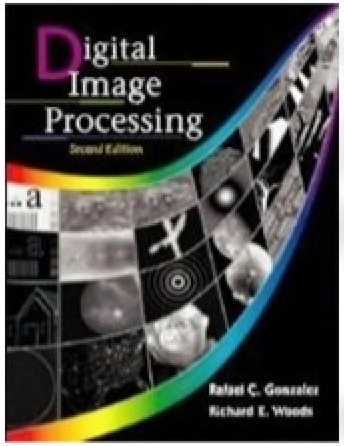
Introduction to Morphological Image Processing

REFERENCES



"Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002

Much of the material that follows is taken from this book

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Introduction
Morphological Processing

- Morphology is concerned with image analysis methods whose outputs describe image content (i.e. extract "meaning" from an image).
- Mathematical morphology is a tool for extracting image components that can be used to represent and describe region shapes such as boundaries and skeletons.
- Morphological methods include filtering, thinning and pruning. These techniques are based on set theory.
- All morphology functions are defined for binary images, but most have natural extension to grayscale images.

Introduction
Morphological Processing

- Introduction to **Morphological Operators**
  - Used generally on binary images, e.g., background subtraction results!
  - Used on gray value images, if viewed as a stack to binary images.
- Good for, e.g.,
  - Noise removal in background
  - Removal of holes in foreground / background

Basic Concepts of Set Theory

- A set is specified by the elements between two braces: { }.
- The elements of the sets are the coordinates (x,y) of pixels representing objects or other features in an image.

Basic Concepts of Set Theory

Let A be a set in 2D image space Z^2:

- If $a = (a_1, a_2)$ is an element of A, then $a \in A$
- If a is not an element of A, then $a \notin A$
- Empty set is a set with no elements and is denoted by $\emptyset$
- If every element of a set A is also an element of another set B, then A is said to be a subset of B, denoted as $A \subseteq B$
- The union of two sets A and B, denoted by $C = A \cup B$
- The intersection of two sets A and B, denoted by $C = A \cap B$
- Two sets A and B are said to be disjoint, if they have no common elements. This is denoted by $A \cap B = \emptyset$

Basic Concepts of Set Theory

- The *complement* of a set A is the set of elements not contained in A.

  This is denoted by $A^c = \{w \mid w \notin A\}$

- The *difference* of two sets A and B, denoted $A - B$, is defined as

  $A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$

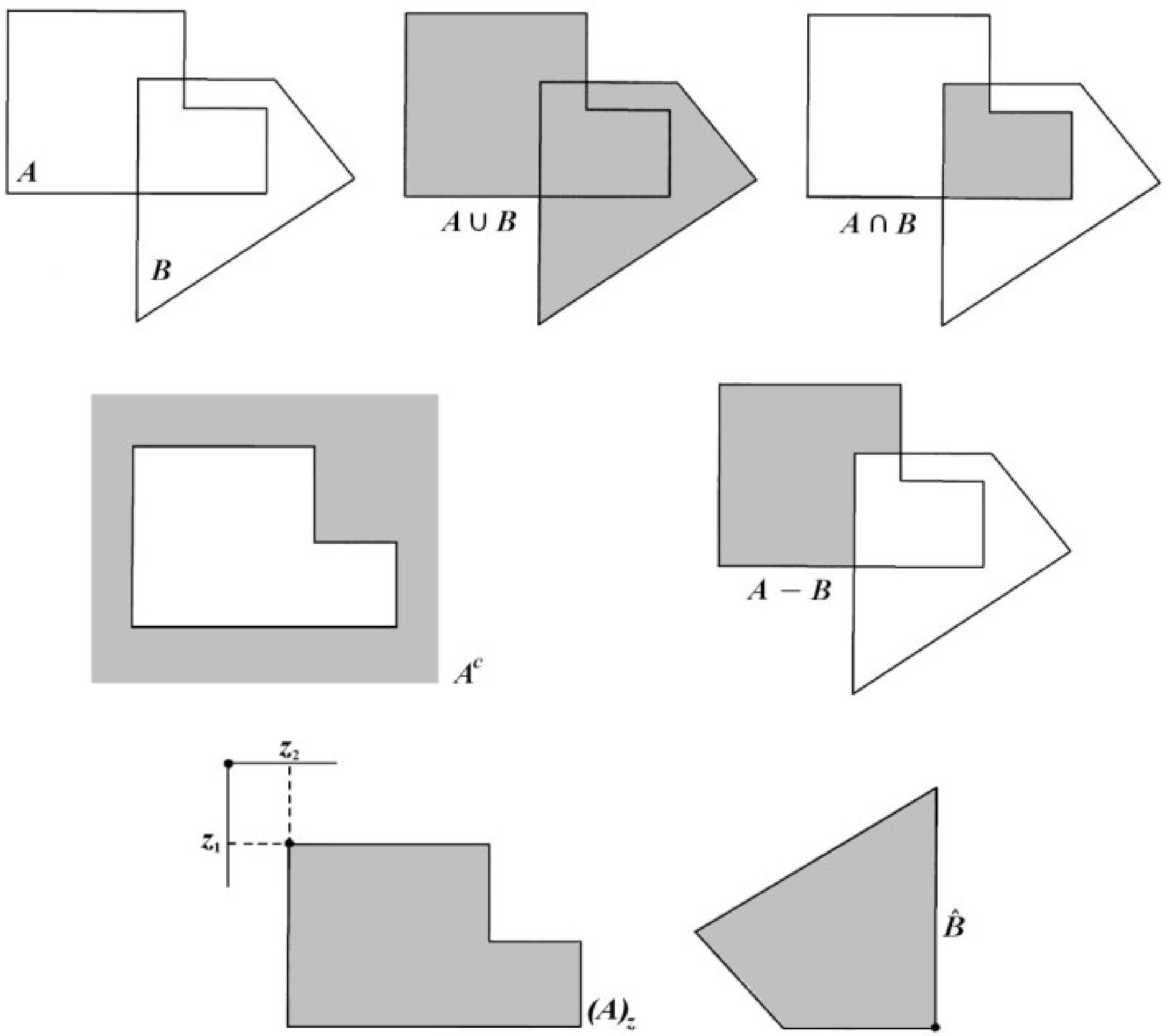
- The *reflection* of set B, denoted $\hat{B}$, is defined as

