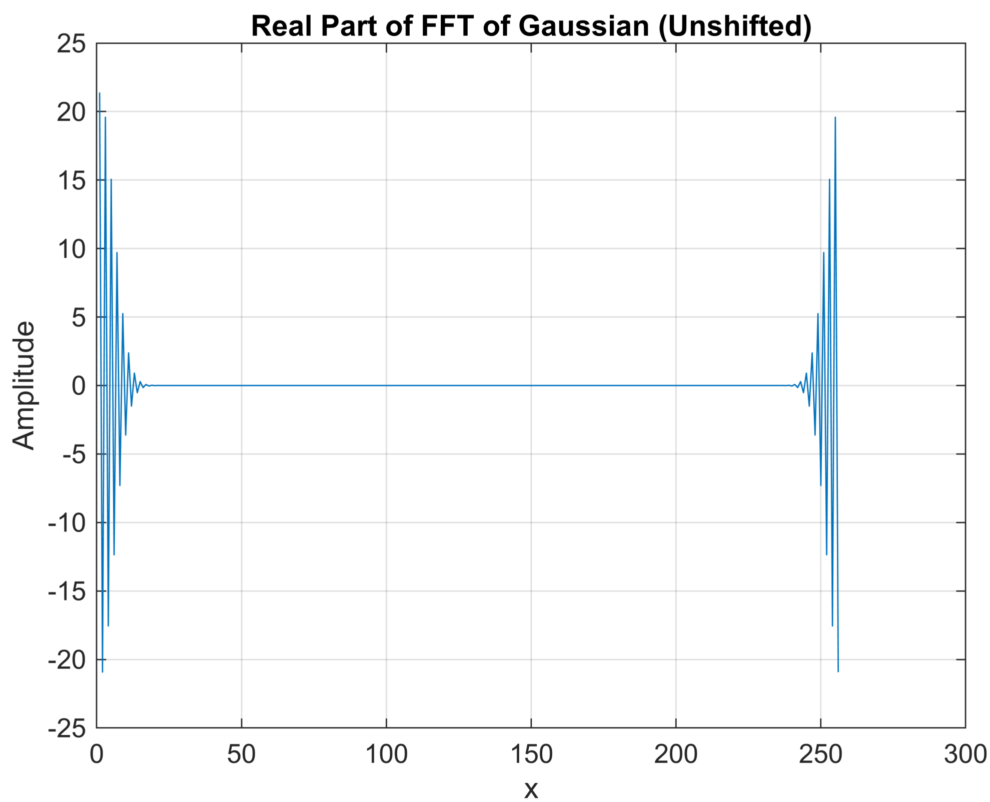
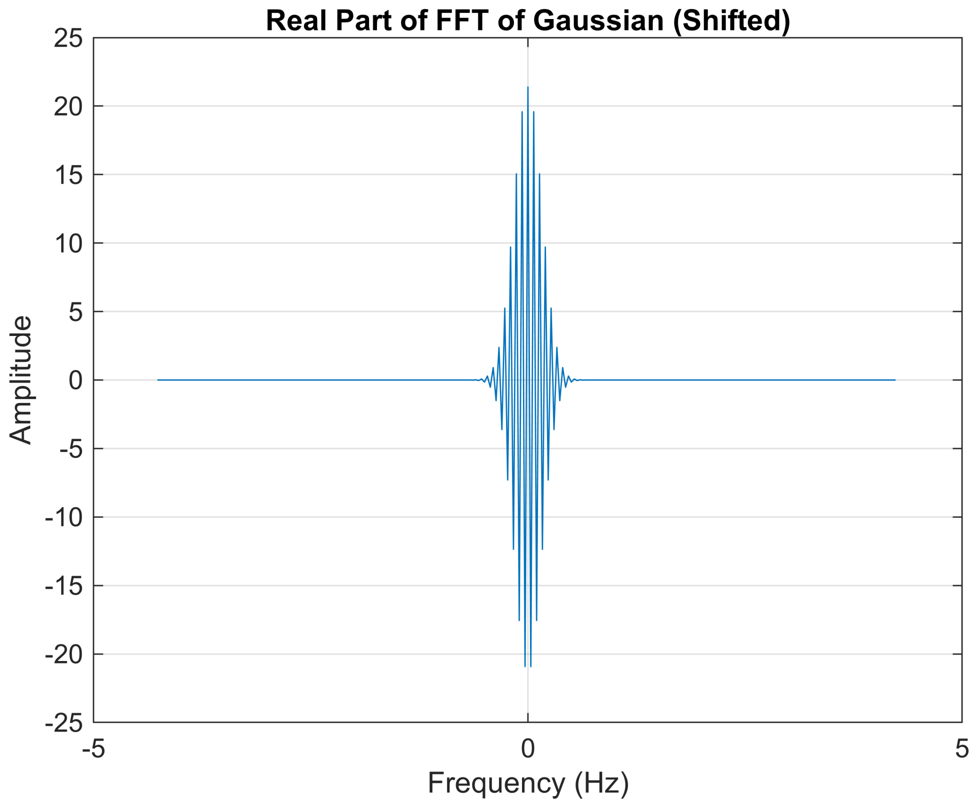
```
% 1.

x = linspace(-15, 15, 257);
x = x(1:256);
y = exp(-0.5 * x.^2);

Y = fft(y);

figure;
plot(real(Y));
title('Real Part of FFT of Gaussian (Unshifted)');
xlabel('x');
ylabel('Amplitude');
grid on;
```




```
Y_shifted = fftshift(Y);
N = length(y);
fs = N / 30;
f = (-N/2 : N/2-1) * (fs/N);
figure;
plot(f, real(Y_shifted));
title('Real Part of FFT of Gaussian (Shifted)');
xlabel('Frequency (Hz)');
ylabel('Amplitude');
grid on;
```




```
fprintf('The Fourier transform of a Gaussian is a Gaussian.\n');
```

```
The Fourier transform of a Gaussian is a Gaussian.
```

```
fprintf('The reason the initial plot doesn''t look right is due to how the FFT
algorithm organizes its output.\n\n');
```

```
The reason the initial plot doesn't look right is due to how the FFT algorithm organizes its output.
```

```
fprintf('The output is not ordered from most negative to most positive frequency.
Instead, it is arranged as:\n');
```

```
The output is not ordered from most negative to most positive frequency. Instead, it is arranged as:
```

```
fprintf('The negative frequency components, which ''wrap around'' to the end of the
array.\n\n');
```

```
The negative frequency components, which 'wrap around' to the end of the array.
```

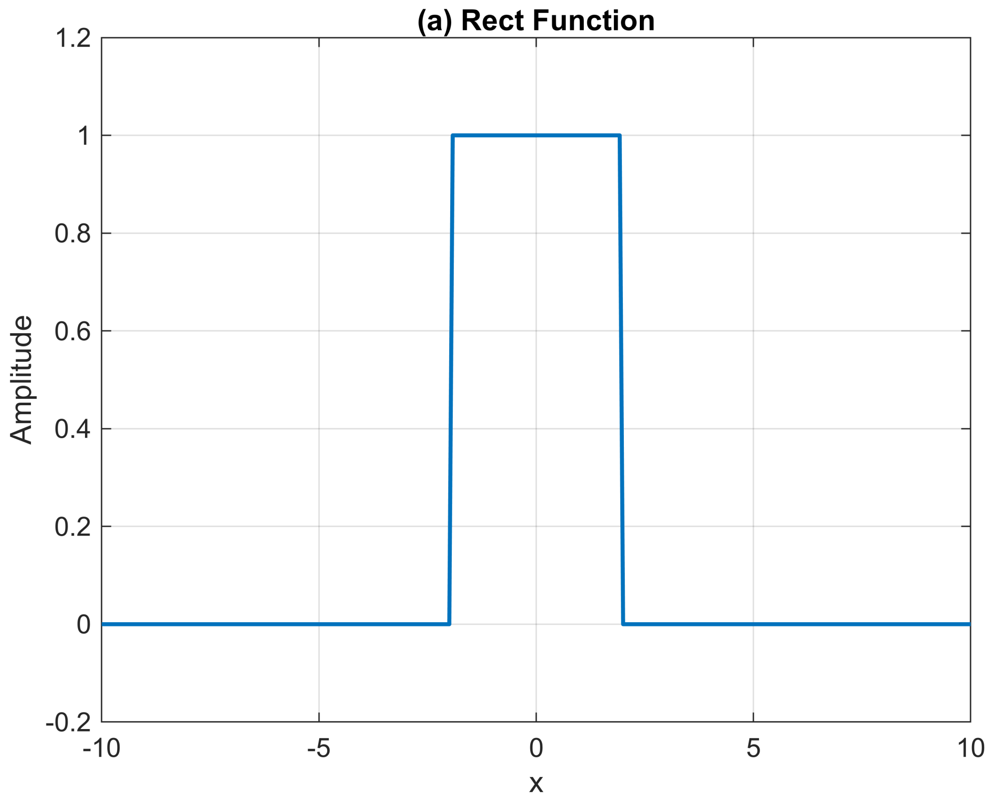
```
fprintf('The fix is to use the fftshift() function. This function rearranges the
array to place\n');
```

```
The fix is to use the fftshift() function. This function rearranges the array to place
```

