STRUCTURE

Figures:

Abstract

This article focuses on the decomposition of the solutions to MFDEs into sums of "future" solutions and "backward" solutions through numerical methods. We take into account form equations $x'(t) = ax(t) + bx(t-1) + cx(t+1)$ and develop a numerical approach, which results in the desired decomposition and propagation of the solution using a central difference approximation. We have illustrative examples to show our method's success and to show its current limitations.

INTRODUCTION

Analytical Solutions

Many problems have defined solutions, which are evident once the problem is defined.

A number of logical steps to calculate a precise result.

For example, given a particular arithmetical task, such as addition and subtraction, you know which operation to use.

A number of methods are available in the linear algebra, which can be used to factor a matrix, depending upon whether its properties are quadratic, rectangular, real or imaginary, etc.

This can be extended broadly towards software technology, where problems arise over and over again and can be solved by a design pattern that works well, regardless of the specifics of your application. Like the pattern of the visitor to perform an operation on every item in the list.

Some problems are well defined and an analytic solution are provided in applied machine learning.[1]

For example, a simple, repeatable and (practically) identical method is the method of transforming a categorical variable into one hot encoding irrespective of how many integer values it contains.

Unfortunately, most of the problems in machine learning we care about do not have analytical solutions.

Numerical Methods

Linking branches of general mathematics

Analytical mathematics and computers, usually in finding some solutions

The presentation of math solving problems, where they are

The result we get. It is only the result of a successful procedure

Approximate, it means the percentage of error that we need to calculate, and our presence in the presence of finding

Error finding find the true solution (which means finding an approximation

The error, or the magnitude of the error, has a value that the error does not exceed.) If a task is summarized

Numerical analysis in solving the approximate problem of touching the line straightening path

That is the fastest method, but the derivative off must be analytically calculated (x). Furthermore, the method cannot converge to the desired root. We can graphically or by using the Taylor series from Newton's Method. We would like to build a sequence again$\boldsymbol{x_0}, \boldsymbol{x_1}, \boldsymbol{x_2}, \ldots$

that converges to the root x = r. Consider the $(\boldsymbol{x_{n+1}})$member of this sequence, and Taylor series expand $\boldsymbol{f}(\boldsymbol{x_{n+1}})$ about the point xn. We have

$$f(x_{n+1}) = f(x_n) + (x_{n+1} - x_n)f'(x_n) + \cdots$$

To determine xn+1, we drop the higher-order terms in the Taylor series, and assume $f(x_{n+1}) = 0$. Solving $(x_{n+1}) =$, we have

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Why to study Numerical Methods?

a. A History Review:

- Computing begins at around 3,000 BC with Chinese society's device ABACUS that was used only for arithmetical operations.
- John Napier rules for calculating an early 16th century logarithmic problem.
- Blaise Pascal's accounting machine called Pascal Calculator.
- Loom jacquard data storage facility in the 18th century at punched card.
- 18th century: Charles Babbage's revolutionary invention of Difference Engine.
- All advanced/newest computers are based on this principle.

b. Present context:

Computers now are great instruments for computing numbers that play an essential part in solving mathematical, physical and engineering problems in real life.

But they will be useless without a basic understanding of engineering problems.

Know the expressions to solve triangular matrix using forward and backward substituting techniques and the FLOPS required for solving it.[2][3]

Forward Substitution:

Consider a set of equations in a matrix form $Ax = b$, where A is a lower triangular matrix with non-zero diagonal elements. The equation is re-written in full matrix form as

$$\begin{bmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

It can be solved using the following expressions

$$\begin{aligned} &x_1 = b_1/a_{11} \\ &x_2 = (b_2 - a_{21}x_1)/a_{22} \\ &x_3 = (b_3 - a_{31}x_1 - a_{32}x_2)/a_{33} \\ &\vdots \\ &x_m = b_m - a_{m1}x_1 - a_{m2}x_2 - \cdots - a_{m,m-1}x_{m-1}\big)/a_{mm} \end{aligned}$$