  $\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$

- The *translation* of set A by point $z = (z_1, z_2)$, denoted $(A)_z$ is

  defined as $(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$

Basic Concepts of Set Theory



Logic Operations Involving Binary Images

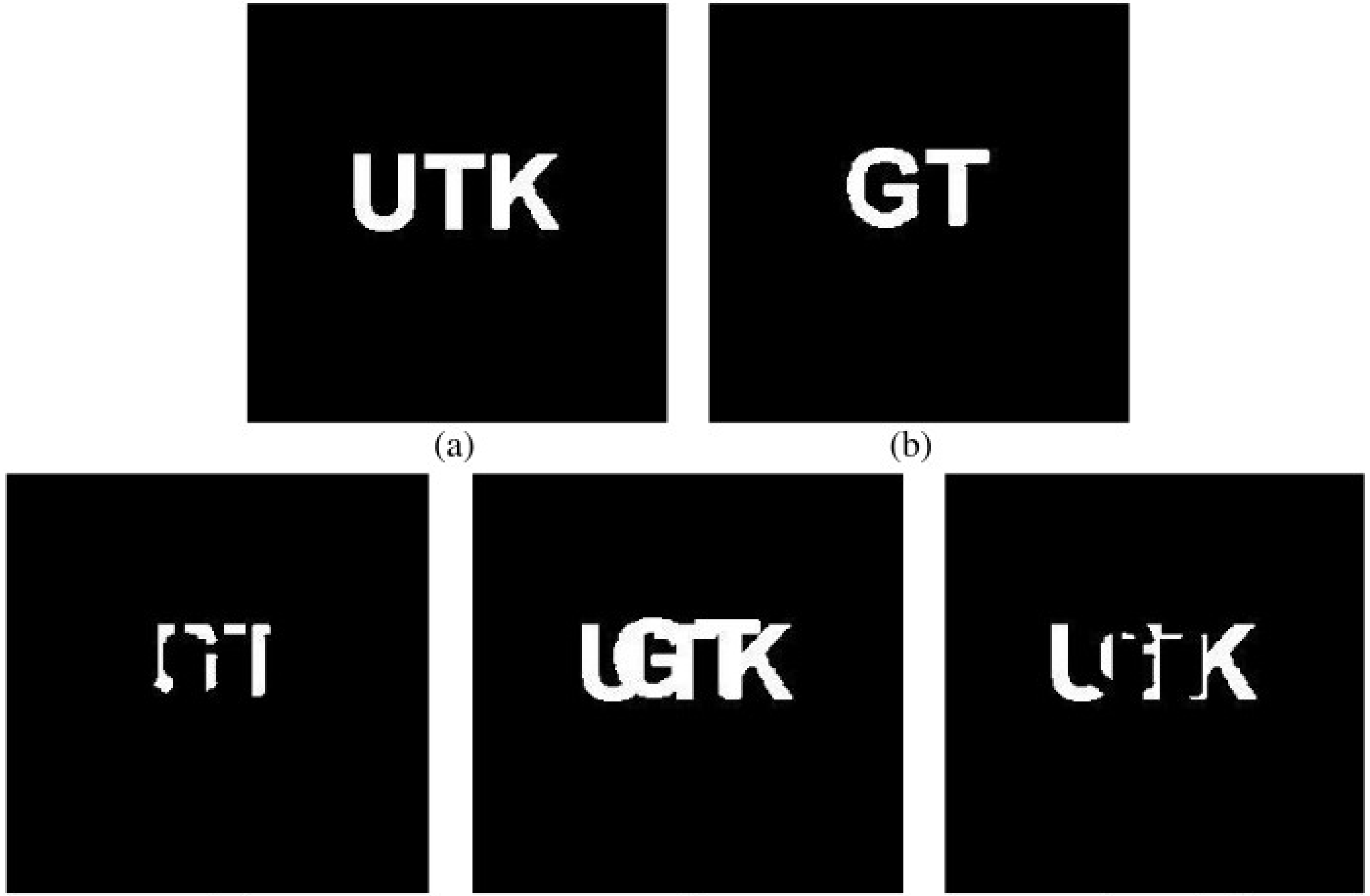
A binary image is an image whose pixel values are 0 (representing black) or 1 (representing white, i.e. 255).

