LUT optimization of Fast Fourier Transform

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Abstract

Fast Fourier Transform (FFT) remains of a great importance due to its substantial role in the field of signal processing and imagery. In this report, multiple designs of 8 point FFT algorithm is proposed. The developed architecture was implemented using an FPGA . Though, the material resources of the FPGA are limited, particularly the integrated DSP blocks, different approaches are used during the Verilog description with the aim to reduce the necessary number of LUTs. The experimental validation was done using ISIM simulation tool, where the numerical synthesis and the post and route described in Verilog was realized using ISE Design Suite 14.7. The FFT modules of all the implementations were tested using a python script with various corner input test vectors.

1 Cooley-Tukey algorithm

1.1 Overview

Cooley–Tukey algorithm re-expresses the discrete Fourier transform (DFT) of an arbitrary composite size $N = N_1 N_2$ in terms of N_1 smaller DFTs of sizes N_2 , recursively, to reduce the computation time to $\mathbf{O}(N \log N)$ for highly composite N.

1.2 Calculation

This is a direct implementation of Cooley-Turkey Algorithm without any modifications , and for a 8 point FFT , it involves 2 multiplications for a single output calculation. The equations involving the calculation of FFT in shown below.

```
\begin{split} t_1 &= D(0) + D(4); m_3 = D(0) - D(4); \\ t_2 &= D(6) + D(2); m_6 = j * (D(6) - D(2)); \\ t_3 &= D(1) + D(5); t_4 = D(1) - D(5); \\ t_5 &= D(3) + D(7); t_6 = D(3) - D(7); \\ t_8 &= t_5 + t_3; m_5 = j * (t_5 - t_3); \\ t_7 &= t_1 + t_2; m_2 = t_1 - t_2; \end{split}
```

```
\begin{split} m_0 &= t_7 + t_8; m_1 = t_7 - t_8; \\ m_4 &= sin(\pi/4) * (t_4 - t_6); m_7 = -j * sin(\pi/4) * (t_4 + t_6); \\ s_1 &= m_3 + m_4; s2 = m_3 - m_4; \\ s_3 &= m_6 + m_7; s4 = m_6 - m_7; \\ DO(0) &= m_0; DO(4) = m_1; \\ DO(1) &= s_1 + s_3; DO(7) = s_1 - s_3; \\ DO(2) &= m_2 + m_5; DO(6) = m_2 - m_5; \\ DO(5) &= s_2 + s_4; DO(3) = s_2 - s_4; \end{split}
```

where D and DO are input and output arrays of the complex data t_1, \ldots, t_8 , $m_1, \ldots, m_7, s_1, \ldots, s_4$ are the intermediate complex results. As we see the algorithm contains only 2 multiplications to the untrivial coefficient $\sin(\pi/4) = 0.7071$, and 22 real additions and subtractions. The multiplication to a coefficient j means the negation the imaginary part and swapping real and imaginary parts.

