STAB52: An Introduction to Probability

Joshua Concon

University of Toronto Scarborough – Summer 2017

Pre-reqs are MATA37, which is Calculus for Mathematical Sciences II. Instructor is Dr. Mahinda Samarakoon. I highly recommend sitting at the front since he likes to teach with the board and can have a bit of trouble projecting his voice in large lecture halls.

Contents

1 Lecture 1 - Wednesday, May 3, 2017

1.1 What is probability?

He wanted us to think about what it really was. i.e. What does it mean for something to have a 50% probability? If a coin landing on heads has a 50% probability, it doesn't guarantee that such an event would occur if you flipped a coin twice.

After a few guesses, he gave us a definition.

1.2 Relative Frequency Definition of Probability

Definition: Relative Frequency

Consider the case where an experiment is performed n times.

Let $|A|$ be the number of trials resulting in 'event' A.

The **Relative Frequency** of $A = \frac{|A|}{n} = \gamma_n$

This is the Probability of A when n is large γ_n by itself is not an accurate definition of the probability of A occurring, so we take the limit as n goes to infinity and then we have such:

$$\lim_{n \to \infty} \gamma_n$$

This definition is difficult to use in most cases but relatively easy to understand. So here is a definition that is easier to do calculations with:

1.3 Formal Definition of Probability

He doesn't actually get into the definition of probability this lecture, instead he goes through a few terms that we need to define before we get to this definition.

- <u>Sample Space (S)</u>: the set of all possible outcomes in an experiment. Size of S is denoted by $n(S)$ or $\#S$

ex. Tossing a coin once: $S = \{H, T\}$, $n(S) = 2$

ex. Rolling a 6-sided die: $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$

ex. Tossing 2 different coins: $S = \{HH, HT, TH, TT\}$, $n(S) = 4$

- Events: Subsets of a sample space

  ex. Experiment rolling a die, $S = \{1, 2, 3, 4, 5, 6\}$
  $A = \{1, 2\}$, $A \subseteq S$, so A is an event of S.
  $B = \{5, 7\}$, $B \nsubseteq S$, so B is not an event of S.

  You can also describe events with words.

  C = the result is an odd number $= \{1, 3, 5\}$, $C \subseteq S$, so C is an event of S.

  An event can be the entire sample space and the null set.

  $D = S \subseteq S$, so D is an event of S.
  $E = \varnothing \subseteq S$, so E is an event of S.

- Operations on Events (Unless specified, valid for all events A, B)

  1. Where $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2\}$, $B = \{2, 4, 5\}$
     $A \cup B$: elements in A or B
     ex. $A \cup B = \{1, 2, 4, 5\}$
  2. Where $S = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2\}$, $B = \{2, 4, 5\}$
     $A \cap B$: elements in A and B
     ex. $A \cap B = \{2\}$
     And for $C = \{5, 6\}$
     $A \cap C = \varnothing$

Recall: So essentially all of the logic laws from CSCA67 hold for these sets as well.
i.e.
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

3. $(A \cap B)^c = (A^c \cup B^c)$ (where A^c is the complement of A, which is the elements in S that is not in A)
   ex.
   Consider $A = \{1, 2\}$ and $S = \{1, 2, 3, 4, 5, 6\}$
   $A^c = S - A = \{3, 4, 5, 6\}$
4. $(A \cup B)^c = (A^c \cap B^c)$
5. $A \cup \varnothing = A$
6. $A \cap \varnothing = \varnothing$

- Event Space: Is a set of all the possible events of S (i.e All the possible subsets of the set S). Denoted P
  ex. The Event Space of the set $\{1, 2, 3\}$ is
  {{ 1 },{ 2 },{ 3 },{ 1,2 },{ 2,3 },{ 1,3 },{ 1,2,3 }}

2 Lecture 2 - Friday, May 5, 2017

2.1 Formal Definition of Probability

Definition: Probability Measure Function

A function $P : S \longmapsto [0, 1]$ defined on sample space S is called a **Probability Measure Function**. It satisfies the following:

1. $0 \geq P(A) \geq 1$
2. $P(S) = 1$ where S is a sample space
3. For disjoint events $A_1, ..., A_n$ (disjoint means where $A_i \cap A_k = \varnothing$ $\forall i, k \in \mathbb{N}$ $1 \geq i < k \geq n$)
4. $P(\varnothing) = 0$