The usual set operations of complement, union, intersection, and difference can be defined easily in terms of the corresponding logic operations NOT, OR and AND.

For example:

- Intersection operation $\cap$ is implemented by AND operation
- Union operation $\cup$ is implemented by OR operation

Logic Operations Involving Binary Images



(a) (b)

a & b   a | b   a – b = a & b^c

Structuring Element

- A morphological operation is based on the use of a filter-like binary pattern called the structuring element of the operation.

- Structuring element is represented by a matrix of 0s and 1s; for simplicity, the zero entries are often omitted.

- The origin of structuring element must be clearly identified

Structuring Element

Line

$$\begin{array}{ccccc} & & & & 1 \\ & & & 1 & \\ & & \boxed{1} & & \\ & 1 & & & \\ 1 & & & & \end{array} \qquad \begin{array}{c} 1 \\ \boxed{1} \\ 1 \end{array}$$

Diamond

$$\begin{array}{ccc} 0 & 1 & 0 \\ 1 & \boxed{1} & 1 \\ 0 & 1 & 0 \end{array}$$

Non -Symmetric

$$\begin{array}{ccccccc} & & & & & & 1 \\ 1 & 1 & \boxed{1} & 1 & 1 & 1 & \\ 1 & 1 & & & & & \\ 1 & & & & & & \end{array} \quad \xrightarrow{\text{Reflection on origin}} \quad \begin{array}{ccccccc} & & & & & & 1 \\ & & & & & 1 & 1 \\ & 1 & 1 & 1 & \boxed{1} & 1 & 1 \\ 1 & & & & & & \end{array}$$

Fitting and Hitting

When a SE is placed in a binary image, each of the pixels in the SE is associated with the corresponding pixel of the neighborhood under the structuring element.

Similar to the process of convolution, however the operation is logical rather than arithmetic

Fitting and Hitting

Fitting:

The SE is said to fit an image if, for each of its pixels that is set to 1, the corresponding image pixel is also 1

Hitting :

The SE is said to hit, or intersect an image if, for any of its pixels that is set to 1, the corresponding image pixel is also 1

Dilation

Dilation is an operation used to grow or thicken objects in binary images. The dilation of a binary image A by a structuring element B is defined as:

$$A \oplus B = \{ z : (\hat{B})_z \cap A \neq \emptyset \}$$

This equation is based on obtaining the reflection of B about its origin and translating (shifting) this reflection by z.

Then, the dilation of A by B is the set of all structuring element origin locations where the reflected and translated B overlaps with A by at least one element.

