Discrete Assignment

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Problem Statement

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

1 Solution

Parameter	Value	Description
x(0)	a	First term
x(2)	b	Third term
x(1)	\sqrt{ab}	Second term
r	-	Common ratio
n	-	Given variable

Table 1: Input parameters table

Consider a GP as in Table 1

$$\therefore \frac{(x(0))^{n+1} + (x(2))^{n+1}}{(x(0))^n + (x(2))^n} = x(1)$$
 (1)

$$\begin{array}{ccc}
 & (x(0))^n + (x(2))^n \\
 & \Rightarrow \frac{(x(0))^{n+1} + (x(0)r^2)^{n+1}}{(x(0))^n + (x(0)r^2)^n} = x(0)r \\
 & \Rightarrow \frac{(x(0))^{n+1} (r^{2n+2} + 1)}{(x(0))^n (r^{2n} + 1)} = x(0)r \\
 & \Rightarrow r^{2n+2} + 1 = r^{2n+1} + r
\end{array} \tag{3}$$

$$\implies \frac{(x(0))^{n+1}(r^{2n+2}+1)}{(x(0))^n(r^{2n}+1)} = x(0)r \tag{3}$$

$$\implies r^{2n+2} + 1 = r^{2n+1} + r \tag{4}$$

$$\Rightarrow r^{2n+2}(r-1) = r - 1 \tag{5}$$
$$\Rightarrow r^{2n+1} = r^0 \tag{6}$$

$$\implies r^{2n+1} = r^0 \tag{6}$$

$$\implies n + \frac{1}{2} = 0 \tag{7}$$

$$\implies n = -\frac{1}{2} \tag{8}$$