The calculation requires a floating-point operation per second (FLOPS) from the DSP implementation point of view computation of x_3 one division only. x_2 requires three multiplications – 1 multiplication, 1 separation and 1 subtraction, and x_3 requires 5 FLOPS – two multiplications, 1 subtraction and 1 division. This requires FLOPS $(2n - 1)$ for the x_{mm} calculation.[5]

the overall FLOPS required for forward substitution is

$\mathbf{1 + 3 + 5 + \cdots + (2m - 1) = m^2}$ FLOPS.

Backward substitution:

Consider a set of matrix equations $Ax = b$, where A is an upper triangular matrix with diagonal elements that are not zero. The equation is re-written as complete matrix.

$$\begin{bmatrix} \boldsymbol{a_{11}} & \cdots & \boldsymbol{a_{1,m-1}} & \boldsymbol{a_{1m}} \\ \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{a_{m-1,m-1}} & \boldsymbol{a_{m-1,m}} \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{a_{mm}} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_1} \\ \vdots \\ \boldsymbol{x_{m-1}} \\ \boldsymbol{x_m} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b_1} \\ \vdots \\ \boldsymbol{b_{m-1}} \\ \boldsymbol{b_m} \end{bmatrix}$$

Solved using the following algorithm

$$\begin{aligned} \boldsymbol{x_m} &= \boldsymbol{b_m / a_{mm}} \\ \boldsymbol{x_{m-1}} &= \boldsymbol{\left(b_{m-1} - a_{m-1,m} x_m\right)/a_{m-1,m-1}} \\ \boldsymbol{x_{m-2}} &= \boldsymbol{\left(b_{m-2} - a_{m-2,m-1} x_{m-1} - a_{m-2,m} x_m\right)/a_{m-2,m-2}} \\ &\vdots \\ \boldsymbol{x_1} &= \boldsymbol{b_1 - a_{12} x_2 - a_{13} x_3 - \cdots - a_{1m} x_m)/a_{11}} \end{aligned}$$

This one also requires m^2 FLOPS.

Example 1. Consider the following wave equation:

1. $$\frac{\partial^2 w(x,t)}{\partial t^2} - 4\frac{\partial^2 w(x,t)}{\partial x^2} = 0, 0 \leqslant x \leqslant 1, 0 < t$$

with the boundary conditions

$$w(0,t) = w(1,t) = 0, 0 < t$$

and initial conditions

2. $$w(x,0) = \sin(\pi x), 0 \leqslant x \leqslant 1, \quad \frac{\partial w(x,0)}{\partial t} = 0, 0 \leqslant x \leqslant 1$$

Taking the differential transform of Eq. (1), then

3. $$(k+2)(k+1)W(i,k+2) = 4(i+2)(i+1)W(i+2,k).$$

From the initial condition given by Eq. (2)

$$w(x,0)=\sum_{i=0}^{\infty} W(i,0)x^i=\sin(\pi x)=\sum_{i=1,3,\ldots}^{\infty}\frac{(-1)^{(i-1)/2}}{i!}\pi^i x^i$$

the corresponding spectra can be obtained as follows:

$$W(i,0)=\begin{cases}0, & \text{for } i \text{ is even}\\ \frac{(-1)^{\frac{i-1}{2}}}{i!}\pi^i, & \text{for } i \text{ is odd}\end{cases}$$

and from Eq. (2) it can be obtained

$$4.\ \frac{\partial w(x,0)}{\partial t}=\sum_{i=0}^{\infty} W(i,1)x^i=0.$$

Hence

$$5.\ W(i,1)=0.$$

Substituting Eq. (3) and (4) to Eq. (5), all spectra can be found as $W(i,k)=$

$$\begin{cases}0, & \text{for } i \text{ is even or } k \text{ is odd,}\\ \frac{2^k(-1)^{\frac{i+k-1}{2}}}{i!k!}\pi^{i+k}, & \text{for } i \text{ is odd or } k \text{ is even.}\end{cases}$$