1.3 Implementation

The below code implements the Cooley–Tukey algorithm . inp1,...inp8 are the 16 bit signed floating point inputs of Q format and out1-real , out1-imag, ...,out8-real,out8-imag are the real and imaginary outputs in 16 bit Q format of the FFT8 module.clk , rst are the clock , reset inputs respectively , and output-stb is the output strobe , which is enabled once the output is calculated.

```
'timescale 1ns / 1ps
1
2
   module fft8(
3
           input signed [15:0] inp1,
4
           input signed [15:0] inp2,
           input signed [15:0] inp3,
           input signed [15:0] inp4,
           input signed [15:0] inp5,
           input signed [15:0] inp6,
9
           input signed [15:0] inp7,
10
            input signed [15:0] inp8,
11
            input clk,
12
            input rst,
13
            output signed [15:0] out1_real,
14
            output signed [15:0] out1_imag,
15
            output signed [15:0] out2_real,
16
            output signed [15:0] out2_imag,
17
            output signed [15:0] out3_real,
18
            output signed [15:0] out3_imag,
19
            output signed [15:0] out4_real,
20
            output signed [15:0] out4_imag,
21
            output signed [15:0] out5_real,
22
            output signed [15:0] out5_imag,
23
```

```
output signed [15:0] out6_real,
24
            output signed [15:0] out6_imag,
25
            output signed [15:0] out7_real,
26
            output signed [15:0] out7_imag,
27
            output signed [15:0] out8_real,
            output signed [15:0] out8_imag,
29
            output out_stb
30
   );
31
32
            localparam signed sin_45 = 16'b00000000_10110101;
33
            localparam signed sin_315 = 16'b11111111_01001011;
35
            reg signed [31:0] t1_46,t2_46;
36
            reg signed [15:0] t1,t2,t3,t4,t5,t6,t7,t8,m0,m1,m2,m3,m4;
37
            reg signed [15:0] m7_imag,s1,s2,s3_imag,s4_imag,m5_imag,m6_imag;
38
            reg output_stb;
39
40
            initial
41
                     begin
42
                              output_stb = 1'b0;
43
                     end
44
45
            always @( posedge clk )
46
                     begin
47
                              if (rst == 1'b1)
48
                                       begin
49
                                               output_stb = 1'b0;
50
                                       end
51
                              else
52
                                      begin
53
                                               t1 = inp1 + inp5;
54
                                               t2 = inp7 + inp3;
55
                                               t3 = inp2 + inp6;
56
                                               t5 = inp4 + inp8;
57
                                               m3 = inp1 - inp5;
58
                                               m6_{imag} = inp7 - inp3;
59
                                               t4 = inp2 - inp6;
60
                                               t6 = inp4 - inp8;
61
                                               t8 = t5 + t3;
62
                                               t7 = t1 + t2;
63
                                               m0 = t7 + t8;
64
                                               t1_46 = sin_45 * (t4 - t6);
65
                                               m4 = t1_46 [23:8];
66
                                               m5_{imag} = t5 - t3;
67
                                               m2 = t1 - t2;
69
                                               m1 = t7 - t8;
70
                                               t2_46 = sin_315 * (t4 + t6);
                                               m7_{imag} = t2_{46} [23:8];
71
                                               s1 = m3 + m4;
72
                                               s2 = m3 - m4;
73
```

```
s3_imag = m6_imag + m7_imag;
74
                                           s4_imag = m6_imag - m7_imag;
75
                                           output_stb = 1'b1;
76
                                   end
77
                   end
           assign out1_real = m0;
           assign out1_imag = 16'b0000000000000000;
80
           assign out2_real = s1;
81
           assign out2_imag = s3_imag;
82
           assign out3_real = m2;
           assign out3_imag = m5_imag;
           assign out4_real = s2;
85
           assign out4_imag = ~s4_imag + 1'b1;
86
           assign out5_real = m1;
87
           88
           assign out6_real = s2;
89
           assign out6_imag = s4_imag;
           assign out7_real = m2;
           assign out7_imag = ~m5_imag + 1'b1;
           assign out8_real = s1;
93
           assign out8_imag = ~s3_imag + 1'b1;
94
           assign out_stb = output_stb;
95
   endmodule
96
```

The above code with Test Bench is available in FFT8_implementation_1 directory of the project GIT repository.

1.4 Analysis

This algorithm calculates the different stages of the Butterfly structure in parallel and hence total number of operations is as follows.

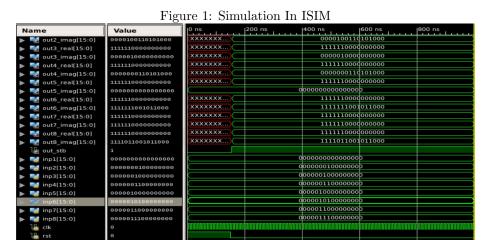
- Multiplications: 2
- Additions and Subtractions: 22

since both the multiplications happen in parallel , total 2 multipliers are used in this block to evaluate , hence improving speed. The simulation of this algorithm is shown in **Figure 1**.

2 Modified Cooley-Tukey algorithm

2.1 Overview

Cooley–Tukey algorithm for 8 point FFT involves 2 multiplication , hence using 2 multiplicative blocks as seen in the first implementation. In this modified algorithm , the multiplications are pipelined to run one by one , by reusing the same multiplier.



Calculation

The calculations of various intermediate complex numbers is divided into groups and executed group by group. The equation involving different groups are mentioned below.

