Q: The Fourier cosine series of a function is given by: $f(x) = \sum_{n=0}^{\infty} f_n \cos nx$. For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is

Solution:

Parameter	Description
$f(x)$	Function
f_n	Coefficient of $\cos nx$ in Fourier series

TABLE I

INPUT PARAMETERS TABLE

$$\cos^4 x = \cos^2 \cos^2 \tag{1}$$

$$= \left(\frac{1 + \cos 2x}{2}\right)\left(\frac{1 + \cos 2x}{2}\right) \tag{2}$$

$$= \frac{1}{4}\left(1 + 2\cos(2x) + \cos^2(2x)\right) \tag{3}$$

$$= \frac{1}{4}\left(1 + 2\cos 2x + \frac{(1 + \cos 4x)}{2}\right) \tag{4}$$

$$= \frac{1}{4} + \frac{\cos 2x}{2} + \frac{1}{8} + \frac{\cos 4x}{8} \tag{5}$$

$$= \frac{3}{8} + \frac{\cos 2x}{2} + \frac{\cos 4x}{8} \tag{6}$$

$$\therefore f_4 = \frac{1}{8}, \tag{7}$$

$$f_5 = 0 \tag{8}$$

$$\implies f_4 + f_5 = 0.125 \tag{9}$$