

# CSCC24: Principles of Programming Languages

## Notes

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## Contents

<b>1</b>	<b>Thursday, January 11, 2017</b>	<b>5</b>
1.1	The Root Cause of this Course . . . . .	5
1.1.1	Higher-order Functions on Aggregates . . . . .	5
1.1.2	Combining Forms . . . . .	6
1.1.3	Example Topic: Evaluation Order . . . . .	7
1.1.4	Example Topic: Scheme Macros . . . . .	7
1.1.5	Dynamic and Static Typing . . . . .	7
1.1.6	Parametric Polymorphism . . . . .	8
1.1.7	What is "Powerful"? – The Tradeoff . . . . .	9
<b>2</b>	<b>Thursday, January 18, 2018</b>	<b>10</b>
2.1	Racket . . . . .	10
2.2	Basic Data Types . . . . .	10
2.3	Procedures and Functions . . . . .	10
2.4	Boolean Operations . . . . .	10
2.5	Number operations . . . . .	10
2.6	Equality . . . . .	11
2.7	Definitions . . . . .	11

2.8	Anonymous Functions . . . . .	12
2.9	Conditionals . . . . .	12
2.9.1	and,or as conditionals . . . . .	12
2.10	Local bindings . . . . .	13
2.10.1	Recursive local bindings . . . . .	13
2.11	Recommended Code Layout . . . . .	13
<b>3</b>	<b>Thursday, January 25, 2018</b>	<b>15</b>
3.1	Pairs and Lists . . . . .	15
3.2	User-Defined Records . . . . .	15
3.3	Pattern Matching . . . . .	15
3.4	Input and Output . . . . .	16
3.4.1	ports . . . . .	16
3.5	Sequencing . . . . .	16
3.6	Mutable Variables . . . . .	16
3.7	map . . . . .	17
3.8	filter . . . . .	17
<b>4</b>	<b>Thursday, February 1, 2018</b>	<b>18</b>
4.1	Scheme (cont'd) . . . . .	18
4.1.1	foldl . . . . .	18
4.1.2	foldr . . . . .	18
4.1.3	Procedure-Call Stack . . . . .	19
4.1.4	Non-Tail Calls and Tail Calls . . . . .	19
4.2	Haskell . . . . .	19
4.2.1	Expressions and Types . . . . .	19
4.2.2	Definitions . . . . .	21
4.2.3	Function Applications . . . . .	21
<b>5</b>	<b>Thursday, February 8, 2018</b>	<b>22</b>
5.1	Haskell (cont'd) . . . . .	22
5.1.1	Local Definitions For Expressions . . . . .	22
5.1.2	Local Definitions For Definitions . . . . .	22
5.1.3	Pattern Matching . . . . .	22
5.1.4	Guards . . . . .	23
5.1.5	Local Definitions under Patterns and Guards . . . . .	23
5.1.6	List Comprehension . . . . .	23
5.1.7	Algebraic Data Types . . . . .	24

5.1.8	Parametric Polymorphism . . . . .	24
5.1.9	Type-Class Polymorphism . . . . .	24
<b>6</b>	<b>Thursday, February 15, 2018</b>	<b>26</b>
6.1	Haskell (cont'd) . . . . .	26
6.1.1	Constraint Instances . . . . .	26
6.1.2	User-Defined Class . . . . .	26
6.1.3	Auto-Generating Instance Implementations . . . . .	27
6.1.4	Haskell's Number System . . . . .	27
6.1.5	Functor . . . . .	27
6.1.6	Applicatives . . . . .	28
<b>7</b>	<b>Thursday, March 1, 2018</b>	<b>29</b>
7.1	Haskell (cont'd) . . . . .	29
7.1.1	Monads . . . . .	29
7.1.2	Haskell I/O System . . . . .	31
7.2	Syntax . . . . .	31
7.2.1	Context-Free Grammar (CFG) . . . . .	31
7.2.2	Derivation (aka Generation) . . . . .	32
7.2.3	Backus-Naur Form (BNF) . . . . .	32
7.2.4	Parse Tree (aka Derivation Tree) . . . . .	33
7.2.5	Ambiguous Grammar . . . . .	34
7.2.6	Unambiguous Grammar Example . . . . .	34
7.2.7	Left Recursive vs Right Recursive . . . . .	34
7.2.8	Recursive Descent Parsing . . . . .	35
7.2.9	Recursive Descent Parser Example . . . . .	35
<b>8</b>	<b>Thursday, March 8, 2018</b>	<b>36</b>
8.1	ParserLib.hs . . . . .	36
8.1.1	Parser Implementation . . . . .	40
8.1.2	anyChar . . . . .	40
8.1.3	satisfy and char . . . . .	40
8.1.4	eof . . . . .	41
8.1.5	empty, many, and some . . . . .	41
8.1.6	whitespace, terminal, integer, identifier . . . . .	41
8.1.7	chainl1, chainr1 . . . . .	41
8.1.8	between . . . . .	41

<b>9</b>	<b>Thursday, March 15, 2018</b>	<b>42</b>
9.1	Num.hs . . . . .	42
9.1.1	Purpose . . . . .	43
9.1.2	Plus function parsers from strings "exprParserL" and "exprParserL2" . . . . .	43
9.1.3	expression Parsing from strings . . . . .	44
9.2	NumVar.hs . . . . .	45
9.2.1	Purpose and interp implementation . . . . .	45
9.2.2	interpM implementation . . . . .	46

# 1 Thursday, January 11, 2017

The purpose of this course is to see the trade-offs between various features in programming languages. This course exists because different programming languages have different features, for example, Java has both class-based OOP and auto-garbage collection while C has neither, but C has union types that Java doesn't have. This means rewriting code into a different language isn't necessarily easy. There may be large semantic differences

## 1.1 The Root Cause of this Course

A guy named "John Backus" gave a lecture for the acceptance for the Turing Award in 1977. He addressed the question, "Can programming be liberated from the von Neumann style?"

Languages then had been only superficial enhancements to the CPU writing 1 word onto memory at a time i.e:

```
1 s := s + a[i]
```

Backus proposed a new direction for programming languages:

- Higher order functions that work on aggregates (a whole list, an array, a dictionary, etc...)
- Combining forms, for example, function composition ( $g \circ f$ )
- Reasoning by algebra, for example, the associative law for a function
- If you need a state, use coarse-grained state transitions rather than changing only one word at a time. (So passing an old state into a stateless function that does a lot and returns an answer and a new state.)

### 1.1.1 Higher-order Functions on Aggregates

Note that the notation to apply a function to several parameters is:  
(Haskell)

```
1 f x y z
```

(Scheme:)

```
1 (f x y z)
```

So in Haskell:

```
1 fmap f [x0, x1, ...]
```

will compute

```
1 [f x0, f x1, ...]
```

And

```
1 fmap abs [3,-1,4]
```

Computes

```
1 [3,-1,4]
```

And

```
1 foldr (+) 0 [3,1,4]
```

Computes

```
1 3+(1+(4+0))
```

Note 2 points:

- "on aggregates" means to work on a whole list at once (such as an array or some "container")
- "Higher-order functions" means that some parameters are functions, so different combinations makes the language more customizable.