2.2.1 Group 1

2.2

```
\begin{split} t_1 &= D(0) + D(4); \\ m_3 &= D(0) - D(4); \\ t_2 &= D(6) + D(2); \\ m_6 &= j*(D(6) - D(2)); \\ t_3 &= D(1) + D(5); \\ t_4 &= D(1) - D(5); \\ t_5 &= D(3) + D(7); \\ t_6 &= D(3) - D(7); \\ t_8 &= t_5 + t_3; \\ m_5 &= j*(t_5 - t_3); \\ t_7 &= t_1 + t_2; \\ m_2 &= t_1 - t_2; \\ m_0 &= t_7 + t_8; \\ m_1 &= t_7 - t_8; \\ m_4 &= \sin(\pi/4)*(t_4 - t_6); \end{split}
```

2.2.2 Group 2

$$m_7 = -j * sin(\pi/4) * (t_4 + t_6);$$

 $s_1 = m_3 + m_4;$
 $s_2 = m_3 - m_4;$

```
\begin{array}{l} s_3 = m_6 + m_7; \\ s4 = m_6 - m_7; \\ DO(0) = m_0; \\ DO(4) = m_1; \\ DO(1) = s_1 + s_3; \\ DO(7) = s_1 - s_3; \\ DO(2) = m_2 + m_5; \\ DO(6) = m_2 - m_5; \\ DO(5) = s_2 + s_4; \\ DO(3) = s_2 - s_4; \end{array}
```

where D and DO are input and output arrays of the complex data t_1, \ldots, t_8 , $m_1, \ldots, m_7, s_1, \ldots, s_4$ are the intermediate complex results. As we see the algorithm contains only 2 multiplications to the untrivial coefficient $\sin(\pi/4) = 0.7071$, and 22 real additions and subtractions. The multiplication to a coefficient j means the negation the imaginary part and swapping real and imaginary parts.

2.3 Implementation

The below code implements the Modified Cooley–Tukey algorithm . inp1,...inp8 are the 16 bit signed floating point inputs of Q format and out1-real , out1-imag, ...,out8-real,out8-imag are the real and imaginary outputs in 16 bit Q format of the FFT8 module.clk , rst are the clock , reset inputs respectively , and output-stb is the output strobe , which is enabled once the output is calculated. multiplier module takes inp1, inp2 16 bit signed floating point inputs of Q format and outputs the product of the inputs to out.The module multiplier is initialized once and reused twice in the main FFT8 block .

```
'timescale 1ns / 1ps
   module multiplier(
3
        input signed [15:0] inp1,
4
        input signed [15:0] inp2,
5
        output signed[15:0] out
6
            reg signed [31:0] inp12;
            always @ ( * )
                     begin
10
                              inp12 = inp1*inp2;
11
                     end
12
            assign out = inp12 [23:8];
13
14
   endmodule
15
16
   module fft8(
17
            input signed [15:0] inp1,
18
```

```
input signed [15:0] inp2,
19
            input signed [15:0] inp3,
20
            input signed [15:0] inp4,
21
            input signed [15:0] inp5,
22
            input signed [15:0] inp6,
23
            input signed [15:0] inp7,
24
            input signed [15:0] inp8,
25
            input clk,
26
            input rst,
27
            output signed [15:0] out1_real,
28
            output signed [15:0] out1_imag,
29
            output signed [15:0] out2_real,
30
            output signed [15:0] out2_imag,
31
            output signed [15:0] out3_real,
32
            output signed [15:0] out3_imag,
33
            output signed [15:0] out4_real,
34
            output signed [15:0] out4_imag,
35
            output signed [15:0] out5_real,
36
            output signed [15:0] out5_imag,
37
            output signed [15:0] out6_real,
38
            output signed [15:0] out6_imag,
39
            output signed [15:0] out7_real,
40
            output signed [15:0] out7_imag,
41
            output signed [15:0] out8_real,
42
            output signed [15:0] out8_imag,
43
            output out_stb
44
   );
45
46
            localparam signed sin_45 = 16'b00000000_10110101;
47
            localparam signed sin_315 = 16'b111111111_01001011;
48
            localparam signed sf = 2.0**-8.0;
49
            reg signed [31:0] t1_46,t2_46;
51
            reg signed [15:0] t1,t2,t3,t4,t5,t6,t7,t8,m0,m1,m2,m3,m4;
52
            reg signed [15:0] m7_imag,s1,s2,s3_imag,s4_imag,m5_imag,m6_imag;
53
            reg [15:0] mult_inp1,mult_inp2;
54
            wire [15:0] mult_out;
55
            reg [1:0] stage;
56
