Infumtial Statistics ←

1. Z test
2. t test
3. Z test propotion population
4. Chi Square Test
5. ANNOVA (F Test)

Sample DATA ⟶ Population Data.

⇒ Hypothesis Testing

(Q) A factory has a machine that fills 80ml of Baby medicines in a bottle. An employee believes the average amount of baby medicine is not 80ml. Using 40 samples, he measures the average amount dispersed by the machine to be 78ml with a standard deviation of 2.5

(a) State Null & Alternate hypothesis

(b) At 95% C.I, is there enough evidence to support whether machine is working properly or not.

{Z test or t test}

Ans) $\mu = 80ml$ $n = 40$ $\bar{x} = 78$ $s = 2.5$

① $H_0 : \mu = 80$ {Null Hypothesis} ✓

$H_1 : \mu \neq 80$ {Alternate Hypothesis}. ✓

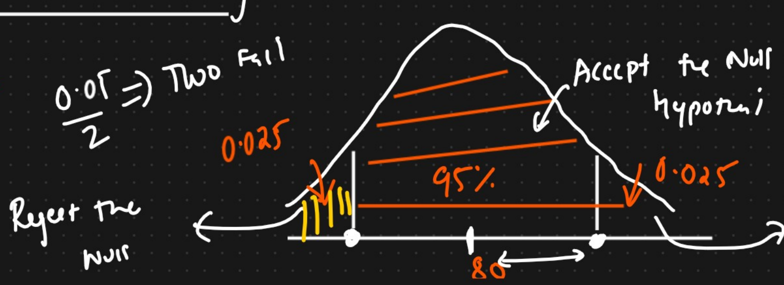
Z test $SV = 1 - CI$

② Step 2 : $\alpha = 0.05$ $C.I = 95\%$ $\alpha = 1 - 0.95 = 0.05$

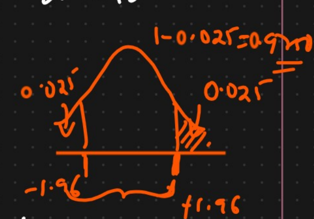
③ Decision Boundary $\boxed{50ml} \leftrightarrow \boxed{150ml}$

One Tail or 2 tail

$\alpha = 0.05$ $\frac{0.05}{2} \Rightarrow$ Two Tail



2 Tail Test



→ Reject the Null hypothesis

Hypothesis: -1.96 $+1.96$ $1-0.025 = 0.9750$

Z table

④ Calculate Test Statistics

$n=1 \Rightarrow$ No

$\boxed{n=40} \Rightarrow$ Sample size

$$Z = \frac{\bar{x}-\mu}{\boxed{S/\sqrt{n}}} \Rightarrow 5.6$$

$$= \frac{78-80}{2.5/\sqrt{40}} = \frac{-2\times\sqrt{40}}{2.5} = \frac{-2}{2.5}\times 6.32 = \boxed{-5.05}$$

↓ Standard Deviation }.

⑤ Conclusions

Decision Rule: If $Z=-5.05$ is less than -1.96 or greater than 1.96, then reject the Null hypothesis with 95% C.I

Reject the Null Hypothesis { There is some fault in the machine }

Machine is not working properly

(*) A complain was registered, the boys in a Government School are underfed. Average weight of the boys of age 10 is 32 kgs with S.D = 9kgs. A Sample of 25 boys were selected from the Government School and the average weight was found to be 29.5 kgs? With C.I = 95%. Check Whether it is True or False?

Conditions For Z-test ✓

① We know the population s.d $\boxed{\text{OR}}$

② We do not know the population s.d but our sample is large $n \geq 30$

Conditions for T test ✓

① We do not know the population variance or std

② Our sample size is small $n < 30$.

$H_0 : \mu = 32$ {Not underfed} ← weight has decreased

$H_1 : \mu \neq 32$

$$Z\text{-score} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{29.5 - 32}{9 / \sqrt{25}} = -1.39$$

$1 > -1.39$

Conclusion : $-1.39 > -1.96$ So Accept the Null Hypothesis 95% C.I

Conclusion : Students are not underfed

(4) A factory manufactures cars with a warranty of 5 years or more on the engine and transmission. An engineer believes that the engine or transmission will malfunction in less than 5 years. He tests a sample of 40 cars and finds the average time to be 4.8 years with a standard deviation of 0.50. (1) State the null & alternate hypothesis

(2) At a 2% significance level, is there enough evidence to support the idea that the warranty should be revised?

