Gate Assignment

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Problem Statement

A finite impulse response (FIR) filter has only two non-zero samples in its impulse response $h[n]$, namely $h[0] = h[1] = 1$. The Discrete Time Fourier Transform (DTFT) of $h[n]$ equals $H(e^{j\omega})$, as a function of the normalized angular frequency ω. For the range $|\omega| \leq \pi$, $|H(e^{j\omega})|$ is equal to

(A) $2\left|\cos(\omega)\right|$

(B) $2\left|\sin(\omega)\right|$

(C) $2\left|\cos(\frac{\omega}{2})\right|$

(D) $2\left|\sin(\frac{\omega}{2})\right|$

(GATE BM 2023)

Solution

Parameter	**Value**	**Description**
$h[n]$	-	impulse response
$h[0]$	1	impulse response at $n = 0$
$h[1]$	1	impulse response at $n = 1$
ω	$-\pi \leq \omega \leq \pi$	normalized frequency
$H(e^{j\omega})$	$\sum_{n=0}^{M} h[n]e^{-jn\omega}$	frequency response

Table 1: Input Parameters Table

From Table 1,

$$
\begin{aligned}
H(e^{j\omega}) &= h[0] + h[1]e^{-j\omega} + h[2]e^{-2j\omega} + ... + h[M]e^{-Mj\omega} && (1)\\
&= 1 + e^{-j\omega} && (2)\\
\left|H(e^{j\omega})\right| &= \sqrt{(1+\cos{(j\omega)})^2 + (\sin{(-j\omega)})^2} && (3)\\
&= \sqrt{1 + (\cos{(j\omega)})^2 + 2\cos{(j\omega)} + (\sin{(j\omega)})^2} && (4)\\
&= \sqrt{2(1+\cos{(j\omega)})} && (5)\\
&= 2\left|\cos{(\frac{j\omega}{2})}\right| && (6)
\end{aligned}
$$