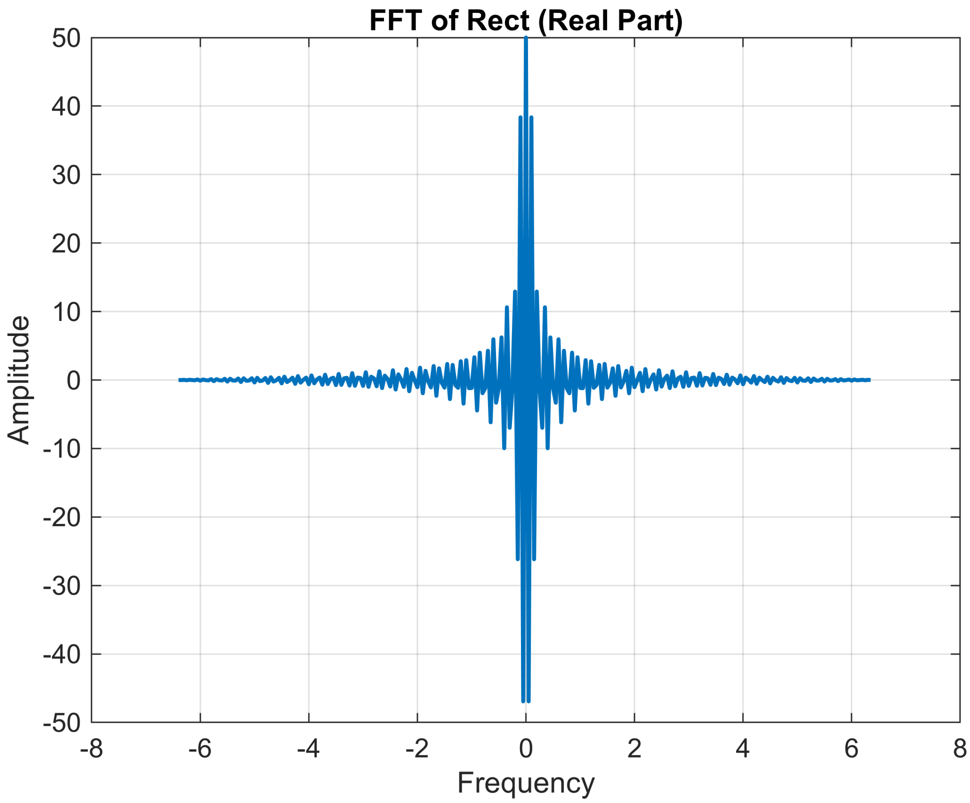
```
fprintf('the zero-frequency component in the center, which is the conventional way
to view a frequency spectrum.\n');
```

the zero-frequency component in the center, which is the conventional way to view a frequency spectrum.

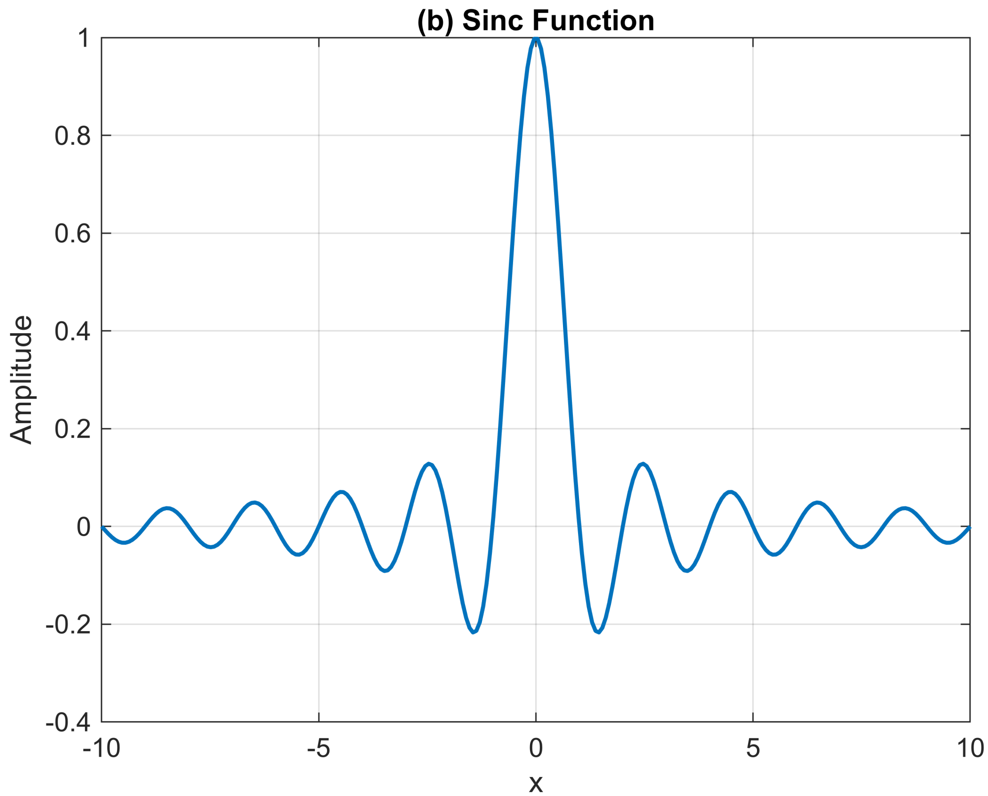
```
% 2.

N = 256;
x = linspace(-10, 10, N);
fs = N / 20;
f = (-N/2 : N/2-1) * (fs/N);

y_rect = zeros(1, N);
y_rect(abs(x) < 2) = 1;
Y_rect = fftshift(fft(y_rect));

y_sinc = sinc(x);
Y_sinc = fftshift(fft(y_sinc));

y_const = ones(1, N);
Y_const = fftshift(fft(y_const));

figure;
plot(x, y_rect, 'LineWidth', 1.5);
title('(a) Rect Function');
xlabel('x');
ylabel('Amplitude');
grid on;
ylim([-0.2, 1.2]);
```



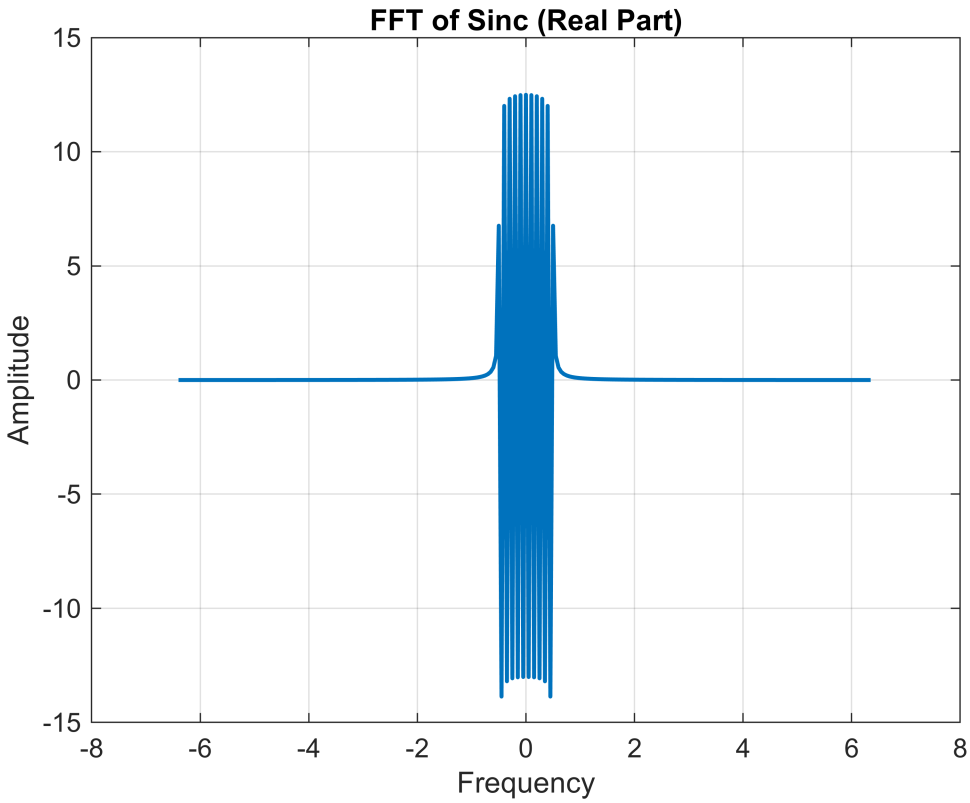

```
figure;
plot(f, real(Y_rect), 'LineWidth', 1.5);
title('FFT of Rect (Real Part)');
xlabel('Frequency');
ylabel('Amplitude');
grid on;
```



