Raman-Nath and Bragg diffraction

Sunday, October 20, 2024 1:49 PM

I.

A. Paraxial wave equation

B. Diffraction geometry.

a.)

$$\omega = \frac{2\pi c}{\lambda_0}$$

$n_b \rightarrow$ refractive medium index

$$\lambda = \frac{\lambda_0}{n_b}$$

$v \approx 10^3 \, m/s$

$$\boxed{\Lambda = \frac{v}{f}}$$

difference in velocity compensates

then they will be in micrometers

Consider, $\lambda_0 = 500nm$

$n_b = 1.5$

$$\lambda = \frac{500}{1.5} = 333 \, nm$$

$v = 10^3 m/s$

$f = 100 \, MHz$

$$\Lambda = \frac{10^3 m/s}{100 \times 10^6 \, Hz}$$

$\boxed{\Lambda = 10 \, \mu m}$ They are comparable,

b.)

$$ik_0 \Delta n(x) \mathcal{E} = ik_0 \Delta n_0 \cos(kx) \mathcal{E}$$

$k = 2\pi/\Lambda$

$$\Delta n(x) = \Delta n_0 \cos(kx)$$

$K_{inc} = 0$

grating modulates field and introduces new wavevector components

$$k_x = \pm k$$

$$|\vec{k}_\pm| = \sqrt{k_x^2 + k_z^2}$$

$$= k_b$$

$$= \frac{2\pi n_b}{\lambda_0}$$

Diffraction angles:

$\theta_\pm$

$$\sin(\theta_\pm) = \frac{k_x}{k_b}$$

$$= \pm \frac{k}{k_b} = \pm \frac{\lambda_0}{n_b \Lambda}$$

$$\boxed{\theta_\pm = \pm \sin^{-1}\left(\frac{\lambda_0}{n_b \Lambda}\right)}$$

II. Raman Nath diffraction.

a.)

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i}{2k_b}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\mathcal{E} + ik_0 \Delta n(x,y) \mathcal{E}$$

$$k_b = \frac{n_b \omega}{c}$$

$\Delta n(x) = \Delta n_0 \cos(kx)$

$K_{inc} = 0$

$$\frac{\partial \mathcal{E}}{\partial z} = ik_0 \Delta n(x) \mathcal{E}$$

$$\frac{\partial \mathcal{E}}{\partial z} = ik_0 \Delta n_0 \cos(kx) \mathcal{E}$$

$\mathcal{E}(x,z) = e^{i\phi(x,z)}$

$\phi(x,z)$ is the phase

then

$$i\frac{\partial \phi}{\partial z}e^{i\phi(x,z)} = ik_0 \Delta n_0 \cos(kx)e^{i\phi(x,z)}$$

integrating w.r.t z

$$\frac{\partial \phi}{\partial z} = k_0 \Delta n_0 \cos(kx)$$

$$\phi(x,z) = k_0 \Delta n_0 \cos(kx)\, z$$

$$\mathcal{E}(x,z) = e^{ik_0 \Delta n_0 z \cos(kx)}$$

$\xi = \frac{z}{L}$ $\quad$ L is length of medium

$\eta = k_0 \Delta n_0 L$ $\quad$ (Raman-Nath parameter)

$$\boxed{\mathcal{E}(x,z) = e^{i\eta\xi\cos(kx)}}$$

b.)

$$e^{is\cos(\theta)} = \sum_{j=-\infty}^{\infty} i^j J_j(s)\, e^{ij\theta}$$

$s = \eta\xi$ $\quad$ $(\eta\xi = k_0 \Delta n_0 z)$

$\theta = kx$

$$e^{i\eta\xi\cos(kx)} = \sum_{j=-\infty}^{\infty} i^j J_j(\eta\xi)\, e^{ijkx}$$

superposition of diffraction orders.

E-fild in terms of

$$\mathcal{E}(x,z) = \sum_{j=-\infty}^{\infty} i^j J_j(\eta\xi)\, e^{ijkx}$$

defining $c_j(\xi)$

$$c_j(\xi) = i^j J_j(\eta\xi)$$

$$\mathcal{E}(x,z) = \sum_{j=-\infty}^{\infty} c_j(\xi)\, e^{ijkx}$$

$k_x = jk$

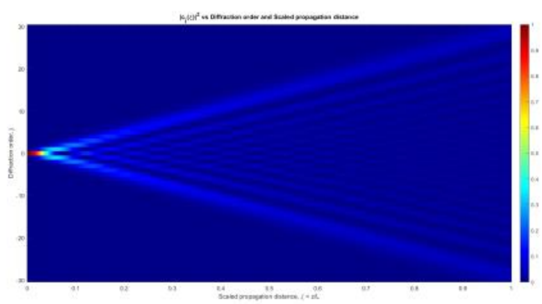
$k_b = \frac{2\pi n_b}{\lambda_0} \Rightarrow \sin(\theta_j) = \frac{k_x}{k_b} = \frac{jk}{k_b} = \frac{j\lambda_0}{n_b\Lambda}$

