GATE 22 CE-30

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Question

Q: The Fourier cosine series of a function is given by: $f(x) = \sum_{n=0}^{\infty} f_n \cos nx$. For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is

Input Parameters Table

Parameter	Value	Description
$f(x)$	-	Function
f_n	-	Coefficient of $\cos nx$ in Fourier series
C_n	-	Coefficient of $e^{\frac{-jn2\pi t}{T}}$ in Fourier series

Table: Input Parameters Table

Solution

$$C_n = \frac{1}{T}\int_0^T \cos^4(t)e^{\frac{-jn2\pi t}{T}}\,dt \tag{1}$$

$$C_n = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \cos^4(x)\cos(nx)\,dx + 0 \tag{2}$$

$$C_4 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} (\cos x)^4\cos(4x)\,dx \tag{3}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\int_0^{\frac{\pi}{2}} \cos(4x)\,dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2x)\cos(4x)\,dx + \frac{1}{8}\int_0^{\frac{\pi}{2}} \cos(4x)^2\,dx\right) \tag{4}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\frac{1}{4}\sin(4x)\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\left[\frac{1}{6}\sin(6x) + \frac{1}{2}\sin(2x)\right]\Big|_0^{\frac{\pi}{2}} + \frac{1}{8}\frac{1}{2}\left[x + \frac{1}{8}\sin(8x)\right]\Big|_0^{\frac{\pi}{2}}\right) \tag{5}$$

$$= \frac{1}{16} \tag{6}$$

$$C_5 = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} (\cos x)^4 \cos(5x)\,dx \tag{7}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\int_0^{\frac{\pi}{2}} \cos(5x)\,dx + \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2x)\cos(5x)\,dx + \frac{1}{8}\int_0^{\frac{\pi}{2}} \cos(4x)\cos(5x)\,dx\right) \tag{8}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\frac{1}{5}\sin(5x)\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\left[\frac{1}{7}\sin(7x) + \frac{1}{3}\sin(3x)\right]\Big|_0^{\frac{\pi}{2}} + \frac{1}{2}\left[\frac{1}{9}\sin(9x) + \sin(x)\right]\Big|_0^{\frac{\pi}{2}}\right) \tag{9}$$