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1								

Fully Matched :	1
Some Match:	1
No Match:	0

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0							

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1						

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1					

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1	1				

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1	1	1			

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1	1	1	1		

Dilation

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	1	0	1	1	1	1	1	1	

Dilation

1
1
$\boxed{1}$
1
1

Structuring element

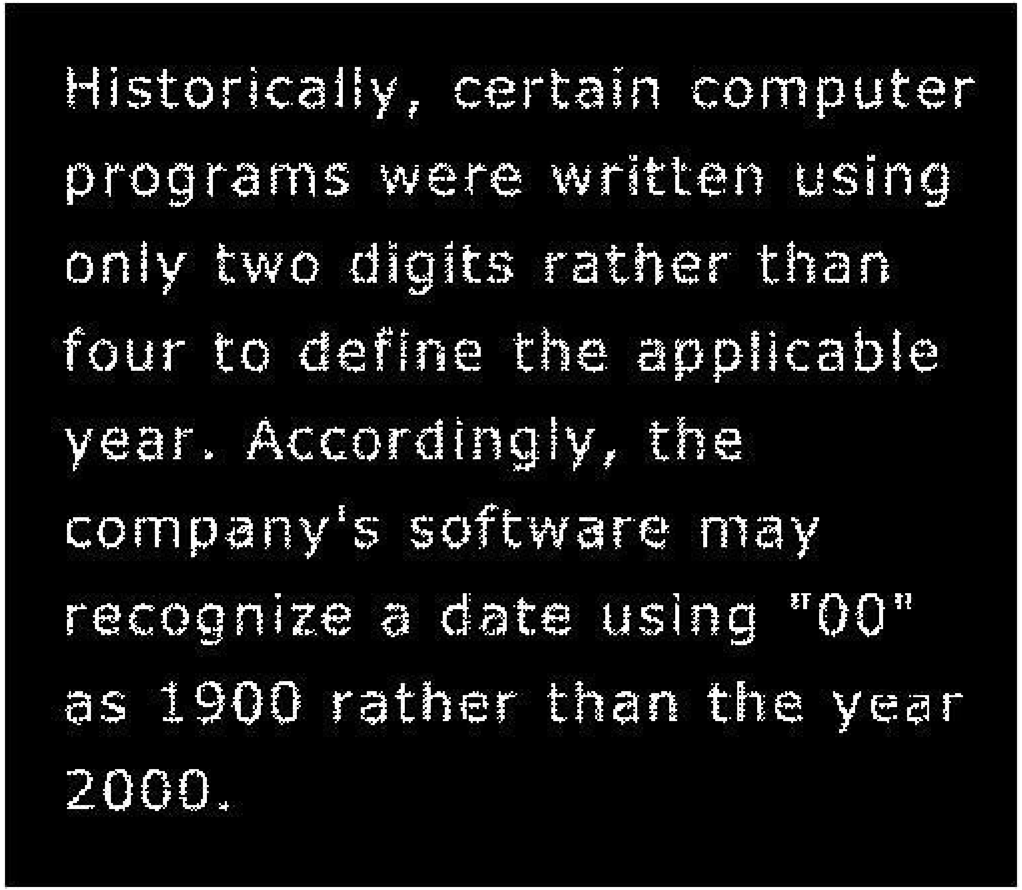
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Binary image

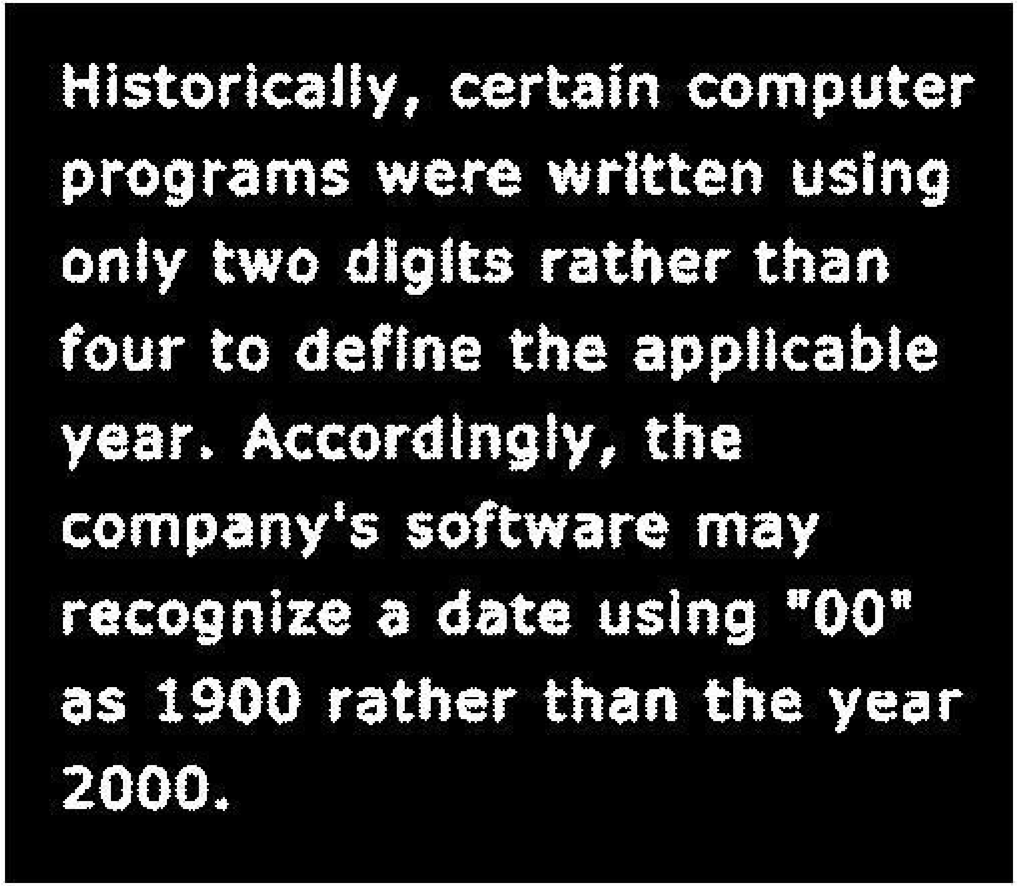
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	*1*	*1*	*1*	*1*	*1*	*1*	0
0	0	0	0	*1*	*1*	*1*	*1*	*1*	*1*	*1*	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	*1*	*1*	0
0	0	*1*	**1**	**1**	**1**	**1**	**1**	**1**	*1*	0	0
0	*1*	*1*	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	*1*	*1*	*1*	*1*	*1*	*1*	*1*	0	0	0	0
0	*1*	*1*	*1*	*1*	*1*	*1*	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$A \oplus B$

