Discrete Assignment

Mohana Eppala EE23BTECH11018

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Problem Statement

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

1 Solution

Parameter	Value	Description
a	_	First number
b	-	Second number
n	?	Given variable

Table 1: Input parameters table

Geometric mean of a and b: \sqrt{ab}

$$\therefore \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab} \tag{1}$$

$$\implies a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}}$$
 (2)

$$\implies a^{n+1} - a^{n+\frac{1}{2}}b^{\frac{1}{2}} = a^{\frac{1}{2}}b^{n+\frac{1}{2}} - b^{n+1} \tag{3}$$

$$\implies a^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) \tag{4}$$

$$\implies a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}} \tag{5}$$

$$\implies \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = \left(\frac{a}{b}\right)^0 \tag{6}$$

$$\implies n + \frac{1}{2} = 0 \tag{7}$$

$$\implies n = \frac{1}{2} \tag{8}$$