Java and MATLAB have the former but lack the latter.

### 1.1.2 Combining Forms

An obvious example is function composition ( $g \circ f$ ).

In Haskell, this is:

```
1 g . f
```

And in Racket (Scheme) this is:

```
1 compose g f
```

For example, the following code computes the 1-norm of your vector.

```
1 foldr (+) 0 . fmap abs
```

There are other combining forms. There is another example in Haskell.

```
1 (f &&& g) x = (f x, g x)
```

The point is that you can combine functions to perform compound tasks, and this type of language is not about shorter code (although it has that side effect), but about working with building blocks.

### 1.1.3 Example Topic: Evaluation Order

You can define your own logical "and" in Scheme

```
1 (define (my-and b c) (if b c #f))
2 (my-and #f (list-ref '(#t #f #t) 10))
```

The second line fails in Scheme, but if typed in the Haskell version, succeeds.

In most languages, parameters are evaluated before passed into the bodies of functions. In Haskell however, parameters are passed as is. Because of this, in Haskell, many short circuiting operators and control constructs are user-definable, and therefore, very customizable.

### 1.1.4 Example Topic: Scheme Macros

Scheme offers a macro system for user-defined constructs:

```
1 (define-syntax-rule (my-and b c) (if b c #f))
```

Now if we run the following code, it succeeds.

```
1 (my-and #f (list-ref '(#t #f #t) 10))
```

The explanation for this is that this is a macro expansion in Scheme, so the parameters are copy-pasted into the macros. This means that there is a downside, for example:

```
1 (define-syntax-rule (double x) (+ x x))
2 (double (* 3 4))
```

The second line spawns two copies of `(* 3 4)` and performs redundant work, while Haskell's version does not. The Upside is that Scheme's macro system offers other flexibilities not shown in this lecture.

### 1.1.5 Dynamic and Static Typing

In Scheme:

```

1  (if #f 0 (+ 0 "hello"))
2  (if #t 0 (+ 0 "hello"))

```

The first line fails but the second line succeeds. This is because Types are checked dynamically. When running the program, only the code that is actually run is checked.

In Haskell, the following line fails:

```

1  if True then 0 else 0 + "hello"

```

The reason for this is because types are checked statically, without running, over all the code. (If this code is compiled, then at compile time, if interpreted, then at load time, etc.) So the error of adding 0 to "hello".

Food for thought, Java is both compiled and interpreted.

### 1.1.6 Parametric Polymorphism

In Haskell, we define:

```

2  trio x = [x, x, x]

```

The inferred type is:

```

1  a -> [a]

```

This is analogous to Java's

```

1  <T> LinkedList<T> trio(<T> x)

```

Note: That the following 2 lines are both legal if we have trio defined

```

1  trio 0
2  trio "hello"

```

"Parametric" means: Supposed you have defined  $d$  of type  $a \mapsto [a]$ , Then you would need one test to know what it does. Say we test  $dTrue$  and the answer has length 2. Then we can deduce that  $dx$  returns  $[x, x]$  for all  $x$ .

The basic explanation for this is that  $d$  cannot vary behaviour by types. Haskell allows type-determined behaviour, but the function type will look like:

```

1  Foo a => a -> [a]

```



**1.1.7 What is "Powerful"? – The Tradeoff**

"Macro systems, dynamic typing, ... are powerful." This refers to the flexibility for the implementer or the original author.

"Static typing, parametric polymorphism... are powerful." This refers to the predictability for the user or the maintainer.

Programming is a dialectic class struggle between the user and the implementer. Or between the maintainer and the original author.

## 2 Thursday, January 18, 2018

### 2.1 Racket

We won't be using just Scheme, we'll be using Racket which is a version of Scheme. Racket is a platform for implementing and using many languages, and Scheme is one of those that come out of the box.

Racket's version of scheme is somewhat different from the standards with regards to function names, and some features. We will cover Racket, but note that these examples and features may fail for standard Scheme.

### 2.2 Basic Data Types

```
1 #t, #f ;booleans
2 42 ;numbers, can be ints, rational, floats, complex
3 "hello" ; strings
4 #\h ; this is a char of just the letter h
5 'Chrome ;this is a symbol
```

**Symbols** Symbols are user-defined atomic values. You think of a name, put a single quote in front. Symbols are not strings, you can't perform string operations onto them.

### 2.3 Procedures and Functions

For example:

```
1 (sin (/ 0.2 2)) ; sine of 1 over 10
```

### 2.4 Boolean Operations

```
1 (not expr)
2 (and expr expr)
3 (or expr expr)
4 (or) ; gives #f
5 (and) ; gives #t
6 (boolean? expr) ; tests if you have a boolean
```

### 2.5 Number operations

```

1  +
2  -
3  *
4  /
5  max
6  min
7  -
8  <
9  >
10 <=
11 >=
12 ; these operations can all take multiple operands

```

```

1  number?
2  complex?
3  real?
4  rational?
5  ; these functions test if a number is a certain type

```

## 2.6 Equality

There are 3 types of equalities.

```

1  eq?

```

This one is Good for booleans and symbols, uses pointer inequality for aggregates like strings and lists, but has complicated rules for numbers.

```

1  eqv?

```

This one has complicated rules for numbers as well, and different from eq? as it treats the floating-points NaN and signed zero differently.

```

1  equal?

```

This one is for structural equality for most aggregates. So comparing contents of an aggregate.

## 2.7 Definitions

You can define functions and constants, recursion is allowed.

```

1  ; A constant
2  (define my-width/height (/ 4 3))
3  ; a function with 2 parameters
4  (define (my-log base x)
5    (/ (log x) (log base)))

```

## 2.8 Anonymous Functions

Basically a function without a name, you can either write lambda or use  $\lambda$  in your code.

Example of a function:

```
1 (lambda (base x) (/ (log x) (log base)))
```

Example of the same function being used

```
1 ((lambda (base x) (/ (log x) (log base))) 128 2)
```

So in this case, base passed in as 128 and x as 2

## 2.9 Conditionals

If-then-else conditions look like the following

```
1 (if test then-expr else-expr)
```

Test can be non-boolean, and this will be treated as true.

Multiple conditions are as such:

```
1 (cond
2   [(> x y) (sin x)]
3   [(< x y) (cos y)]
4   [else 0])
```

This is if  $x > y$  then  $\sin x$  else if  $x < y$  then  $\cos y$  else 0. Test results can be non-boolean, which are treated as true. You can obtain the result of a test to return it.

```
1 (cond
2   [(+ 4 2) => (lambda (x) (* x x))]
3   [else 0])
```

This gives 36.

### 2.9.1 and, or as conditionals

**and** evaluates all of its operands from left to right and stops as soon as `#f` operand is read, otherwise the last expression is the answer.