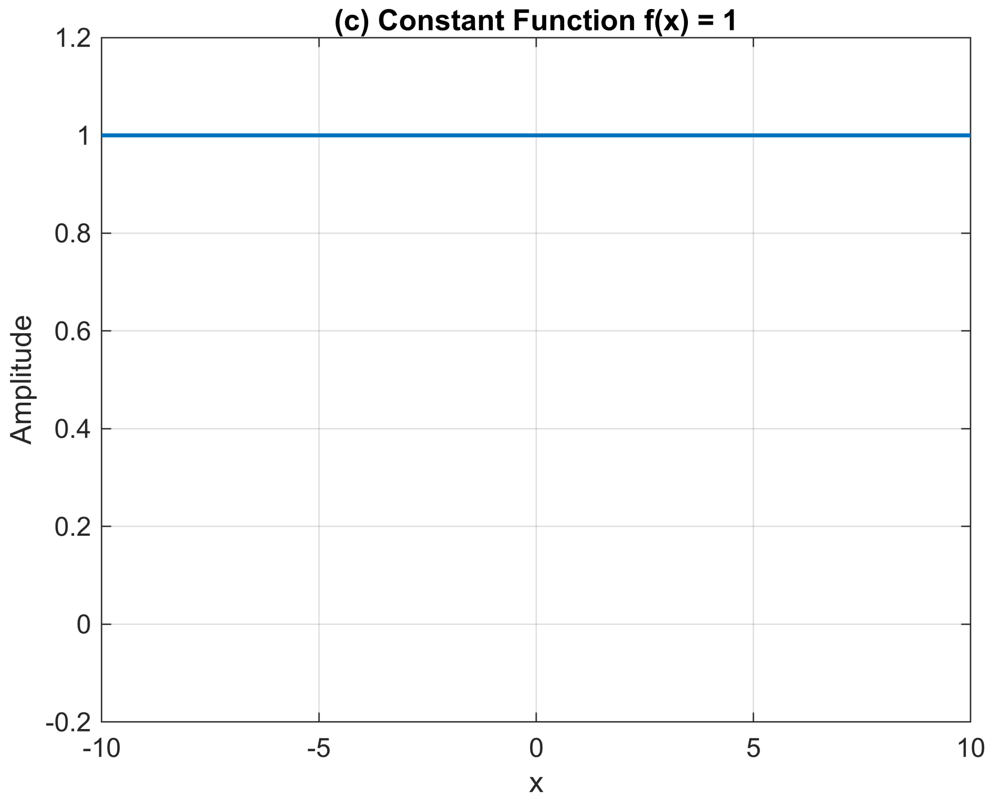

```
figure;
plot(x, y_sinc, 'LineWidth', 1.5);
title('(b) Sinc Function');
xlabel('x');
ylabel('Amplitude');
grid on;
```



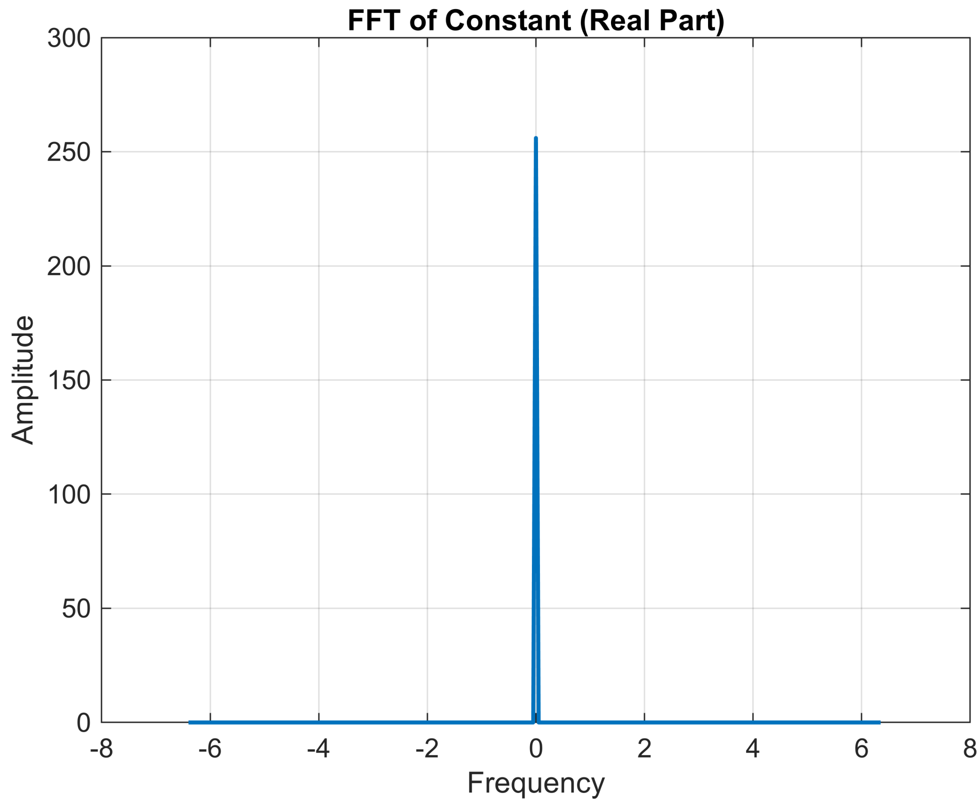

```
figure;
plot(f, real(Y_sinc), 'LineWidth', 1.5);
title('FFT of Sinc (Real Part)');
xlabel('Frequency');
ylabel('Amplitude');
grid on;
```




```
figure;
plot(x, y_const, 'LineWidth', 1.5);
title('(c) Constant Function f(x) = 1');
xlabel('x');
ylabel('Amplitude');
grid on;
ylim([-0.2, 1.2]);
```




```
figure;
plot(f, real(Y_const), 'LineWidth', 1.5);
title('FFT of Constant (Real Part)');
xlabel('Frequency');
ylabel('Amplitude');
grid on;
```




```
fprintf('The Fourier transform of a Rect function is a Sinc function (sin(x)/
x).\n');
```

```
The Fourier transform of a Rect function is a Sinc function (sin(x)/x).
```

```
fprintf('The resulting FFT plot shows a main lobe at the zero frequency with
decaying side lobes, which is characteristic of a Sinc.\n\n');
```

```
The resulting FFT plot shows a main lobe at the zero frequency with decaying side lobes, which is characteristic of
```

```
fprintf('The Fourier transform of a Sinc function is a Rect function.\n');
```

```
The Fourier transform of a Sinc function is a Rect function.
```

```
fprintf('The FFT plot shows that the Sinc function is composed of a narrow band of
low-frequency components.\n\n');
```

```
The FFT plot shows that the Sinc function is composed of a narrow band of low-frequency components.
```

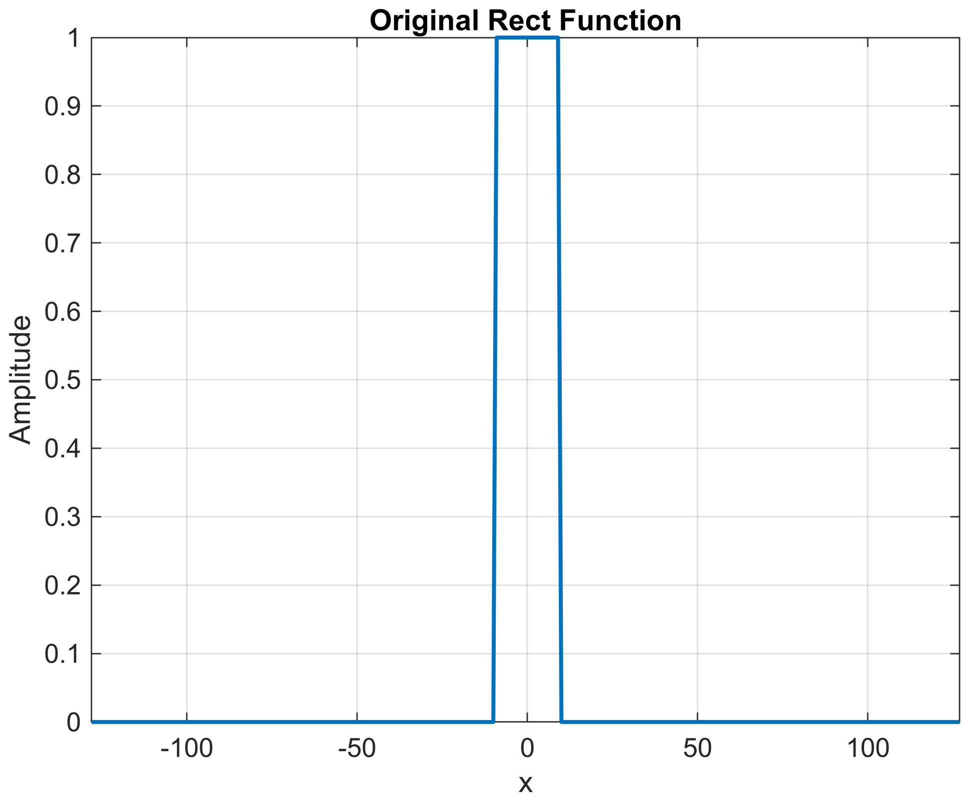
```
fprintf('A constant signal has no change, so its frequency is zero. Its Fourier
transform is a single spike at the zero frequency.\n');
```

```
A constant signal has no change, so its frequency is zero. Its Fourier transform is a single spike at the zero frequ
```

