

# GATE 22 CE-30

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# Question

Q: The Fourier cosine series of a function is given by:

$f(x) = \sum_{n=0}^{\infty} f_n \cos nx$ . For  $f(x) = \cos^4 x$ , the numerical value of  $(f_4 + f_5)$  is

# Input Parameters Table

| Parameter | Value | Description  |
|-----------|-------|--|
| $f(x)$    | -     | Function   |
| $f_n$     | -     | Coefficient of $\cos nx$ in Fourier series                 |
| $C_n$     | -     | Coefficient of $e^{\frac{-jn2\pi t}{T}}$ in Fourier series |

Table: Input Parameters Table

$$C_n = \frac{1}{T} \int_0^T \cos^4(x) e^{\frac{-jn2\pi t}{T}} dx \quad (1)$$

$$C_n = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos^4(x) \cos(nx) dx + 0 \quad (2)$$

$$C_4 = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} (\cos x)^4 \cos(4x) dx \quad (3)$$

$$= \frac{2}{\pi} \left( \frac{3}{8} \int_0^{\frac{\pi}{2}} \cos(4x) dx + \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos(2x) \cos(4x) dx + \frac{1}{8} \int_0^{\frac{\pi}{2}} \cos(4x)^2 dx \right) \quad (4)$$

$$= \frac{2}{\pi} \left( \frac{3}{8} \frac{1}{4} \sin(4x) \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \left[ \frac{1}{6} \sin(6x) + \frac{1}{2} \sin(2x) \right] \Big|_0^{\frac{\pi}{2}} + \frac{1}{8} \frac{1}{2} \left[ x + \frac{1}{8} \sin(8x) \right] \Big|_0^{\frac{\pi}{2}} \right) \quad (5)$$

$$= \frac{1}{16} \quad (6)$$

$$C_5 = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} (\cos x)^4 \cos(5x) dx \quad (7)$$

$$= \frac{2}{\pi} \left( \frac{3}{8} \int_0^{\frac{\pi}{2}} \cos(5x) dx + \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos(2x) \cos(5x) dx + \frac{1}{8} \int_0^{\frac{\pi}{2}} \cos(4x) \cos(5x) dx \right) \quad (8)$$

$$= \frac{2}{\pi} \left( \frac{3}{8} \frac{1}{5} \sin(5x) \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \left[ \frac{1}{7} \sin(7x) + \frac{1}{3} \sin(3x) \right] \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \left[ \frac{1}{9} \sin(9x) + \sin(x) \right] \Big|_0^{\frac{\pi}{2}} \right) \quad (9)$$

$$= 0 \quad (10)$$

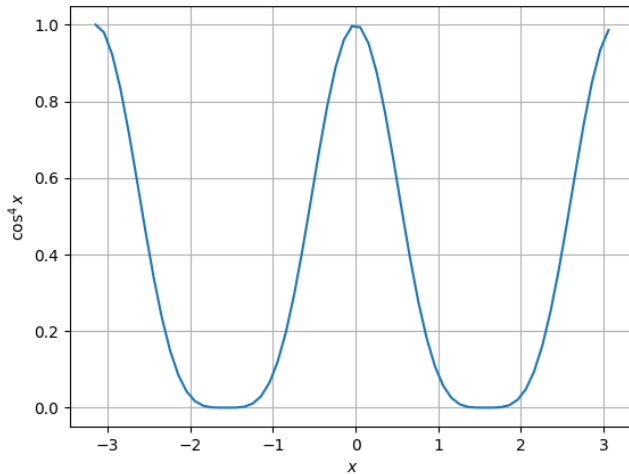
Since the function is even,

$$\cos(\omega t) = \frac{1}{2}(e^{j\omega t} + e^{-j\omega t}) \quad (11)$$

$$f_n = 2C_n \quad (12)$$

$$\therefore f_4 + f_5 = 0.125 \quad (13)$$

# Plot



# C Code

```
1 #include <stdio.h>
2 #include <math.h>
3 #include "mylib.h"
4 float f(float x){
5     return pow(cos(x) , 4);
6 }
7 int main() {
8     // Define the range and step size
9     float start = -M_PI;
10    float stop = M_PI;
11    float step = 0.1;
12
13    // Calculate the number of values in the range
14    int num_values = (stop - start) / step + 1;
15
16    // Allocate arrays to store the generated values
17    float x_values[num_values];
18    float y_values[num_values];
19    float(*func)(float);
20    func = f;
21    // Call the linspace function
22    linspace(start, stop, step, x_values, y_values, num_values, func);
23    //Call save function
24    save(num_values, x_values, y_values);
25 }
```

# Library Code

```
1 #include "mylib.h"
2 #include <stdio.h>
3 #include <math.h>
4 void linspace(float start, float stop, float step, float* x_values, float* y_values, int num_values, float(*func)(float)){
5     for(int i = 0; i < num_values; ++i){
6         x_values[i] = start + i * step;
7         y_values[i] = func(x_values[i]);
8     }
9     void save(int x, float* x_values, float* y_values){
10         //Save data to a file
11         FILE* file = fopen("output.dat", "w");
12
13         if (file != NULL) {
14             for (int i = 0; i < x; ++i) {
15                 fprintf(file, "%f %f\n", x_values[i], y_values[i]);
16             }
17             fclose(file);
18             printf("Data saved to 'output.dat'.\n");
19         } else {
20             printf("Error opening file for writing.\n");
21         }
22     }
23 }
24 void app(int x, float* x_values, float* y_values){
25     //Save data to a file
26     FILE* file = fopen("output.dat", "a");
27
28     if (file != NULL) {
29         for (int i = 0; i < x; ++i) {
30             fprintf(file, "%f %f\n", x_values[i], y_values[i]);
31         }
32
33         fclose(file);
34         printf("Data saved to 'output.dat'.\n");
35     } else {
36         printf("Error opening file for writing.\n");
37     }
38 }
```



# Python Code

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 # Load data from the "output.dat" file using numpy's loadtxt
5 data = np.loadtxt("output.dat")
6
7 # Extract n_values and y_values from the data
8 x_values = data[:, 0].astype(float)
9 y_values = data[:, 1].astype(float)
10
11 # Plot
12 plt.plot(x_values, y_values)
13 plt.xlabel(r'$x$')
14 plt.ylabel(r'$\cos^4 x$')
15 plt.grid(True)
16 plt.savefig('../figs/fig1.png')
```