**or** evaluates all of its operands from left to right and stops as soon as a non `#f` operand is read, and that becomes the answer, otherwise the answer is `#f`

## 2.10 Local bindings

Local definitions for use in just one expression

```
1 (let ([x expr1]
2      [y expr2])
3      (+ x y (* 2 x y)))
```

This means to compute  $x + y + 2xy$  where  $x = expr1$  and  $y = expr2$ . These 2 expressions cannot see the others variables, only the global ones outside of their scope.

```
1 (let ([x 3]
2      (let ([x (* 3 3)]) ; (* 3 3)
3          x))
```

This results in 9 and is not recursive. Let\* allows later bindings to see earlier bindings

```
1 (let* ([x 5]
2       [y (+ x 1)]) ; (+ 5 1)
3       (+ x y (* 2 x y)))
```

### 2.10.1 Recursive local bindings

letrec allows more recursive bindings

```
1 (letrec ([fac
2          (lambda (n)
3            (if (= n 0) 1 (* n (fac (- n 1)))))])
4  [even
5    (lambda (n)
6      (or (= n 0) (not (odd (- n 1)))))])
7  [odd
8    (lambda (n)
9      (not (even (- n 1)))))])
10 (even (fac 5)))
```

This returns whether or not factorial 5 is even. So true.

## 2.11 Recommended Code Layout

- Open parentheses then immediately first word
- Procedure definition: Body starts on new line, indented
- Long expression: Parts start on new lines, indented

- Closing parentheses not on new lines

## 3 Thursday, January 25, 2018

### 3.1 Pairs and Lists

A cons cell is a 2-tuple pair and has the following syntax:

```
1 cons(x y)
```

Essentially a pair of pointers. Has special support for lists.

```
1 '() ; an empty list
2 (list x y z) = (cons x (cons y (cons z '())))
3 '(42 "hi" Chrome)
4 ; Chrome here will be the symbol 'Chrome
```

For a cons cell, you can use car to access the first field and cdr to access the second field.

### 3.2 User-Defined Records

```
1 (struct dim (width height))
```

This creates a new record type with 2 fields

```
1 (dim 4 7) ; this constructs a value of this type
2 dim? ; this tests for this type
3 dim-width
4 dim-height
5 ; these are the field accessors
```

We can also use struct-copy to clone a record while replacing some values:

```
1 (define d1 (dim 4 7))
2 (define d2 (struct-copy dim d1 [width 5]))
3 ; d2 is (dim 5 7)
```

### 3.3 Pattern Matching

You can test for a literal, cons cell, or a record type, can get their content as well.

```
1 (struct dim (width height))
2
3 (define (foo x)
4   (match x
5     ['() 'nada]
6     [(cons b _) b]
7     [(dim w h) (* w h)]))
```

```

8 (foo '()) ; returns 'nada
9 (foo '(1 2 3)) ; returns 1
10 (foo (dim 4 7)) ; returns 28

```

### 3.4 Input and Output

We can print with `display`, `printf` and `displayln`

```

1 (display 5)
2 (newline)
3 (displayln 5)
4 (printf "yes" "price" 5)

```

```

1 (read-line) ; this reads a line
2 (read-string 10) ; this reads up to the upper bound.
3 ; if it reaches the end of the file, it returns eof, which you can use eq? or eof-
  object? to test
4 ; for stderr, eprintf is like printf but goes to stderr

```

#### 3.4.1 ports

Racket has ports, analogous to Java Reader/Writer – behind it can be file, string, network connection, message queue, user-defined, etc.

### 3.5 Sequencing

If we want to evaluate multiple expressions in the order we specify

```

1 (begin
2   (display ln "Please enter your name")
3   (read-line)) ; this returns the last expression
4
5 (begin0 expr1 expr2) ; this returns the first expression, but the others are still
  evaluated.
6
7 (when (> x 0) expr1 expr2 ...)
8 ; if true, evaluates the expressions, returns what the last one returns, if false,
  returns #<void>

```

### 3.6 Mutable Variables



```
1 (define v 5)
2 (define (f x) (+ x v))
3 (f 0) ; this gives 5
4 (set! v 6)
5 (f 0) ; this gives 6
```

Mutable pairs, list, strings, arrays, etc. are also available. Use mutation judiciously, is not that necessary.

### 3.7 map

Takes in a function and a list and applies the function to every element in that list

```
1 (map f (list x y z)) = (list (f x) (f y) (f z))
```

### 3.8 filter

filter takes in a boolean function and a list (A) and returns a list of the items in the list A that satisfy the boolean function

```
1 (filter number? '(9 "4" 0 "1" "6" 5)) = '(9 0 5)
```

## 4 Thursday, February 1, 2018

### 4.1 Scheme (cont'd)

#### 4.1.1 foldl

Consider the problem of summing an entire list and multiplying an entire list. Summing requires us to add up all the elements of the list plus 0 for the first item. Multiplying requires us to multiply up all the elements of the list times 1 for the first item. This is the motivation.

So we define foldl as

```
1 (define (foldl binop a lst)
2   (match lst
3     [?( ) a]
4     [( cons hd tl) (foldl binop (binop a hd) tl )]))
```

So Intuitively,

```
1 (foldl binop a (list x y z))
```

Looks like

$$(((a + x) + y) + z)$$

where + is where binop is being performed

#### 4.1.2 foldr

Basically in the opposite direction of foldl, so if

```
1 (foldr binop a (list x y z))
```

Looks like

$$(z + (y + (x + a)))$$

where + is where binop is being performed, then

```
1 (foldr binop a (list x y z))
```

Looks like

$$(((a + x) + y) + z)$$

where + is where binop is being performed, then

### 4.1.3 Procedure-Call Stack

Consider the following:

```
1 (define (f n) (... (f (- n 1)) ...))
2 (displayln (+ (f 4) (f 1) (f 6)))
```

The Control-flow jumps to into  $f$  when it's called and later knows where to return to after the recursive calls. This is done because a stack is used to remember where to return to in recursion, called a **Call Stack**. The Benefit of this is that it supports recursion, but it comes at a price of occupying  $\theta(1)$  space while the stack is being used.

### 4.1.4 Non-Tail Calls and Tail Calls

**Non-Tail Calls**, are if you still have to do your own processing or computation after getting the results from another function, so basically the results are not returned right away.

For example, for the following function:

```
1 (define (my -sum lst)
2   (match lst [(?) 0]
3     [(cons hd tl) (+ hd (my -sum tl))]))
```

Takes  $\theta(n)$  space if the list length is  $n$ .

A Tail call is the exact opposite, there is no computation after getting the results back from a called function and the function returns the value right away. The complexity of this is  $O(1)$  under Tail-Calling optimization in Scheme.

Tail-Calling optimization isn't in every language, Java and Python don't have this.

## 4.2 Haskell

### 4.2.1 Expressions and Types

**Characters** chars are denoted with single quotes

**Tuples** Not the same as cons

() is a special type, called the unit type, used as a return value for functions that don't return anything

**Lists** Lists are implemented as such:

```
1 3 : ( 1 : ( 4 : [] ) ) = [3,1,4]
```

So a list of length one is an item with an empty list and the colon inbetween separates the items

```
1 [1] = 1 : []
```

Note that the following list has a type `[[integer]]`

```
1 [[3,1,4], [10,20], []]
```

So as for now, arbitrary list nesting is not supported, so basically

```
1 [1, [3]]
```

is not supported (yet).

Note that because of static typing, every item must be the same type, we can't mix integers with floats in the same list.