            reg output_stb;
57
58
            multiplier mult (.inp1(mult_inp1), .inp2(mult_inp2), .out(mult_out));
59
60
            initial
61
62
                    begin
                             stage = 2'b00;
64
                             output_stb = 1'b0;
                    end
65
66
            always @( posedge clk)
67
                    begin
```

```
if (rst == 1'b1)
69
                                        begin
70
                                                 output_stb = 1'b0;
71
                                        end
72
                               if (stage == 2'b00 && rst == 1'b0)
                                        begin
74
                                                 $display("stage-1");
75
                                                 t1 = inp1 + inp5;
76
                                                 t2 = inp7 + inp3;
77
                                                 t3 = inp2 + inp6;
78
                                                 t5 = inp4 + inp8;
                                                 m3 = inp1 - inp5;
80
                                                 m6_{imag} = inp7 - inp3;
81
                                                 t4 = inp2 - inp6;
82
                                                 t6 = inp4 - inp8;
83
                                                 t8 = t5 + t3;
84
                                                 t7 = t1 + t2;
85
                                                 m0 = t7 + t8;
86
                                                 mult_inp1 = t4 - t6;
87
                                                 mult_inp2 = sin_45;
88
                                                 stage = 2'b01;
89
                                        end
90
                               else if (stage == 2'b01)
91
                                        begin
                                                 $display("stage-2");
93
                                                 m4 = mult_out;
94
                                                 m5_{imag} = t5 - t3;
95
                                                 m2 = t1 - t2;
96
                                                 m1 = t7 - t8;
97
                                                 mult_inp1 = t4 + t6;
98
                                                 mult_inp2 = sin_315;
                                                 stage = 2'b10;
100
                                        end
101
                               else if (stage == 2'b10)
102
                                        begin
103
                                                 m7_imag = mult_out;
104
                                                 s1 = m3 + m4;
105
                                                 s2 = m3 - m4;
106
                                                 s3_imag = m6_imag + m7_imag;
107
                                                 s4_imag = m6_imag - m7_imag;
108
                                                 stage = 2'b00;
109
                                                 output_stb = 1'b1;
110
                                        end
111
                      end
112
             assign out_stb = output_stb;
113
114
             assign out1_real = m0;
             assign out1_imag = 16'b000000000000000;
115
             assign out2_real = s1;
116
             assign out2_imag = s3_imag;
117
             assign out3_real = m2;
118
```

```
assign out3_imag = m5_imag;
119
           assign out4_real = s2;
120
           assign out4_imag = ~s4_imag + 1'b1;
121
           assign out5_real = m1;
122
           assign out6_real = s2;
124
                  out6_imag = s4_imag;
125
           assign out7_real = m2;
126
           assign out7_imag = ~m5_imag + 1'b1;
127
           assign out8_real = s1;
128
           assign out8_imag = ~s3_imag + 1'b1;
   endmodule
130
```

