Discrete Assignment

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Problem Statement

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

1 Solution

Parameter	Value	Description
$x(0)$	a	First term
$x(2)$	b	Third term
$x(1)$	$\sqrt{ab} = \frac{a^{n+1}+b^{n+1}}{a^n+b^n}$	Second term
r	$\sqrt{\frac{b}{a}}$	Common ratio
n	-	Given variable
$x(k)$	ar^k	General term

Table 1: Input parameters table

Consider a GP as in Table 1,

$$\therefore \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = x(1) \tag{1}$$

$$\implies a^{n+1}+b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}} \tag{2}$$

$$\implies a^{n+1} - a^{n+\frac{1}{2}}b^{\frac{1}{2}} = a^{\frac{1}{2}}b^{n+\frac{1}{2}} - b^{n+1} \tag{3}$$

$$\implies a^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) \tag{4}$$

$$\implies a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}} \tag{5}$$

$$\implies (\frac{a}{b})^{n+\frac{1}{2}} = (\frac{a}{b})^0 \tag{6}$$

$$\implies n + \frac{1}{2} = 0 \tag{7}$$

$$\implies n = -\frac{1}{2} \tag{8}$$

From Table 1,

$$X(z) = \sum_{k=-\infty}^{\infty} (ar^k)u(k) \tag{9}$$

$$= a(\frac{1}{1-r})(\frac{1}{1-z^{-1}}) \tag{10}$$

$$= a(\frac{1}{1-\sqrt{(\frac{b}{a})}}(\frac{1}{1-z^{-1}}) \tag{11}$$