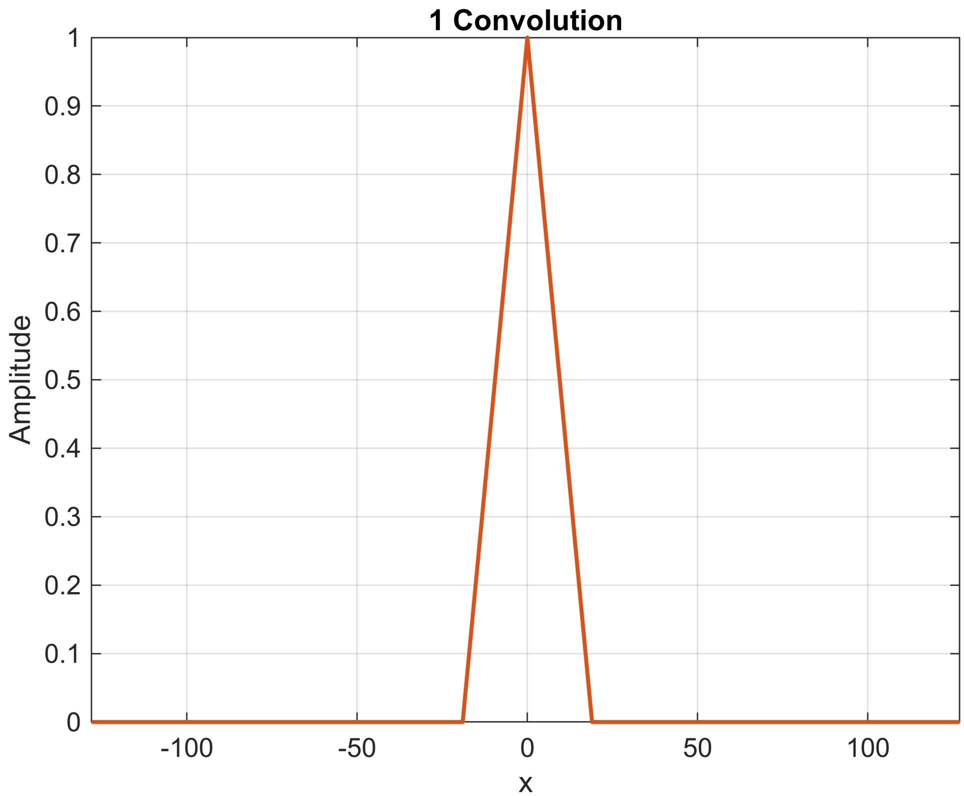
```
fprintf('In the discrete case, this is a spike at the center of the DFT,
representing a Dirac delta function.\n');
```

```
In the discrete case, this is a spike at the center of the DFT, representing a Dirac delta function.
```

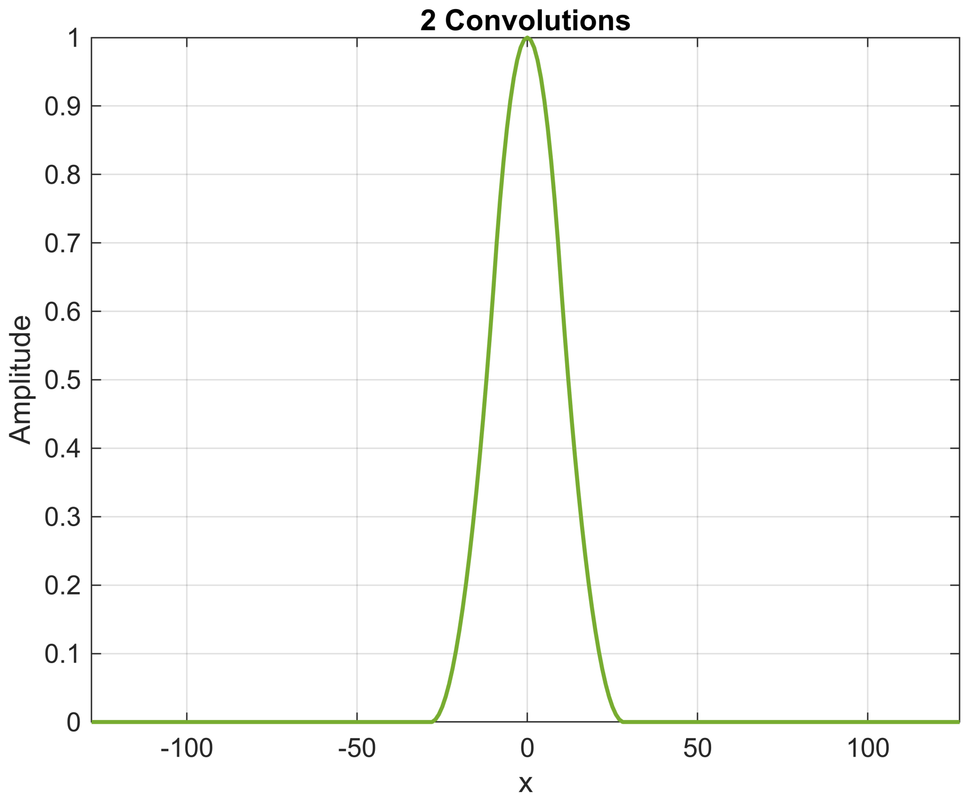
```
% 3.



N = 256;
x = linspace(-N/2, N/2 - 1, N);

rect_width = 20;
y_rect = zeros(1, N);
y_rect(abs(x) < rect_width/2) = 1;
Y_rect_fft = fft(fftshift(y_rect));

conv1_fft = Y_rect_fft .^ 2;
result1 = ifft(conv1_fft);
conv2_fft = Y_rect_fft .^ 3;
result2 = ifft(conv2_fft);
conv3_fft = Y_rect_fft .^ 4;
result3 = ifft(conv3_fft);

figure;
plot(x, y_rect, 'LineWidth', 1.5);
title('Original Rect Function');
xlabel('x');
ylabel('Amplitude');
grid on;
axis tight;
```



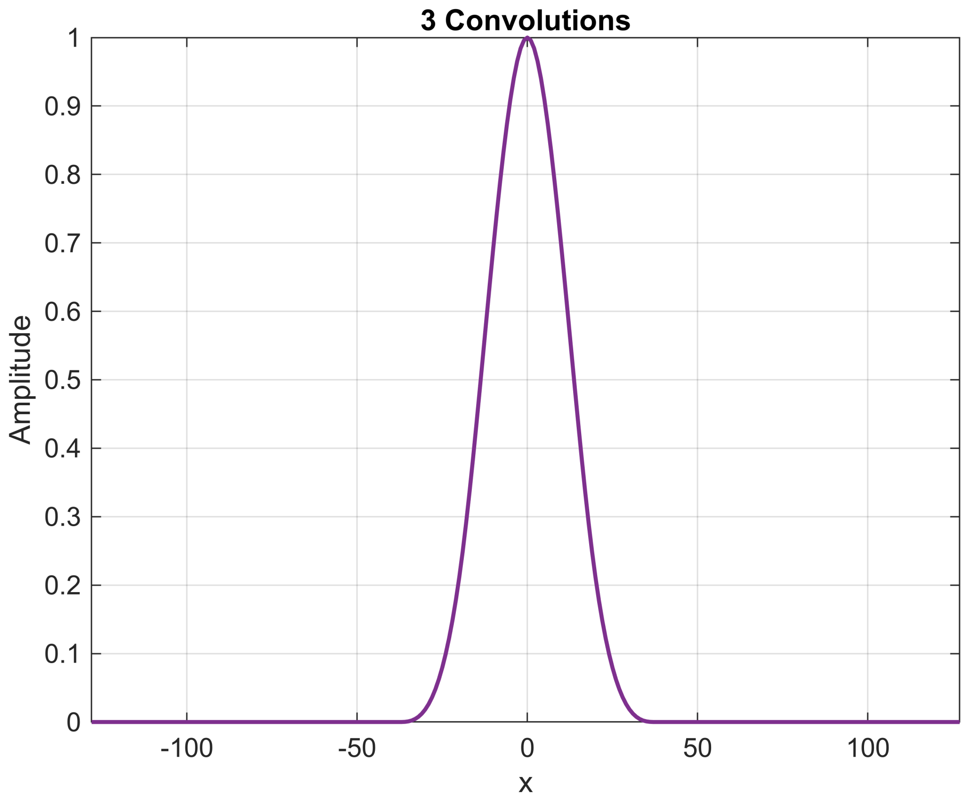

```
figure;
plot(x, fftshift(real(result1))/max(real(result1)), 'LineWidth', 1.5, 'Color',
'#D95319');
title('1 Convolution');
xlabel('x');
ylabel('Amplitude');
grid on;
axis tight;
```




```
figure;
plot(x, fftshift(real(result2))/max(real(result2)), 'LineWidth', 1.5, 'Color',
'#77AC30');
title('2 Convolutions');
xlabel('x');
ylabel('Amplitude');
grid on;
axis tight;
```




```
figure;
plot(x, fftshift(real(result3))/max(real(result3)), 'LineWidth', 1.5, 'Color',
'#7E2F8E');
title('3 Convolutions');
xlabel('x');
ylabel('Amplitude');
grid on;
axis tight;
```




```
fprintf('The theorem states that convolution in the spatial domain is equivalent to
simple point-wise multiplication in the frequency domain.\n');
```

```
The theorem states that convolution in the spatial domain is equivalent to simple point-wise multiplication in the f
```

```
fprintf('F{f(x) * g(x)} = F(w) * G(w)\n');
```

