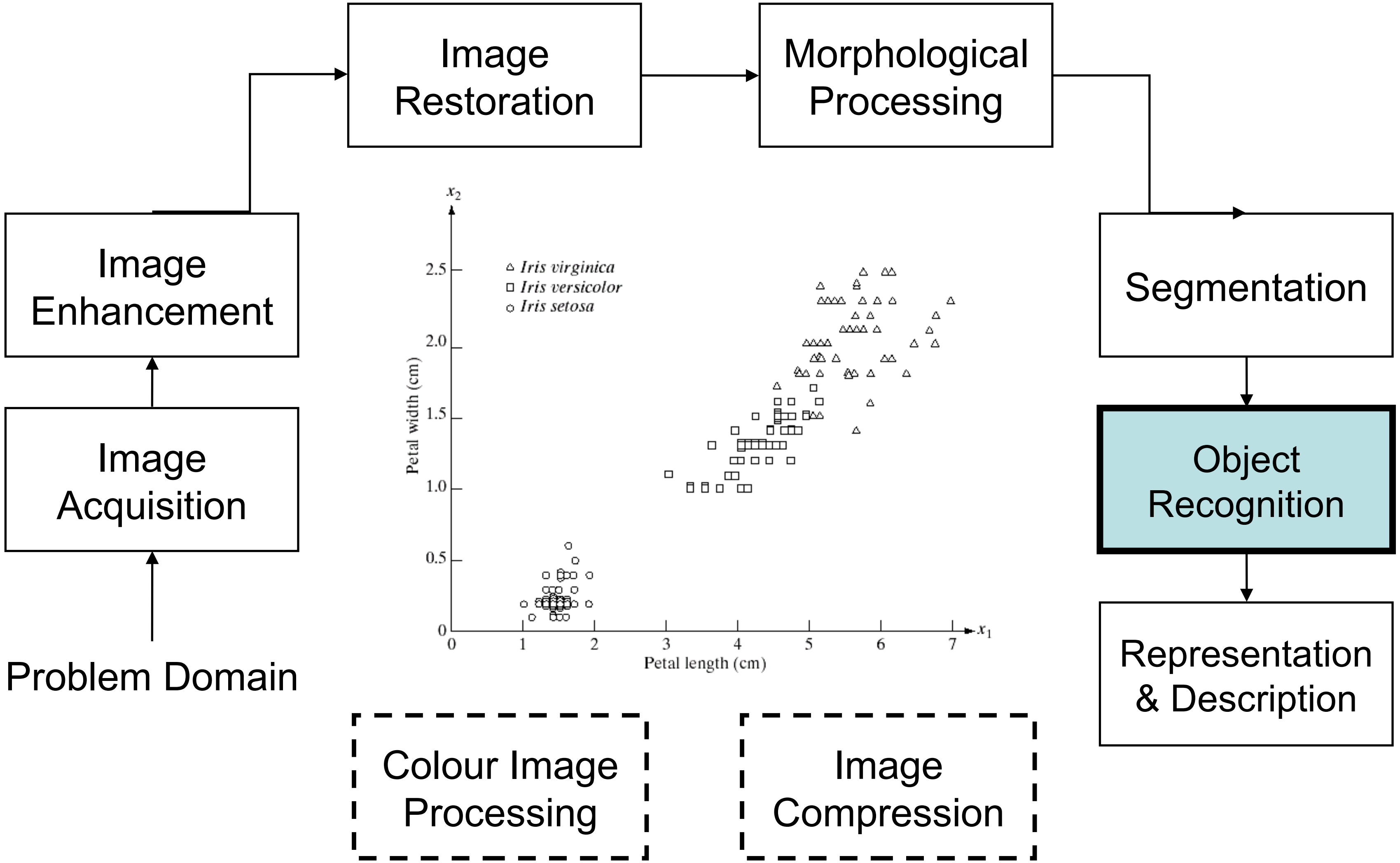


Key Stages in Digital Image Processing: Segmentation

- Broadly defined: breaking an image into its constituent parts

- In general, one of the most difficult tasks in image processing
  - Good segmentation simplifies the rest of the problem
  - Poor segmentation make the task impossible
- Output is usually raw pixel data: may represent region boundaries, points in the region itself, etc.
  - Boundary representation can be useful when the focus is on external shape characteristics (e.g. corners, rounded edges, etc.)
  - Region representation is appropriate when the focus is on internal properties (e.g. texture or skeletal shape)

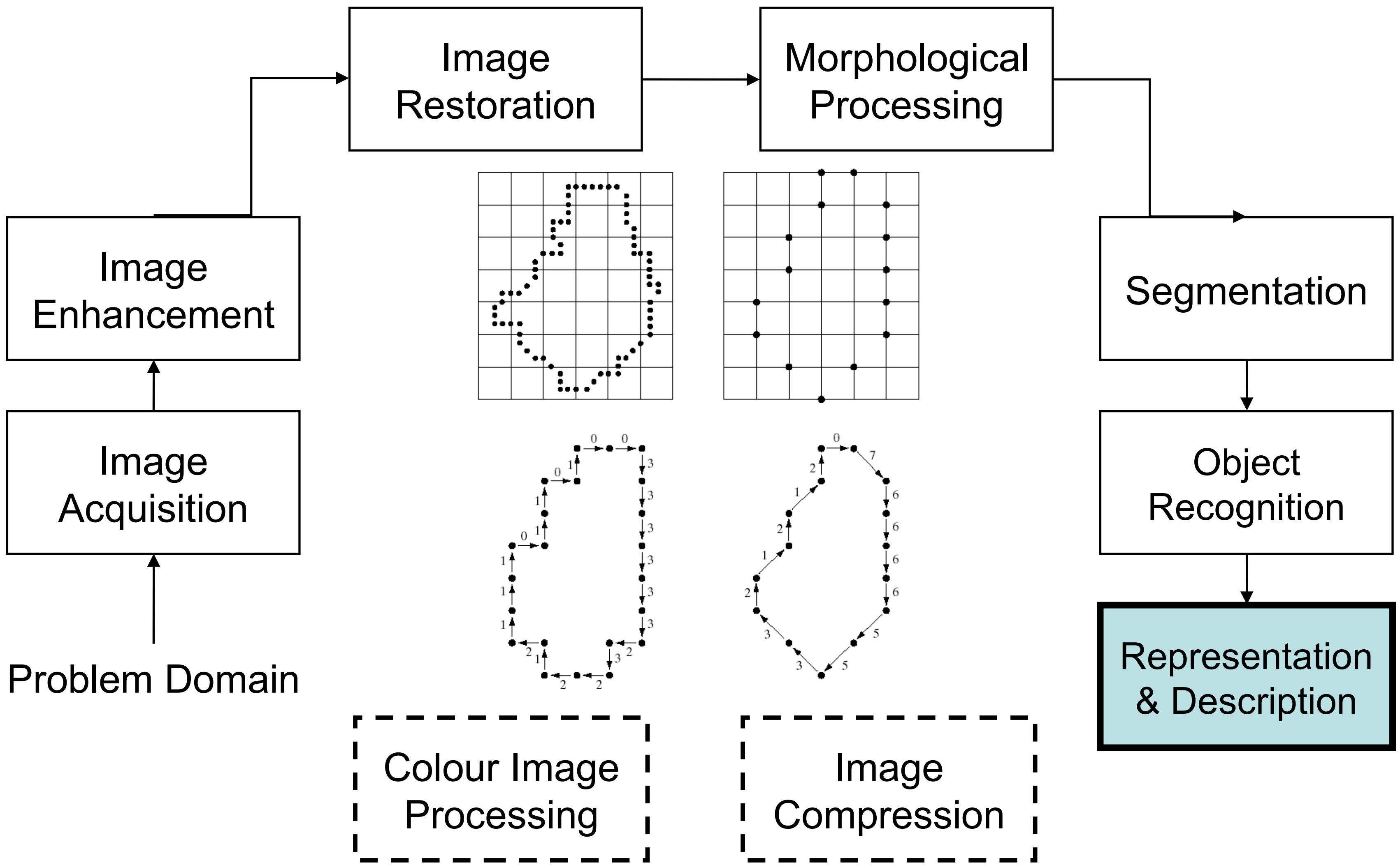
Key Stages in Digital Image Processing: Object Recognition



Key Stages in Digital Image Processing: Object Recognition

Recognition is the process that assigns a label, such as, "vehicle" to an object based on its descriptors.

Key Stages in Digital Image Processing: Representation & Description

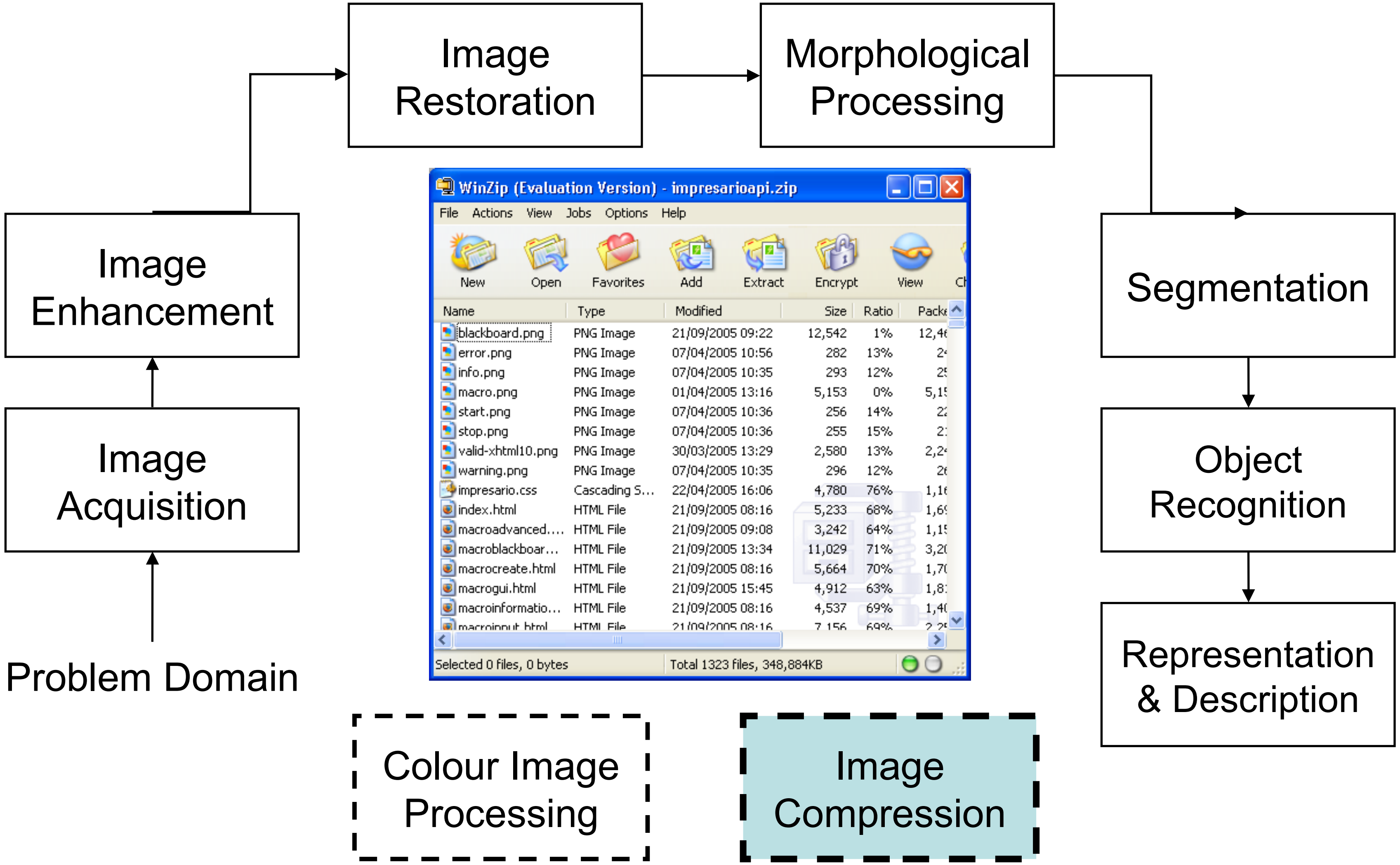


Key Stages in Digital Image Processing: Representation and Description

- Representation: transforming raw data into a form suitable for computer processing

- Description (also called feature extraction) deals with extracting features that result in some quantitative information of interest or features which are basic for differentiating one class of objects from another

