

Discrete 11.9.3.27

EE23BTECH11018 - E.Mohana*

Question

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b .

Input Parameters Table

Parameter	Value	Description
$x(0)$	a	First term
$x(2)$	b	Third term
$x(1)$	$\sqrt{ab} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$	Second term
r	$\sqrt{\frac{b}{a}}$	Common ratio
n	-	Given variable
$x(k)$	$ar^k u(k)$	General term

Table: Input parameters table

Solution

Consider a GP as in Table 1,

$$\therefore \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = x(1) \quad (1)$$

$$\implies a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}} \quad (2)$$

$$\implies a^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) \quad (3)$$

$$\implies \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = \left(\frac{a}{b}\right)^0 \quad (4)$$

$$\implies n = -\frac{1}{2} \quad (5)$$

From Table 1,

$$X(z) = \frac{a}{1 - \left(\sqrt{\frac{b}{a}}\right)z^{-1}} \quad |z| > \left|\sqrt{\frac{b}{a}}\right| \quad (6)$$