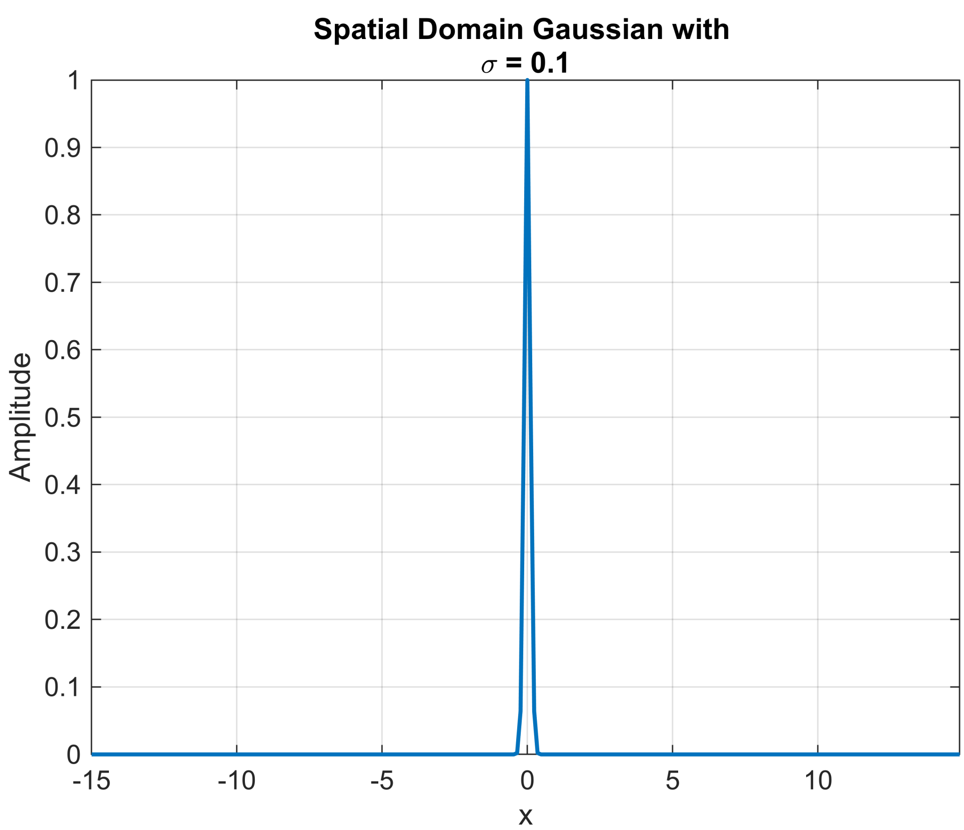
```
F{f(x) * g(x)} = F(w) * G(w)
```

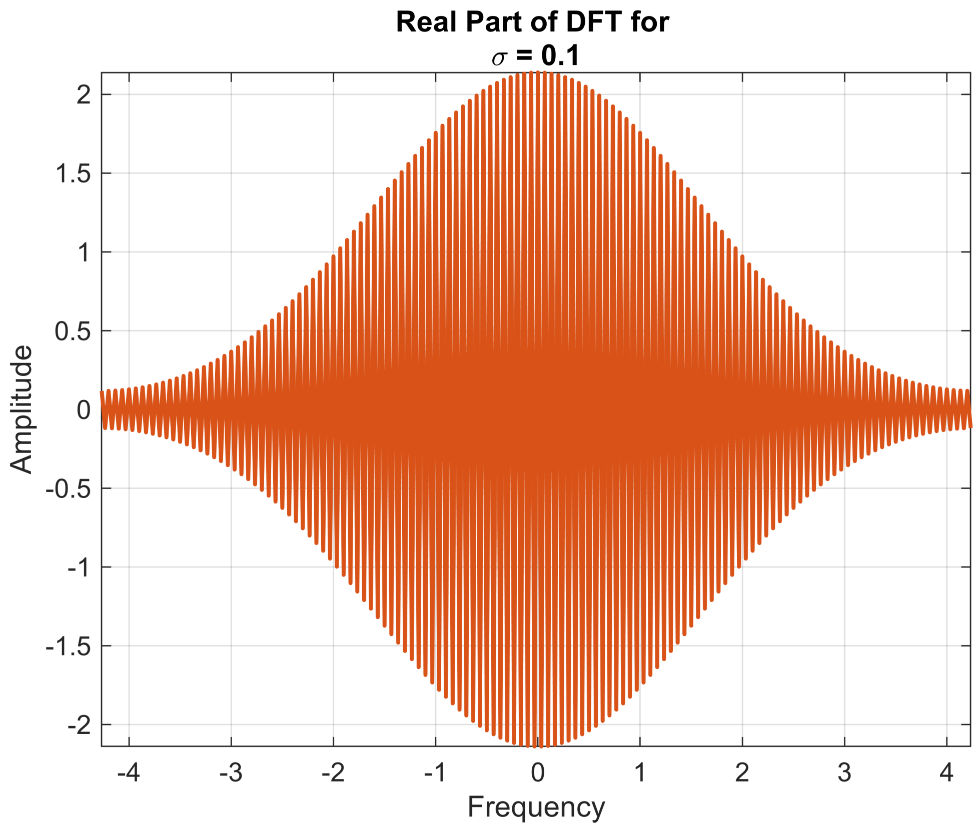
```
fprintf('This behavior is a demonstration of the Central Limit Theorem, which
states that repeated convolutions will cause a function to approach a Gaussian
shape.\n');
```

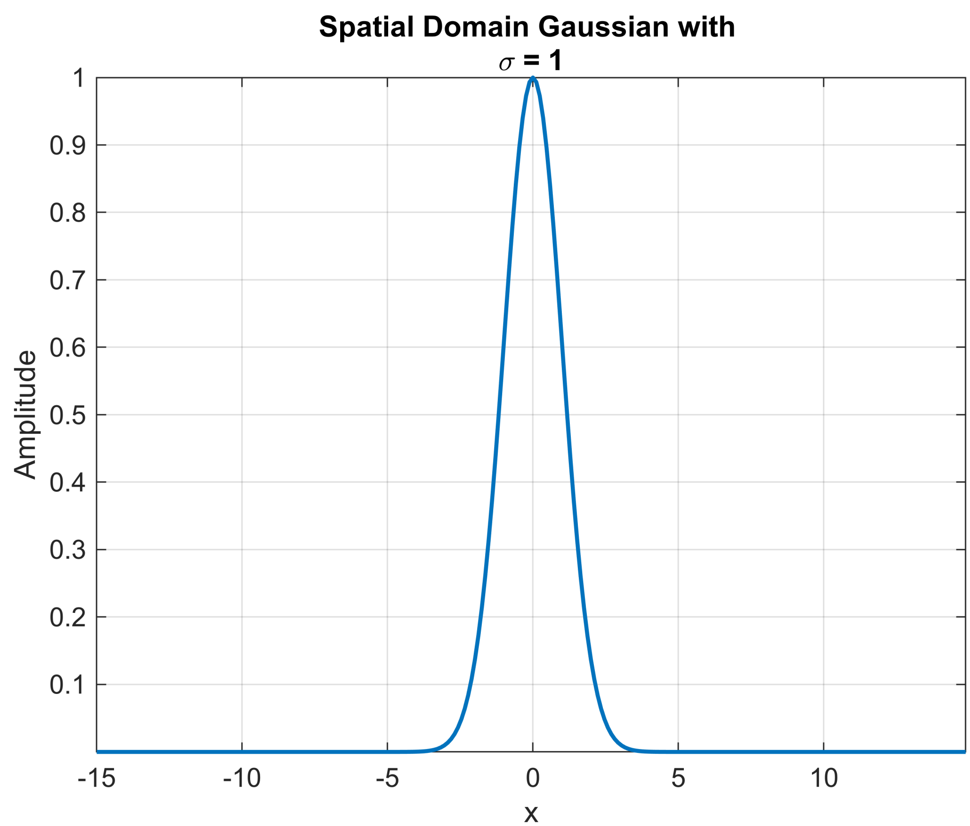
```
This behavior is a demonstration of the Central Limit Theorem, which states that repeated convolutions will cause a
```