$$\theta_j \approx \frac{j\lambda_0}{n_b\Lambda}$$

$$|c_j(\xi)|^2 = |J_j(\eta\xi)|^2$$

$$\boxed{\mathcal{E}(x,z) = \sum_{j=-\infty}^{\infty} i^j J_j(\eta\xi)\, e^{ijkx}}$$

$$\theta_j = \frac{j\lambda_0}{n_b\Lambda}$$

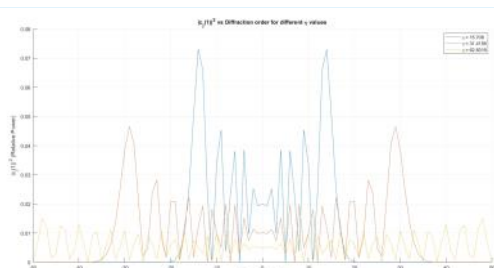


c.)

MATLAB

d.)

MATLAB



- $Q = \frac{k^2 L}{k_b} = \frac{2\pi \lambda_0 f^2 L}{v^2}$

$Q << 1$ $\quad$ low acoustic frequencies

$f < 10\,MHz$

Diffraction primarily affects the beam without significant spreading.

III. Intermediate diffraction Regime.

a.)

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i}{2k_b}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\mathcal{E} + ik_0 \Delta n(x,y)\mathcal{E}$$

$$\Delta n(x) = \Delta n_0 \cos(kx)$$

$$\Delta n(x) = \Delta n_0 \cos(kx)$$

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i}{2k_b}\frac{\partial^2 \mathcal{E}}{\partial x^2} + ik_0 \Delta n_0 \cos(kx)\mathcal{E}$$

$$\mathcal{E}(x,z) = \sum_{j=-\infty}^{\infty} c_j(z)\, e^{i(k_{inc}+jk)x}$$

$k = 2\pi/\Lambda$

Substituting in paraxial eqn.

$$\frac{\partial}{\partial z}\left(\sum_j c_j(z)\, e^{i(k_{inc}+jk)x}\right) = \frac{i}{2k_b}\frac{\partial^2}{\partial x^2}\left(\sum_j c_j(z)\, e^{i(k_{inc}+jk)x}\right)$$
$$+ ik_0 \Delta n_0 \cos(kx) \sum_j c_j(z)\, e^{i(k_{inc}+jk)x}$$

$$\frac{\partial^2}{\partial x^2}\left(c_j(z)\, e^{i(k_{inc}+jk)x}\right) = -(k_{inc}+jk)^2 c_j(z)\, e^{i(k_{inc}+jk)x}$$

$$\sum_j -\frac{i}{2k_b}(k_{inc}+jk)^2 c_j(z)\, e^{i(k_{inc}+jk)x}$$

$$\cos(kx) = \frac{1}{2}\left(e^{ikx} + e^{-ikx}\right)$$

Substituting in to ref. ind. term

$$ik_0 \Delta n_0 \cos(kx)\mathcal{E} = \frac{ik_0\Delta n_0}{2}\sum_j\left(c_j(z)\, e^{i(k_{inc}+(j+1)k)x} + c_j(z)\, e^{i(k_{inc}+(j-1)k)x}\right)$$

Combining terms

$$\sum_j \frac{\partial c_j}{\partial z} e^{i(k_{inc}+jk)x} = \sum_j\left[-\frac{i}{2k_b}(k_{inc}+jk)^2 c_j + \frac{ik_0\Delta n_0}{2}(c_{j+1}+c_{j-1})\right]e^{i(k_{inc}+jk)x}$$

we get coupled mode equations.

$$\frac{\partial c_j}{\partial z} = -\frac{i}{2k_b}(k_{inc}+jk)^2 c_j + \frac{ik_0\Delta n_0}{2}(c_{j+1}+c_{j-1})$$

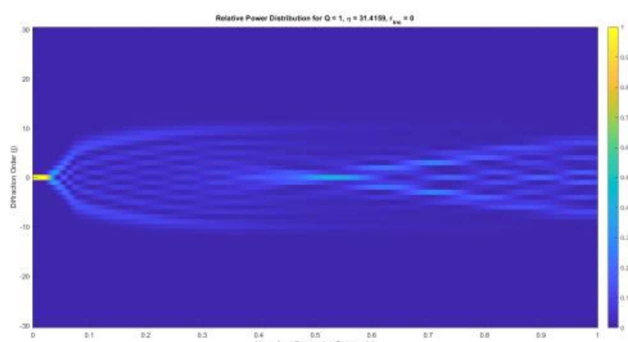
introducing, $\xi = z/L$

$Q = \frac{k^2 L}{k_b}$

$\eta = k_0 \Delta n_0 L$ $\Rightarrow$ $\boxed{\frac{dc_j}{d\xi} = -\frac{iQ}{2}(r_{inc}+j)^2 c_j + \frac{i\eta}{2}(c_{j+1}+c_{j-1})}$

$r_{inc} = \frac{k_{inc}}{k}$

Raman-Nath equation.



b.)