Therefore, the closed form of the solution can be easily written as

$$\begin{aligned} w(x,t)&=\sum_{i=0}^{\infty}\sum_{k=0}^{\infty} W(i,k)x^i t^k=\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\frac{2^k}{k!i!}(-1)^{(i+k-1)/2}\pi^{i+k}x^i t^k\\ &=\left(\sum_{i=1,3,\ldots}^{\infty}\frac{1}{i!}(-1)^{(i-1)/2}(\pi x)^i\right)\left(\sum_{k=0,2,\ldots}^{\infty}\frac{1}{k!}(-1)^{(k/2)}(2\pi t)^k\right)\\ &=\sin(\pi x)\cos(2\pi t).\end{aligned}$$

Now let us use this decomposition method to calculate the approximate solution of this linear differential equation If this method is addressed by Eq. (1).

$$w_{tt}=4w_{\mathrm{x}}$$

is found. In operator form, this equation is formed as

$$6.\ L_t w=4L_x w,$$

Where

$$L_t = \frac{\partial^2}{\partial t^2}, L_x = \frac{\partial^2}{\partial x^2}$$

On the other hand, using the integral operator is shown as below

$$L_t^{-1}(\cdot) = \int_0^t \int_0^t (\cdot)\mathrm{d}t\mathrm{d}t$$

and applying both sides of Eq. (6)

$$7.\ \ w(x,t) = w(x,0) + tw_t(x,0) + 4L_t^{-1}(L_x w).$$

Here the main point is that the solution of the decomposition method is in the form of

$$8.\ \ w(x,t) = \textstyle\sum_{n=0}^{\infty} w_n(x,t).$$

Substituting Eq. (7) in (8)

$$\sum_{n=0}^{\infty} w_n(x,t) = w(x,0) + tw_t(x,0) + 4L_t^{-1}\left(L_x \sum_{n=0}^{\infty} w_n\right)$$

is found. Thus, approximate solution can be obtained as follows:

$$9.\ \ \begin{aligned} w_0 &= w(x,0) + tw_t(x,0) \\ w_{n+1} &= 4L_t^{-1}(L_x w_n), n \geqslant 0 \end{aligned}$$

On the other hand, from Eq. (9)

$$\begin{aligned}
w_0 &= \sin(\pi x) \\
w_1 &= -2\pi^2 t^2 \sin(\pi x) \\
w_2 &= \frac{2}{3}\pi^4 t^4 \sin(\pi x) \\
w_3 &= -\frac{4}{45}\pi^6 t^6 \sin(\pi x) \\
w_4 &= \frac{2}{315}\pi^8 t^8 \sin(\pi x) \\
w_5 &= -\frac{4}{14{,}175}\pi^{10} t^{10} \sin(\pi x) \\
w_6 &= \frac{4}{467{,}775}\pi^{12} t^{12} \sin(\pi x)
\end{aligned}$$

are found. Thus, the approximate solution of the given equation is obtained as

$$w(x,t) = \sin(\pi x)\left(1 - 2\pi^2 t^2 + \frac{2}{3}\pi^4 t^4 - \frac{4}{45}\pi^6 t^6 + \frac{2}{315}\pi^8 t^8 - \frac{4}{14{,}175}\pi^{10} t^{10} + \frac{4}{467{,}775}\pi^{12} t^{12} - \cdots\right) = \sin(\pi x)\cos(2\pi t)$$

The analytic solution an approximate solution is by coincidence exactly identical to the given equation.