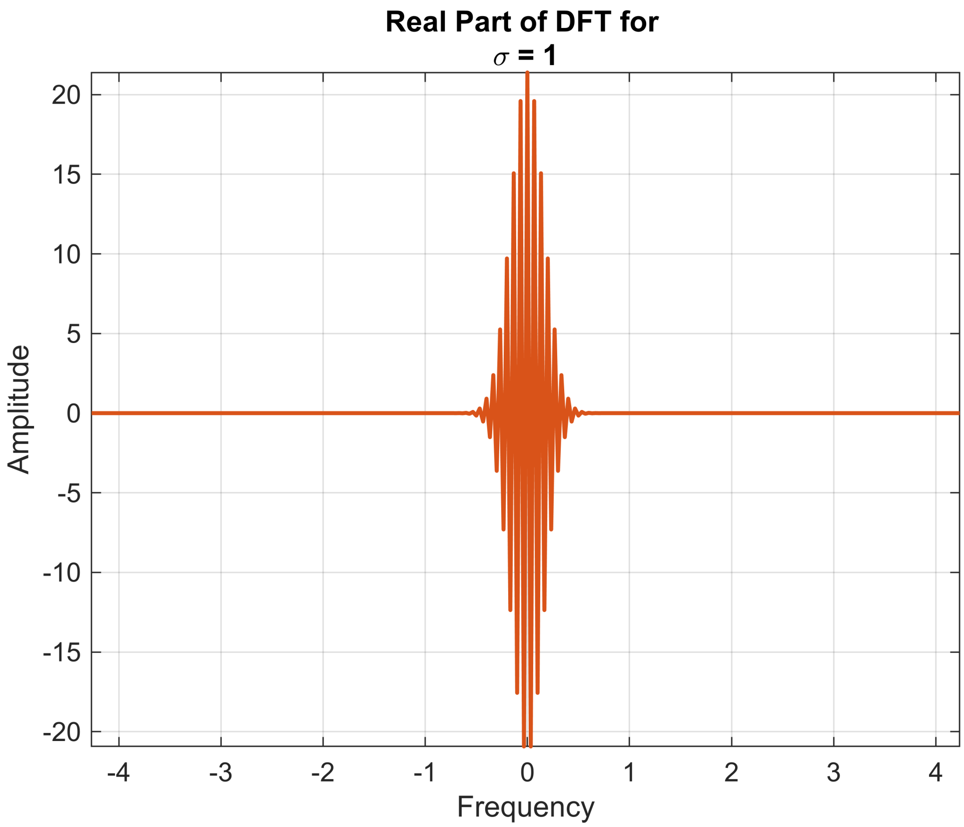
```
% 4.


N = 256;
x = linspace(-15, 15, 257);
x = x(1:N);
fs = N / 30;
f = (-N/2 : N/2-1) * (fs/N);
sigmas = [0.1, 1, 10];
titles = ["\sigma = 0.1", "\sigma = 1", "\sigma = 10"];
```

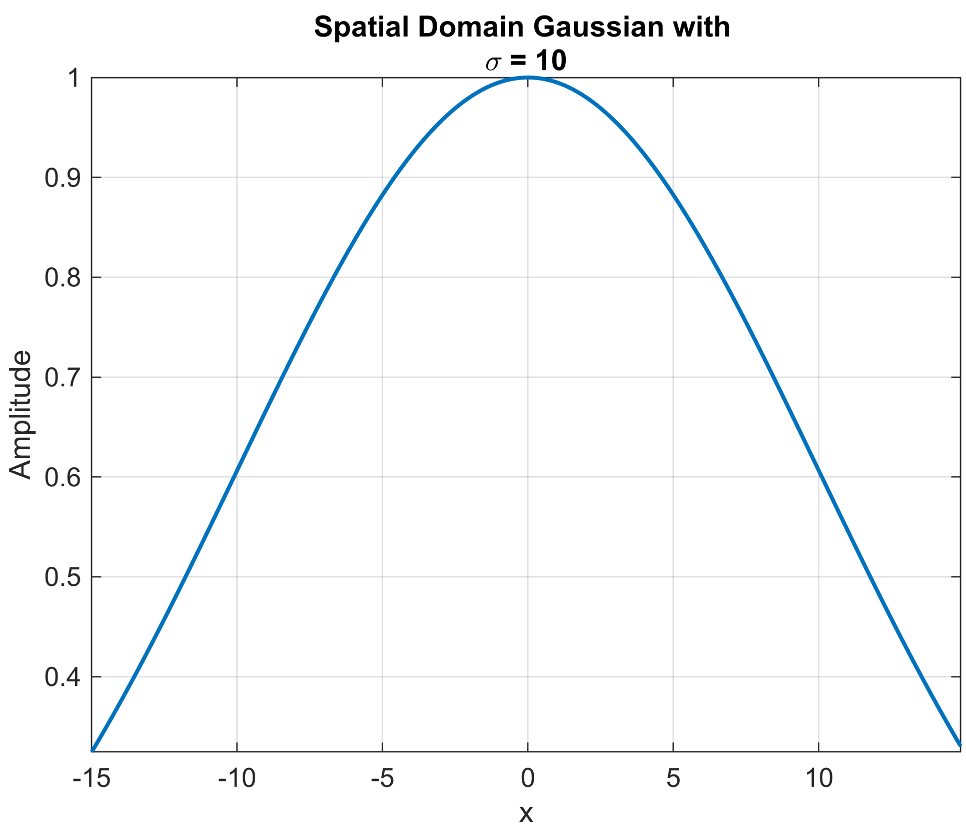
```
for i = 1:length(sigmas)
    sigma = sigmas(i);
    y_gauss = exp(-x.^2 / (2 * sigma^2));
    Y_gauss_fft = fftshift(fft(y_gauss));

    figure;
    plot(x, y_gauss, 'LineWidth', 1.5);
    title(['Spatial Domain Gaussian with ', titles(i)]);
    xlabel('x');
    ylabel('Amplitude');
    grid on;
    axis tight;

    figure;
    plot(f, real(Y_gauss_fft), 'LineWidth', 1.5, 'Color', '#D95319');
    title(['Real Part of DFT for ', titles(i)]);
    xlabel('Frequency');
    ylabel('Amplitude');
    grid on;
    axis tight;
end
```

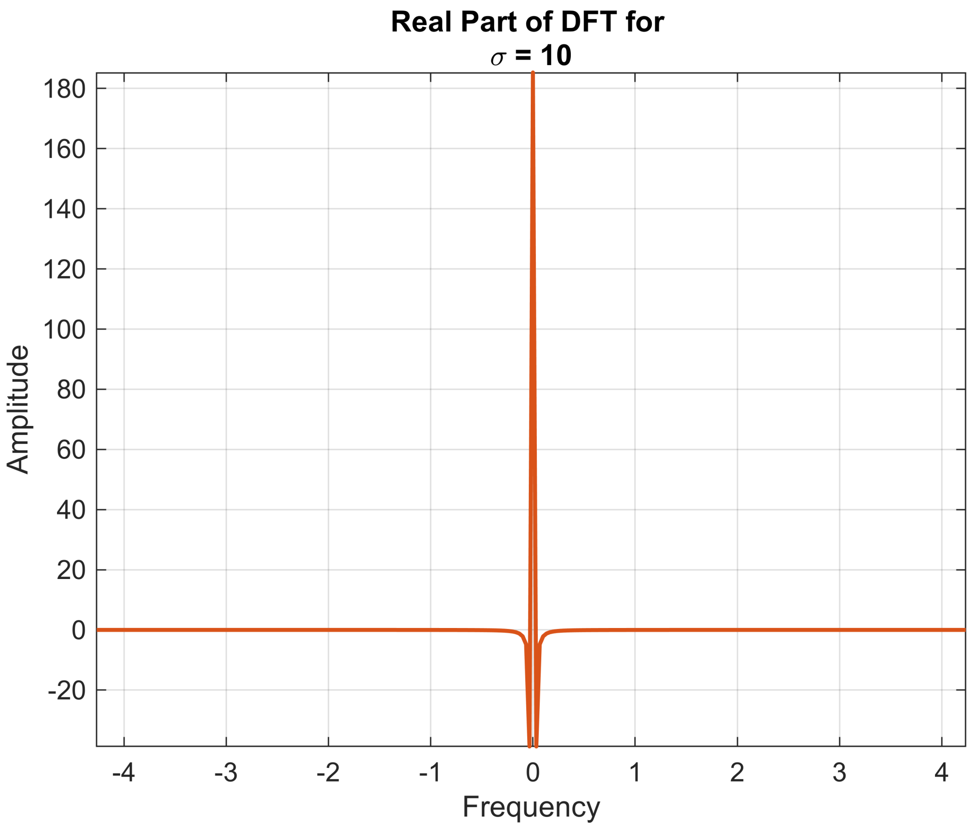



Real Part of DFT for
σ = 0.1
Amplitude
Frequency
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-4
-3
-2
-1
0
1
2
3
4


