CSCC63: Computability and Complexity
Lecture Notes

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1 LEC 1: Monday, January 6, 2020

In this course, we want to explore the limitations of computation that go beyond processing power.
Consider the following problems:

1. Given $x, y \in \mathbb{Z}$, what is $x^2 + y^3$?

2. Given $x, y, z \in \mathbb{Z}$, is $z = x^2 + y^3$ true?

3. Given x, z, does there exist a y such that $z = x^2 + y^3$?

Note that the first problem is not a **decision problem**, because it's solution is not yes/no or true/false, but the other two are. This course will mostly revolve around decision problems.

Decision Problem A problem whose answer is Yes or No.
Now, consider the following. We'll define $f(x, y, z)$ as a program that solves the second problem. So the second problem returns true iff this function returns true and vice versa.
Now, we'd call (x, y, z) the input to the program f, but we want to separate the problem from the program. So we'll call the (x, y, z) an **instance** of the second problem.

Instance the input of a problem

Yes-Instance the input of a decision problem that returns a yes

No-Instance the input of a decision problem that returns a no

If we wanted to work with graphs, we'd have to encode a graph in a computer as an adjacency matrix, which we can turn into a string (which is computer readable). If we encode a graph G, we refer to its encoded format as $\langle G \rangle$

Note that there are some things we cannot represent in a computer, like π

Algorithm A finite sequence of logical steps that always terminates.