Dilation



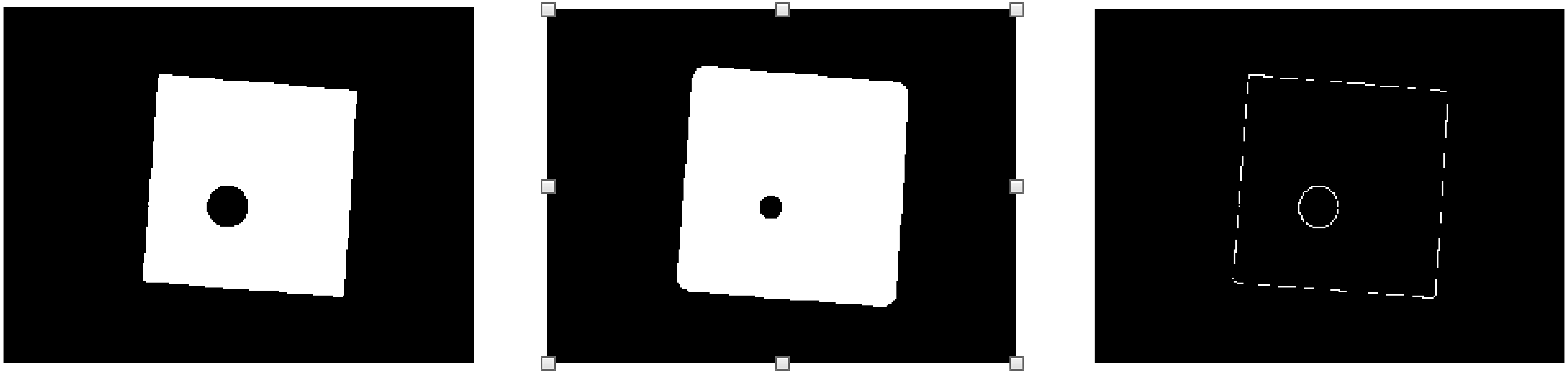
Broken-text binary image



Dilated image

Dilation

- Dilate input image
- Subtract input image from dilated image
- Edges remain!



Erosion

Erosion is used to shrink or thin objects in binary images. The erosion of a binary image A by a structuring element B is defined as:

$$A \ominus B = \{ z \ : \ (B)_z \cap A^c \neq \emptyset \}$$

The erosion of A by B is the set of all structuring element origin locations where the translated B does not overlap with the background of A.

Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0								

Fully Matched :	1
Some Match:	0
No Match:	0

Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0							

Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0						

Erosion

Input image	1	0	0	0	1	1	1	0	1	1

Structuring Element	1	1	1

Output Image		0	0	0	0					

Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1				

Erosion

Input image	1	0	0	0	1	1	1	0	1	1
Structuring Element						1	1	1		
Output Image		0	0	0	0	1	0			

Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1	0	0		

Erosion

Input image

1	0	0	0	1	1	1	0	1	1

Structuring Element

1	1	1

Output Image

	0	0	0	0	1	0	0	0	

Erosion

$$\begin{matrix}1\\ \boxed{1}\\ 1\end{matrix}$$

Structuring element

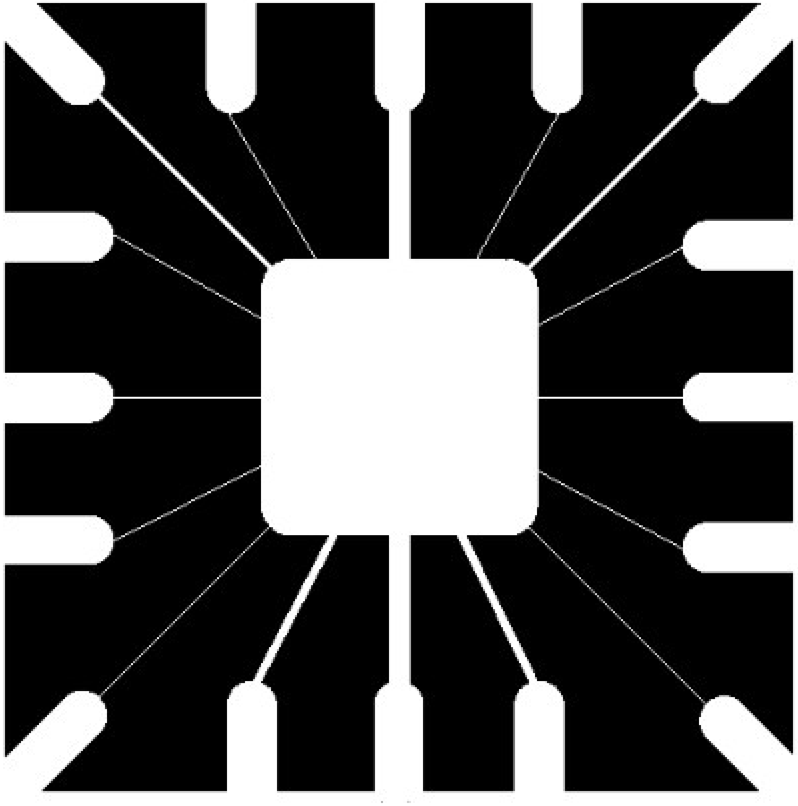
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

Binary image

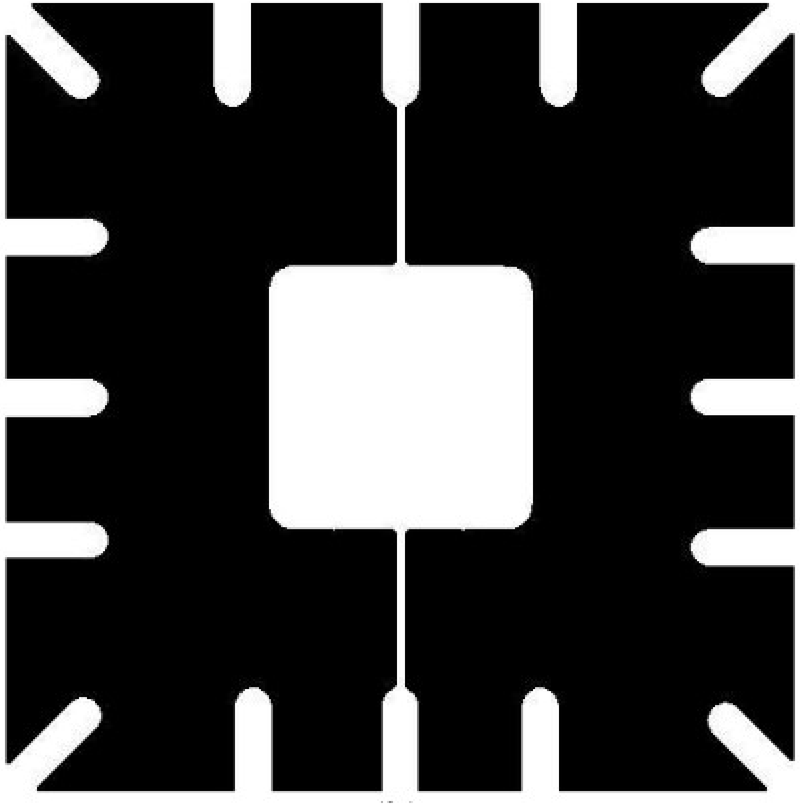
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	*0*	*0*	*0*	*0*	*0*	*0*	0	0	0
0	0	0	**1**	**1**	**1**	**1**	**1**	**1**	0	0	0
0	0	0	*0*	*0*	*0*	*0*	*0*	*0*	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$A \ominus B$