The above code with Test Bench is available in FFT8_implementation_2 directory of the project GIT repository.

2.4 Analysis

This algorithm calculates the intermediate values in FFT block serially hence reusing the multipliers. Since multipliers are reused , the input bandwidth reduces by half from previous implementation , since 2 clock cycles are needed to perform 2 multiplications one by one. since both the multiplications happen in serial , total 1 multiplier is only used in this block to evaluate , hence reducing speed . Number of LUTs used is increased since multiplexers are implemented in LUT now to reuse the multiplier by changing inputs in alternate clock cycles . The simulation of this algorithm is shown in **Figure 2**.

3 Modified Cooley–Tukey algorithm using LUT based Multiplier

3.1 Overview

Cooley–Tukey algorithm for 8 point FFT involves 2 multiplication , hence using 2 multiplicative blocks as seen in the first implementation. In this modified algorithm , the multiplications are pipelined to run one by one , and using a LUT based 16 bit signed multiplier , reducing number of dedicated multiplier blocks , but increasing number of LUTs used.

3.2 Implementation

The below code implements the Modified Cooley–Tukey algorithm using LUT based multiplier. inp1,...inp8 are the 16 bit signed floating point inputs of Q format and out1-real , out1-imag, ...,out8-real,out8-imag are the real and imaginary outputs in 16 bit Q format of the FFT8 module.clk , rst are the clock , reset inputs respectively , and output-stb is the output strobe , which is enabled once the output is calculated. multiplier module takes inp1, inp2 16 bit signed floating point inputs of Q format and outputs the product of the inputs to out.The module multiplier is initialized once and reused twice in the main FFT8 block . The multiplier module takes input of 2 Q format number , and using Shift-Add algorithm , it multiplies the numbers and outputs every 16 clock cycles.

```
'timescale 1ns / 1ps
2
3
   module multiplier(
       input signed [15:0] inp1,
5
       input signed [15:0] inp2,
             input rst,
             input clk,
       output signed [15:0] out,
9
             output out_stb
10
   );
11
      localparam sf = 2.0**-8.0;
12
            reg [29:0] inp12;
13
            reg [14:0] input_1;
            reg [14:0] input_2;
15
            reg output_stb;
16
            reg out_sign;
17
            integer counter;
18
            assign out_stb = output_stb;
            initial
                     begin
21
                              output_stb = 1'b0;
22
```

```
inp12 = 32,b0;
23
                              counter = 0;
24
                     end
25
26
   always @ ( posedge clk )
   begin
   if (rst == 1'b1)
29
            begin
30
            output_stb = 1'b0;
31
            inp12 = 30'b0;
32
            counter = 0;
33
            end
34
   else if (output_stb == 1'b0 && counter < 15)</pre>
35
            begin
36
            if (counter == 0)
37
            begin
38
            out_sign = (inp1[15] && ~ inp2 [15]) + (inp2[15] && ~ inp1 [15]);
39
            input_1 = inp1[15] == 1'b0 ? inp1[14:0] : ~inp1[14:0] +1'b1;
            input_2 = inp2[15] == 1'b0 ? inp2[14:0] : ~inp2[14:0]+1'b1;
41
42
            if(input_1[counter] == 1' b1)
43
            begin
44
            inp12[29:14] = inp12[29:14] + input_2[14:0];
45
46
            end
            inp12 = inp12 >> 1;
47
            counter = counter + 1;
48
49
   else if (counter >= 15)
50
            begin
51
                     output_stb = 1'b1;
52
53
            end
   end
   assign out = {out_sign,inp12[22:7]};
   endmodule
57
58
59
   module fft8(
            input signed [15:0] inp1,
61
            input signed [15:0] inp2,
62
            input signed [15:0] inp3,
63
            input signed [15:0] inp4,
64
            input signed [15:0] inp5,
65
            input signed [15:0] inp6,
            input signed [15:0] inp7,
68
            input signed [15:0] inp8,
            input clk,
69
            input rst,
70
            output signed [15:0] out1_real,
71
            output signed [15:0] out1_imag,
72
```

```
output signed [15:0] out2_real,
73
             output signed [15:0] out2_imag,
74
             output signed [15:0] out3_real,
75
             output signed [15:0] out3_imag,
76
             output signed [15:0] out4_real,
77
             output signed [15:0] out4_imag,
78
             output signed [15:0] out5_real,
79
             output signed [15:0] out5_imag,
80
             output signed [15:0] out6_real,
81
             output signed [15:0] out6_imag,
82
             output signed [15:0] out7_real,
             output signed [15:0] out7_imag,
84
             output signed [15:0] out8_real,
85
             output signed [15:0] out8_imag,
86
             output out_stb
87
   );
88
89
             localparam signed sin_45 = 16'b00000000_10110101;
90
             localparam signed sin_315 = 16'b11111111_01001011;
91
             localparam signed sf = 2.0**-8.0;
92
93