Key Stages in Digital Image Processing: Image Compression

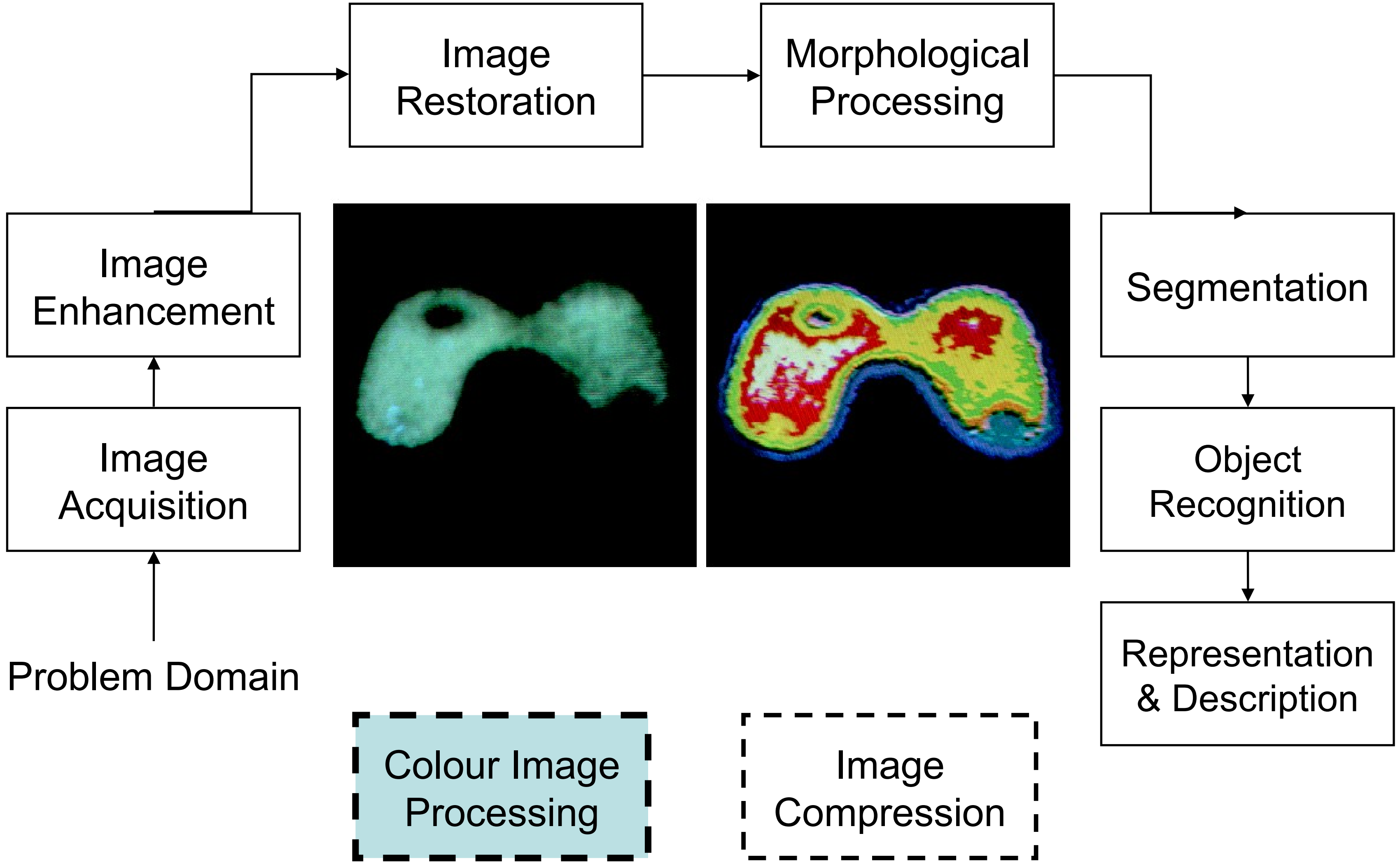


Key Stages in Digital Image Processing: Image Compression

- Compression deals with techniques for reducing the storage required to save an image or the bandwidth to transmit it.

- Particularly in the uses of internet it is very much necessary to compress data.

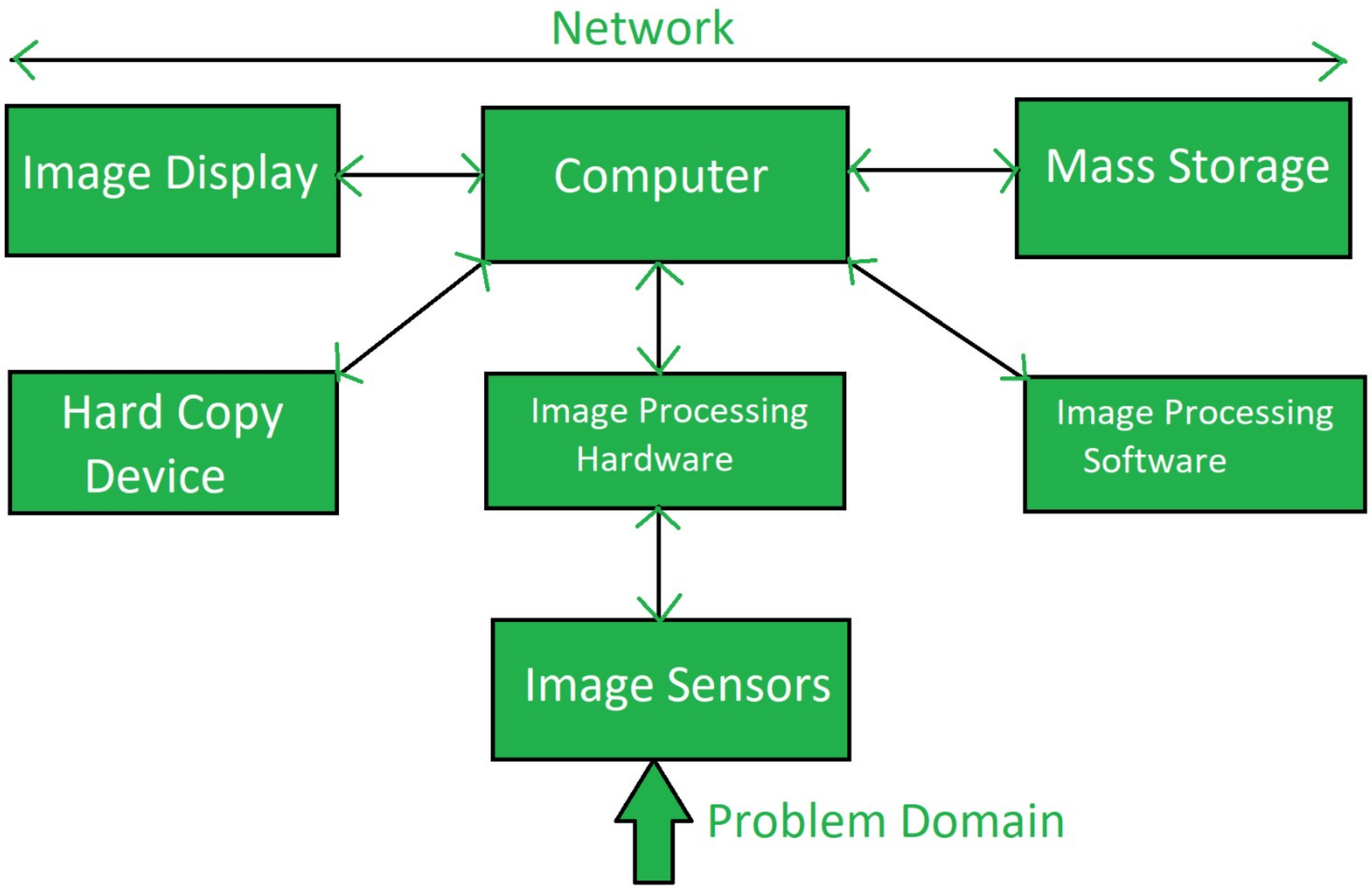
Key Stages in Digital Image Processing: Colour Image Processing



Key Stages in Digital Image Processing: Colour Image Processing

- Color image processing is an area that has been gaining its importance because of the significant increase in the use of digital images over the Internet.
- This may include color modeling and processing in a digital domain etc.

Elements in Digital Image Processing



Assignment

Explain the Elements used in Digital Image Processing

Human Visual System

- The best vision model we have!
- Knowledge of how images form in the eye can help us with processing digital images
- We will take just a whirlwind tour of the human visual system

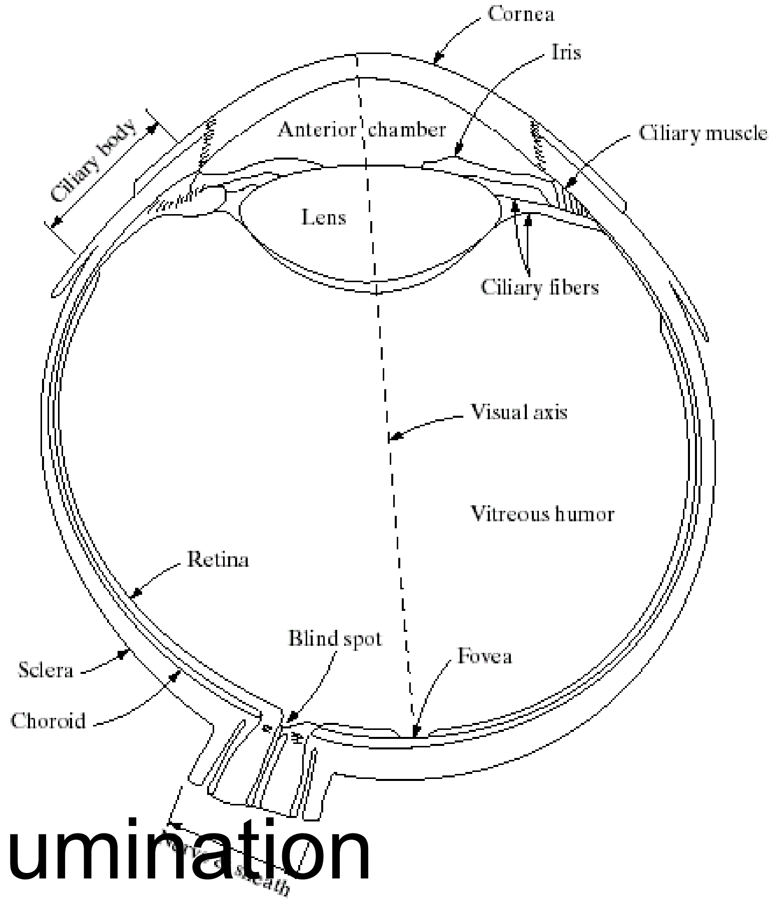
Structure Of The Human Eye

The lens focuses light from objects onto the retina

The retina is covered with light receptors called *cones* (6-7 million) and *rods* (75-150 million)

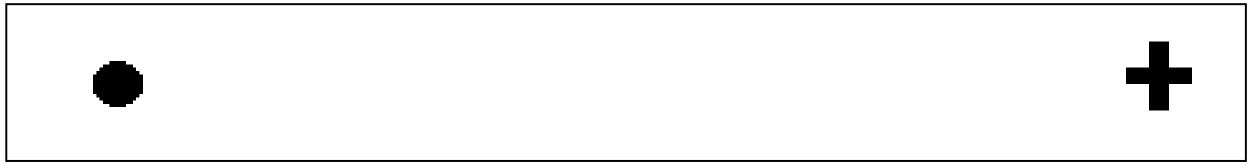
Cones are concentrated around the fovea and are very sensitive to colour

Rods are more spread out and are sensitive to low levels of illumination



Blind-Spot Experiment

Draw an image similar to that below on a piece of paper (the dot and cross are about 6 inches apart)



Close your right eye and focus on the cross with your left eye

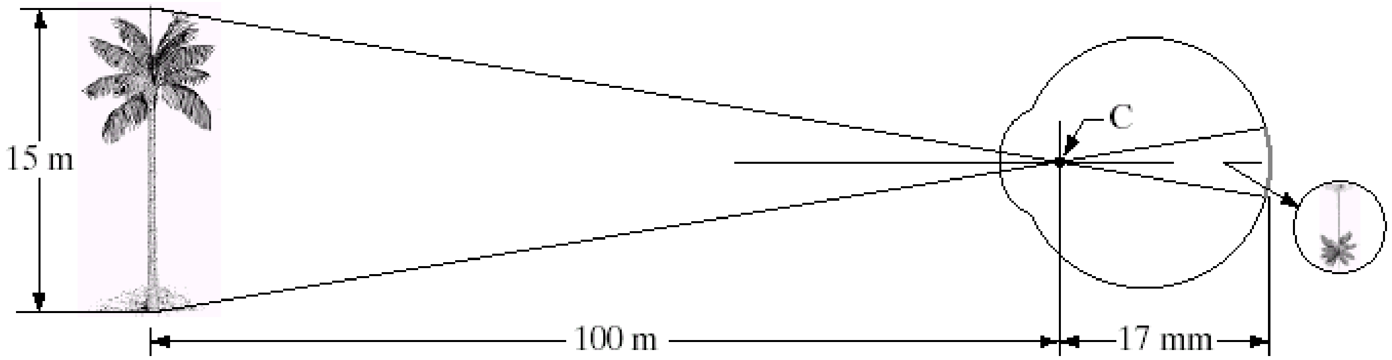
Hold the image about 20 inches away from your face and move it slowly towards you

The dot should disappear!

