

A finite impulse response (FIR) filter has only two non-zero samples in its impulse response $h[n]$, namely $h[0] = h[1] = 1$. The Discrete Time Fourier Transform (DTFT) of $h[n]$ equals $H(e^{j\omega})$, as a function of the normalized angular frequency ω . For the range $|\omega| \leq \pi$, $|H(e^{j\omega})|$ is equal to

- (A) $2|\cos(\omega)|$
- (B) $2|\sin(\omega)|$
- (C) $2\left|\cos\left(\frac{\omega}{2}\right)\right|$
- (D) $2\left|\sin\left(\frac{\omega}{2}\right)\right|$

(GATE BM 2023)

Solution:

Parameter	Value	Description
$h[n]$	-	impulse response
$h[0]$	1	impulse response at $n = 0$
$h[1]$	1	impulse response at $n = 1$
ω	$-\pi \leq \omega \leq \pi$	normalized frequency
$H(e^{j\omega})$	$\sum_{n=0}^M h[n]e^{-jn\omega}$	frequency response

TABLE I

INPUT PARAMETERS TABLE

From Table I,

$$H(e^{j\omega}) = 1 + e^{-j\omega} \quad (1)$$

$$= e^{-\frac{j\omega}{2}} (e^{\frac{j\omega}{2}} + e^{-\frac{j\omega}{2}}) \quad (2)$$

$$= e^{-\frac{j\omega}{2}} (2\cos\left(\frac{\omega}{2}\right)) \quad (3)$$

$$|H(e^{j\omega})| = 2 \left| \cos\left(\frac{\omega}{2}\right) \right| \quad (4)$$