$n = 40 \quad \bar{x} = 4.8 \quad S = 0.50$

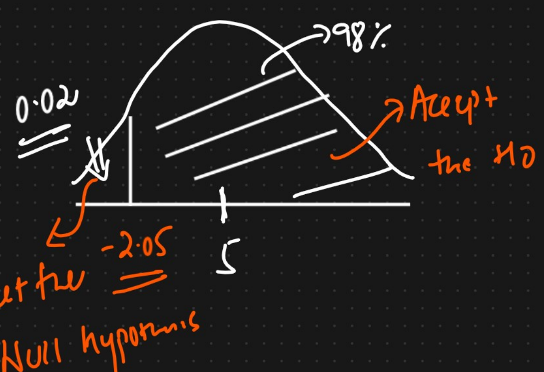
$\alpha = 0.02$

Ans) $H_0 : \mu \geq 5$

$H_1 : \mu < 5$ {Alternate Hypothesis}

Step 2 $\quad \alpha = 0.02 \quad\quad CI = 98\%$

Z table



1 tail Test

$-2.52 < -2.05$

$$Z_{score} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = -2.52$$

Reject the Null Hypothesis

Conclusion : Warranty needs to be revised.

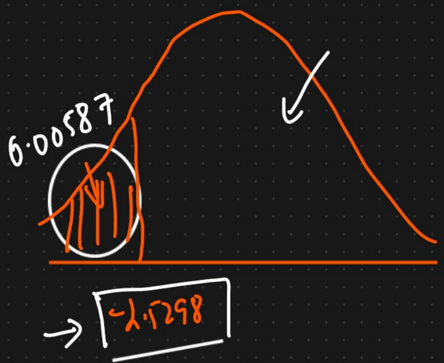
$\frac{\boxed{4}}{\boxed{4.8}}$ ⇒ Company

P-value

⇓

(1) Z-score ⟷ Sd

Z-score = -2.5298



1 - 0.00587 = 0.99413

$\alpha = 0.02$

p-value = 0.00587

$\boxed{\text{p-value} < \alpha}$

Reject the Null hypothesis

② In the population the average IQ is 100 with a standard deviation of 15. A team of scientists wants to test a new medication to see if it has a +ve or -ve effect, or no effect at all.

A sample of 30 participants who have taken the medication has a mean of 140. Did the medication affect Intelligence? $\left\{ \begin{array}{c} 95\% \\ \Downarrow \\ C.I \end{array} \right\}$

Ans) $\sigma = 15$ $\quad n = 30$ $\quad \bar{x} = 140$ $\quad \alpha = 0.05$ $\quad C.I = 95\%$

$1 - 0.95 = 0.05$

Step 1

Null Hypothesis $H_0 : \mu = 100$

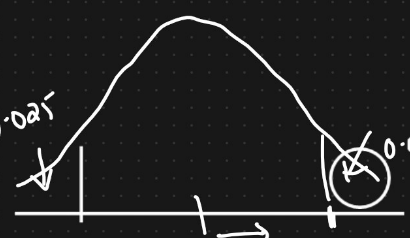
Alternate Hypothesis $H_1 : \mu \neq 100$

Step 2 : $\alpha = 0.05$ $\quad C.I = 0.95$

$1 - 0.025 = 0.9750$

Step 3

Two Tail Test



0.025 $\quad$ 0.025

-1.96 $\quad$ +1.96 $\hookrightarrow$ Z table.

Z test

Step 4 :

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{140 - 100}{15/\sqrt{30}} = 14.60$$

Conclusion

$\Rightarrow$ Reject the Null hypothesis

$14.6 > 1.96$ $\Rightarrow$ The medication had some affect

+ve effect

1) The average weight of all residents in a town XYZ is 168 pounds. A nutritionist believes the true mean to be different. She measured the weight of 36 individuals and found the mean to be 169.5 pounds with a standard deviation of 3.9

(a) Null & Alternate Hypothesis

(b) 95%. Is there enough evidence to discard the null hypothesis?

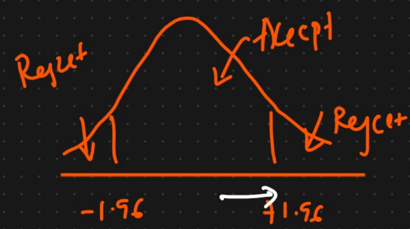
Ans). $\bar{x} = 169.5$ $\quad s = 3.9$ $\quad n = 36$ $\quad \mu = 168$

① $H_0: \mu = 168$

$H_1: \mu \neq 168$

② $\alpha = 0.05$

③ Decision Boundary



④ Z score: $\dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

$= \dfrac{169.5 - 168}{3.9/\sqrt{36}}$

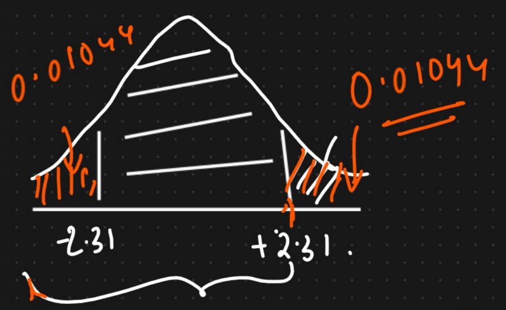
$= 2.31$ //.

$2.31 > 1.96$ {Reject Null Hypothesis}.