Image Formation In The Eye

Muscles within the eye can be used to change the shape of the lens allowing us focus on objects that are near or far away

An image is focused onto the retina causing rods and cones to become excited which ultimately send signals to the brain

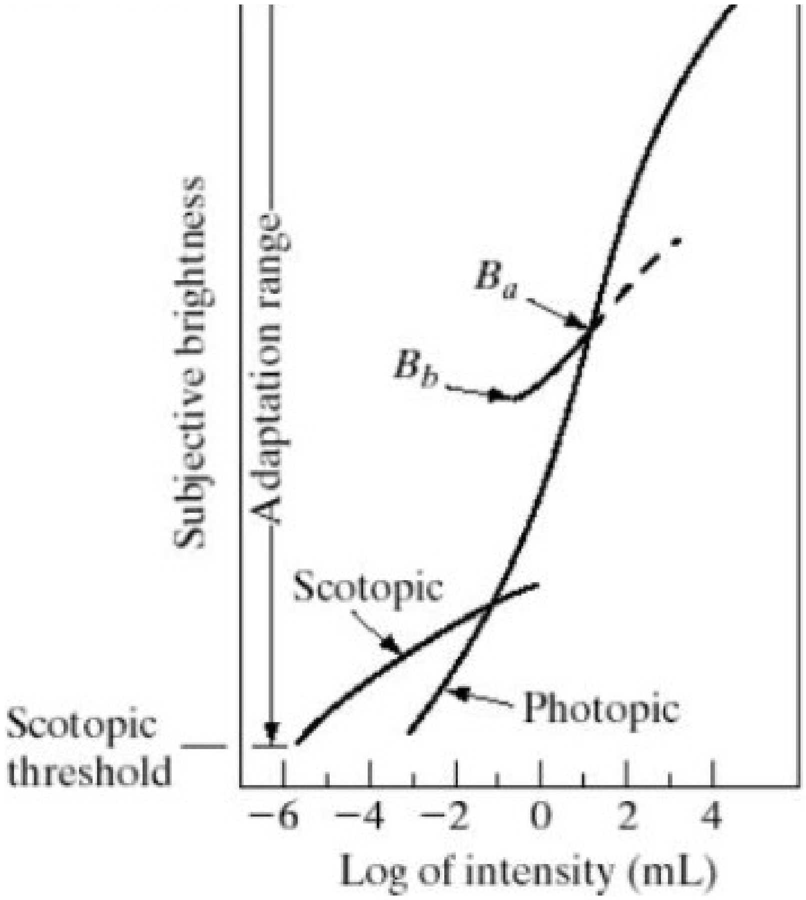


Brightness Adaptation & Discrimination

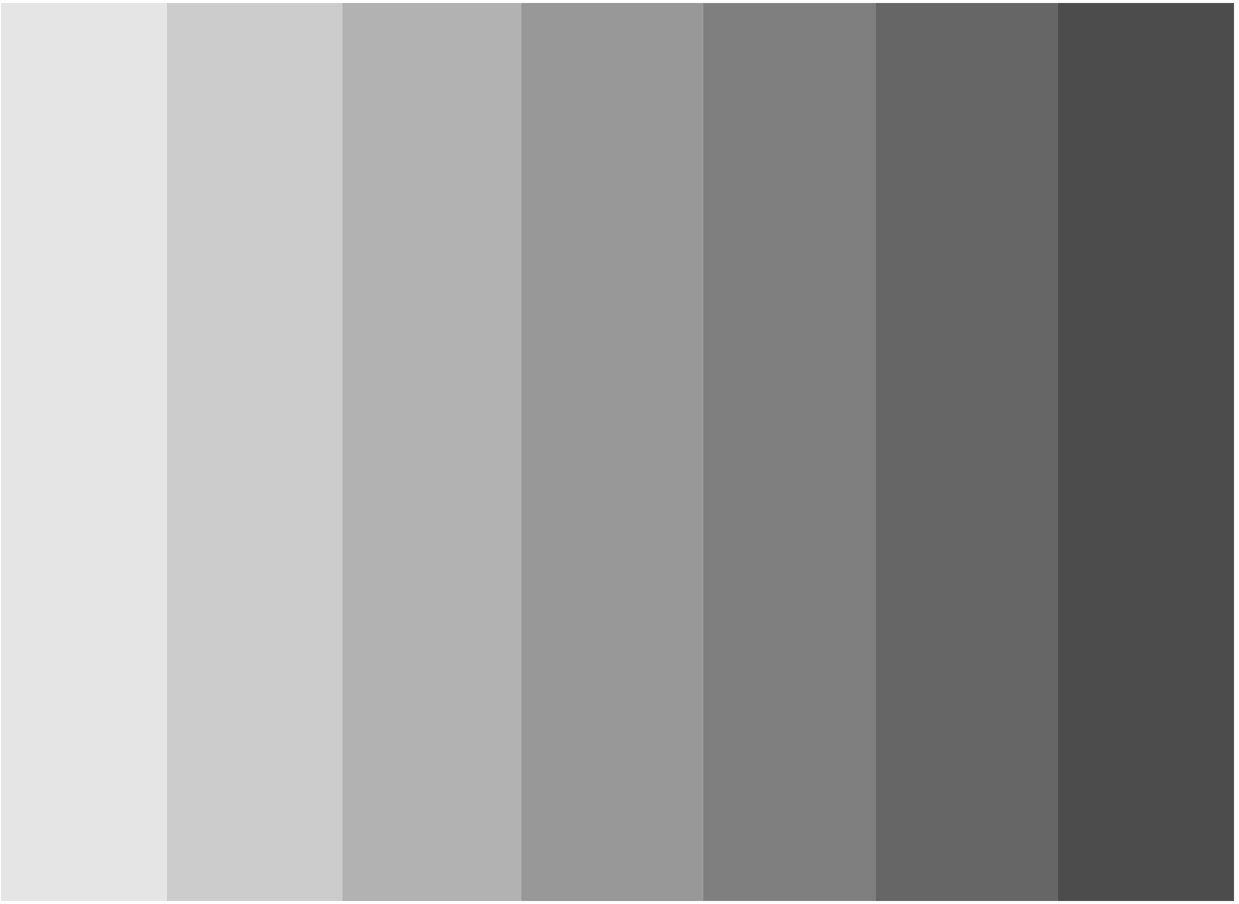
The range of light intensity levels to which the human visual system can adapt is enormous—on the order of 10^{10} from the scotopic threshold to the glare limit.

However, at any one time we can only discriminate between a much smaller number *brightness adaptation*

Similarly, the *perceived intensity* of a region is related to the light intensities of the regions surrounding it