             reg signed [31:0] t1_46,t2_46;
94
             reg signed [15:0] t1,t2,t3,t4,t5,t6,t7,t8,m0,m1,m2,m3,m4;
95
             \tt ref \  \, signed \  \, [15:0] \  \, m5\_imag\,, m6\_imag\,, m7\_imag\,, s1\,, s2\,, s3\_imag\,, s4\_imag\,;
             reg [15:0] mult_inp1, mult_inp2;
97
             wire [15:0] mult_out;
98
             reg [1:0] stage;
99
             reg output_stb,mult_rst;
100
             wire mult_stb;
101
102
             multiplier mult (
103
                      .inp1(mult_inp1),
104
                      .inp2(mult_inp2),
105
                      .rst(mult_rst),
106
                      .out(mult_out),
107
                      .clk(clk),
108
                      .out_stb(mult_stb)
109
             );
110
111
             initial
112
                      begin
113
                               stage = 2'b00;
114
                               output_stb = 1'b0;
115
                               mult_rst = 1'b1;
116
117
                      end
118
             always @( posedge clk)
119
                      begin
120
                               if (rst == 1'b1)
121
                                         begin
122
```

```
output_stb = 1'b0;
123
                                                 mult_rst = 1'b1;
124
                                        end
125
                               if (stage == 2'b00 && rst == 1'b0 && output_stb == 1'b0)
126
127
                                        begin
                                                 t1 = inp1 + inp5;
128
                                                 t2 = inp7 + inp3;
129
                                                 t3 = inp2 + inp6;
130
                                                 t5 = inp4 + inp8;
131
                                                 m3 = inp1 - inp5;
132
                                                 m6\_imag = inp7 - inp3;
                                                 t4 = inp2 - inp6;
134
                                                 t6 = inp4 - inp8;
135
                                                 t8 = t5 + t3;
136
                                                 t7 = t1 + t2;
137
                                                 m0 = t7 + t8;
138
                                                 mult_inp1 = t4 - t6;
139
                                                 mult_inp2 = sin_45;
140
                                                 stage = 2'b01;
141
                                                 mult_rst = 1'b0;
142
                                        end
143
                               else if (stage == 2'b01 && mult_stb == 1'b1)
144
                                        begin
145
                                                 m4 = mult_out;
146
                                                 mult_rst = 1'b1;
147
                                                 stage = 2'b10;
148
149
                               else if (stage == 2'b10)
150
                                        begin
151
                                                 m5_{imag} = t5 - t3;
152
                                                 m2 = t1 - t2;
153
                                                 m1 = t7 - t8;
154
                                                 mult_inp1 = t4 + t6;
155
                                                 mult_inp2 = sin_315;
156
                                                 mult_rst = 1'b0;
157
                                                 stage = 2'b11;
158
                                        end
159
                               else if (stage == 2'b11 && mult_stb == 1'b1)
160
                                        begin
161
                                                 m7_imag = mult_out;
162
                                                 s1 = m3 + m4;
163
                                                 s2 = m3 - m4;
164
                                                 s3_imag = m6_imag + m7_imag;
165
                                                 s4_imag = m6_imag - m7_imag;
166
                                                 mult_rst = 1'b1;
167
168
                                                 stage = 2'b00;
                                                 output_stb = 1'b1;
169
                                        end
170
                      end
171
             assign out_stb = output_stb;
172
```

```
assign out1_real = m0;
173
           assign out1_imag = 16'b0000000000000000;
174
           assign out2_real = s1;
175
           assign out2_imag = s3_imag;
176
           assign out3_real = m2;
           assign out3_imag = m5_imag;
178
           assign out4_real = s2;
179
           assign out4_imag = ~s4_imag + 1'b1;
180
           assign out5_real = m1;
181
           182
           assign out6_real = s2;
           assign out6_imag = s4_imag;
184
           assign out7_real = m2;
185
           assign out7_imag = ~m5_imag + 1'b1;
186
           assign out8_real = s1;
187
           assign out8_imag = ~s3_imag + 1'b1;
188
   endmodule
189
```

