

Q: The Fourier cosine series of a function is given by: $f(x) = \sum_{n=0}^{\infty} f_n \cos nx$. For $f(x) = \cos^4 x$, the numerical value of $(f_4 + f_5)$ is

Solution:

Parameter	Value	Description
$f(x)$	-	Function
f_n	-	Coefficient of $\cos nx$ in Fourier series
C_n	-	Coefficient of $e^{\frac{-jn2\pi t}{T}}$ in Fourier series

TABLE I
INPUT PARAMETERS TABLE

$$C_n = \frac{1}{T} \int_0^T \cos^4(x) e^{\frac{-jn2\pi t}{T}} dx \quad (1)$$

$$C_n = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \cos^4(x) \cos(nx) dx + 0 \quad (2)$$

$$C_4 = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} (\cos x)^4 \cos(4x) dx \quad (3)$$

$$= \frac{2}{\pi} \left(\frac{3}{8} \int_0^{\frac{\pi}{2}} \cos(4x) dx + \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos(2x) \cos(4x) dx + \frac{1}{8} \int_0^{\frac{\pi}{2}} \cos(4x)^2 dx \right) \quad (4)$$

$$= \frac{2}{\pi} \left(\frac{3}{8} \frac{1}{4} \sin(4x) \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \left[\frac{1}{6} \sin(6x) + \frac{1}{2} \sin(2x) \right] \Big|_0^{\frac{\pi}{2}} + \frac{1}{8} \frac{1}{2} \left[x + \frac{1}{8} \sin(8x) \right] \Big|_0^{\frac{\pi}{2}} \right) \quad (5)$$

$$= \frac{1}{16} \quad (6)$$

$$C_5 = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} (\cos x)^4 \cos(5x) dx \quad (7)$$

$$= \frac{2}{\pi} \left(\frac{3}{8} \int_0^{\frac{\pi}{2}} \cos(5x) dx + \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos(2x) \cos(5x) dx + \frac{1}{8} \int_0^{\frac{\pi}{2}} \cos(4x) \cos(5x) dx \right) \quad (8)$$

$$= \frac{2}{\pi} \left(\frac{3}{8} \frac{1}{5} \sin(5x) \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \left[\frac{1}{7} \sin(7x) + \frac{1}{3} \sin(3x) \right] \Big|_0^{\frac{\pi}{2}} + \frac{1}{2} \left[\frac{1}{9} \sin(9x) + \sin(x) \right] \Big|_0^{\frac{\pi}{2}} \right) \quad (9)$$

$$= 0 \quad (10)$$

Since the function is even,

$$f_n = 2C_n \quad (11)$$

$$\therefore f_4 + f_5 = 0.125 \quad (12)$$