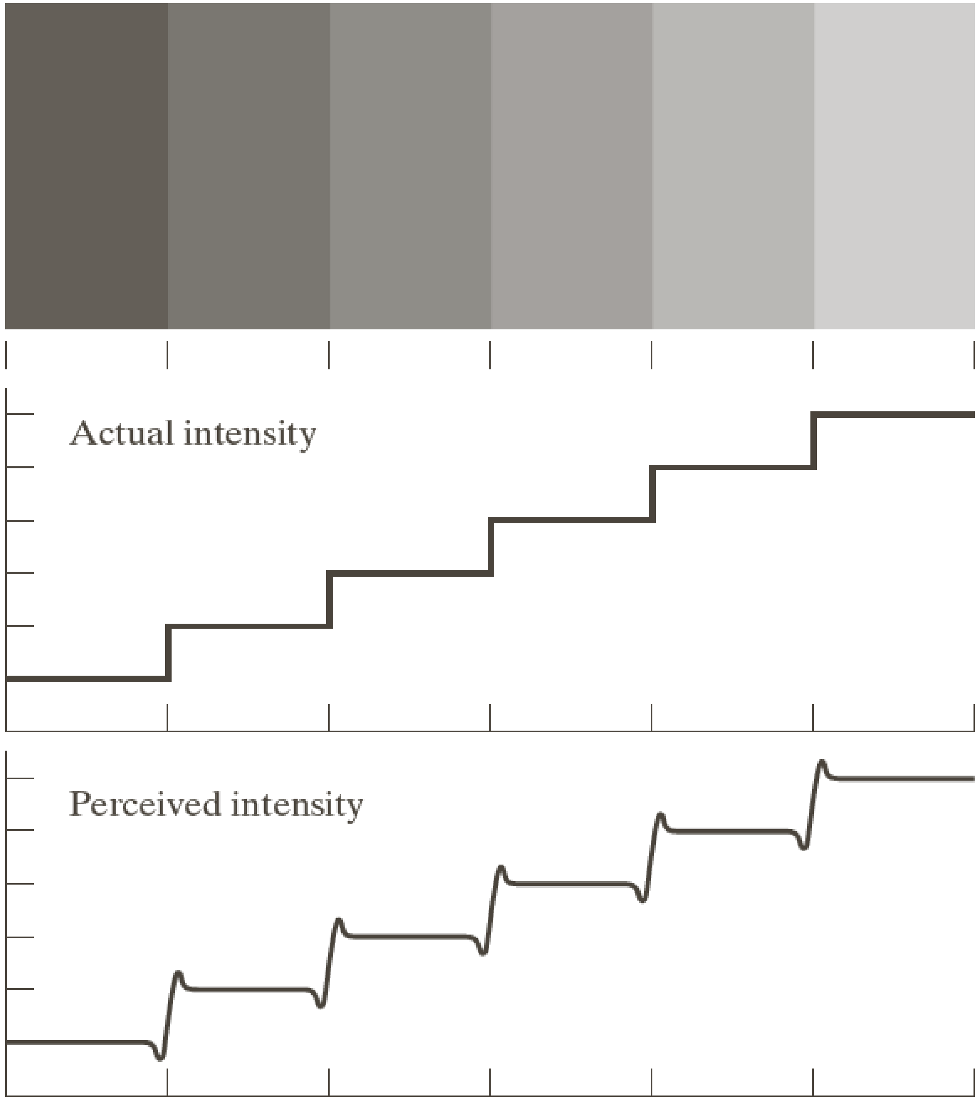


Brightness Adaptation & Discrimination

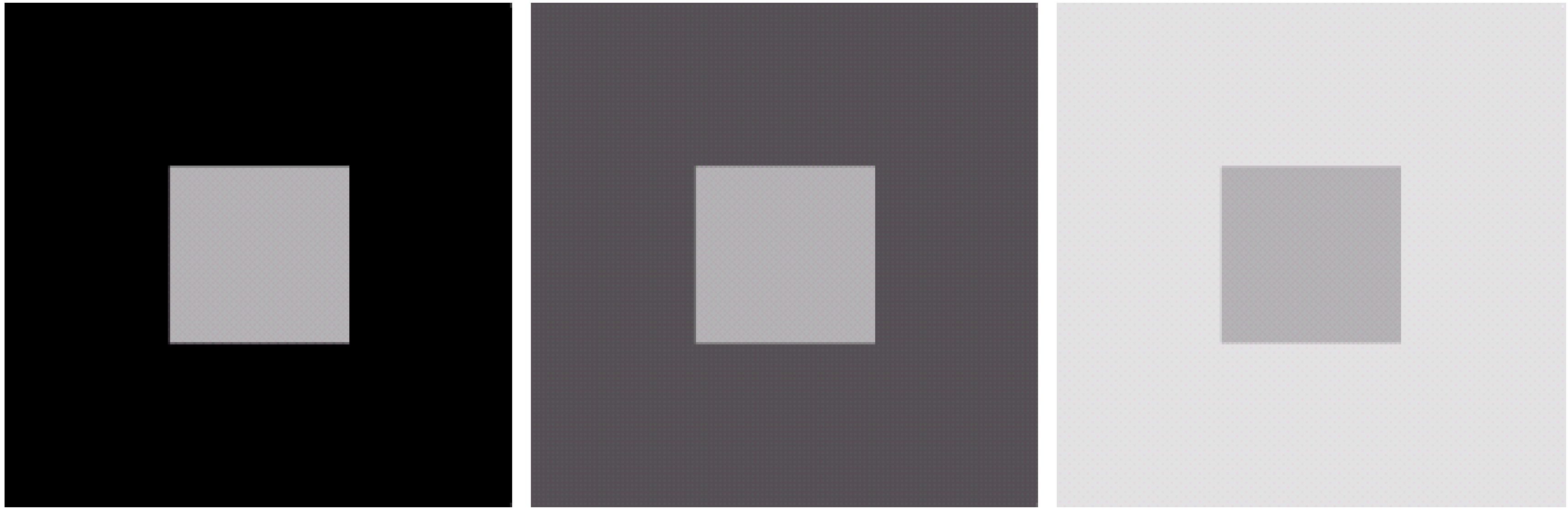


An example of Mach bands

Brightness Adaptation & Discrimination

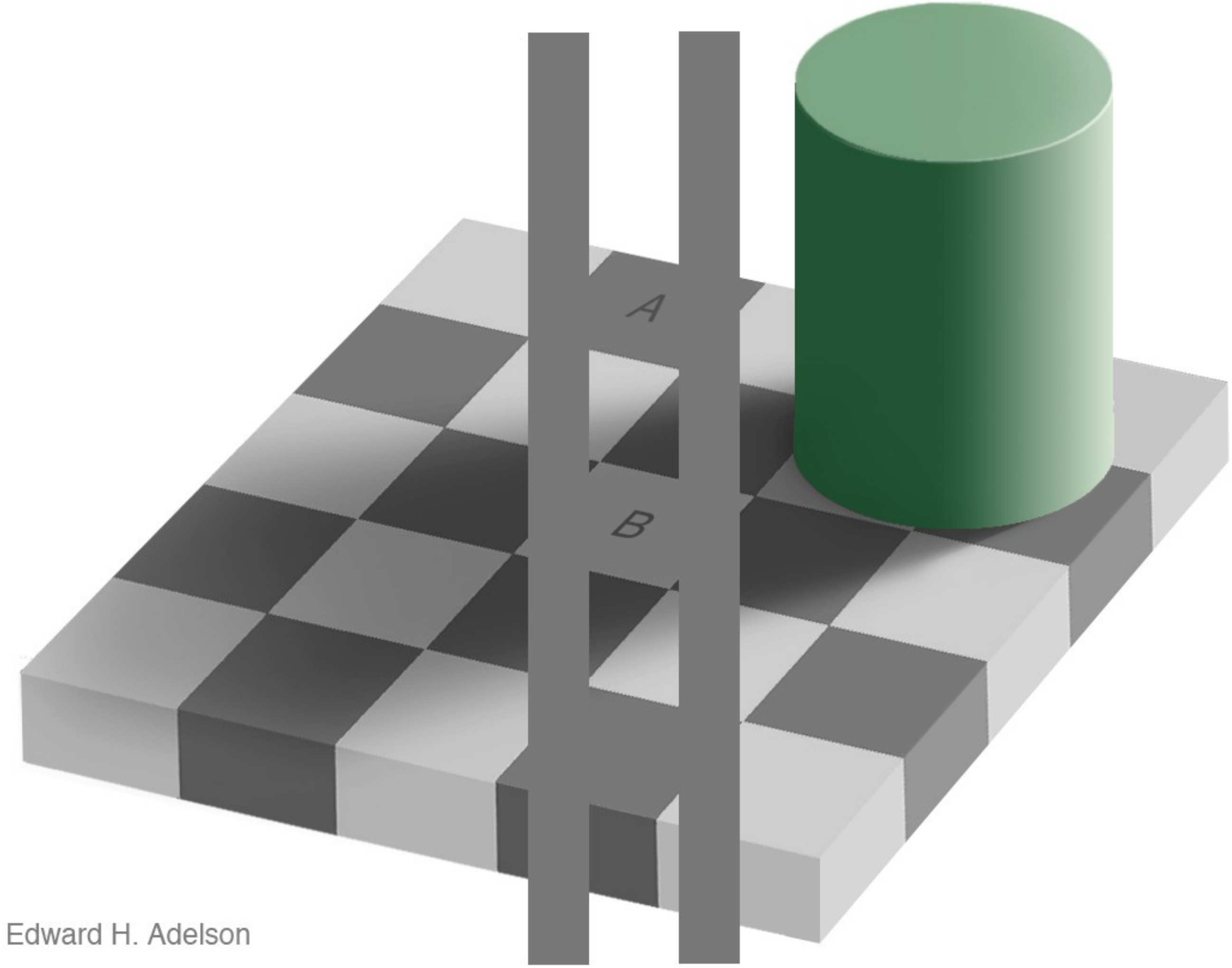


Brightness Adaptation & Discriminatio



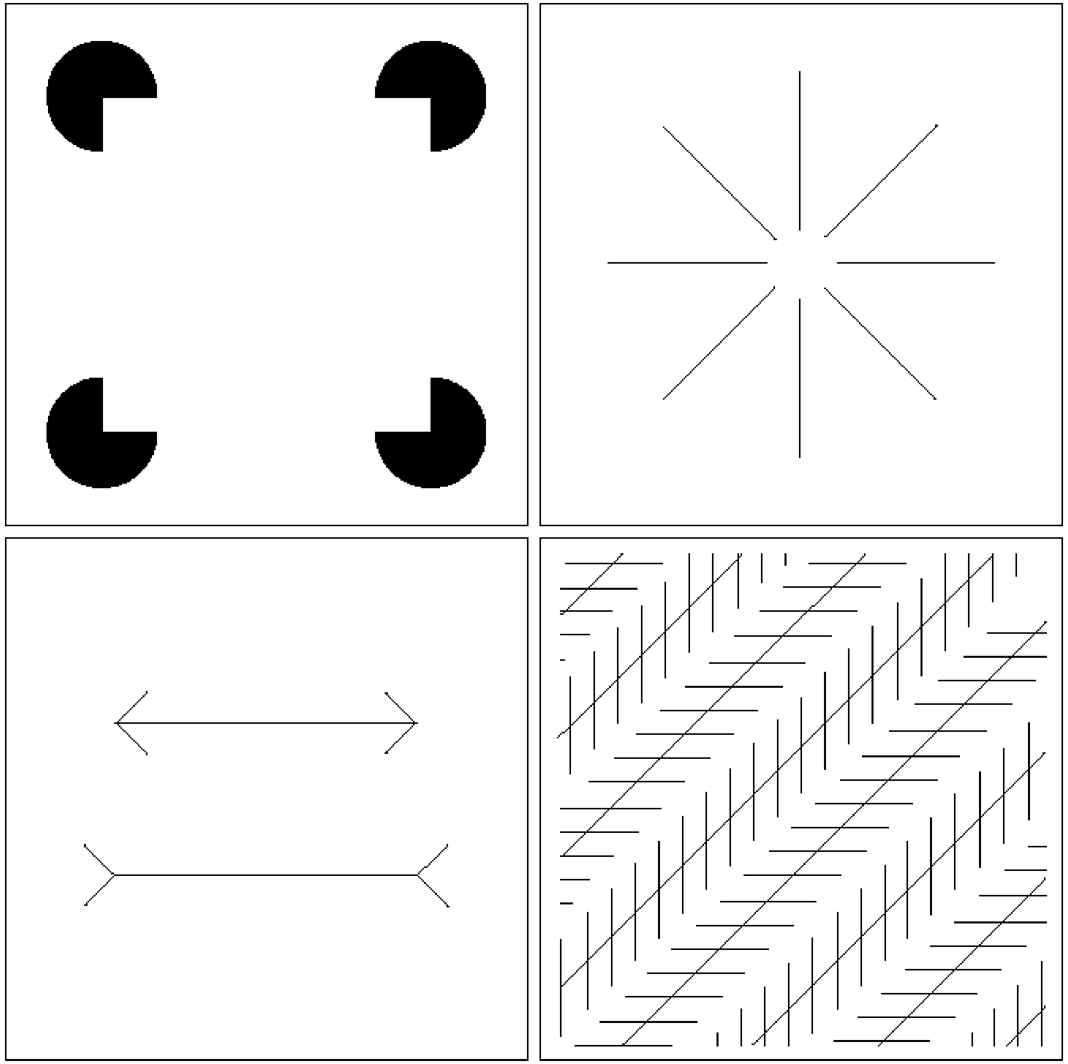
An example of *simultaneous contrast*

Brightness Adaptation & Discrimination



Optical Illusions

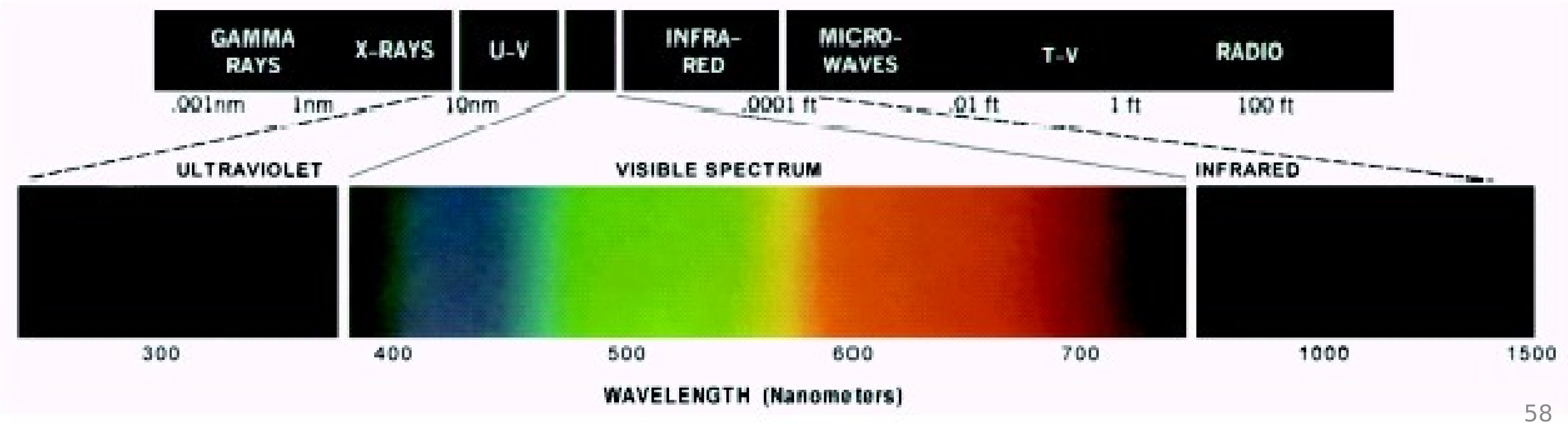
Our visual systems play lots of interesting tricks on us



Light And The Electromagnetic Spectrum

Light is just a particular part of the electromagnetic spectrum that can be sensed by the human eye

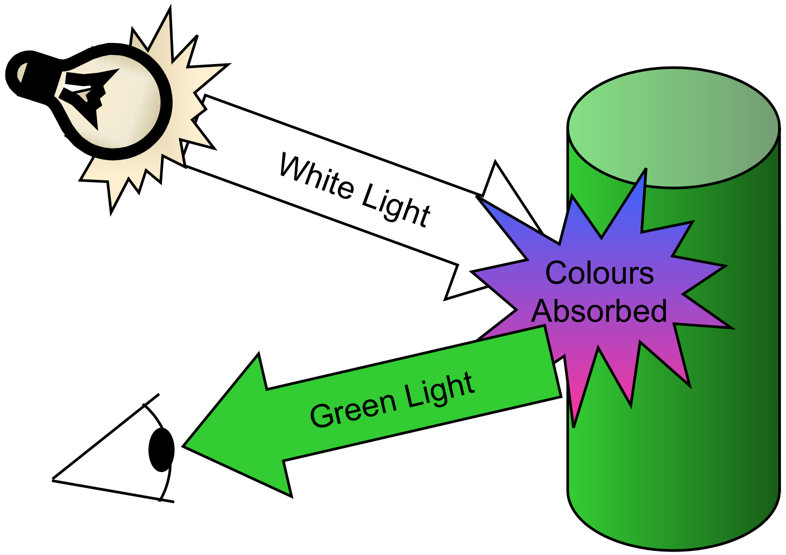
The electromagnetic spectrum is split up according to the wavelengths of different forms of energy



Reflected Light

The colours that we perceive are determined by the nature of the light reflected from an object

For example, if white light is shone onto a green object most wavelengths are absorbed, while green light is reflected from the object



Sampling, Quantisation And Resolution

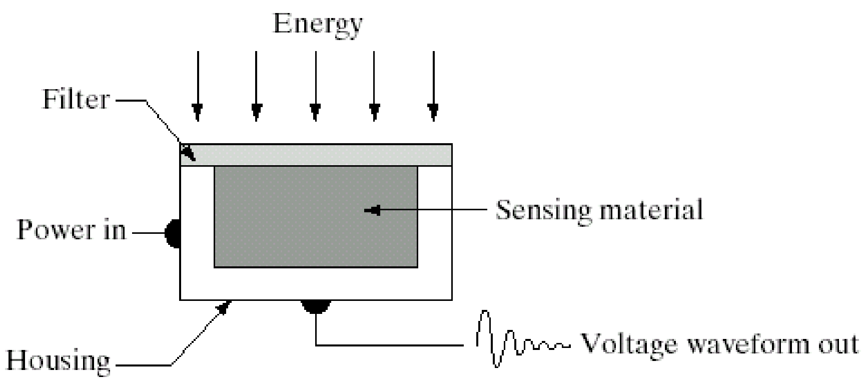
In the following slides we will consider what is involved in capturing a digital image of a real-world scene

- Image sensing and representation
- Sampling and quantisation
- Resolution

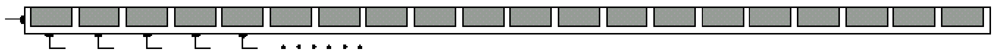
Image Sensing

Incoming energy lands on a sensor material responsive to that type of energy and this generates a voltage

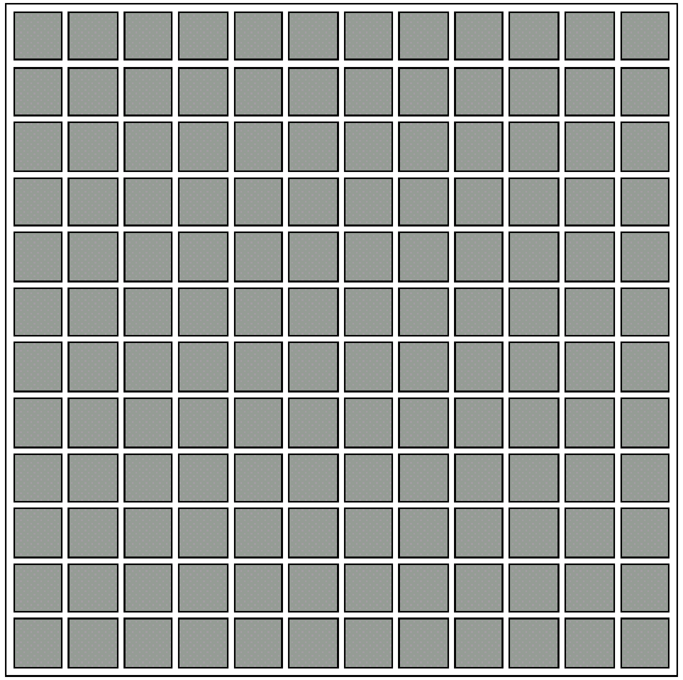
Collections of sensors are arranged to capture images



Imaging Sensor



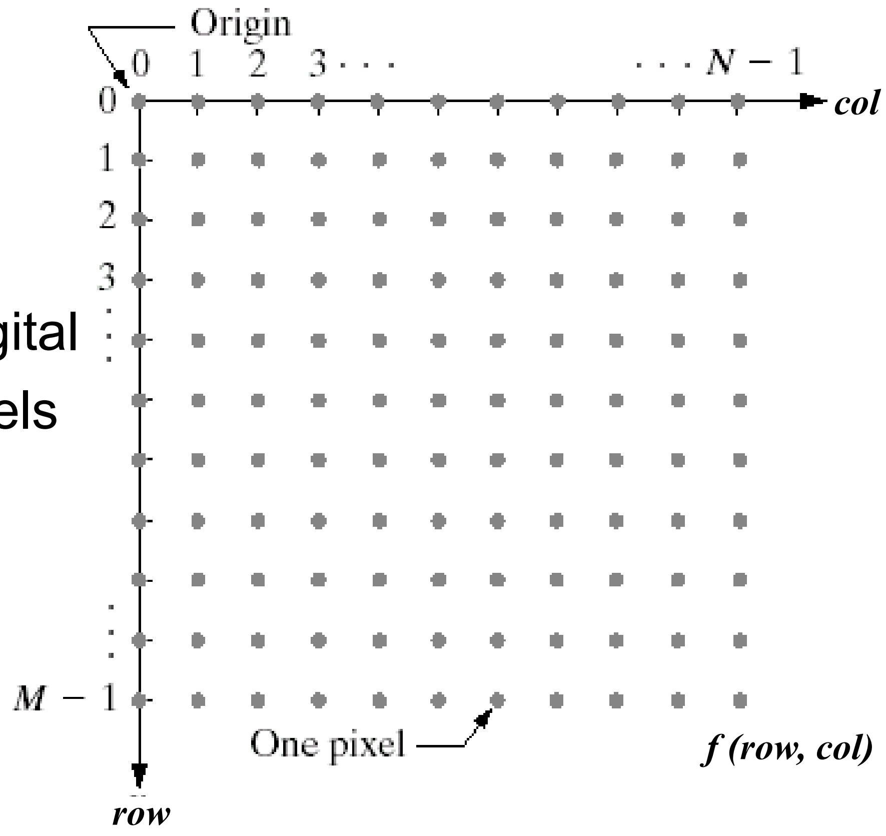
Line of Image Sensors



Array of Image Sensors

Image Representation

A Digital image is an image f(x, y) that is discrete in both the spatial co-ordinates and its brightness. It is represented by the 2D integer array or series of 2D array for each colour band.