Spatial Domain Gaussian with
σ = 1
Amplitude
x
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-15
-10
-5
0
5
10


Real Part of DFT for
σ = 1
Amplitude
Frequency
20
15
10
5
0
-5
-10
-15
-20
-4
-3
-2
-1
0
1
2
3
4


Spatial Domain Gaussian with
σ = 10
Amplitude
x
1
0.9
0.8
0.7
0.6
0.5
0.4
-15
-10
-5
0
5
10




```
fprintf('A very narrow, sharp Gaussian in space is composed of a very wide range of frequencies. Its DFT is a wide Gaussian.\n\n');
```

```
A very narrow, sharp Gaussian in space is composed of a very wide range of frequencies. Its DFT is a wide Gaussian.
```

```
fprintf('Large Sigma (10) - The ''Weird'' Result:\n');
```

```
Large Sigma (10) - The 'Weird' Result:
```

```
fprintf('A Gaussian with a very large sigma is too wide for our finite domain of [-15, 15].\n');
```

```
A Gaussian with a very large sigma is too wide for our finite domain of [-15, 15].
```

```
fprintf('The tails of the function that extend beyond the boundaries ''wrap around'' to the other side. This is called Aliasing.\n');
```

```
The tails of the function that extend beyond the boundaries 'wrap around' to the other side. This is called Aliasing
```

```
fprintf('The resulting DFT is a very narrow Gaussian but with added high-frequency ripples or oscillations.\n');
```

```
The resulting DFT is a very narrow Gaussian but with added high-frequency ripples or oscillations.
```

```
% 5.
```

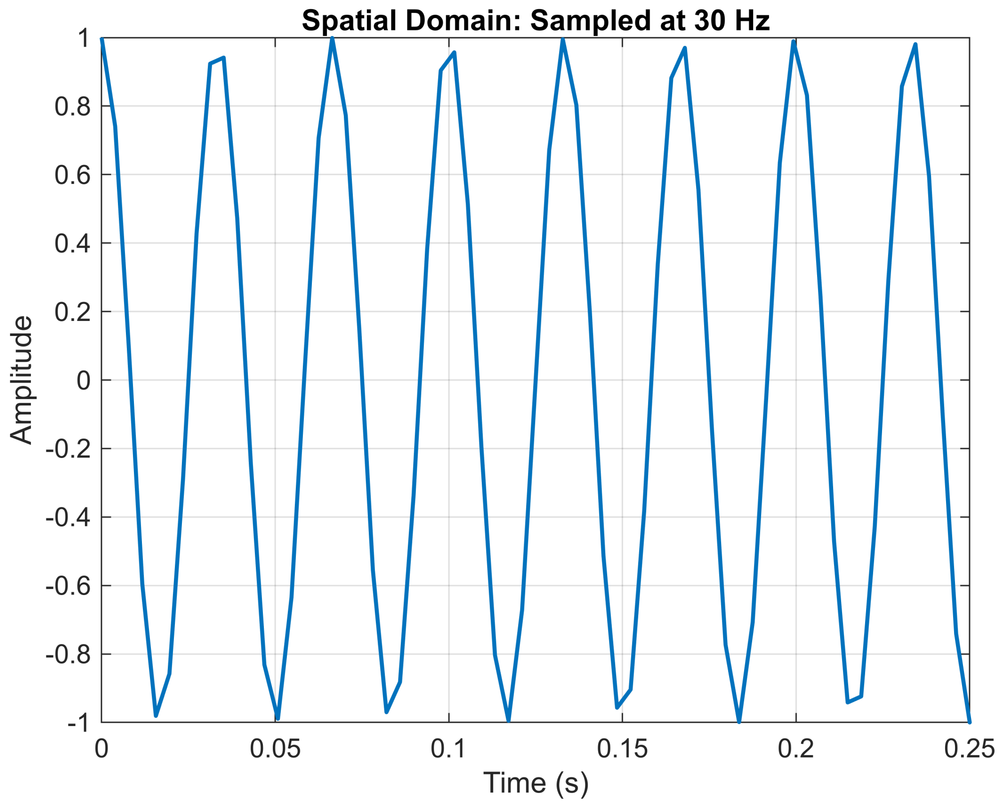
```
N = 256;
T = 1;
fs = N / T;
t = (0:N-1) / fs;
f_nyquist = fs / 2;
fprintf('Sampling Frequency: %.f Hz\n', fs);
```

```
Sampling Frequency: 256 Hz
```

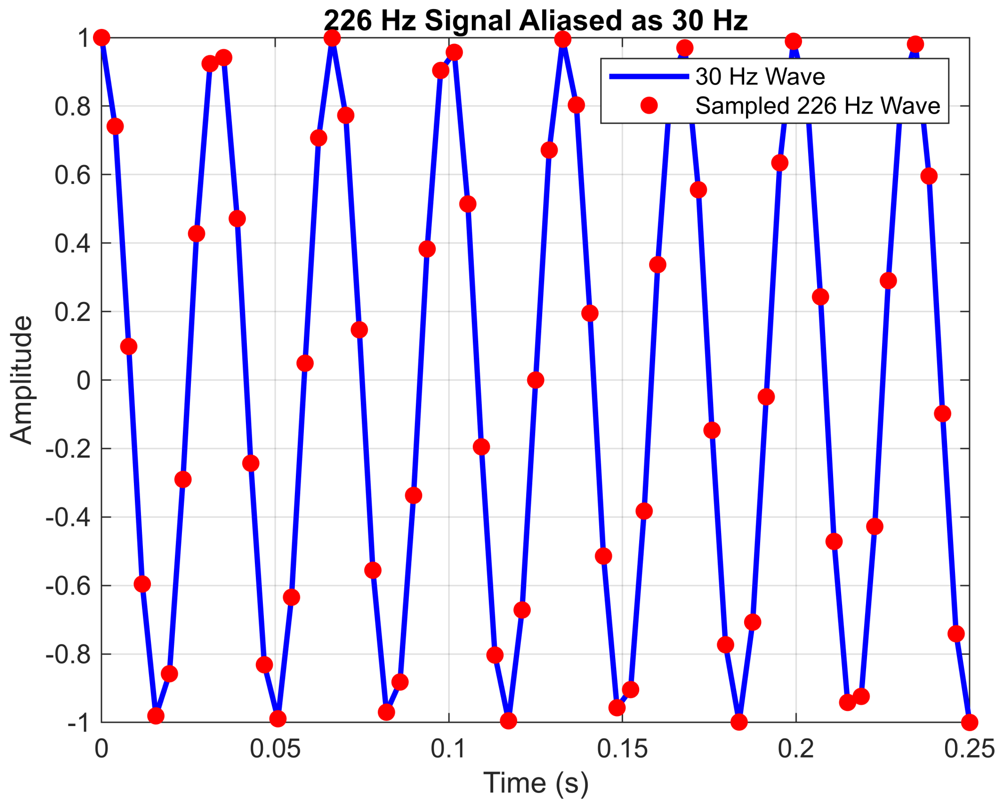
```
fprintf('Nyquist Frequency: %.f Hz\n', f_nyquist);
```

```
Nyquist Frequency: 128 Hz
```