As can be seen in Table 1 of this chart, it is clearly apparent that By using the method of differential transformation and domain decomposition the partial differential equation solution is calculated (ADM),as shown in Fig. 1 and 2, in the area as shown in Fig. 2. Since it is possible to say;[5]

Table .1 Different approximate solutions for Example .1:

x	t	Exact solutions	Approximate solutions with DTM	Approximate solutions with ADM
−1.0	−1.0	0.0000000000	0.0000000000	0.0000000000
−0.8	−0.8	−0.1816356321	−0.1570247070	−0.1570247070
−0.6	−0.6	0.7694208839	0.6326088724	0.6326088724
−0.4	−0.4	0.7694208846	0.7669073581	0.7669073581
−0.2	−0.2	−0.1816356317	−0.1816372037	−0.1816372037
0.0	0.0	0.0000000000	0.0000000000	0.0000000000
0.2	0.2	0.1816356317	0.1816372037	0.1816372037
0.4	0.4	−0.7694208846	−0.7669073581	−0.7669073581
0.6	0.6	−0.7694208839	−0.6326088724	−0.6326088724
0.8	0.8	0.1816356321	0.1570247070	0.1570247070
1.0	1.0	0.0000000000	0.0000000000	0.0000000000

Fig. 1. The analytic solution of Example. 1

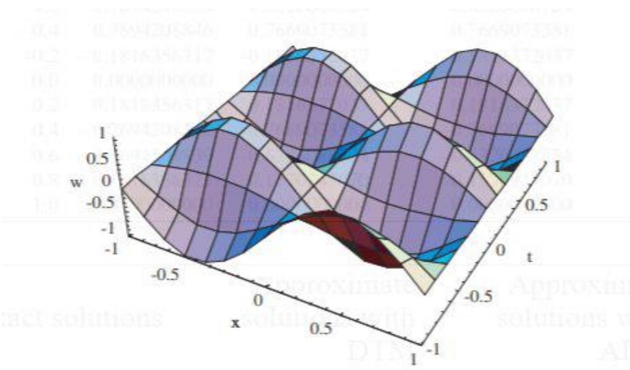
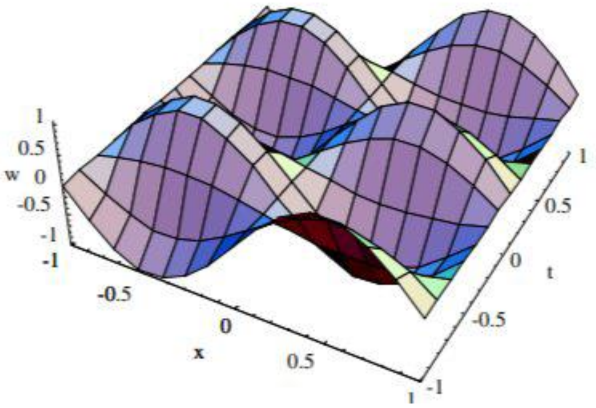


Fig. 2. The approximate solution with A domain's decomposition method of



Example .1.

Example 2. Consider the following Laplace equation:

$$1.\ \ u_{xx} + u_{yv}$$

with the boundary conditions

$$2.\ \ u_y(x,0) = \cos\ x, u(x,0) = 0$$

Taking the differential transform of (1),then

$$(k+1)(k+2)U(k+2,h) + (h+1)(h+2)U(k,h+2)$$

and also

$$U(k+2,h) = -\frac{(h+1)(h+2)U(k,h+2)}{(k+1)(k+2)}$$

From the boundary conditions Eq. (2), all spectra can be found that

$$U(k,1) = \begin{cases} \frac{(-1)^{k+1}}{k!}, & k \text{ even} \\ 0, & k \text{ odd} \end{cases}$$

And

$$3.\ \ U(k,h) = \begin{cases} \frac{(-1)^{k+1}}{k!h!}, & k \text{ even and } h \text{ odd} \\ 0, & \text{otherwise} \end{cases}$$