Before we discuss image acquisition recall that a digital image is composed of M rows and N columns of pixels each storing a value

Pixel values are most often grey levels in the range 0-255(black-white)

Image Representation

$$f(x,y) \approx \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,M-1) \\ f(1,0) & f(1,1) & \dots & f(1,M-1) \\ . & & & \\ f(N-1,0) & f(N-1,1) & \dots & f(N-1,M-1) \end{bmatrix}$$

Image Sampling And Quantisation

To convert a continuous image f(x, y) into digital form, we have to sample the function in both co-ordinates and amplitude

Sampling

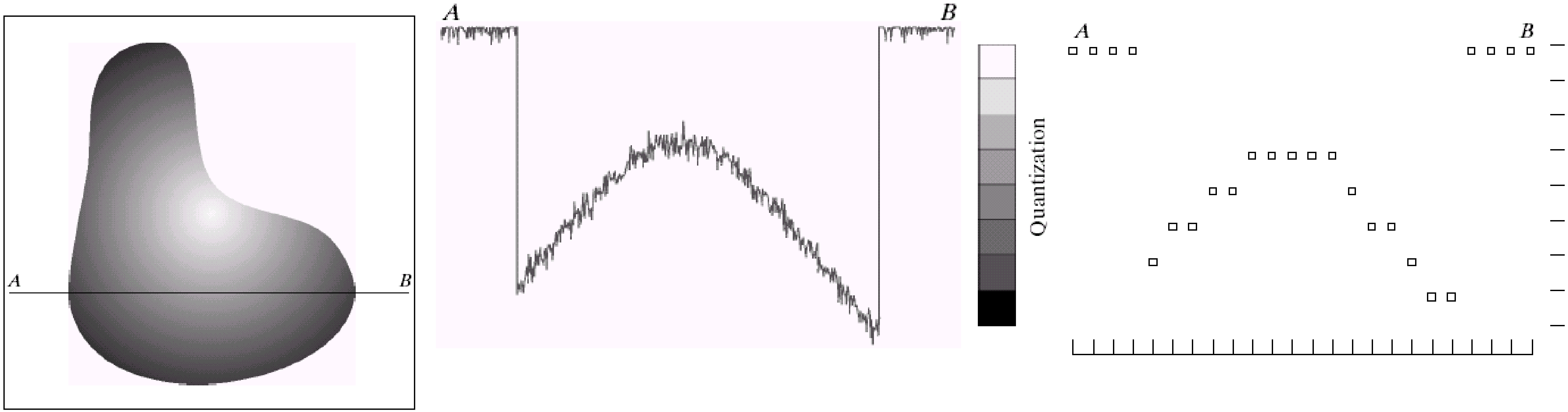
- Digitizing the co-ordinate value is called sampling.
- Determines the spatial resolution of the digitized images.

Quantization

- Digitizing the amplitude value is called quantization.
- Determines the number of gray levels in the digitized images.

Image Sampling And Quantisation

A digital sensor can only measure a limited number of **samples** at a **discrete** set of energy levels



Remember that a digital image is always only an **approximation** of a real world scene

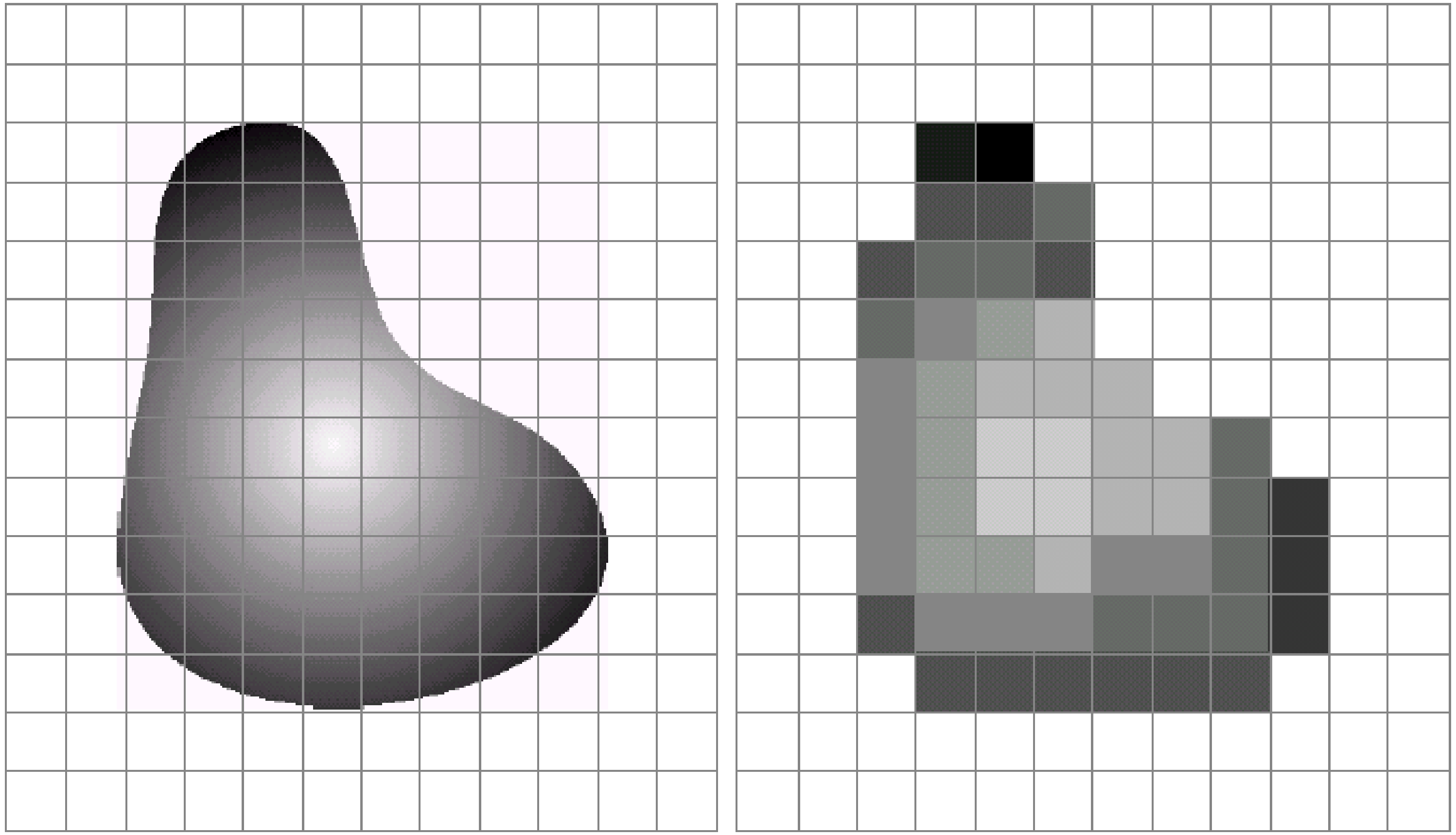
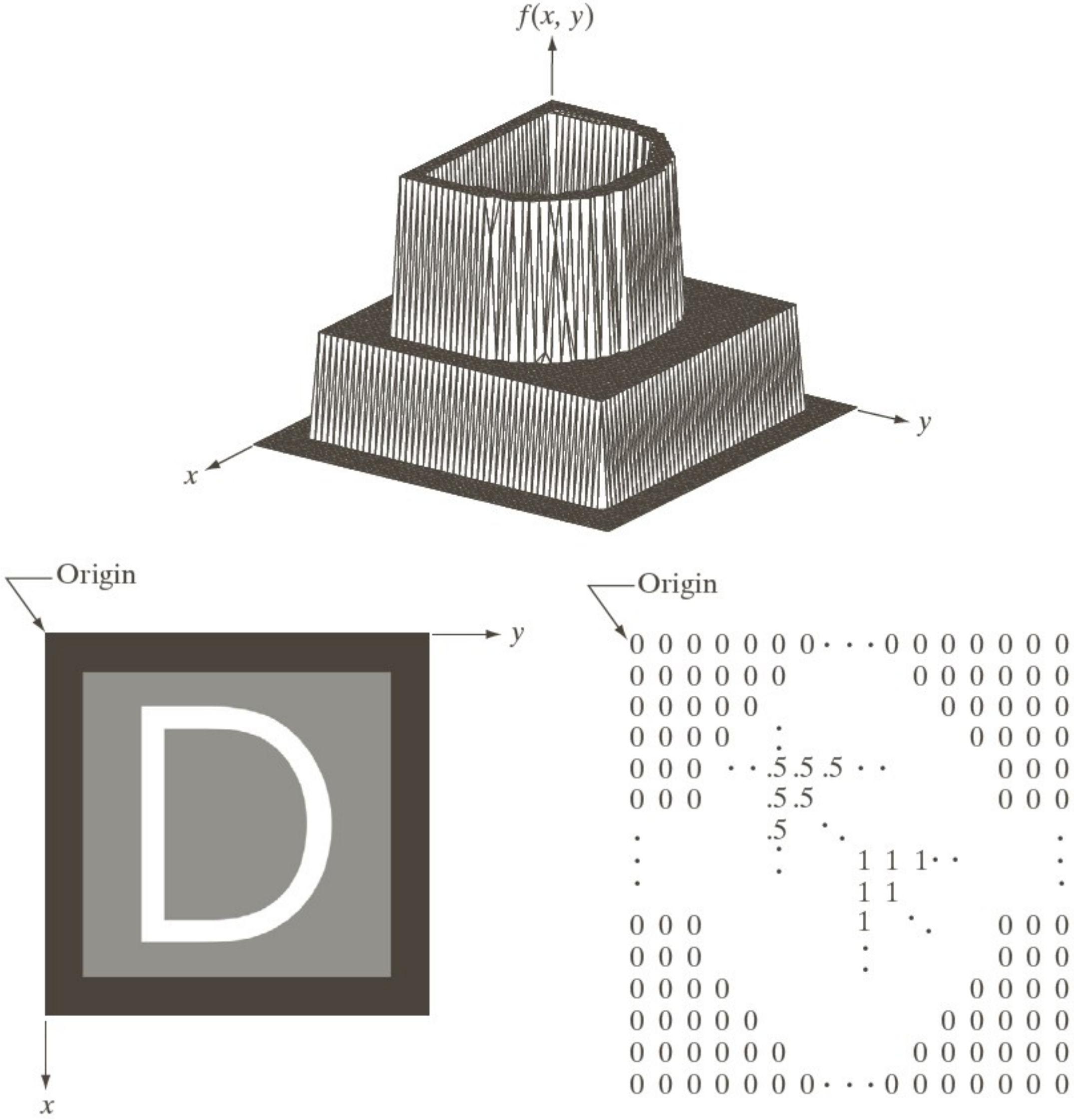


Image Representation



Spatial Resolution

Spatial resolution states that the clarity of an image cannot be determined by the pixel resolution. The number of pixels in an image does not matter.

Or in other way we can define spatial resolution as the number of independent pixels values per inch.