Erosion



Binary image

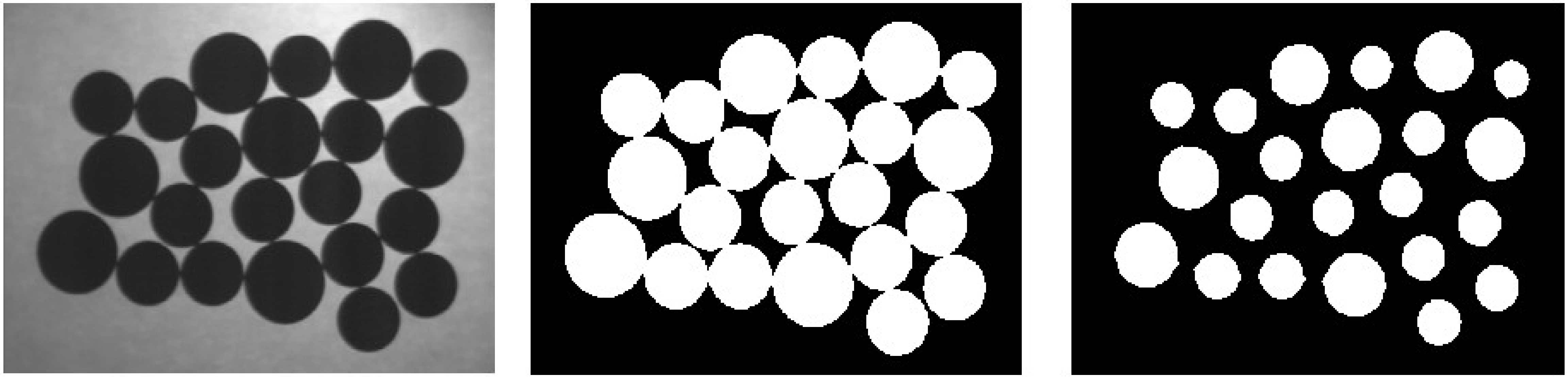


Eroded image.

Erosion

Counting coins is difficult because they touch each other!

Solution: Binarization and Erosion separates them!



Combining Dilation & Erosion - Opening Morphology

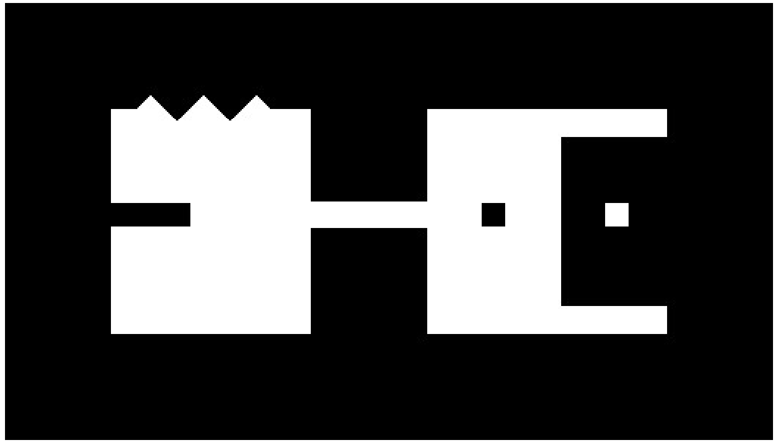
The opening operation erodes an image and then dilates the eroded image using the same structuring element for both operations, i.e.

$$A \circ B = (A \ominus B) \oplus B$$

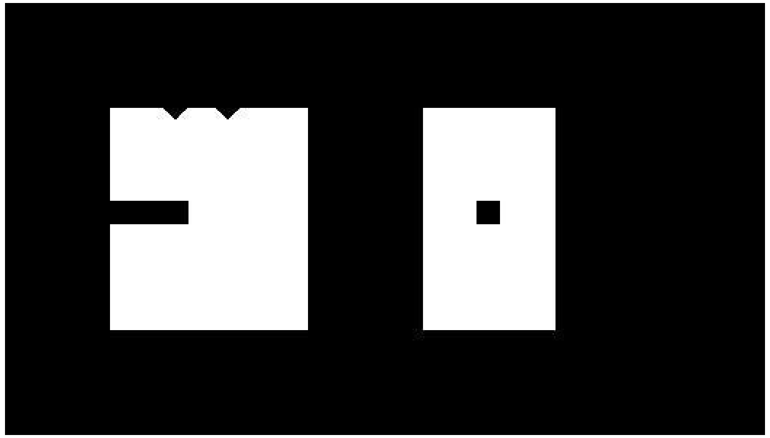
where A is the original image and B is the structuring element.