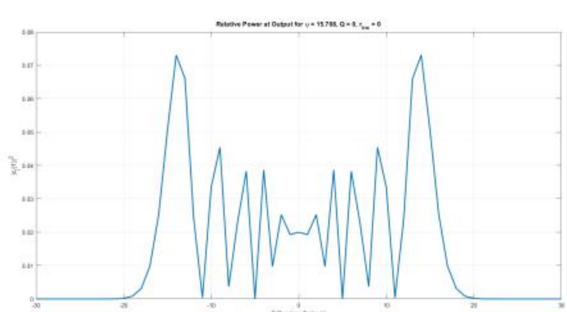
MATLAB.

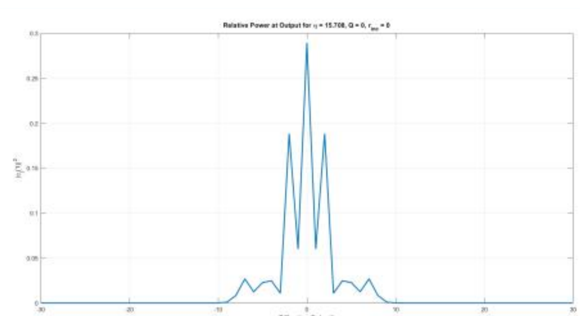
c.)

MATLAB.

d.)

MATLAB





iv Bragg diffraction:

a.)

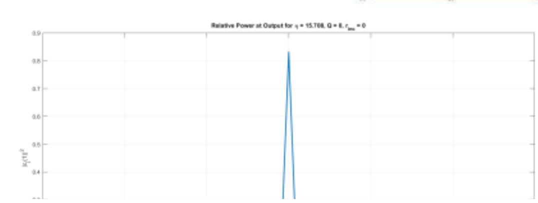
$k_0 = 2\pi/\lambda$

$k_0 \sin(\theta_{inc}) \rightarrow$ Transverse Comp.

$k = 2\pi/\Lambda$

orders, $\sin(\theta_j) = \sin(\theta_{inc}) + \frac{j\lambda}{\Lambda}$

$j = 0, -1$



orders, $\sin(\theta_j) = \sin(\theta_{inc}) + \frac{j\lambda}{\Lambda}$

$j = 0, -1$

$r_{inc} = \frac{K_{inc}}{k} = 0.5$

$K_{inc} = k_0 \sin(\theta_{inc})$

for phase matching condition

$K_{inc} = k/2$

then,

$k_0 \sin(\theta_{inc}) = \frac{2\pi}{2\Lambda}$

$= \pi/\Lambda$

$\sin(\theta_{inc}) = \frac{\pi/\Lambda}{k_0}$

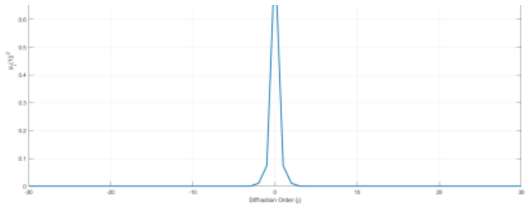
$= \frac{\lambda}{2\Lambda}$

Two diffraction orders are typically phase-matched when $\sin(\theta_{inc}) = \lambda/2\Lambda$. diffraction from grating is maximed between

$j = 0, -1$ orders

causing Bragg diffraction.

$Q >> 1$ $j = -2, -3, ..$ no phase-matching.



b.)

$\frac{da}{d\xi} = \frac{i\eta}{2} b$

$\frac{db}{d\xi} = \frac{i\eta}{2} a$

$\frac{d^2a}{d\xi^2} = \frac{i\eta}{2}\left(\frac{i\eta}{2}a\right)$

$= -\frac{\eta^2}{4} a$

general solution is,

$a(\xi) = A\cos\left(\frac{\eta\xi}{2}\right) + B\sin\left(\frac{\eta\xi}{2}\right)$

Now, $b(\xi) = \frac{2}{i\eta}\frac{da}{d\xi}$

$\frac{da}{d\xi} = -A\frac{\eta}{2}\sin\left(\frac{\eta\xi}{2}\right) + B\frac{\eta}{2}\cos\left(\frac{\eta\xi}{2}\right)$

$b(\xi) = \frac{2}{i\eta}\left(-A\frac{\eta}{2}\sin\left(\frac{\eta\xi}{2}\right) + B\frac{\eta}{2}\cos\left(\frac{\eta\xi}{2}\right)\right)$

$= -iA\sin\left(\frac{\eta\xi}{2}\right) + iB\cos\left(\frac{\eta\xi}{2}\right)$

$\therefore \xi = 0$

$a(0) = 1$

$b(0) = 0$ $\Rightarrow$ At $\xi = 0$; $a(0) = 1$

$j = 0, -1$

$a(0) = A\cos(0) + B\sin(0)$

$= A$

At $\xi = 0$; $b(0) = 0$

$b(0) = iA\sin(0) + iB\cos(0)$

$= iB$

$\therefore A = 1$; $B = 0$

then, $a(\xi) = \cos\left(\frac{\eta\xi}{2}\right)$

$b(\xi) = i\sin\left(\frac{\eta\xi}{2}\right)$

c.)

MATLAB.