**Strings** are a list of chars. The downside of this is that it uses a huge amount of memory, as it's stored as a linked list, and each node and pointer takes up a linear amount of space.

**Keyword: Just** If you type

```
1 Just 'C'
```

The variable is either `Nothing` or `Char`

**Nothing** Is the empty type, can be any type

**"Left 'C'"** Can either be of type `Char Bool`, `Char Int`, ... all we know that the left variable is a character.

**"Right False"** Can either be of type `Char bool`, `Int bool`, ... all we know that the right variable is a boolean

**anonymous functions** Ex.

```
1 \x -> x >= 'C'
```

Has the type `Char -> Bool` with `char` as the domain and `bool` as the codomain

### 4.2.2 Definitions

We can define expressions to variables and vice versa, for example, in the following, we are defining "ten" to be "1+2+3+4" and binding "1+2+3+4" to "ten":

```
1 ten = 1 + 2 + 3 + 4
```

There is also pattern binding, with tuples, but that will be shown later

Functions can also be defined:

```
1 square x = x * x
2 nand a b = not (a && b)
```

We can also define type signatures for the definitions as such

```
1 ten, four :: Integer
```

But Haskell is written such that the type signature can be separated from the definition, so you don't have to put them in the same few lines, they just have to be in the same file.

### 4.2.3 Function Applications

If you insert one parameter to a function that takes 2 parameters, that function will return a function of 1 parameter. This is how Haskell does multiple parameters

## 5 Thursday, February 8, 2018

### 5.1 Haskell (cont'd)

#### 5.1.1 Local Definitions For Expressions

```

1  let x = 4 + 5
2      y = 4 - 5
3  in x+y+2*x*y

```

Layout Version above. Braced Version below

#### 5.1.2 Local Definitions For Definitions

```

1  foo u v = x + y + 2*x*y
2      where -- this where refers to the statement in line 1
3          x = u + v
4          y = u - v

```

#### 5.1.3 Pattern Matching

Can be done for expressions or function definitions as well as pattern binding.

```

1  -- expression case
2  case expr of
3      [] -> 0
4      42 : xs -> foo xs
5      x : xs -> x + foo xs

```

In this example, the expression *expr* would evaluate to 0 if it was an empty list, have foo applied to its tail if it started with 42 as its first index and x plus foo applied to its tail for any other list.

```

1  -- function definition case examples
2
3  mySum [] = 0
4  mySum (x : xs) = x + mySum xs
5
6  nand False _ = True
7  nand True False = True
8  nand True True = False

```

```

1  -- pattern binding case
2  [a, b, c] = take 3 someList
3  -- a = take
4  -- b = 3
5  -- c = someList

```

### 5.1.4 Guards

Guards are extra conditions imposed on patterns.

```

1  -- expression case
2  case expr of
3    [] -> 0
4    x : xs | x < 0 -> x + foo xs
5            | x > 2 -> x - foo xs
6            | True -> x * foo xs
7
8  -- definition case
9  foo [] = 0
10 foo (x : xs) | x < 0 -> x + foo xs
11              | x > 2 -> x - foo xs
12              | True -> x * foo xs

```

Instead of True for the edge case, can also use "otherwise".

### 5.1.5 Local Definitions under Patterns and Guards

```

1  foo :: Either String Integer -> Integer
2  foo (Left str) | suffix > "albert" = 42
3                | otherwise = 24
4                where
5                  suffix = drop 10 str
6
7  foo (Right x) | x > 0 = 2*y
8                | x < 0 = y
9                | otherwise = 0
10             where
11               y = div 1000 x

```

Note that the first where belongs to the foo where the input is a Left str, the second where belongs to the foo where the input is a Right integer and if  $x = 0$ , then  $y$  will not be calculated.

### 5.1.6 List Comprehension

```

1  [x + y | x <- [10,20,30], x > 10, y <- [4,5] ]
2  -- this results in
3  --[20+4, 20+5, 30+4, 30+5]

```

We can use pattern matching as well

```

1  [x+3 | Just x <= [Just 10, Nothing, Just 30]]
2  -- this results in
3  -- [10+3, 30+3]

```

There is also a range notation we can use:

```
1 [1...5]  -- this is the same as [1,2,3,4,5]
```

### 5.1.7 Algebraic Data Types

```
1 data MyType = Nada | Duplet Double String | Uno Integer
```

Nada, Duplet, Uno are data constructors. They must start with uppercase letters. They form expressions and patterns.

The following is an example function that takes in "MyType":

```
1 plus1 :: MyType -> MyType
2 plus1 Nada = Nada
3 plus1 (Duplet r s) = Duplet (r+1) s
4 plus1 (Uno i) = Uno (i+1)
```

List, unit, tuple, Maybe, and either are algebraic data types from the standard library.

Recursive definitions are ok too, like the following:

```
1 data Stack = Button | Push Int Stack
```

### 5.1.8 Parametric Polymorphism

consider the following function type contract

```
1 map :: (a -> b) -> [a] -> [b]
```

Here both a and b are type variables. They start with lowercase letters (actual data types are capitalized, like Bool). The user chooses what types to use for a and b, and the implementer cannot choose what type of a and b, and must let their function work for all types of a and b.

Algebraic data types can be parameterized by type variables too, for example:

```
1 data Either a b = Left a | Right b
2 -- We can generalize the previous stack example to
3 data Stack a = Bottom | Push a (Stack a)
```

### 5.1.9 Type-Class Polymorphism

We notice that comparison functions like



```

1  (==)
2  (<)

```

cannot use completely general it its polymorphism, they must take in items as input that are two of the same type of class that can be compared.

So how does Haskell pull these off?

Haskell uses a **type class** that declares overloaded operations. So the example from before:

```

1  (==)
2  (<)

```

must take in two inputs, lets call them  $a$  and  $b$ , where they are both of type  $Eq\ a$ , which is a class that lets the two inputs be compared.

However, note that classes are not the same as types.  $Eq$  is not a type,  $Bool$  is not a subclass.

So if we were to type this out:

```

1  (==) :: Eq a => a -> a -> Bool
2  - "Eq a" is a "class constraint"

```

In this example, the user chooses what type to use for  $a$ , but that chosen type must be an instance of  $Eq$ .

Note that constraints propagate down the dependency chain:

```

1  (==) :: Eq a => a -> a -> Bool
2
3  eq3 :: Eq a => a -> a -> a -> Bool
4  eq3 x y z = x==y && y==z

```

## 6 Thursday, February 15, 2018

### 6.1 Haskell (cont'd)

#### 6.1.1 Constraint Instances

What if you are comparing instances inside of a data structure (let's say, a list for example)?