Combining Dilation & Erosion - Opening Morphology

The opening operation is used to remove regions of an object that cannot contain the structuring element, smooth objects contours, and breaks thin connections



Original binary image



Result of opening with square structuring element of size 20 pixels

Combining Dilation & Erosion - Closing Morphology

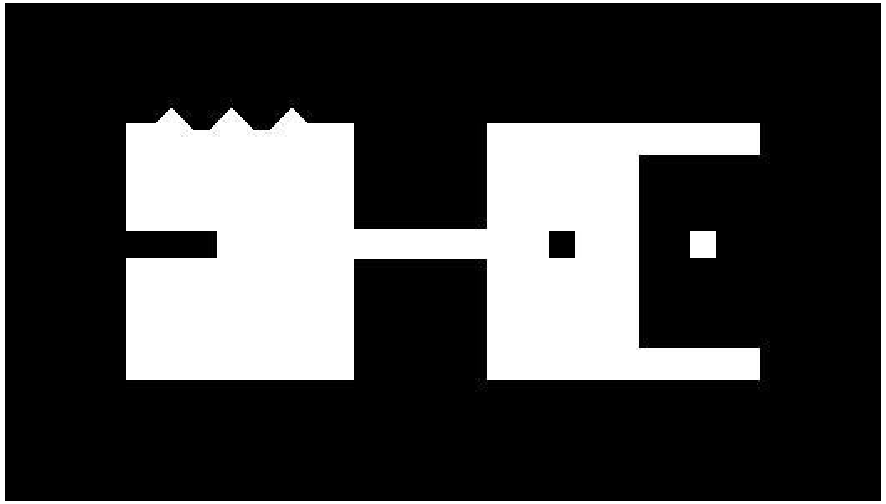
The closing operation dilates an image and then erodes the dilated image using the same structuring element for both operations, i.e.

$$A \bullet B = (A \oplus B) \ominus B$$

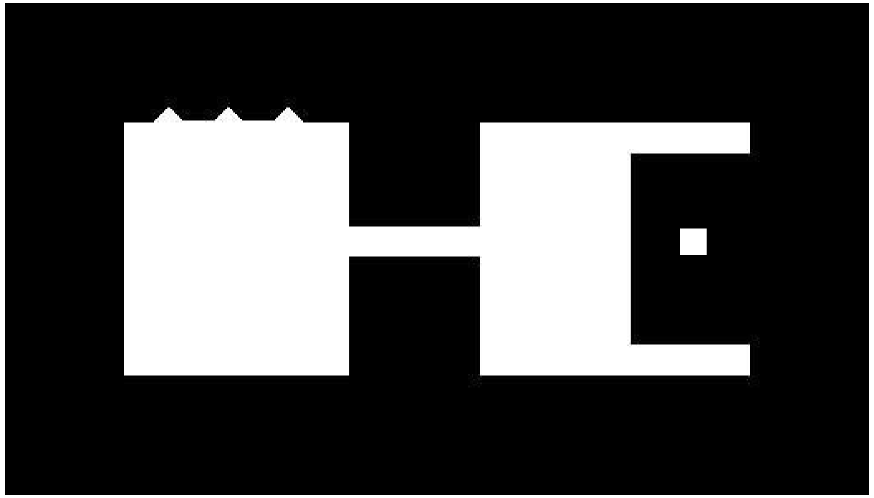
where A is the original image and B is the structuring element.

Combining Dilation & Erosion - Closing Morphology

The closing operation fills holes that are smaller than the structuring element, joins narrow breaks, fills gaps in contours, and smoothes objects contours



Original binary image.



Result of closing with square structuring element of size 20 pixels.

Combining Opening & Closing Morphology

The noise was removed by opening the image, but this process introduced numerous gaps in the ridges of the fingerprint.

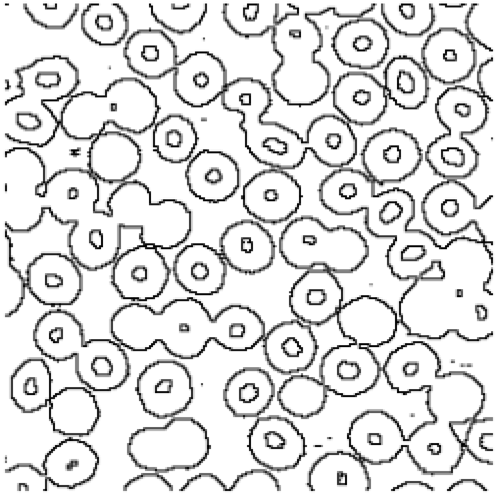
These gaps can be filled by following the opening with a closing operation

Figure 11.8 (a) Noisy fingerprint. (b) Result of opening (a) with square structuring element of size 3 pixels. (c) Result of closing (b) with the same structuring element.

Skeletonization

- Morphological skeletonization reduces a region to its minimum number of connected “1”-valued pixels
- Sometimes known as thinning
- Connected image regions must thin to connected line structures. The thinning operation should preserve the connectivity of the original.

Extracted Boundaries



Skeletonization