Using Eq. (3), then the closed form of the solution is

$$4.\ \ u(x,y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} U(k,h)x^k y^h = \sum_{k=0}^{\infty} (-1)^{k+1} \frac{x^{2k}}{(2k)!} \sum_{h=0}^{\infty} \frac{y^{2h+1}}{(2h+1)!}$$

Eq. (4) is series expansion of exact solution. Now let us solve the equation by using the decomposition method. Since

$$5.\ \ L_x = \frac{\partial^2}{\partial x^2}, L_y = \frac{\partial^2}{\partial y^2}, L_y^{-1} = \int_0^y \int_0^y (\cdot)\mathrm{d}y\mathrm{d}y$$

SO

$$L_y u = -L_x u, u = \sum_{n=0}^{\infty} u_n$$

is found. If L_y^{-1} is applied to the both sides of the equation, then

$$6.\ \ L_y^{-1} L_y u = -L_y^{-1} L_x u$$

Using Eqs. (6), (5) and (1) thus

$$u(x,y) - u(x,0) - y u_y(x,0) = -L_y^{-1}\left[\frac{\partial^2}{\partial x^2} u\right]$$

is found. And u (x, y) can be reformed as

$$u(x,y) = u(x,0) + y u_y(x,0) - L_y^{-1} L_x u$$

On the other hand, if u_0 is considered as below

$$u_0 = u(x,0) + y u_y(x,0)$$

Then

$$u_{n+1} = -L_y^{-1} L_x u_n, n \geqslant 0$$

is calculated.

From this point,

$u_0 = \mathrm{y}\cos\ x$

$u_1 = -\int_0^y \int_0^y \left[\frac{\partial^2}{\partial x^2} u_0\right] \mathrm{d}y\mathrm{d}y = \int_0^y \int_0^y (\mathrm{y}\cos\ x)\mathrm{d}y\mathrm{d}y = \frac{y^3}{3!}\cos\ x$

$u_2 = -\int_0^y \int_0^y \left[\frac{\partial^2}{\partial x^2} u_1\right] \mathrm{d}y\mathrm{d}y = \int_0^y \int_0^y \left[\frac{y^3}{3!}\cos x\right] \mathrm{d}y\mathrm{d}y = \frac{y^5}{5!}\cos\ x,$

$u_3 = \frac{y^7}{7!}\cos\ x, u_4 = \frac{y^9}{9!}\cos\ x, \ldots$

are found. If these values substitute in equation which is written below

$$u = \sum_{n=0}^{\infty} u_n = u_0 + u_1 + u_2 + \cdots$$

then the solution which is

$$u = \cos\ x\left(y + \frac{y^3}{3!} + \frac{y^5}{5!} + \frac{y^7}{7!} + \frac{y^9}{9!} + \cdots\right)$$

is obtained. So, the analytic solution of this equation is $u = \cos\ x \sinh\ y$

.

Table 2 Different approximate solutions for Example 2

x	y	Exact solutions	Approximate solutions with DTM and ADM
−5.0	−2.0	−1.0288031500	−1.0287882160
−4.0	−1.6	1.5527748380	1.5527719100
−3.0	−1.2	1.4943554150	1.4943552300
−2.0	−0.8	0.3695824950	0.3695824941
−1.0	−0.4	−0.2219304288	−0.2219304288
0.0	0.0	0.0000000000	0.0000000000
1.0	0.4	0.2219304288	0.2219304288
2.0	0.8	−0.3695824950	−0.3695824941
3.0	1.2	−1.4943554150	−1.4943552300
4.0	1.6	−1.5527748380	−1.5527719100
5.0	2.0	1.0288031500	1.0287882160