Vision specialists will often talk about pixel size

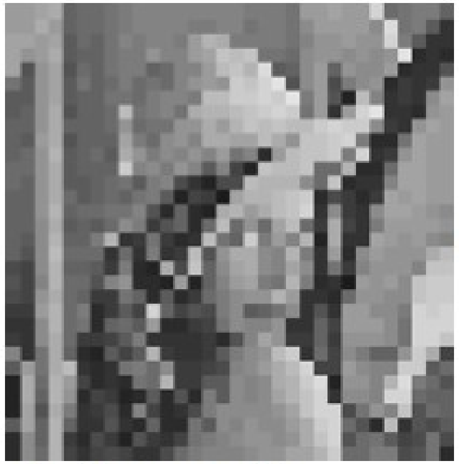
Graphic designers will talk about *dots per inch* (DPI)

Effects of reducing spatial resolution

256x256

128x128

64x64



32x32

Pixel replication occurs as resolution is decreased

Intensity Level Resolution

Intensity level resolution refers to the number of intensity levels used to represent the image

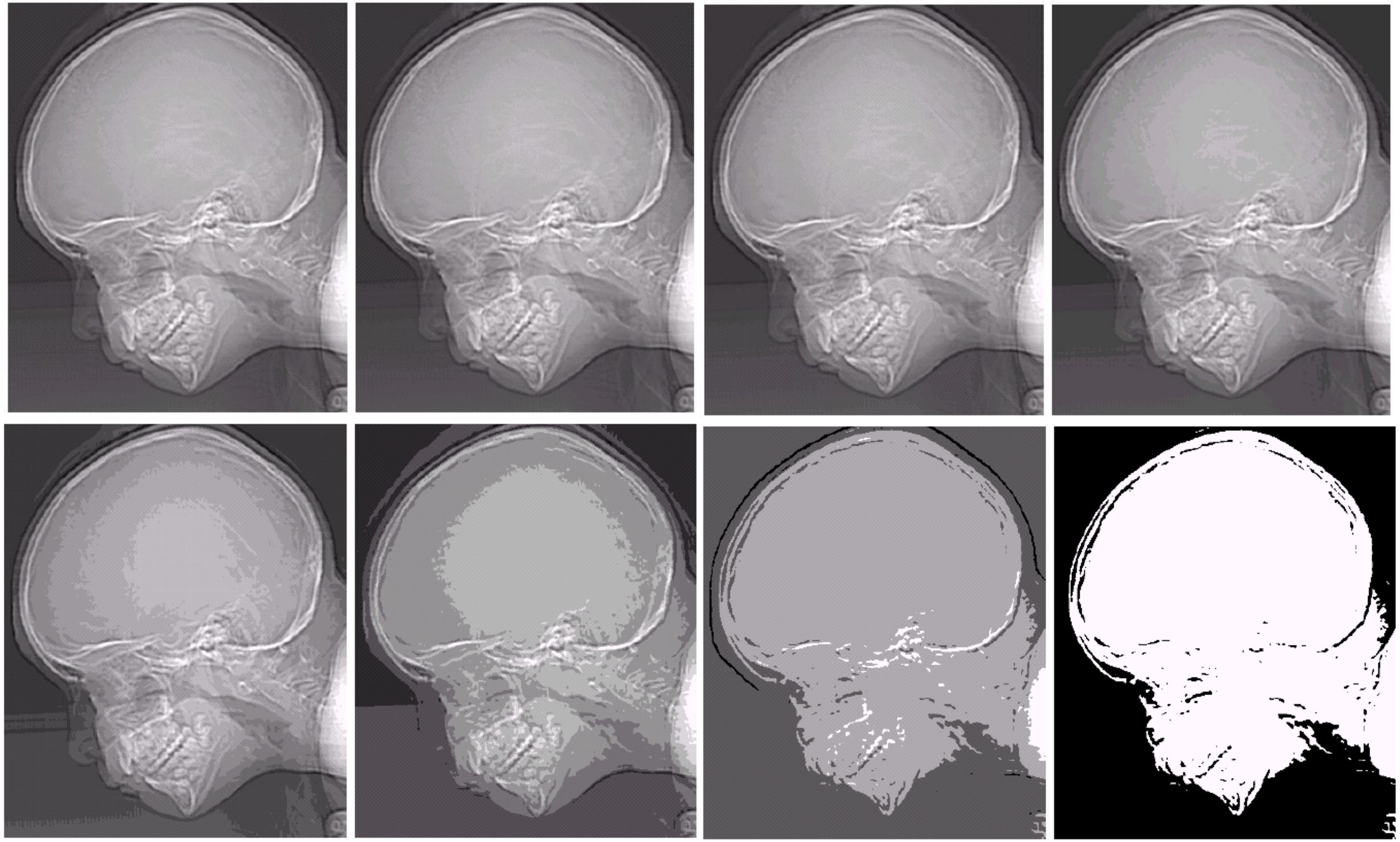
The more intensity levels used, the finer the level of detail discernable in an image

Intensity level resolution is usually given in terms of the number of bits used to store each intensity level

Number of Bits	**Number of Intensity Levels**	**Examples**
1	2	0, 1
2	4	00, 01, 10, 11
4	16	0000, 0101, 1111
8	256	00110011, 01010101
16	65,536	1010101010101010

Intensity Level Resolution

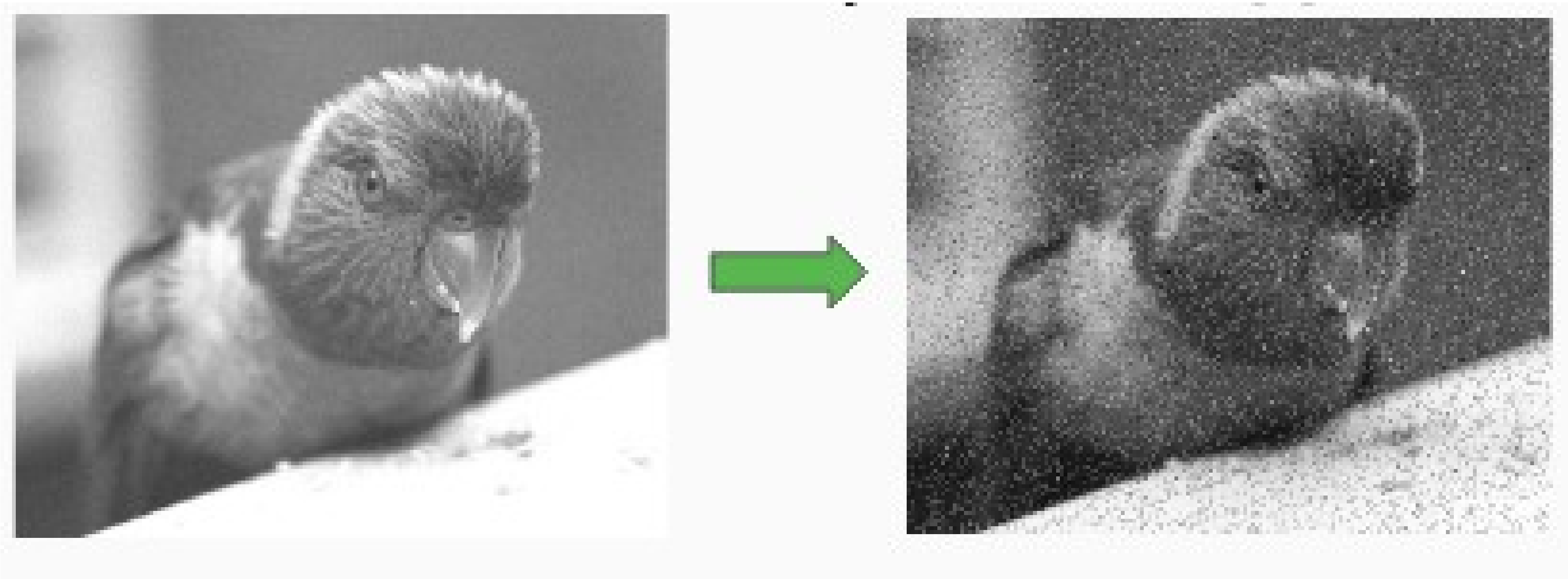
256 grey levels (8 bits per pixel) | 128 grey levels (7 bpp) | 64 grey levels (6 bpp) | 32 grey levels (5 bpp)



16 grey levels (4 bpp) | 8 grey levels (3 bpp) | 4 grey levels (2 bpp) | 2 grey levels (1 bpp)

Noise

- Image noise is random variation of brightness or color information in the images captured.
- It is degradation in image signal caused by external sources



Resolution: How Much Is Enough?

- The big question with resolution is always *how much is enough*?
- This all depends on what is in the image and what you would like to do with it
- Key questions include
  - Does the image look aesthetically pleasing?
  - Can you see what you need to see within the image?

Resolution: How Much Is Enough?

The picture on the right is fine for counting the number of cars, but not for reading the number plate

Intensity Level Resolution (cont…)

Low Detail

Medium Detail

High Detail

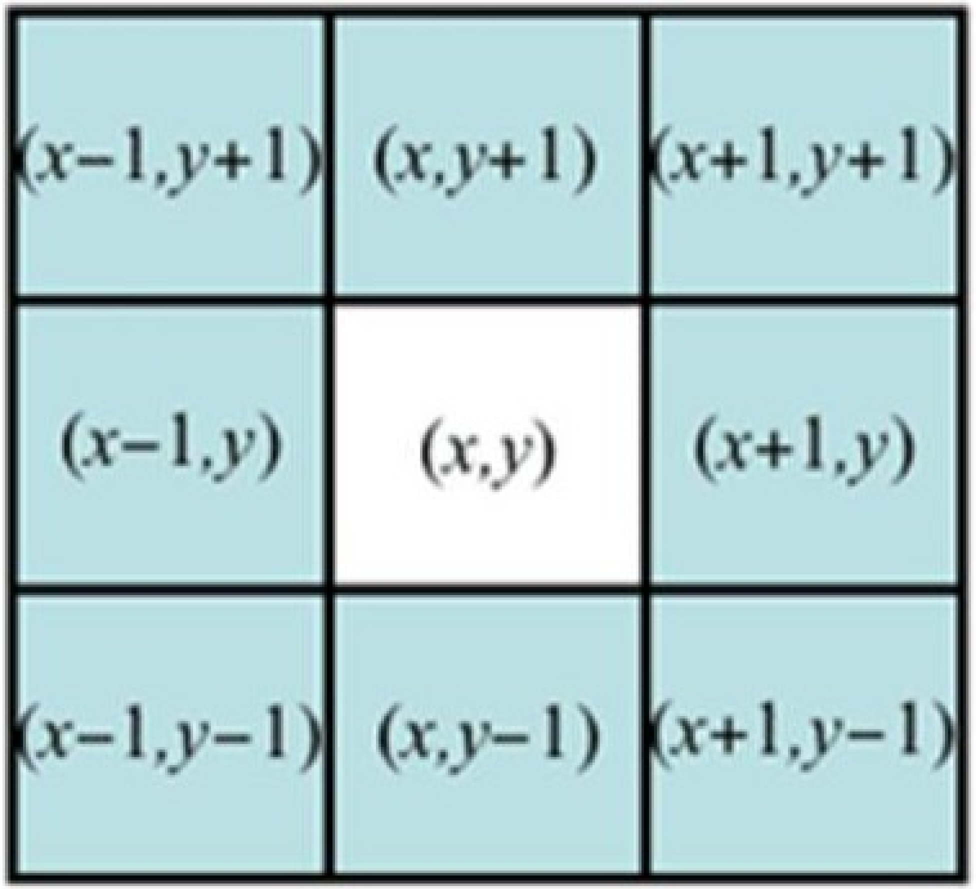
Basic relationships between pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries
- Distance Measure

Neighbourhood

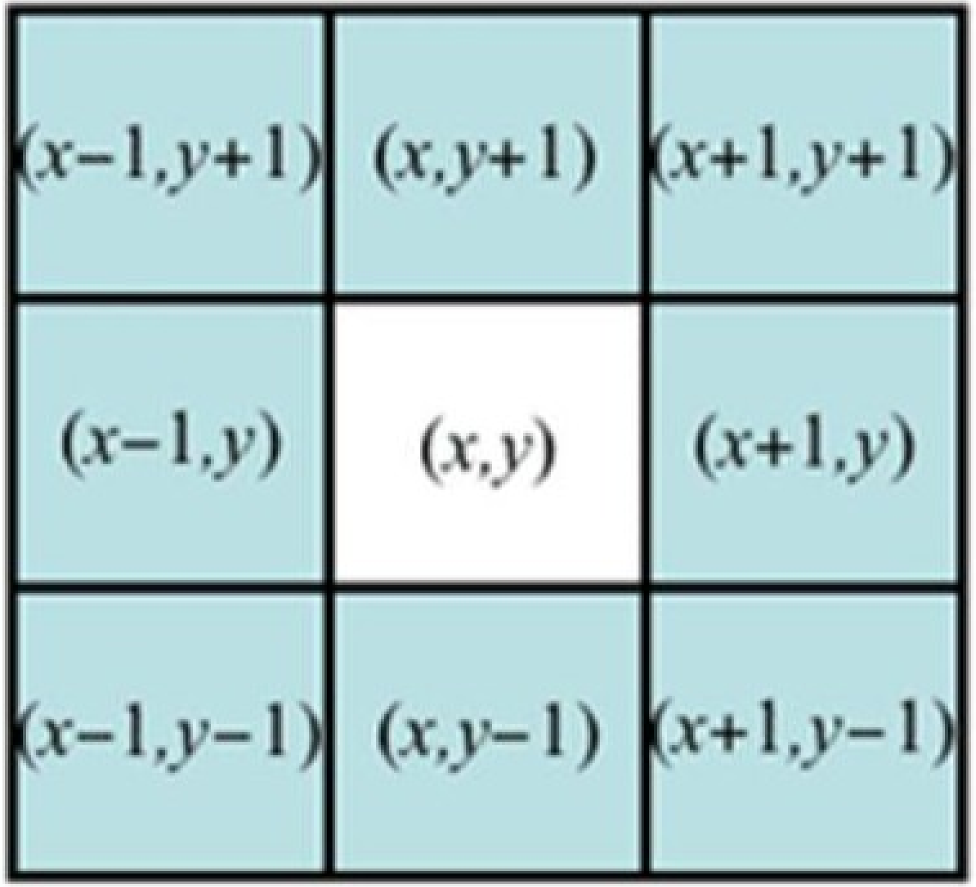
N_4 (p) : 4-neighbors of p(x,y)

1. Any pixel p(x, y) has two vertical and two horizontal neighbors, given by (x+1,y), (x-1, y), (x, y+1), (x, y-1)
2. This set of pixels are called the 4-neighbors of P, and is denoted by $N_4(P)$
3. Each of them is at a unit distance from P