P-value

$1 - 0.02088 = 0.97912$

$1 - 0.98956 = 0.01044$



In 2 tail test

P-value $= 0.01044 + 0.01044$

$= \boxed{0.02088}$ $\qquad \alpha = 0.05$

$0.02 < 0.05$ Reject Null Hypothesis

(4) A company manufactures bikes batteries with an average life span of 2 year or more years. An Engineer believes this value to be less. Using 10 Samples, he measures the average life span to be 1.8 years with a standard deviation of 0.15.

a) State the Null and Alternate Hypothesis?

b) At a 99% C.I, is there enough evidence to discard the H_0?

(1)

Ans) $H_0 = \mu \geq 2$

$H_1 : \mu < 2.$

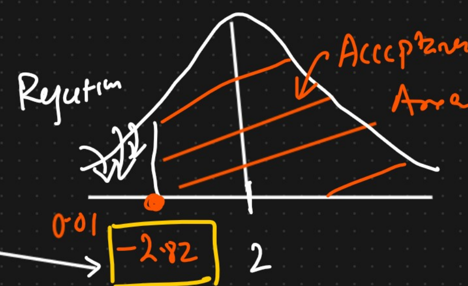
$n=10 \quad \mu=2 \quad \bar{x}=1.8 \quad s=0.15$

{T test}.

(2) $\alpha = 0.01 \qquad \alpha = 1 - C.I = 0.01$

(3) Degree of freedom $\Rightarrow n-1 = 10-1 = 9$

(4) Decision



(4) Calculate t-test Statistics

$\boxed{n=10}$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{1.8 - 2}{0.15/\sqrt{10}} = -4.216$$

(5) $-4.216 < -2.82$ {Reject the Null Hypothesis}.

Z test with proportions

(1) A tech company believes that the percentage of residents in town XYZ that owns a cell phone is 70%. A marketing manager believes that this value to be different. He conducts a survey of 200 individuals and found that 130 responded yes owning a cell phone

$n=1$

$$\frac{\boxed{x_i} - \mu}{\sigma/\sqrt{n}}$$

(a) State the Null and Alternate Hypothesis?

(b) At a 95% C.I, is there enough evidence to reject the Null Hypothesis?

Null Hypothesis

$$\left.\begin{aligned} H_0 &: P_0 = 70\% \\ H_1 &: P_0 \neq 70\% \end{aligned}\right\}$$

$n = 200 \qquad x = 130$

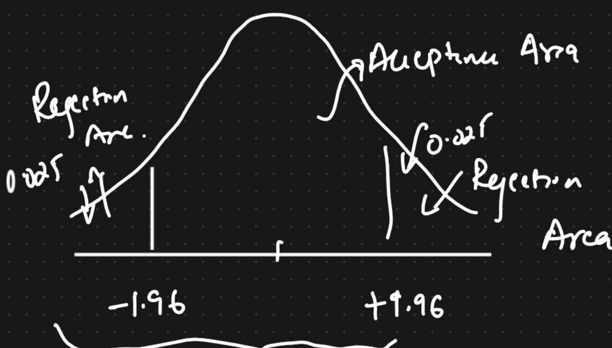
$$\hat{P} = \frac{x}{n} = \frac{130}{200} = 0.65$$

$P_0 \qquad q_0 = 1 - P_0 = 1 - 0.7 = 0.3$

(2) $\alpha = 0.05$

$1 - 0.025 = 0.9750$

(3)

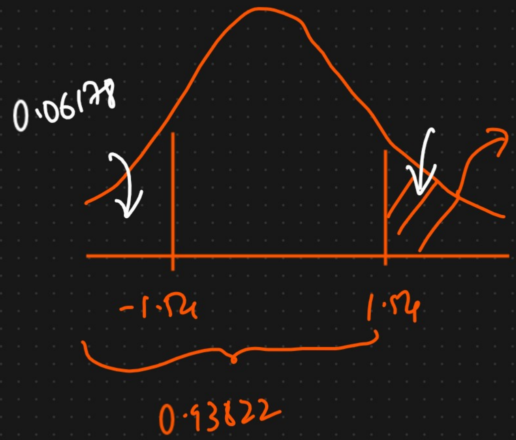


(4) Z-test with proportion

$$\boxed{Z_{test} = \frac{\hat{P} - P_0}{\sqrt{\frac{P_0 q_0}{n}}}}$$

$$= \frac{0.65 - 0.70}{\sqrt{\frac{0.7 \times 0.3}{200}}} \approx -1.54$$

$-1.54 > -1.96$ {Accept the Null Hypothesis}.



$1 - 0.93822 = 0.06178$

$$P\text{-value} = 0.06178 + 0.06178$$
$$= \underline{\underline{0.12356}}$$

P-value > significance-value → Accept the Null Hypothesis.

$0.12356 > 0.05$

④ A car company believes that the percentage of residents in City ABC that owns a vehicle is 60% or less. A sales manager disagrees with this. He conducts a hypothesis testing surveying 250 residents and found that 170 responded yes to owning a vehicle.

(a) State the Null & Alternate Hypothesis

(b) At 10% significance level, is there enough evidence to support the idea that vehicle ownership in City ABC is 60% or less?