A finite impulse response (FIR) filter has only two non-zero samples in its impulse response $h[n]$, namely $h[0] = h[1] = 1$. The Discrete Time Fourier Transform (DTFT) of $h[n]$ equals $H(e^{j\omega})$, as a function of the normalized angular frequency ω. For the range $|\omega| \leq \pi$, $\left|H(e^{j\omega})\right|$ is equal to

(A) $2\,|\cos(\omega)|$
(B) $2\,|\sin(\omega)|$
(C) $2\left|\cos(\frac{\omega}{2})\right|$
(D) $2\left|\sin(\frac{\omega}{2})\right|$

(GATE BM 2023)

Solution:

Parameter	Value	Description
$h[n]$	-	impulse response
$h[0]$	1	impulse response at $n = 0$
$h[1]$	1	impulse response at $n = 1$
ω	$-\pi \leq \omega \leq \pi$	normalized frequency
$H(e^{j\omega})$	$\sum_{n=0}^{M} h[n]e^{-jn\omega}$	frequency response

TABLE I
Input Parameters Table

From Table I,

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-2j\omega} + ... + h[M]e^{-Mj\omega} \tag{1}$$

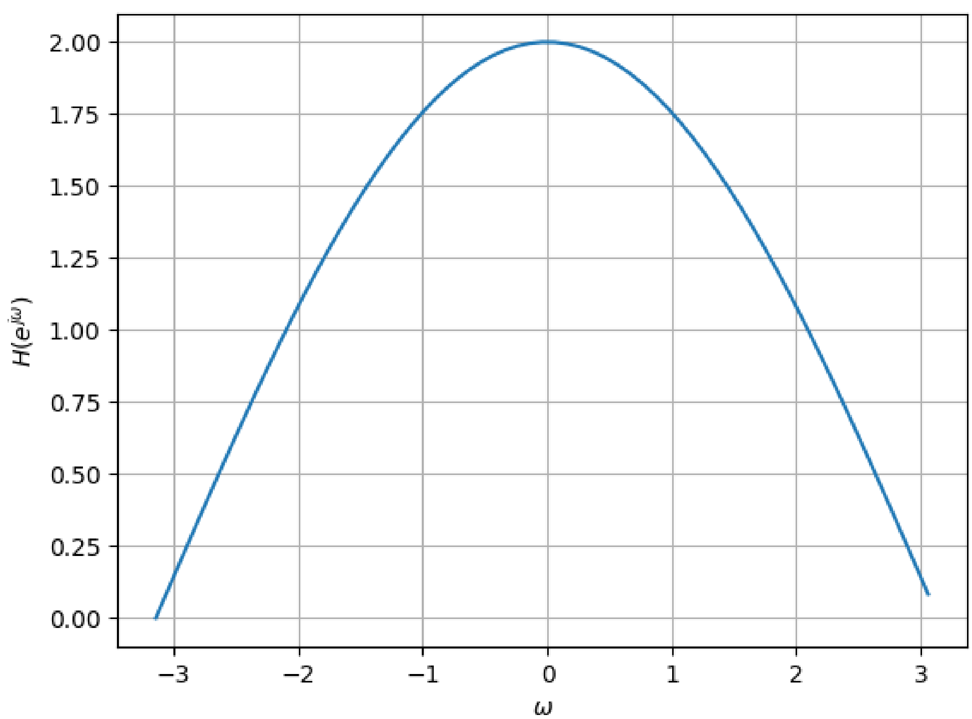
$$= 1 + e^{-j\omega} \tag{2}$$

$$\left|H(e^{j\omega})\right| = \sqrt{(1 + \cos(j\omega))^2 + (\sin(-j\omega))^2} \tag{3}$$

$$= \sqrt{1 + (\cos(j\omega))^2 + 2\cos(j\omega) + (\sin(j\omega))^2} \tag{4}$$

$$= \sqrt{2(1 + \cos(j\omega))} \tag{5}$$

$$= 2\left|\cos\left(\frac{j\omega}{2}\right)\right| \tag{6}$$


2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
H(e^{jω})
−3
−2
−1
0
1
2
3
ω