Well we can do this:

```

1 instance Eq a => Eq [a] where
2   [] == []      = True
3   (x:xs) == (y:ys) = x==y && xs == ys
4   _ == _        = False

```

Constraints propagate down the dependency chain, including other instance implementations

#### 6.1.2 User-Defined Class

```

1 class ADT a where
2   tag :: a -> String
3
4 instance ADT (Either a b) where
5   tag (Left _) = "Left"
6   tag (Right _) = "Right"
7
8 instance ADT MyType where
9   tag Nada = "Nada"
10  tag (Duplet _ _) = "Duplet"
11  tag (Uno _) = "Uno"

```

Classes in Haskell are similar to Java's Interfaces in the sense that you implement the classes' functions for each instance as well.

```

1 class Eq a => Ord a where
2   (<), (<=), (>), (>=) :: a -> a -> Bool
3   compare :: a -> a -> Ordering
4 data Ordering = LT | EQ | GT
5
6 -- The "Eq a =>" here means that
7 -- Every Ord instance is also an EQ instance
8 -- (Superclass, subclass)

```

For implementers of type and instances, these superclasses must be specified, but for users, they can use `Ord` without mentioning `Eq`.

### 6.1.3 Auto-Generating Instance Implementations

The compiler is willing to write some instance code for you, for select standard classes: (`Eq`, `Ord`, `Enum` (but no fields allowed), `Show`, and a few others.

```
1 data MyType = Nada | Duplet Double String | Uno Integer
2   deriving (Eq, Ord, Show)
3 data Browser = FireFox | Chrome | Edge | Safari
4   deriving (Eq, Ord, Show)
```

### 6.1.4 Haskell's Number System

You can't use doubles and integers together for number operators, you have to convert one so that they are the same (Can use `fromIntegral` to convert an integer into a double).

### 6.1.5 Functor

```
1 fmap_List :: (a -> b) -> [a] -> [b]
2 fmap_List = map
3
4 fmap_Maybe :: (a -> b) -> Maybe a -> Maybe b
5 fmap_Maybe f Nothing = Nothing
6 fmap_Maybe f (Just a) = Just (f a)
7
8 fmap_Either :: (a->b) -> Either e a -> Either e b
9 fmap_Either f (Left e) = Left e
10 fmap_Either f (Right a) = Right (f a)
```

The pattern here is that there is a  $f : a \mapsto b$  induces a corresponding  $Fa \mapsto Fb$  where  $F$  is a parameterized type. There is a class for that.

```
1 class Functor f where
2   fmap :: (a->b) -> f a -> f b
```

So this function generalizes the previous examples above.

Every instance of `Functor` should satisfy:

```
1 fmap id xs = xs
2 fmap g (fmap f xs) = fmap (g . f) xs
```

`fmap` also has an infix alias of `< $ >`, for example:

```
1 sin <$> [1,2,3]
```

### 6.1.6 Applicatives

You now become ambitious. You ask: What if you have a binary operator, and two lists, ...

```
1 listCross :: (a -> b -> c) -> [a] -> [b] -> [c]
2
3 maybeBoth :: (a -> b -> c) -> Maybe a -> Maybe b -> Maybe c
4 maybeBoth op (Just a) (Just b) = Just (op a b)
5 maybeBoth op _ _ = Nothing
```

And what if you have a ternary operator and three lists?

Can you implement

```
1 ap_List :: [a -> b] -> [a] -> [b]
```

such that, for example:

```
1 ap_List [f,g] [1,2,3] = [f 1, f 2, f 3, g 1, g 2, g 3]
```

Answer:

```
1 ap_List = listCross (\f -> \x -> f x)
```

Equivalently `listCross ($)`

Now can implement ternary too:

```
1 listTernary :: (a->b->c->d)->[a]->[b]->[c]->[d]
2 listTernary ternary as bs cs =
3   ((ternary <$> as) 'ap_List' bs) 'ap_List' cs
```

There is a class for this too.

```
1 class Functor f => Applicative f where
2   pure :: a -> f a
3   (<*>) :: f (a -> b) -> f a -> f b
4
5   -- example instance
6 instance Applicative Maybe where
7   pure a = Just a
8   Just f <*> Just a = Just (f a)
9   _ <*> _ = Nothing
```

Applicative subsumes Functor, so we can implement `fmap` as

```
1 fmap f xs = pure f <*> xs
```

## 7 Thursday, March 1, 2018

### 7.1 Haskell (cont'd)

#### 7.1.1 Monads

So far we have been thinking of List, Maybe, Either,... as data structures (ex. containers). Now we must think of them as programs:

- `foo :: Maybe Int` means: a program that may return a number successfully, or may abort
- `foo :: [Int]` means: a non-deterministic program that returns different numbers in different parallel universes.
- `foo :: Either String Int` means: like Maybe, but if it aborts, it uses Left to tell you an error message

Now we re-read Functor and Applicative from this angle

```

1 fmap abs foo
2 -- this now means to return the absolute value
3 -- of what foo returns
4
5 (+) <$> foo <*> bar
6 -- this now means to return the sum of what
7 -- foo and bar returns

```

But bar does not know what foo returns, or vice versa.

You now become ambitious. Can you combine two programs such that the return value(s) of the 1st is fed to the 2nd so the 2nd can behave independently? Like such:

```

1 bind :: F a -> (a -> F b) -> F b
2 -- so we can have
3 -- prog1st 'bind' prog2nd?

```

We can think of prog2nd as a callback to prog1st.

Other examples would look like the following:

```

1 -- bind for Maybe
2
3 bind_Maybe :: Maybe a -> (a -> Maybe b) -> Maybe b

```

```

4 bind_Maybe Nothing _ = Nothing
5 bind_Maybe (Just a) k = k a
6
7 -- bind for List
8
9 bind_List :: [a] -> (a -> [b]) -> [b]
10 bind_List [] _ = []
11 bind_List (a:as) k = k a ++ bind_List as k

```

There is a class for that too.

```

1 class Applicative f => Monad f where
2   return :: a -> f a
3   (>>=) :: f a -> (a -> f b) -> f b
4
5 -- example instance
6
7 instance Monad [] where
8   return a = [a]
9   as >>= k = concat (map k as)

```

Remark: return and pure should be the same thing. Historically, Monad came first, Applicative came later, thus the redundancy. There is a proposed change to make return an alias of pure.

Monad subsumes Applicative: Can implement ( $\langle * \rangle$ ) as

```

1 fs <*> as = fs >>=
2   \f -> as >>=
3   \a -> return (f a)

```

There are equations for Monad too, such as:

```

1 return a >>= k = k a

```

There is "do-notation" so code looks nicer and the computer emits

```

1 >>= \v ->

```

for you:

```

1 fs <*> as = do
2   f <- fs
3   a <- as
4   return (f a)

```

### 7.1.2 Haskell I/O System

Parametrized type "IO" for all "I/O" commands. Instance of Monad, Applicative, Functor.

```

1 foo : : IO Char
2 -- this means a program that interacts with the outside world, then returns a
   character (or gets stuck forever, or throws an exception).
3
4 putStrLn :: String IO ()
5 getLine  :: IO String
6 -- NOT: getLine :: String

```

Do not think about how to extract the string. Use ( $>>=$ ) to feed it to the next program (callback).

```

1 main = getLine >>= \s -> putStrLn("It's " ++ s)
2 -- OR
3 main = do
4   s <- getLine
5   putStrLn ("It's " ++ s)