$$= 0 \tag{10}$$

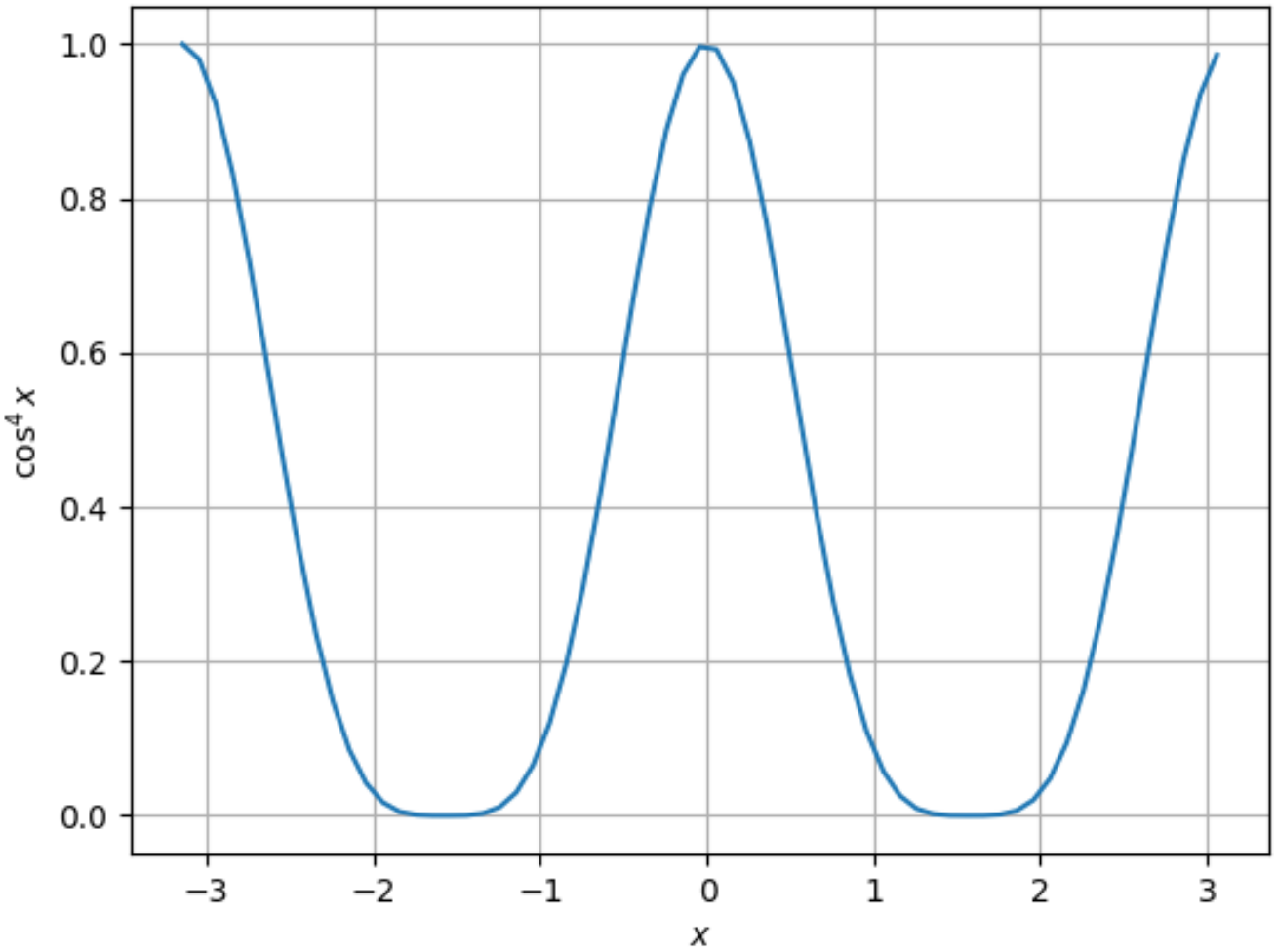
Since the function is even,

$$\cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t}) \tag{11}$$

$$f_n = 2C_n \tag{12}$$

$$\therefore f_4 + f_5 = 0.125 \tag{13}$$

Plot



C Code

```c
#include <stdio.h>
#include <math.h>
#include "mylib.h"
float f(float x){
return pow(cos(x) , 4);
}
int main() {
    // Define the range and step size
    float start = -M_PI;
    float stop = M_PI;
    float step = 0.1;

    // Calculate the number of values in the range
    int num_values = (stop - start) / step + 1;

    // Allocate arrays to store the generated values
    float x_values[num_values];
    float y_values[num_values];
        float(*func)(float);
        func = f;
    // Call the linspace function
    linspace(start, stop, step, x_values, y_values, num_values, func);
    //Call save function
    save(num_values, x_values, y_values);
    }
```

Library Code

```c
#include "mylib.h"
#include <stdio.h>
#include <math.h>
void linspace(float start, float stop, float step, float* x_values, float* y_values, int num_values, float(*func)(float)){
for(int i = 0; i<num_values; ++i){
 x_values[i] = start + i * step;
 y_values[i] = func(x_values[i]);
}}
void save(int x, float* x_values, float* y_values){
//Save data to a file
     FILE* file = fopen("output.dat", "w");

    if (file != NULL) {
        for (int i = 0; i < x; ++i) {
            fprintf(file, "%f %f\n", x_values[i], y_values[i]);
        }

        fclose(file);
        printf("Data saved to 'output.dat'.\n");
    } else {
        printf("Error opening file for writing.\n");
    }
}
void app(int x, float* x_values, float* y_values){
//Save data to a file
     FILE* file = fopen("output.dat", "a");

    if (file != NULL) {
        for (int i = 0; i < x; ++i) {
            fprintf(file, "%f %f\n", x_values[i], y_values[i]);
        }

        fclose(file);
        printf("Data saved to 'output.dat'.\n");
    } else {
        printf("Error opening file for writing.\n");
    }
}
```

Python Code

```python
import numpy as np
import matplotlib.pyplot as plt

# Load data from the "output.dat" file using numpy's loadtxt
data = np.loadtxt("output.dat")

# Extract n_values and y_values from the data
x_values = data[:, 0].astype(float)
y_values = data[:, 1].astype(float)

# Plot
plt.plot(x_values, y_values)
plt.xlabel(r'$x$')
plt.ylabel(r'$\cos^{4}x$')
plt.grid(True)
plt.savefig('../figs/fig1.png')
```