Discrete Assignment

Mohana Eppala EE23BTECH11018

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Problem Statement

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

Solution

Parameter	Value	Description
x(0)	a	First term
x(2)	b	Third term
x(1)	$\sqrt{ab} = \frac{a^{n+1} + b^{n+1}}{a^n + b^n}$	Second term
r	$\sqrt{\frac{b}{a}}$	Common ratio
n	-	Given variable
x(k)	$ar^k u(k)$	General term

Table 1: Input parameters table

Consider a GP as in Table 1,

$$\therefore \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = x(1) \tag{1}$$

$$\implies a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}}$$
 (2)

$$\implies a^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n+\frac{1}{2}}(a^{\frac{1}{2}} - b^{\frac{1}{2}})$$
 (3)

$$\implies \left(\frac{a}{b}\right)^{n+\frac{1}{2}} = \left(\frac{a}{b}\right)^0 \tag{4}$$

$$\implies n = -\frac{1}{2} \tag{5}$$

From Table 1,

$$X(z) = \frac{a}{1 - (\sqrt{\frac{b}{a}})z^{-1}} \quad |z| > \left| \sqrt{\frac{b}{a}} \right| \tag{6}$$