Calculus
Notes

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MAT157 Self-Study

Not entirely sure about pre-requisites. If you find any problems in these notes, feel free to contact me at conconjoshua@gmail.com.

Contents

1 Chapter 1 - Basic Properties of Numbers

Some properties of Numbers

- (Associative law for addition) $a + (b + c) = (a + b) + c$
- (Existence of an additive identity) $a + 0 = 0 + a = a$
- (Existence of additive inverses) $a + (-a) = (-a) + a = 0$
- (Commutative law for addition) $a + b = b + a$
- (Associative law for multiplication) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
- (Existence of a multiplicative identity) $a \cdot 1 = 1 \cdot a = a; 1 \neq 0$
- (Existence of a multiplicative inverses) $a \cdot a^{-1} = a^{-1} \cdot a = 1, \text{ for } a \neq 0$
- (Commutative law for multiplication) $a \cdot b = b \cdot a$
- (Distributive law) $a \cdot (b + c) = a \cdot b + a \cdot c$
- (Trichotomy law) For every number a, one and only one of the following holds
    1. $a = 0$
    2. $a \in \mathbb{R}^+$
    3. $-a \in \mathbb{R}^+$
- (Closure under addition) If a and b are in $\mathbb{R}^+$, then $a + b$ is in $\mathbb{R}^+$
- (Closure under multiplication) If a and b are in $\mathbb{R}^+$, then $a \cdot b$ is in $\mathbb{R}^+$

Absolute Value

$$|x| = \begin{cases} -x \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$$

Theorem

$$|a + b| \leq |a| + |b|$$

2 Chapter 2 - Number of Various Sorts

2.1 Simple Induction

Mathematic induction states that a statement $P(x)$ is true for all natural numbers x provided that

- $P(1)$ is true
- Whenever $P(k)$ is true, $P(k+1)$ is true

2.2 Complete Induction

The properties of complete induction are a bit different, they just have different properties to prove that the natural numbers is in a set A, as shown

1. 1 is in A
2. $k+1$ is in A, if $1, ..., k$ are in A