```

## 7.2 Syntax

### 7.2.1 Context-Free Grammar (CFG)

A context-free grammar looks like this bunch of rules:

$$\begin{aligned}
 E &\rightarrow E + E \\
 M &\rightarrow M \times M \\
 A &\rightarrow 0 \\
 A &\rightarrow (E) \\
 E &\rightarrow M \\
 M &\rightarrow A \\
 A &\rightarrow 1
 \end{aligned}$$

Main idea:

- $E, M, A$  are non-terminal symbols aka variables. When you see them, you apply rules to expand
- $+, \times, 1, 0, (, )$  are terminal symbols. They are the characters you want in your language

### 7.2.2 Derivation (aka Generation)

Derivation is a finite sequence of applying the rules until all non-terminal symbols are gone. Often aim for a specific final string.

$$\begin{aligned} E &\rightarrow M \\ &\rightarrow M \times M \\ &\rightarrow A \times M \\ &\rightarrow 1 \times M \\ &\rightarrow 1 \times A \\ &\rightarrow 1 \times (E) \\ &\rightarrow 1 \times (E + E) \\ &\rightarrow 1 \times (M + E) \\ &\rightarrow 1 \times (A + E) \\ &\rightarrow 1 \times (0 + E) \\ &\rightarrow 1 \times (0 + M) \\ &\rightarrow 1 \times (0 + M \times M) \\ &\rightarrow 1 \times (0 + A \times M) \\ &\rightarrow 1 \times (0 + 1 \times M) \\ &\rightarrow 1 \times (0 + 1 \times A) \\ &\rightarrow 1 \times (0 + 1 \times 1) \end{aligned}$$

Context-free grammars can support: matching parentheses, unlimited nesting.

### 7.2.3 Backus-Naur Form (BNF)

Backus-Naur Form is a computerized, practical notation for CFG



- Surround non-terminal symbols by  $\langle \rangle$ ; allow multi-letter names
- Merge rules with the same LHS
- (Some versions.) Surround terminal strings by single or double quotes.
- use  $::=$  for  $\rightarrow$

Our example grammar in BNF:

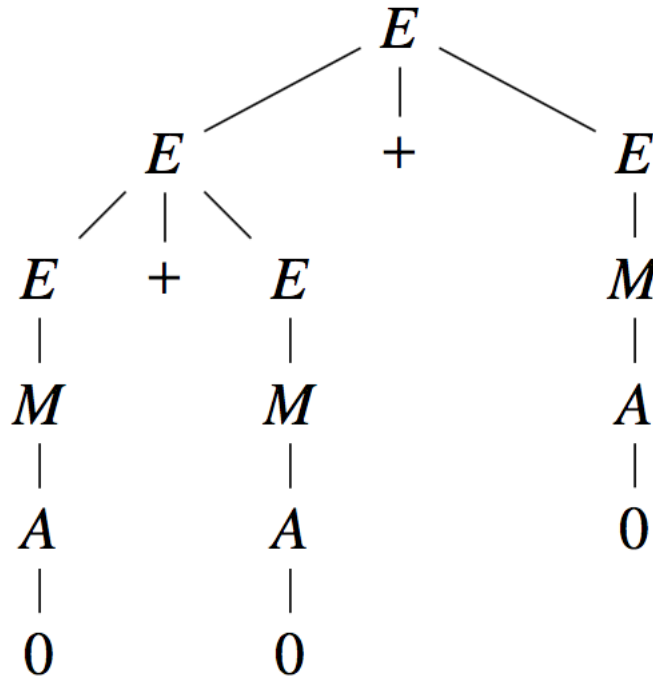
```

1 <expr> ::= <expr> "+" <expr> | <mul>
2 <mul> ::= <mul> "*" <mul> | <atom>
3 <atom> ::= "0" | "1" | "(" <expr> ")"

```

#### 7.2.4 Parse Tree (aka Derivation Tree)

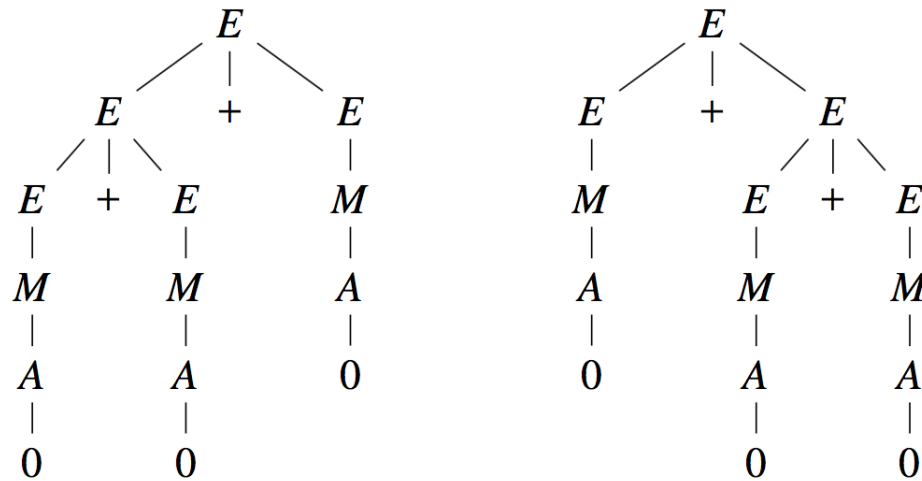
A parse tree aka derivation tree presents a derivation with more structure (tree), less repetition.



This example generates  $0 + 0 + 0$

### 7.2.5 Ambiguous Grammar

Two different trees generate the same  $0 + 0 + 0$



If this happens, the grammar is ambiguous.

We try to design unambiguous grammars.

(Bad news: CFG ambiguity is undecidable)

### 7.2.6 Unambiguous Grammar Example

An unambiguous grammar that generates the same language as our ambiguous grammar example:

```

1 <expr> ::= <expr> "+" <expr> | <mul>
2 <mul> ::= <mul> "*" <mul> | <atom>
3 <atom> ::= "0" | "1" | "(" <expr> ")"
  
```

(Bad news: Equivalence of two CFGs is also undecidable)

### 7.2.7 Left Recursive vs Right Recursive

```

1 <expr> ::= <expr> "+" <mul>
  
```

This is an example of a left recursive rule. The recursion is at the beginning (left).

```
1 <expr> ::= <mul> "+" <expr>
```

This is an example of a right recursive rule. The recursion is at the end (right).

They affect whether infix operators associate to the left or right.

They also affect some parsing algorithms.

### 7.2.8 Recursive Descent Parsing

Recursive descent parsing is a simple strategy for writing a parser.

- Write a procedure for each rule
- Non-terminals on Right Hand Side become procedure calls, possible recursive calls. (Thus "recursive descent". Also "top-down".) (Left-recursive grammars need special treatment.)
- Terminal symbols: Consume input and check
- Alternatives require lookahead and/or backtracking

### 7.2.9 Recursive Descent Parser Example

Example grammar suitable for recursive descent parsing:

```
1 <sub> ::= <atom> "-" <sub> | <atom>
2 <atom> ::= "0" | "1" | "(" <sub> ")"
```

Pseudo-code of recursive descent parser:

```
1 sub:
2   try (atom
3     read; if not "-" then fail
4     sub)
5   if that failed: atom
6
7 atom:
8   read;
9   if "1" or "0": return
10  if "(" : sub
11    read; if not ")" then fail
12  else: fail
```

## 8 Thursday, March 8, 2018

The lectures after this, Albert no longer provides lecture slides, but instead, decides to do everything in haskell files, so I will just be posting the lecture slides here and an explanation.

