Q: The Fourier cosine series of a function is given by: $f(x) = \sum_{n=0}^{\infty} f_n \cos nx$. For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is

Solution:

Parameter	Value	Description
$f(x)$	-	Function
f_n	-	Coefficient of $\cos nx$ in Fourier series

TABLE I

INPUT PARAMETERS TABLE

$$f(x) = \frac{1}{2}a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x) + \dots \tag{1}$$

$$+ b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x) + \dots \tag{2}$$

$$\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \frac{1}{2}a_0\int_{-\pi}^{\pi}\cos(nx)\,dx + a_1\int_{-\pi}^{\pi}\cos(x)\cos(nx)\,dx + a_2\int_{-\pi}^{\pi}\cos(2x)\cos(nx)\,dx + \dots \tag{3}$$

$$+ b_1\int_{-\pi}^{\pi}\sin(x)\cos(nx)\,dx + b_2\int_{-\pi}^{\pi}\sin(2x)\cos(nx)\,dx + \dots \tag{4}$$

$$\int_{-\pi}^{\pi}\sin(mx)\cos(nx)\,dx = 0 \tag{5}$$

$$\int_{-\pi}^{\pi}\sin(mx)\sin(nx)\,dx = \begin{cases}\pi & \text{if} \quad m = n \neq 0\\ 0 & \text{otherwise}\end{cases} \tag{6}$$

$$\int_{-\pi}^{\pi}\cos(mx)\cos(nx)\,dx = \begin{cases}2\pi & \text{if} \quad m = n = 0\\ \pi & \text{if} \quad m = n \neq 0\\ 0 & \text{if} \quad m \neq n\end{cases} \tag{7}$$

$$\therefore f_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \tag{8}$$

$$f_4 = \frac{2}{\pi}\int_0^{\pi}(\cos x)^4\cos(4x)\,dx \tag{9}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\int_0^{\pi}\cos(4x)\,dx + \frac{1}{2}\int_0^{\pi}\cos(2x)\cos(4x)\,dx + \frac{1}{8}\int_0^{\pi}\cos(4x)^2\,dx\right) \tag{10}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\frac{1}{4}\sin(4x)\Big|_0^{\pi} + \frac{1}{2}\left[\frac{1}{6}\sin(6x) + \frac{1}{2}\sin(2x)\right]\Bigg|_0^{\pi} + \frac{1}{8}\frac{1}{2}\left[x + \frac{1}{8}\sin(8x)\right]\Bigg|_0^{\pi}\right) \tag{11}$$

$$= \frac{1}{8} \tag{12}$$

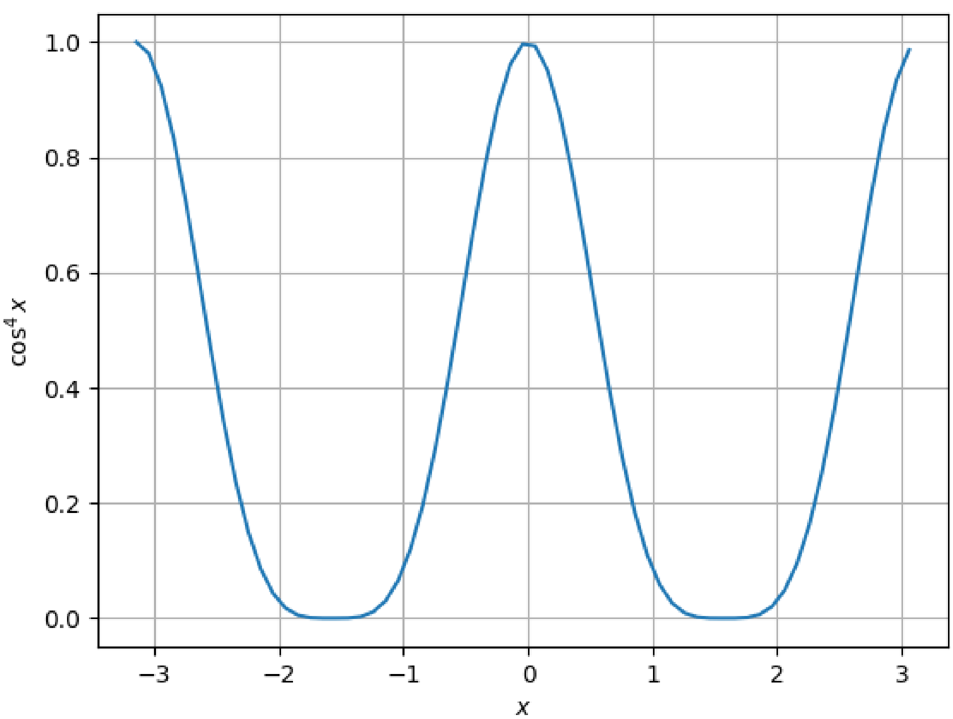
$$f_5 = \frac{2}{\pi}\int_0^{\pi}(\cos x)^4\cos(5x)\,dx \tag{13}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\int_0^{\pi}\cos(5x)\,dx + \frac{1}{2}\int_0^{\pi}\cos(2x)\cos(5x)\,dx + \frac{1}{8}\int_0^{\pi}\cos(4x)\cos(5x)\,dx\right) \tag{14}$$

$$= \frac{2}{\pi}\left(\frac{3}{8}\frac{1}{5}\sin(5x)\Big|_0^{\pi} + \frac{1}{2}\left[\frac{1}{7}\sin(7x) + \frac{1}{3}\sin(3x)\right]\Bigg|_0^{\pi} + \frac{1}{2}\left[\frac{1}{9}\sin(9x) + \sin(x)\right]\Bigg|_0^{\pi}\right) \tag{15}$$

$$= 0 \tag{16}$$

$$\therefore f_4 + f_5 = 0.125 \tag{17}$$


1.0
0.8
0.6
0.4
0.2
0.0
$\cos^4 x$
−3
−2
−1
0
1
2
3
x