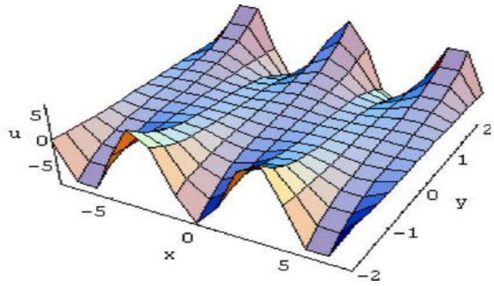


Fig. 3. The analytic solution of Example 2

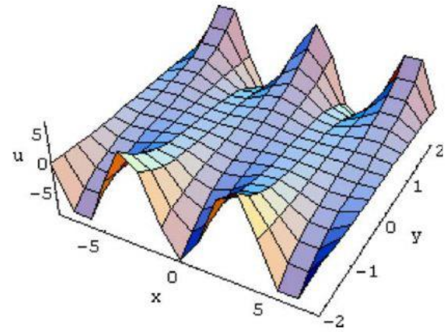


Fig. 4. The approximate solution of Example 2 by using ADM and DTM

On the basis of these values shown in Table 2, the approach is too close to the exact solution. The differences between Figs. 3 and 4 are therefore impossible to understand, as most wanted. Table 3 shows the

The calculation of approximate solutions is made by order expansion 11.

Case Study

Case study is aimed at students of freshwater studies. The objective is for students to examine the factors that affect the dissolution of a new drug coating. Two chemicals engineering students have written this story from the perspective of ChemE 101, a course in fiction. The learning objectives are: 1) describe the factors influencing mass transfers among phases, 2) collect and evaluate mass transfer data, and 3) assess a model to describe mass transfers. The results of the students' case studies are: These general goals are also suitable for a graduate course. A lollipop and the mathematical model are a linear model that shows that the rate of mass transfer is proportional to the gradient of concentration. The rate of the lollipop radius is described in this model. Project for active learning at the beginning of the semester, graduates were given the following problem statement: "Design an encapsulated (your group's choice) drug that provides a suitable dose for a suitable number of hours. Choose the most economical method."

• Consider different kinds of phenomena the problem educational goals were: (microscopic, multiple gradients, maximum gradient, macroscopic-Himmelblau and Bischoff 3).

• Understand what are mathematical and physical models.
• Think of unstable phenomena.
• Develop models covering as many appropriate types of phenomena.
• Developing the maximum spherical co-ordinates gradient model.
• Develop experimental protocol for the simulation of tootsie-pops drug delivery.
• Consider numerical solutions to the problem (ODE vs. PDE). The course was structured to provide students with VBA/EXCEL and MATLAB instruction in a computing lab on Tuesday for the first third of the semester. For the remaining part of the semester, Tuesday was a time for students to either work with instructors to support/mentor or to present specific content (review sessions, presentation skills, lab safety discussion, etc.). Students were grouped at the beginning of the semester. The questions in Table 1 were used to provide heterogeneity of experience, gender and ethnicity in student groups. After

group students, the teachers held an experimental session in which they began studying dissolution cases, to determine whether the linear model for lollipop is appropriate (the students used Tootsie Pops to study a system with an encapsulation material over an internal 'drug' material experimentally. A laboratory designated for the use of students was established.[4]

How you use case studies will depend on the goals, as well as on the format, of your course?

For instance, if the course is extensive, you may use a case study to illustrate and improve the lecture material. (Included in the real life with a particular company or product, for example, is the instructor who is lecturing on marketing principles. In order to talk about a relevant case, you can also consider breaking the class into small groups or pairs in larger class. When your class is smaller and more detailed in a format for discussion, you will be able to explore the perspectives presented more carefully and maybe integrate other educational strategies, such as role playing and debate.
- It is important that you, as a trainer, know all the problems involved, prepare and anticipate the students' potential problems in advance irrespective of the case study in which you use it. Finally, consider who you are and how you can learn from your background, experiences, personalities, etc.[6]

These five steps provide a general framework for conducting case-based discussions, while the use of case studies is subject to several variations: Take time to read the case with students If there are many questions, assign them to students (i.e., what is the nature of the problem which is central to them? What are the possible courses of action? What could be the obstacles?)