### 8.1 ParserLib.hs

Albert provided this really long document called "Pearl.pdf" that explained the concept of parsing, and parsers are essentially built from this one basic parser that "consumes" one character of a string and returns a result based off that input (so think of a parser consuming the  $c$  of a  $(c : cs)$  string, and returning some result based off that input with the rest of the unparsed string  $(cs)$ ).

```

3  -- | Library of parser definition and operations.
4  module ParserLib where
5
6  import Control.Applicative
7  import Data.Char
8  import Data.Functor
9  import Data.List
10
11 newtype Parser a = PsrOf{
12     -- | Function from input string to:
13     --
14     -- * Nothing, if failure (syntax error);
15     -- * Just (unconsumed input, answer), if success.
16     dePsr :: String -> Maybe (String, a)}
17
18 -- Monadic Parsing in Haskell uses [] instead of Maybe to support ambiguous
19 -- grammars and multiple answers.
20
21 -- | Use a parser on an input string.
22 runParser :: Parser a -> String -> Maybe a
23 runParser (PsrOf p) inp = case p inp of
24     Nothing -> Nothing
25     Just (_, a) -> Just a
26     -- OR: fmap (\(_,a) -> a) (p inp)
27
28 -- | Read a character and return. Failure if input is empty.
29 anyChar :: Parser Char
30 anyChar = PsrOf p

```

```

31 where
32   p "" = Nothing
33   p (c:cs) = Just (cs, c)
34
35 -- | Read a character and check against the given character.
36 char :: Char -> Parser Char
37 -- char wanted = PsrOf p
38 -- where
39 --   p (c:cs) | c == wanted = Just (cs, c)
40 --   p _ = Nothing
41 char wanted = satisfy (\c -> c == wanted) -- (== wanted)
42
43 -- | Read a character and check against the given predicate.
44 satisfy :: (Char -> Bool) -> Parser Char
45 satisfy pred = PsrOf p
46 where
47   p (c:cs) | pred c = Just (cs, c)
48   p _ = Nothing
49
50 -- | Expect the input to be empty.
51 eof :: Parser ()
52 eof = PsrOf p
53 where
54   p "" = Just ("", ())
55   p _ = Nothing
56
57
58 -- | Read and check against a given string.
59 string :: String -> Parser String
60 string wanted = PsrOf p
61 where
62   p inp = case stripPrefix wanted inp of
63     Nothing -> Nothing
64     Just suffix -> Just (suffix, wanted)
65     -- Refactor this!
66
67 -- But you have to compose smaller parsers to build larger parsers and to return
68 -- more interesting answers, e.g., abstract syntax trees.
69 --
70 -- This is what fmap, pure, <*>, >>= are for. And there are more...
71
72 instance Functor Parser where
73   -- fmap :: (a -> b) -> Parser a -> Parser b
74   fmap f (PsrOf p) = PsrOf q
75     -- (\inp -> fmap (\(rest, a) -> (rest, f a)) (p inp))

```

```

76     where
77         q inp = case p inp of
78             Nothing -> Nothing
79             Just (rest, a) -> Just (rest, f a)
80             -- fmap (\(rest, a) -> (rest, f a)) (p inp)
81
82 instance Applicative Parser where
83     -- pure :: a -> Parser a
84     pure a = PsrOf (\inp -> Just (inp, a))
85     -- (<*>) :: Parser (a -> b) -> Parser a -> Parser b
86     -- Consider the 1st parser to be stage 1, 2nd parser stage 2.
87     PsrOf p1 <*> PsrOf p2 = PsrOf q
88     where
89         q inp = case p1 inp of
90             Nothing -> Nothing
91             Just (middle, f) ->
92                 case p2 middle of
93                     Nothing -> Nothing
94                     Just (rest, a) -> Just (rest, f a)
95             -- dePsr (fmap f (PsrOf p2)) middle
96
97 instance Alternative Parser where
98     -- empty :: Parser a
99     -- Always fail. The identity for <|> below.
100    empty = PsrOf (\_ -> Nothing)
101    -- (<|>) :: Parser a -> Parser a -> Parser a
102    -- Try the 1st one. If success, done; if failure, do the 2nd one
103    PsrOf p1 <|> PsrOf p2 = PsrOf q
104    where
105        q inp = case p1 inp of
106            j@(Just _) -> j
107            Nothing -> p2 inp
108    -- many :: Parser a -> Parser [a]
109    -- 0 or more times, maximum munch, collect the answers into a list.
110    -- Can use default implementation.
111
112    -- some :: Parser a -> Parser [a]
113    -- 1 or more times, maximum munch, collect the answers into a list.
114    -- Can use default implementation.
115
116 instance Monad Parser where
117     return = pure
118     PsrOf p1 >>= k = PsrOf q
119     where
120         q inp = case p1 inp of

```

```

121         Nothing -> Nothing
122         Just (rest, a) -> dePsr (k a) rest
123
124 -- | Space or newline or tab.
125 whitespace :: Parser Char
126 whitespace = satisfy (\c -> c `elem` ['\t', '\n', ' '])
127
128 -- | Consume zero or more whitespaces, maximum munch.
129 whitespaces :: Parser String
130 whitespaces = many whitespace
131
132 -- | Read and check a terminal string, then skip trailing spaces.
133 terminal :: String -> Parser String
134 terminal wanted = string wanted < * whitespaces
135
136 -- | Read an integer, then skip trailing spaces.
137 integer :: Parser Integer
138 integer = sign < * (read < $> some (satisfy isDigit)) < * whitespaces
139 where
140     sign = (char '-' *> pure negate) <|> pure id
141
142 -- | Read an identifier, then skip trailing spaces. Disallow certain keywords.
143 identifier :: [String] -> Parser String
144 identifier keywords = do
145     c <- satisfy isAlpha
146     cs <- many (satisfy isAlphaNum)
147     whitespaces
148     let str = c:cs
149     if str `elem` keywords then empty else return str
150
151 -- | One or more operands separated by an operator. Apply the operator(s) in a
152 -- left-associative way.
153 chainl1 :: Parser a -- ^ operand parser
154         -> Parser (a -> a -> a) -- ^ operator parser
155         -> Parser a -- ^ evaluated answer
156 chainl1 arg op = do
157     a <- arg
158     more a
159 where
160     more x = do
161         f <- op
162         y <- arg
163         more (f x y)
164     <|>
165     return x

```

```

166
167 -- | One or more operands separated by an operator. Apply the operator(s) in a
168 -- right-associative way.
169 chainr1 :: Parser a          -- ^ operand parser
170         -> Parser (a -> a -> a) -- ^ operator parser
171         -> Parser a          -- ^ evaluated answer
172 chainr1 arg op = do
173     x <- arg
174     ((\f y -> f x y) <$> op <*> chainr1 arg op) <|> return x
175
176 -- | Parse a thing that is wrapped between open and close brackets.
177 between :: Parser open      -- ^ open bracket parser
178         -> Parser close    -- ^ close bracket parser
179         -> Parser a        -- ^ thing parser
180         -> Parser a        -- ^ return the thing parsed
181 between open close p = open *> p <* close

```

### 8.1.1 Parser Implementation

So how he sets up Parsers here is that every parser returns a `Maybe` containing a string of the unconsumed input with *a* being the result, and if it fails at any point, it returns `Nothing`. So Parsers are essentially "eating" a string, performing some functions (maybe, success or fail) and then returning the value in a way that it can be further parsed if necessary, or just `Nothing` if it fails along the way. It's built as a `Monad` so you can keep applying Parsers to a string easily, and this makes it easier to check for failure.