The above code with Test Bench is available in FFT8_implementation_3 directory of the project GIT repository.

3.3 Analysis

This algorithm calculates the intermediate values in FFT block serially hence reusing the multipliers. Since multipliers are reused , the input bandwidth reduces by half from implementation where 2 multiplier blocks are used. Since the multiplier is LUT based , the usage of LUTs increase , but it reduces the amount number of DSP48A1s which are very scarce in FPGAs.

Figure 3: Simulation In ISIM

Name Value 001001101010 (XXXXXXXXXXX (XXXXXXXXXXXX 1111110 out3_imag[15:0] XXXXXXXXXXXXXXX 111111000000000 1111110 ut5_real[15:0] ut5_imag[15:0] 111111000000 t6 real[15:0] 111111000000 xxxxxxxxxxxxxx 11111100101100 (XXXXXXXXXXX 1111110 xxxxxxxxxxxxxx xxxxxxxxxxxxx 1111011001011000 inp1[15:0] inp2[15:0] inp3[15:0] 000000100000 000001100000000

14

4 Validation Script

A python Validation script is written to validate the FFT Verilog Blocks. The python script below uses Numpy library to generate 100 random floating point test inputs and using ISIM command line options simulates all the inputs and validates them with the standard FFT function present in the numpy library of python.

```
2 import numpy as np
 3 from bitstring import Bits
 4 import os
 5 implement=raw_input("design to test(allowed values 1,2,3)")
 6 working_directory=r"/FFT8_implementation_"+str(implement)
  print ("Generating 100 Random Inputs ...")
8
  cmds = "
10 for i in range (100):
     input\_array=np.random.uniform(low=-8.0, high=8.0, size=(8,))
     binary_array = []
     for inp in input_array:
13
       binary_array.append(Bits(int=int(inp*2**8), length=16).bin)
14
      isim force add {/fft8_tb/inp1} %s -radix bin
      isim force add {fft8\_tb/inp2} %s -radix bin isim force add {fft8\_tb/inp3} %s -radix bin
17
18
      isim force add {/fft8_tb/inp4} %s -radix bin
19
      isim force add {/fft8_tb/inp5} %s -radix bin
20
21
      isim force add {/fft8_tb/inp6} %s -radix bin
      isim force add {/fft8_tb/inp7} %s -radix bin isim force add {/fft8_tb/inp8} %s -radix bin
22
      isim force add {/fft8_tb/clk} 1 -radix bin -value 0 -radix
24
25
      bin -time 2500 ps -repeat 5 ns -cancel 1 us
      isim force add {fft8\_tb/rst} 1 -radix bin -cancel 20 ns
26
      isim force add {/fft8_tb/rst} 0 -radix bin -time 20 ps
27
      -cancel 1 us
28
      run
29
30
      dump
     """%tuple(binary_array)
31
     cmds=cmds+cmd
32
os.chdir(working_directory)
35 f=open("inp.test","w")
36 f.write(cmds)
37 f.close()
38 print ("Simulating all the inputs using ISIM...")
39 cmd='"'+working_directory+r'/fft8_tb_isim_beh.exe" < inp.test > out
       .test
40 os.system(cmd)
41 f=open("out.test","r")
42 values = []
43 \text{ inp} = \{\}
44 out={}
45 for line in f.readlines():
46 print line
```

```
if "Signal:" in line:
     out[line.strip().split("{")[1].split("}")[0].split("[")[0].strip
()]=line.strip().split(":")[-1].strip()
       "Variable:" in line:
     inp[line.strip().split("{")[1].split("}")[0].split("[")[0].strip
()]=line.strip().split(":")[-1].strip()
if "{rst}" in line.strip():
     values.append([inp,out])
     inp={}
53
     out={}
54
55
56 f.close()
57
58 print ("Verifying FFT output with the actual values...")
59 correct=0
  count=0
60
61 for val in values:
     count = count + 1
62
     inp = np.asarray([Bits(bin=val[0]['inp'+str(i)]).int/(2.0**8) for
         i in range (1,9)])
     out1 = np.array([Bits(bin=val[1]['out'+str(i)+"_real"]).int
        /(2.0**8) for i in range (1,9)])
     out2 = np.array([Bits(bin=val[1]['out'+str(i)+"_imag"]).int
        /(2.0**8) for i in range (1,9)])
     out=out1 + 1j*out2
     if np.allclose(out, np.fft.fft(inp), 1e-2):
      correct = correct + 1
68
69
70 print(str(correct)+r"/"+str(count)+" Correct ...")
```

4.1 Requirements

- λ =Python 2.7
- bitstring library
- ISIM simulator

4.2 Sample Usage

Clone the GIT repository from the URL given in the project link section, and run validate.py script to validate the designs. The validate.py file is present in the root folder of the GIT repository. Sample usage of the script is shown in figure 4.

5 Project URL

Project GIT repository: https://github.com/Mohankumariitm/Dual-Degree-Project Use Git or checkout with SVN using the web URL.

Figure 4: Python 2.7



6 Design Summary

Below is the resource utilization summary of the above implementations.

Logic Utilization	Implementation 1	Implementation 2	Implementation 3
Number of Slice Registers	47	317	357
Number of Slice LUTs	337	374	456
Number of fully used LUT-FF pairs	47	138	188
Number of BUFG/BUFGCTRLs	1	1	1
Number of DSP48A1s	2	1	0

7 To Do (in upcoming months)

: https://www.embedded.com/design/connectivity/4403178/Doing-Hartley-Smartly

- Implement FFT using Doing-Hartley-Smartly Method, and Analyze the resource usage
- Implement Variations in Doing-Hartley-Smartly Method by reusing resources to reduce the LUTs.