1. Enter the case briefly and give instructions on how to handle it. Clare how you want students to think about this (e.g. "As if you were the head judge, approach this case" or "What will you recommend?" Cut off the steps students want to take in the case analysis (e.g. "First, detect limitations and possibilities in which each character worked. Secondly, evaluate each character's decisions and implications. Finally, why and what else you would do "). Specify this when you want some information to be ignored or concentrated by students. (i.e., "I wish to disregard the political attachment of the characters described and to differentiate them as articulated here on stem cell research.")[11]
2. Establish and monitor groups to ensure that all participants are involved. Breaking the entire course into small groups provides more opportunities

for individual students to participate and interact. However, small groups can drift off the track if you do not provide a structure. It is thus an excellent idea to clarify and specify the Group's work (e.g., "You should identify three possible action paths, outline the advantages and disadvantages of each one from a perspective of public relations."). You may want to specify roles for each group too: for instance, one person can keep the others up and watch the time; A second person could have the role to question and examine the group's assumptions and interpretations more carefully. A third person could have their thinking recorded and their decisions reported to the class. Such members could otherwise have a wide view (e.g., liberal, conservative, libertarian) or talk to the different 'stakeholders' in the case study. ' Have groups come forward with their solutions: When groups know they are responsible for creating something to present to the class (a decision, justification, analysis), they will focus more seriously on the discussion. Write down their findings so that you can come back to them in the discussion that follows.[7]

3. Ask questions of clarification and bring a different level of discussion. A case-based leader has one of the challenges of conducting discussions and analyses deeper without guidance. When the debate takes place, ask students about what their own assumptions should be studied, to justify their claims, etc...

4. Problems related to synthesizing. Ensure that students see what they have learned and take these lessons together at the end of the discussion in different ways. However, the job of synthesizing does not need to be done by the instructor; it may be given to one or more students.

5.Some changes in this generalized method include external studies (both individually and in groups) by students, in order to talk about the problem and to compare the real result of the real-life dilemma with classroom solutions.[9]

Numerical Example

Example .3:

Solution of Triangular Systems

- Consider the nonsingular lower triangular system

Lx = b

where

$$L = \begin{bmatrix} l_{11} & & & \\ l_{21} & l_{22} & & \\ \vdots & & \ddots & \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix}, x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

- 3 × 3 example:

$$\begin{bmatrix} l_{11} & & \\ l_{21} & l_{22} & \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

- In genera

$$x_1 = \frac{b_1}{l_{11}}$$

$$x_i = \frac{1}{l_{ii}}\left(b_i - \sum_{j=1}^{i-1} l_{ij}x_j\right), i = 2, \dots, n$$

- The above is called row-oriented forward substitution because it accesses P L by rows. It is called the inner product form

$$\sum_{j=1}^{i-1} l_{ij}x_j$$

- At the ith step, it requires i $\times/\div$ and $i-1$ $\pm$Total number of flops (floating-point operations) is

$$\sum^{n} i + \sum^{n} (i-1) = n(n+1)/2 + n(n-1)/2 = n^2$$

Implementation of Forward Substitution

- ```
  function [n]=mat square(A)
  [n, m] = size(A);
  if n ~= m
     disp (' Error: the matrix should be squared');
     stop
  end
  return
  ```

- forward row (row-oriented version)
  ```
  function [x]=forward row(L,b)
  [n]=mat square(L);
  l = min(diag(abs(L)));
  if l == 0
  disp('Error: the matrix is singular'); x = [];
  return
  end
  x (1) = b(1)/L(1,1);
  for i = 2: n,
  x(i) = (b(i)-L (i,1: i-1) *(x(1: i-1))')/L(i,i);
  end
  x=x';
  return
  ```