### 8.1.2 anyChar

the `anyChar` function here is a great example here, the `Parser` consumes one char of the string, and if eats nothing (no more string left to parse), it fails, otherwise, it succeeds and formats the output accordingly

### 8.1.3 satisfy and char

For `char`, it uses `satisfy` which uses a function that if the predicate holds true, then the `Parser` succeeds and returns what the `Parser` ate, otherwise, the `Parser` fails. So `char` specifically uses a lambda function that checks if it is the argument "wanted".



#### 8.1.4 eof

This one succeeds if the Parser is called on an empty string (nothing left to eat).

#### 8.1.5 empty, many, and some

Just like what the comments say, the empty Parser fails for any input, the many parser runs a parser as many times as it will succeed and populate a list with the results, and will return the list on the first failure, and the some parser does the exact same thing, but there must be at least one success, while the many parser allows for 0 successes.

#### 8.1.6 whitespace, terminal, integer, identifier

whitespace just has the Parser eat one whitespace character, whitespaces eats whitespace characters with the many parser.

terminal eats all of the whitespaces after a string parser, and identifier eats a string and succeeds if the string is not in the list of provided strings, fails otherwise.

#### 8.1.7 chainl1, chainr1

chainl1 recursively calls the "more" function if it can find more operations and arguments. It coming from the left means that it evaluates it from the left side first (so like this:  $((1 + 2) + 3) + 4 + 5$ ).

chainr1 does the exact same thing but on the right side this time  $5 + (4 + (3 + (1 + 2)))$ .

#### 8.1.8 between

between basically runs its open parser first, then runs the p parser, which result is returned, and then runs the close parser.

## 9 Thursday, March 15, 2018

### 9.1 Num.hs

```
182 module Num where
183
184 import ParserLib
185 import Data.Char
186 import Control.Applicative
187
188 data Expr = N Integer
189           | Plus Expr Expr
190           deriving (Eq, Show)
191
192 interp :: Expr -> Integer
193 interp (N i) = i
194 interp (Plus e1 e2) = interp e1 + interp e2
195
196
197 -- expr ::= { integer "+" } integer
198
199 -- Left associative below:
200 -- E.g., 1+2+3 means (1+2)+3
201
202 exprParserL :: Parser Expr
203 exprParserL = do
204   i <- integer
205   more (N i)
206 where
207   more a = do
208     char '+'
209     j <- integer
210     more (Plus a (N j))
211   <|>
212   return a
213
214 exprParserL2 :: Parser Expr
215 exprParserL2 = chainl1 (fmap N integer) op
216 where
217   op = do char '+'
218         return Plus
219
220 -- Right associative below:
221 -- E.g., 1+2+3 means 1+(2+3)
222
```

```
223 exprParser :: Parser Expr
224 exprParser = do
225   i <- integer
226   (eof *> return (N i)) <|>
227   (do char '+'
228       j <- exprParser
229       return (Plus (N i) j))
230
231 exprParser2 :: Parser Expr
232 exprParser2 = chainr1 (fmap N integer) op
233 where
234   op = do
235     char '+'
236     return Plus
```

### 9.1.1 Purpose

So now that we've done Parsers, we ended up doing an assignment that was about making parsers that turn strings into things like integers and functions, but all in the same format it was given in as a string. Num.hs is essentially about of that into something that actually makes sense.

In this file, we're gonna just be focusing on parsing expressions which are either a Plus of 2 expressions or an integer. We have an interpreter that takes in an integer that represents the number *i* and just return *i* and for the Plus case, we'll interpret the two expressions (which will be integers once interpreted), and then we'll return their sum.

### 9.1.2 Plus function parsers from strings "exprParserL" and "exprParserL2"

If you're reading this and you haven't done the assignment, he also gives an explanation on how to implement a Parser that converts a string into an expression. One implementation that looks like the implementation of chainr1 in ParserLib.hs (because that's essentially what it's trying to do), and the other one utilizes the fact that the "Expr" constructors are also functions. For example, "N" is a function that given an integer "i" returns the data type "(N i)" and "Plus" is a function that given two expressions "a" and "b" will return the data type "(Plus a b)".

### 9.1.3 expression Parsing from strings

And for "exprParser", it is essentially a Parser for turning strings into our expression type. So  $a < | > b$  is the notation for "if Parser a works, return a, otherwise do Parser b" so the two cases are our integer parser that

1. If the string is just an integer "i" (we check this by running eof so that there is no unprocessed input), we return the type "(N i)".
2. If that case does not work or only the integer gets parser'd but the eof fails because there's more stuff, our second case looks for a "+" character and recursively calls itself which would return an integer, it then takes that integer and returns "(Plus (N i) j)". Note that the "i" here comes from the previous case in which eof fails.

## 9.2 NumVar.hs

```

237 module NumVar where
238
239 import           Data.Map (Map)
240 import qualified Data.Map as Map
241 import           Data.Maybe (fromJust)
242
243 data Expr = N Integer
244           | Var String
245           | Plus Expr Expr
246   deriving (Eq, Show)
247
248 interp :: Map String Integer -> Expr -> Integer
249 interp env (N i) = i
250 interp env (Plus e1 e2) = interp env e1 + interp env e2
251 interp env (Var v) = fromJust (Map.lookup v env)
252
253 example = interp (Map.fromList [("x", 5), ("y", 4)])
254           (Plus (N 1) (Plus (Var "x") (Var "y")))
255
256 interpM :: Map String Integer -> Expr -> Maybe Integer
257 interpM env (N i) = Just i
258 interpM env (Plus e1 e2) = fmap (+) (interpM env e1) <*> (interpM env e2)
259 interpM env (Var v) = Map.lookup v env
260
261 exampleM = interpM (Map.fromList [("x", 5), ("y", 4)])
262           (Plus (N 1) (Plus (Var "x") (Var "y")))

```

### 9.2.1 Purpose and interp implementation

This is basically the same as "Num.hs" but introducing environment, which are basically Maps that map strings to integers that are a new argument for the interpreters. So the new case for our interpreter is for a Variable that looks up the string that the var is associated with and just returns whatever it finds.

However, the interp implementation can run into errors, for example, the whole program crashes if it cannot find a variable in the environment (a variable has not been assigned).

### 9.2.2 interpM implementation

The interpM implementation basically turns its output from an integer to a Maybe Integer, so if the variable cannot be found in the map, it just returns an Nothing, which makes the whole expression evaluate to Nothing instead of crashing.