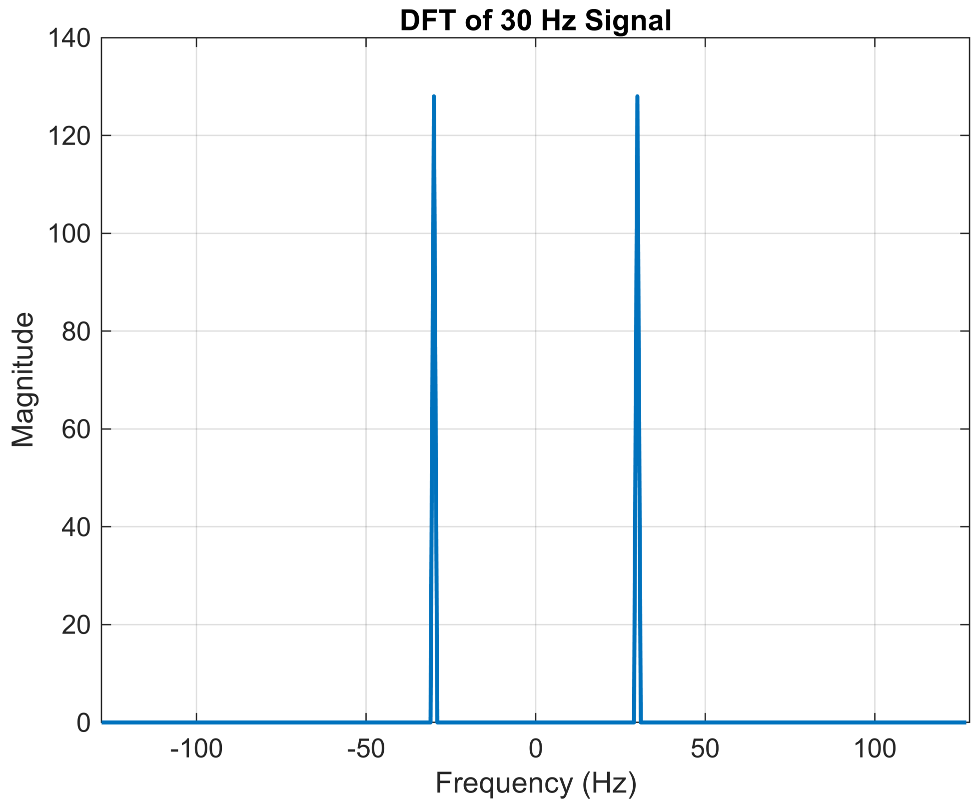
```
f_low = 30;
f_high = fs - f_low;

y_low = cos(2 * pi * f_low * t);
y_high = cos(2 * pi * f_high * t);

figure;
plot(t, y_low, 'LineWidth', 1.5);
title(['Spatial Domain: Sampled at ', num2str(f_low), ' Hz']);
xlabel('Time (s)');
ylabel('Amplitude');
grid on;
xlim([0, 0.25]);
```



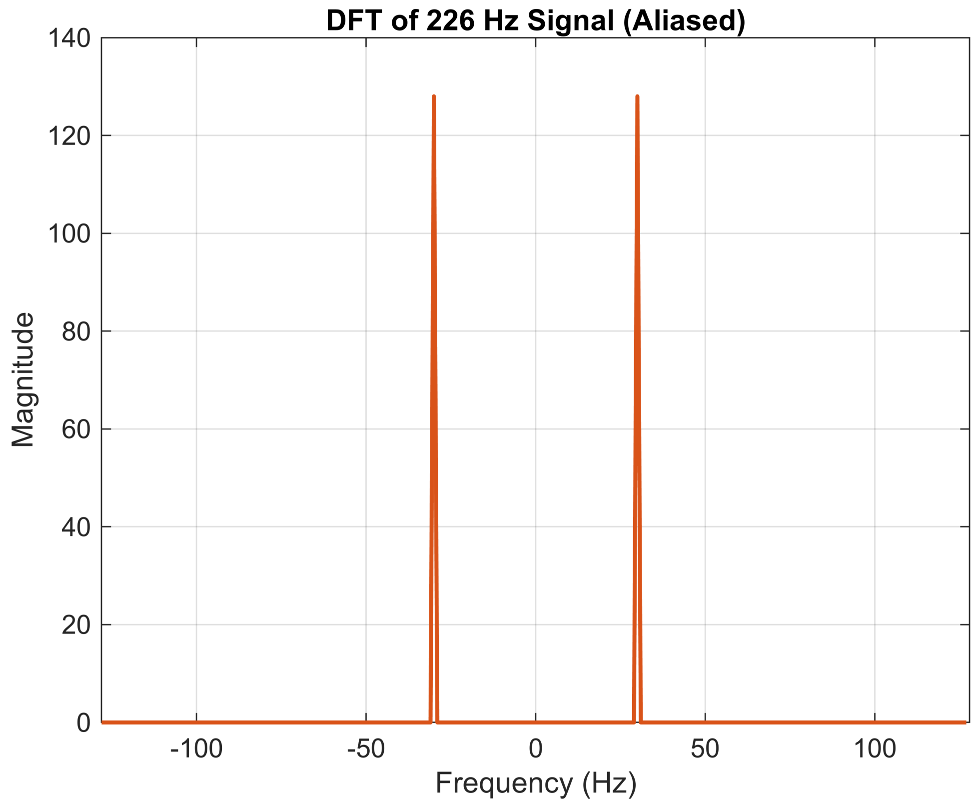

```
figure;
plot(t, y_low, 'b-', 'LineWidth', 2);
hold on;
plot(t, y_high, 'ro', 'MarkerFaceColor', 'r');
hold off;
title(['226 Hz Signal Aliased as 30 Hz']);
xlabel('Time (s)');
ylabel('Amplitude');
legend([num2str(f_low), ' Hz Wave'], ['Sampled ', num2str(f_high), ' Hz Wave']);
grid on;
xlim([0, 0.25]);
```




```
f_axis = (-N/2 : N/2-1) * (fs/N);

Y_low_fft = fftshift(fft(y_low));
figure;
plot(f_axis, abs(Y_low_fft), 'LineWidth', 1.5);
title(['DFT of ', num2str(f_low), ' Hz Signal']);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
grid on;
xlim([-fs/2, fs/2]);
```




```
Y_high_fft = fftshift(fft(y_high));
figure;
plot(f_axis, abs(Y_high_fft), 'LineWidth', 1.5, 'Color', '#D95319');
title(['DFT of ', num2str(f_high), ' Hz Signal (Aliased)']);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
grid on;
xlim([-fs/2, fs/2]);
```




```
fprintf('Aliasing occurs when a signal is sampled too slowly. High frequencies are misrepresented as lower frequencies.\n');
```

```
Aliasing occurs when a signal is sampled too slowly. High frequencies are misrepresented as lower frequencies.
```

```
fprintf('With a sampling rate of 256 Hz, the Nyquist frequency is 128 Hz.\n');
```

```
With a sampling rate of 256 Hz, the Nyquist frequency is 128 Hz.
```

```
fprintf('A 226 Hz signal is above this limit. It aliases to a new frequency of |226 Hz - 256 Hz| = 30 Hz.\n\n');
```

```
A 226 Hz signal is above this limit. It aliases to a new frequency of |226 Hz - 256 Hz| = 30 Hz.
```

```
fprintf('In the SPATIAL domain, the sampled points of the 226 Hz wave fall exactly on top of a true 30 Hz wave.\n');
```

```
In the SPATIAL domain, the sampled points of the 226 Hz wave fall exactly on top of a true 30 Hz wave.
```

```
fprintf('In the FOURIER domain, the DFT of the 226 Hz signal incorrectly shows peaks at +/- 30 Hz, not +/- 226 Hz.\n');
```

```
In the FOURIER domain, the DFT of the 226 Hz signal incorrectly shows peaks at +/- 30 Hz, not +/- 226 Hz.
```

```
fprintf('The high-frequency information has been completely misinterpreted as low-frequency information.\n');
```

```
The high-frequency information has been completely misinterpreted as low-frequency information.
```

```matlab
% 6.


N1 = 65536;
data1 = rand(1, N1);
num_trials = 1000;

tic;
for i = 1:num_trials
    fft(data1);
end
time1 = toc;

avg_time1 = time1 / num_trials;

fprintf('Vector Size: %d (2^16)\n', N1);
```

```
Vector Size: 65536 (2^16)
```

```matlab
fprintf('Total time for %d FFTs: %.4f seconds\n', num_trials, time1);
```

```
Total time for 1000 FFTs: 0.6806 seconds
```

```matlab
fprintf('Average FFT time: %.6f seconds\n\n', avg_time1);
```

```
Average FFT time: 0.000681 seconds
```

```matlab

N2 = 65535;
data2 = rand(1, N2);

tic;
for i = 1:num_trials
    fft(data2);
end
time2 = toc;

avg_time2 = time2 / num_trials;

fprintf('Vector Size: %d\n', N2);
```

```
Vector Size: 65535
```

```matlab
fprintf('Total time for %d FFTs: %.4f seconds\n', num_trials, time2);
```

```
Total time for 1000 FFTs: 2.7442 seconds
```

```matlab
fprintf('Average FFT time: %.6f seconds\n\n', avg_time2);
```

```
Average FFT time: 0.002744 seconds
```

```
speed_difference = avg_time2 / avg_time1;
fprintf('The FFT for N=%d was approximately %.1f times faster than for N=%d.\n',
N1, speed_difference, N2);
```

```
The FFT for N=65536 was approximately 4.0 times faster than for N=65535.
```

```
fprintf('When the input size is not a power of 2 (especially if it has large prime
factors), the algorithm cannot be broken down efficiently.\n');
```

```
When the input size is not a power of 2 (especially if it has large prime factors), the algorithm cannot be broken
```

```
fprintf('For optimal performance, it is good to use input sizes that are a power of
2, often by padding the data with zeros.\n');
```

```
For optimal performance, it is good to use input sizes that are a power of 2, often by padding the data with zeros.
```