N_D (p) : Diagonal Neighbor

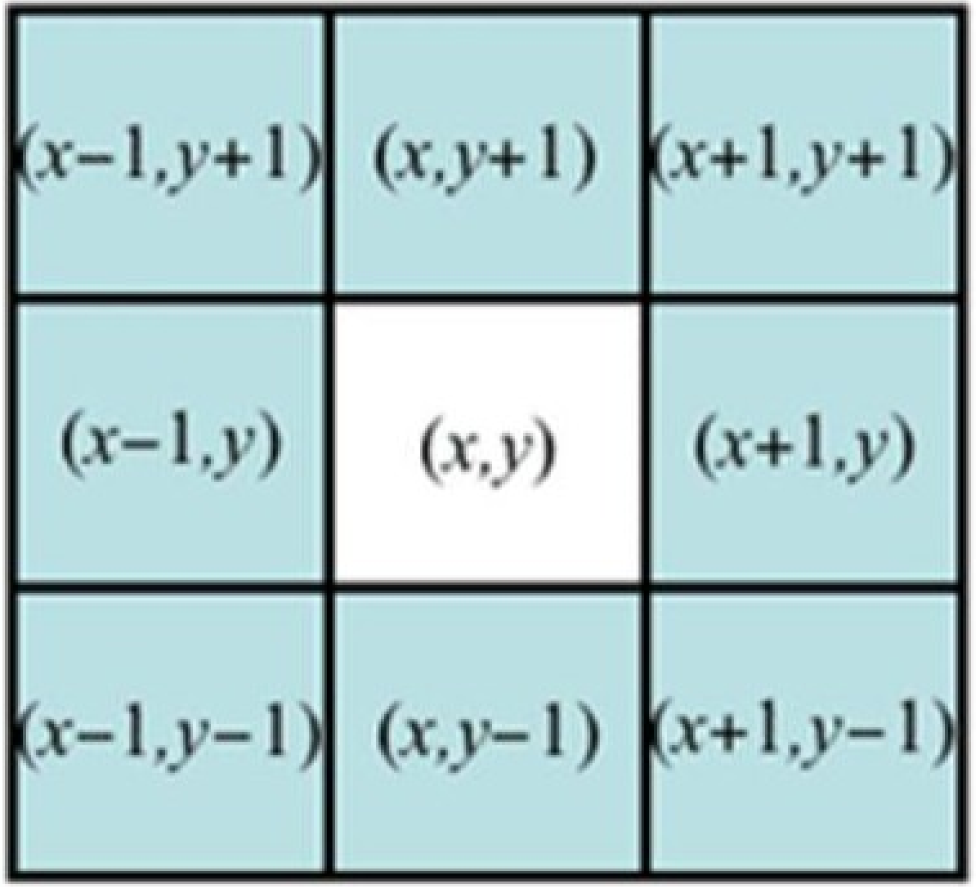
1. 4 diagonal neighbors of P have coordinates: (x+1,y+1), (x+1,y-1), (x-1,y+1), (x-1,y-1)
2. Each of them are at Euclidean distance of 1.414 from P.



Neighbourhood

N_8 (p)

1. N_4(P)and N_D(P) together are called 8-neighbors of P, denoted by N_8(p).
2. $N_8 = N_4 \cup N_D$
3. Some of the points in the N_4, N_D and N_8 may fall outside image when P lies on the border of image.



Adjacency

Two pixels are connected if they are neighbors and their gray levels satisfy some specified criterion of similarity.

Let v be set of gray levels values

- 4-adjacency: Two pixels p and q with values from v are 4-adjacent if q is in the set N_4 (p).

- 8-adjacency: Two pixels p and q with values from v are 8-adjacent if q is in the set N_8 (p).

Adjacency

- m-adjacency (mixed): two pixels p and q with values from v are m-adjacent if:
  - q is in N_4 (p), or
  - q is in N_D (p) and The set $N_4(p) \cap N_4(q)$ has no value from v (No intersection).

Two image subsets S1 and S2 are adjacent if some pixel in S1 is adjacent to S2

Connectivity

- Connectivity is to determine whether the pixels are adjacent in some sense.
- Connectivity is an important concept in establishing boundaries of object and components of regions in an image
- When are two pixels connected?
  - If they are adjacent in some sense (say they are 4-neighbors)
  - and, if their gray levels satisfy similarity (say they are equal)

Example: given a binary image (e.g. gray scale = [0,1]), two pixels may be 4-neighbors but are not considered connected unless they have the same value

0	1
1	1

Connectivity

- Let V be the set of values used to determine connectivity
  - For example, in a binary image, $V=\{1\}$ for the connectivity of pixels with a value of 1
  - In a gray scale image, for the connectivity of pixels with a range of intensity values of, say, 32 to 64, it follows that $V=\{32,33,\ldots,63,64\}$

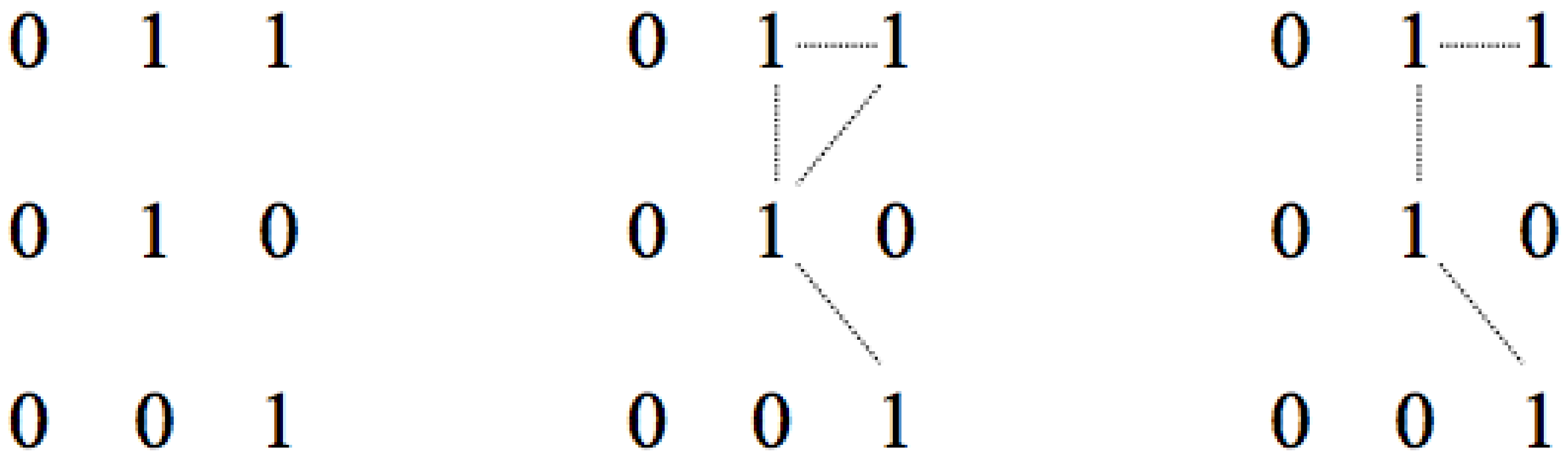
Connectivity

- Consider three types of connectivity
  - 4-connectivity: Pixels p and q with values from V are 4-connected if q is in the set $N_4(p)$
  - 8-connectivity: Pixels p and q with values from V are 8-connected if q is in the set $N_8(p)$
  - m-connectivity (mixed): Pixels p and q with values from V are m-connected if
    - q is in the set $N_4(p)$

    or

    - q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ is empty

Connectivity between pixels



An arrangement of pixels

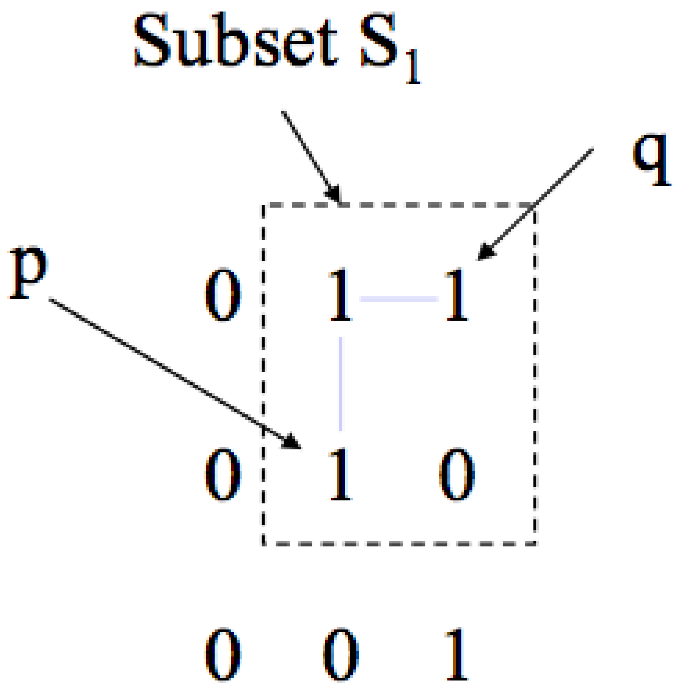
8-connectivity of the pixels $V=\{1\}$

m-connectivity of the pixels $V=\{1\}$

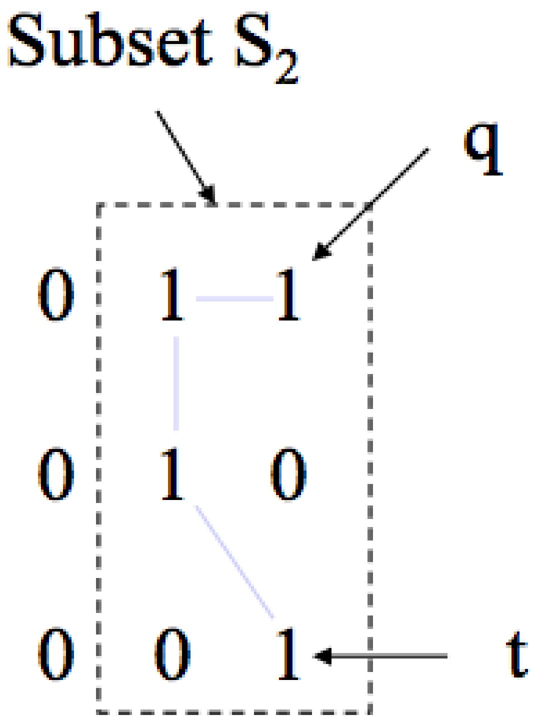
Paths

- A path from *p* at (x,y) to *q* at (s,t) is a sequence of distinct pixels with coordinates (x_0,y_0), (x_1,y_1),....., (x_n,y_n)
  - Where $(x_0,y_0)=(x,y)$ and $(x_n,y_n)=(s,t)$ and
  - (x_i,y_i) is adjacent to (x_{i-1},y_{i-1}) for $1 \le i \le n$
  - n is the *length* of the path
- If *p* and *q* are in *S*, then *p* is connected to *q* in *S* if there is a path from *p* to *q* consisting entirely of pixels in *S*

Example paths



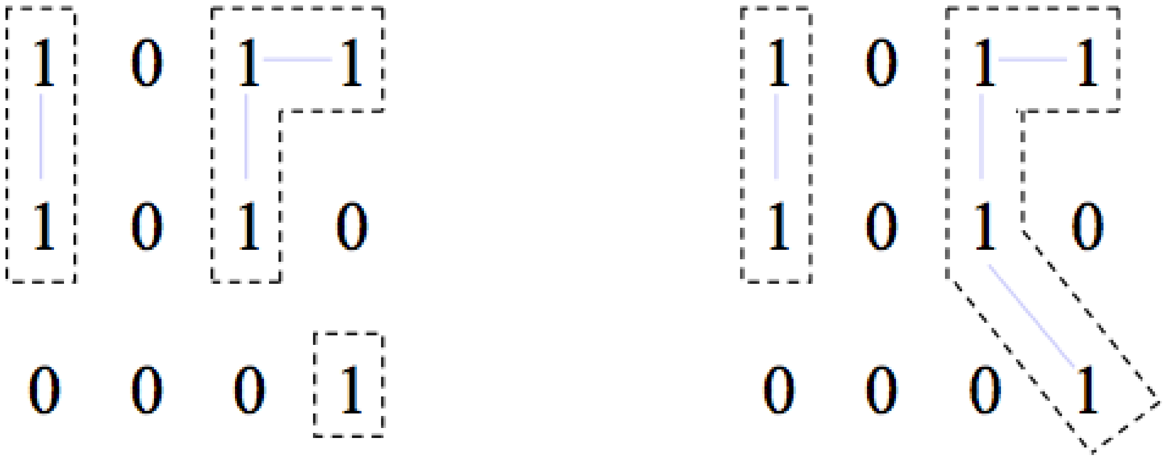
A 4-connected path from p to q (n=2). p and q are connected in S_1



An m-connected path from t to q (n=3). t and q are connected in S_2

Connected components

- For any pixel *p* in *S*, the set of pixels connected to *p* form a connected component of *S*
- Distinct connected components in *S* are said to be *disjoint*

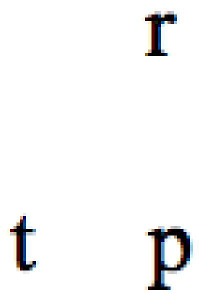


3 4-connected components of S

2 m-connected components of S

Labeling 4-connected components

- Consider scanning an image pixel by pixel from left to right and top to bottom



  - Assume, for the moment, we are interested in 4-connected components
  - Let *p* denote the pixel of interest, and *r* and *t* denote the upper and left neighbors of *p*, respectively
  - The nature of the scanning process assures that *r* and *t* have been encountered (and labelled if 1) by the time *p* is encountered

Regions and Boundaries

Region

- R is a region in an image if R is a connected set
- Two regions, R_i and R_j are adjacent if their union forms a connected set
- Regions that are not adjacent are disjoint
- Region adjacency is defined with respect to both 4-adjacency and 8-adjacency

Boundary

- The boundary of a region R is the set of points (in R) which are adjacent to points in the complement of R
- A boundary is defined in terms of 4- or 8- adjacent

Distance measures

- Given pixels *p*, *q*, and *z* at (*x*,*y*), (*s*,*t*) and (*u*,*v*) respectively, *D* is a *distance function* (or *metric*) if:
  - $D(p,q) \geq 0$ ($D(p,q)=0$ iff $p=q$),
  - $D(p,q) = D(q,p)$, and
  - $D(p,z) \leq D(p,q) + D(q,z)$.
- The *Euclidean distance* between *p* and *q* is given by:

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

- The pixels having distance less than or equal to some value *r* from (x,y) are the points contained in a disk of radius *r* centered at (x,y)

Distance measures

- The D_4 distance (also called the *city block distance*) between *p* and *q* is given by:

$$D_4(p,q) = |x-s| + |y-t|$$

- The pixels having a D_4 distance less than some *r* from (x,y) form a diamond centered at (x,y)
- Example: pixels where $D_4 \leq 2$

$$\begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

Note: Pixels with D_4=1 are the 4-neighbors of (x,y)

Distance measures

- The D_8 distance (also called the *chessboard distance*) between *p* and *q* is given by:

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

- The pixels having a D_8 distance less than some *r* from (x,y) form a square centered at (x,y)
- Example: pixels where D8 $\leq$ 2

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Note: Pixels with D_8=1 are the 8-neighbors of (x,y)

Distance measures and connectivity

- The D_4 distance between two points *p* and *q* is the shortest 4-path between the two points
- The D_8 distance between two points *p* and *q* is the shortest 8-path between the two points
- D_4 and D_8 may be considered, regardless of whether a connected path exists between them, because the definition of these distances involves only the pixel coordinates

Distance measures and m-connectivity

Consider the given arrangement of pixels and assume

- p, p_2 and $p_4 = 1$
- p_1 and p_3 can be 0 or 1

p_3 p_4

p_1 p_2

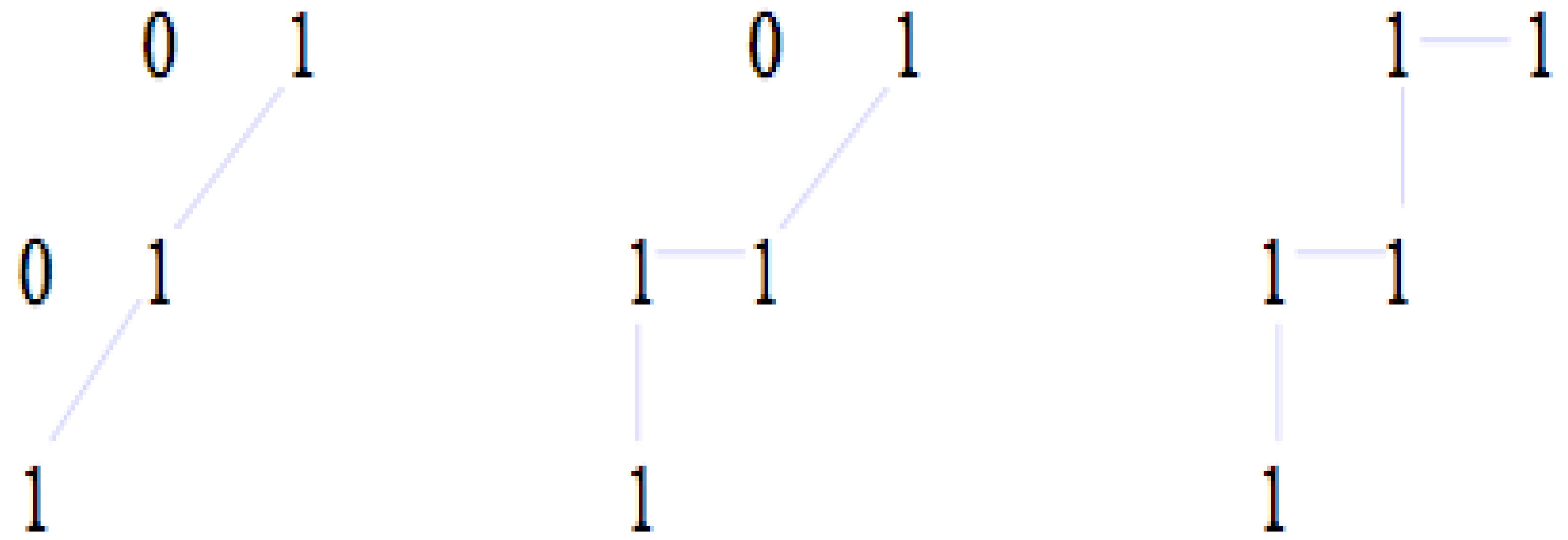
p

If $V=\{1\}$ and p_1 and p_3 are 0, the m-distance (p, p_4) is 2

If either p_1 or p_3 are 1, the m-distance (p, p_4) is 3

If p_1 and p_3 are 1, the m-distance (p, p_4) is 4

M-connectivity example



m-distance(p,p_4)=2 *m-distance*(p,p_4)=3 *m-distance*(p,p_4)=4

Arithmetic & logic operations

- Arithmetic & logic operations on images used extensively in most image processing applications
- May cover the entire image or a subset Arithmetic operation between pixels p and q are defined as:
  - Addition: $(p + q)$

    Used often for image averaging to reduce noise
  - Subtraction: $(p - q)$

    Used often for static background removal
  - Multiplication: $(p * q)$ (or $p\ q$, $p \times q$)

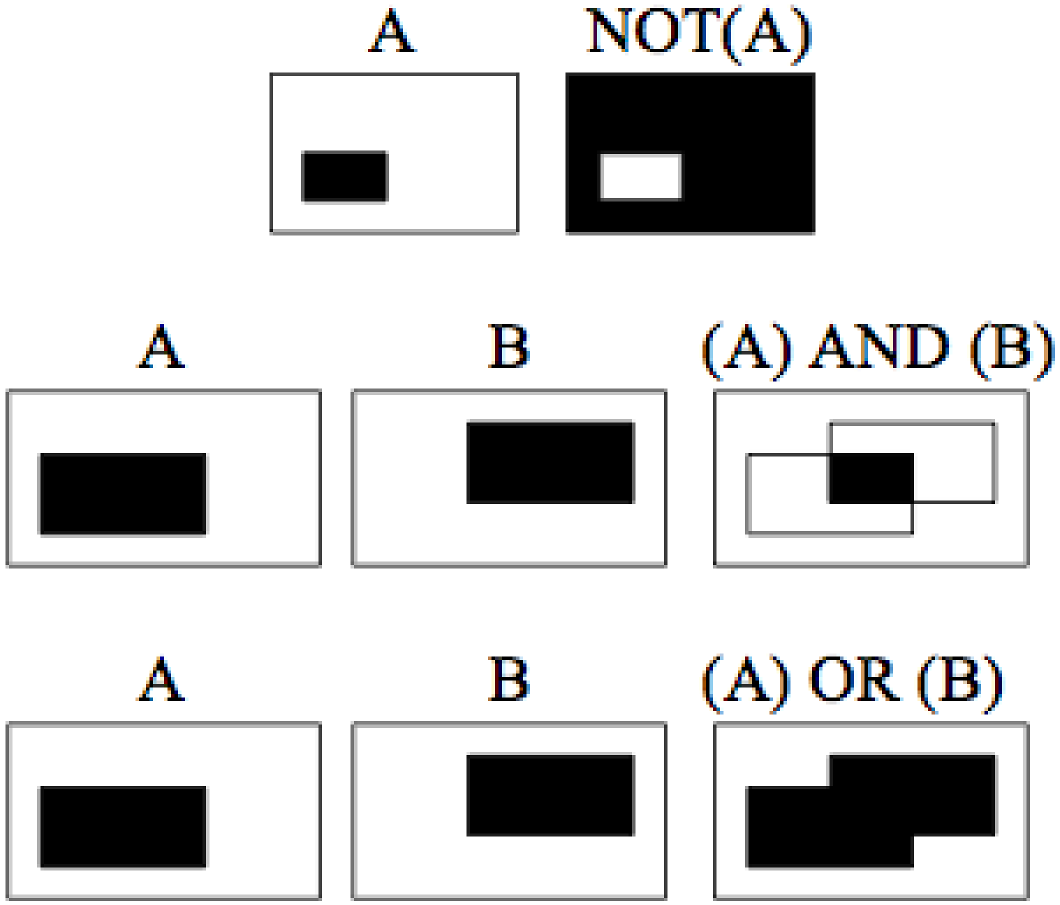
    Used to correct gray-level shading
  - Division: $(p \div q)$ (or p/q)

    As in multiplication

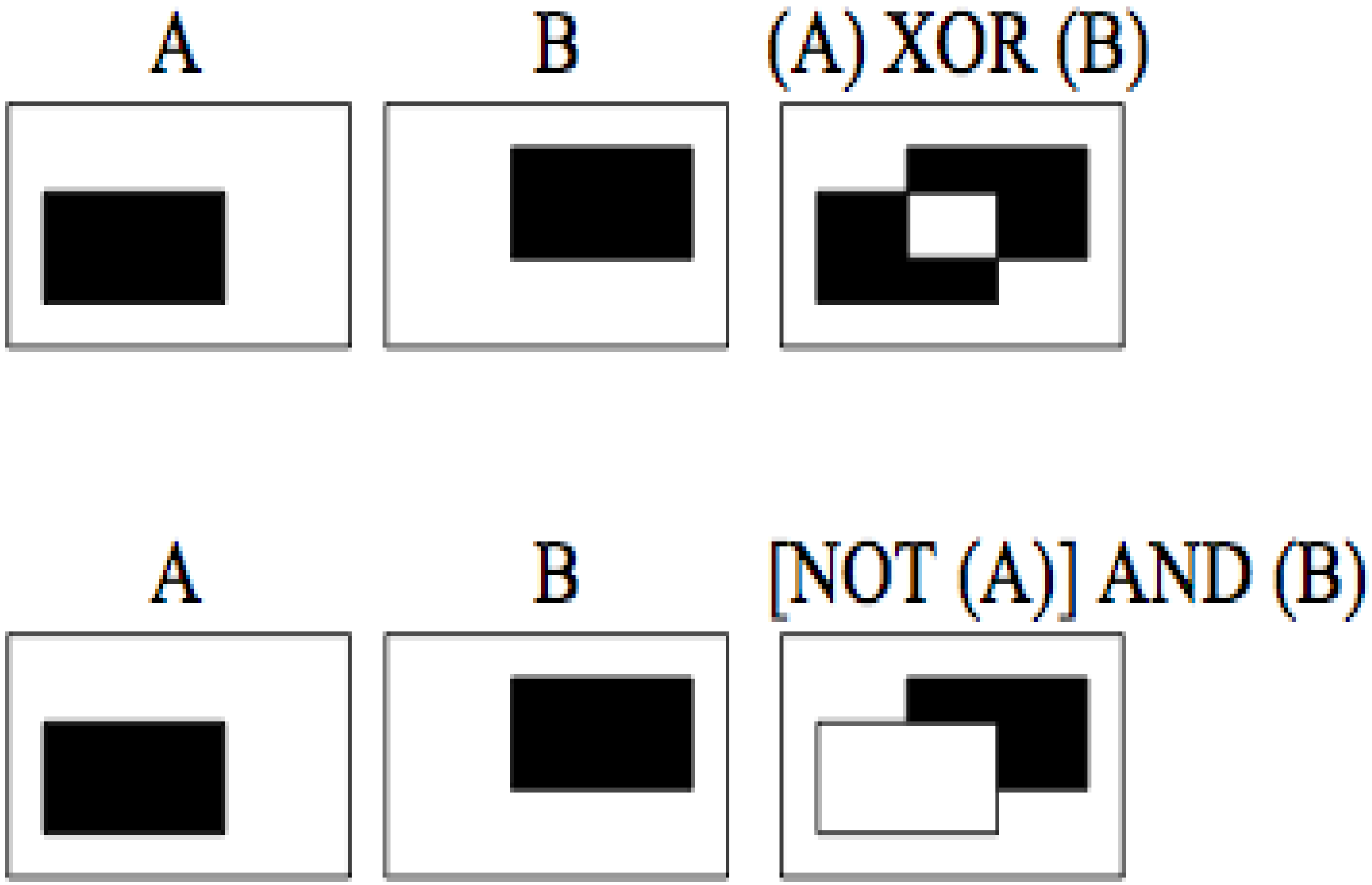
Logic operations

- Arithmetic operation between pixels *p* and *q* are defined as:
  - AND: *p* AND *q* (also p·q)
  - OR: *p* OR *q* (also p+q)
  - COMPLEMENT: NOT*q* (also *q'*)
- Applicable to binary images
- Basic tools in binary image processing, used for:
  - Masking
  - Feature detection
  - Shape analysis

Examples of logic operations



Examples of logic operations



Neighbourhood-Oriented Operations

- Arithmetic and logical operations may take place on a subset of the image
  - Typically neighbourhood oriented
  - Formulated in the context of *mask* operations
- Basic concept:

  Let the value of a pixel be a function of its (current) gray level and the gray level of its neighbors (in some sense)

Neighbourhood-oriented operations

- Consider the following subset of pixels in an image
- Suppose we want to filter the image by replacing the value at Z_5 with the average value of the pixels in a 3x3 region centered around Z_5
- Perform an operation of the form:

$$z = \frac{1}{9}(z_1 + z_2 + \ldots + z_9) = \frac{1}{9}\sum_{i=1}^{9} z_i$$

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

and assign to z_5 the value of z

Neighbourhood-Oriented Operations

- In the more general form, the operation may look like:

$$z = (w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9) = \sum_{i=1}^{9} w_i z_i$$

- This equation is widely used in image processing
- Proper selection of coefficients (weights) allows for operations such as
  - noise reduction
  - region thinning
  - edge detection

Z_1	Z_2	Z_3
Z_4	Z_5	Z_6
Z_7	Z_8	Z_9

W_1	W_2	W_3
W_4	W_5	W_6
W_7	W_8	W_9