- Once we calculate x_i we no longer need b_i. Thus, it is useful to store x over b.[8][12]
  ```

Column-Oriented Forward Substitution

- Partition Lx = b as follows:

$$\begin{bmatrix} l_{11} & 0 \\ \hat{l} & \hat{L} \end{bmatrix} \begin{bmatrix} x_1 \\ \hat{x} \end{bmatrix} = \begin{bmatrix} b_1 \\ \hat{b} \end{bmatrix}, \hat{l} \in \mathbb{R}^{n-1}, \hat{L} \in \mathbb{R}_{(n-1)\times(n-1)}$$

This leads to

$$\begin{aligned} l_{11}x_1 &= b_1 \\ \hat{l}x_1 + \hat{L}\hat{x} &= \hat{b} \\ &= b_1/l_{11} \\ b' &= \hat{b} - \hat{l}x_1 \end{aligned}$$

solve $\hat{L}\hat{x} = b'$ for $\hat{x}(n-1) \times (n-1)$ system

- forward col (column-oriented version)

```
function [b]=forward col(L,b)
% [x]=forward col(L,b)
% >> INPUT <<
% L lower triangular matrix
% b right-hand-side
% >> OUTPUT <<
% x solution of Lx = b
[n]=mat square(L);
l = min(diag(abs(L)));
if l == 0
disp('Error: the matrix is singular'); b = [];
break
end
for j=1:n-1,
b(j)= b(j)/L(j,j);
b(j+1:n)=b(j+1:n)-b(j)*L(j+1:n,j);
end;
b(n) = b(n)/L(n,n);
```

Example .4:

Solution of Triangular Systems. Back substitution.

Solution of Upper Triangular Systems (Row - Oriented Version).

Input: Upper triangular matrix $U \in \mathbb{R}^{n\times n}$,right hand side $b \in \mathbb{R}^n$.

Output: Solution $x \in \mathbb{R}^n$ of $Ux = b$.

Mathematically

$$x_i = \left(b_i - \sum_{j=i+1}^{n} u_{ij}x_j\right)/u_{ii}, \text{ if } u_{ii} \neq 0.$$

MATLAB code that overwrites **b** with the solution to $Ux = b$.

```
if all(diag(u)) == 0
        disp('the matrix is singular')
else
    b(n) = b(n)/U(n,n);
    for i = n-1:-1:1
        b(i)= (b(i) - U(i,i+1:n)*b(i+1:n))/U(i,i);
    end
end
```

Conclusion

-Absolute errors for Example. 1& -Absolute errors for Example.2:

Absolute errors
0.0000000000
0.0246109251
0.1368120115
0.0025135265
0.0000015720
0.0000000000
0.0000015720
0.0025135265
0.1368120115
0.0246109251
0.0000000000

Absolute errors
0.0000149340
0.0000029280
0.0000001850
0.0000000009
0.0000000000
0.0000000000
0.0000000000
0.0000000009
0.0000001850
0.0000029280
0.0000149340

- Inferences

1) Numerical analysis is an algorithm study using numerical approach to mathematical problem analysis (unlike symbolic manipulation) (as distinguished from discrete mathematics).

2) Numerical analysis applies of course in all fields of engineering and physical sciences and elements of scientific computing in life sciences, social science, medicine, business and even the arts were adopted during the 21st century.

3) The growth in computer power has revolutionised the use of realistic mathematical model in science and engineering in order to implement these detailed world models. For instance, in the mechanics of celestial mechanics normal differential equations are found (prediction of the movement of planets and stars and galaxies). For data analysis, linear numerical algebra is important.

4) Numerical methods often depend on formulas for hand interpolations used before the advent of modern computers for the large printed table data. Computers use computers from the mid-20th century, but software algorithms still use many of the same formulas.

5) The figures revert to the earliest mathematical scripts. A Yale Babylon Collection tablet (YBC 7289) gives the square root of 2, the diagonal length of the unit square, a sexagesimal numeric approximation.

6) This long tradition of numerical analysis continues: it provides approximate solutions within specific error limits instead of exact symbolic answers that are only applicable to actual measurements via translation in digits.

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