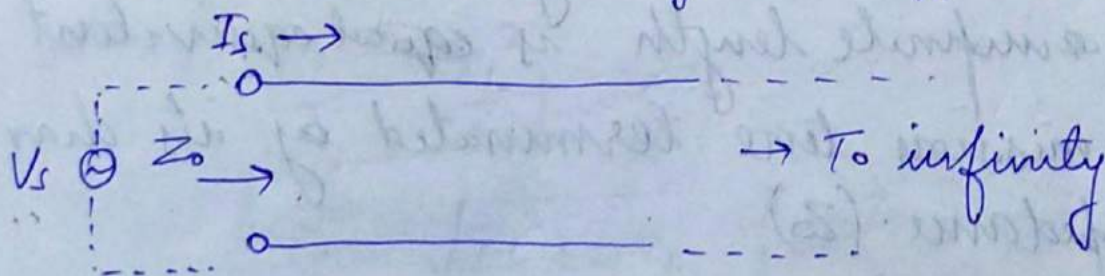


π -section representation of a transmission line section.

Concept of Infinite Line

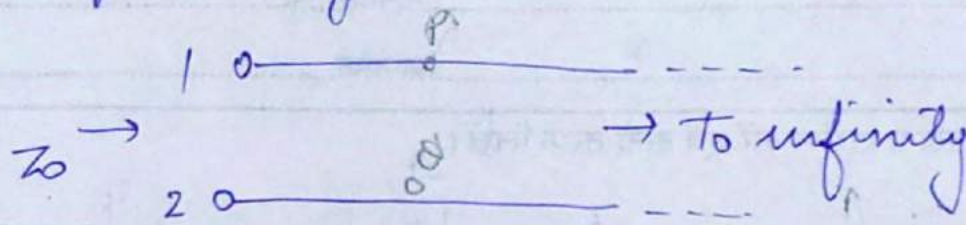
→ When a transmission line is considered to be of infinite length, then there will be no possibility of reflection of signal. As the conductor is of infinite length, so ~~there~~ no electrical signal will ^{reach} its end.



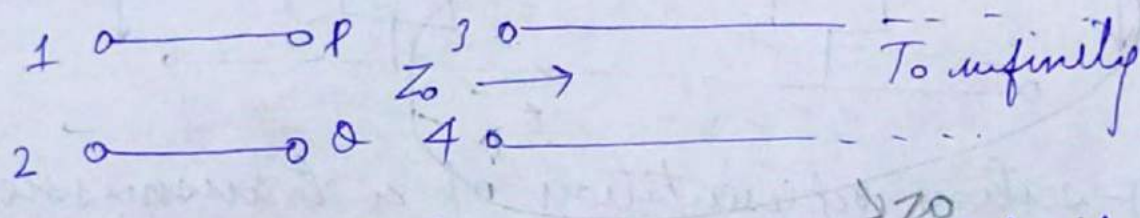
In above figure, a transmission line of infinite length is considered. V_s is the input voltage and I_s is the flow of current in the line due to V_s .

The ratio of V_s and I_s will be the input impedance of the line.

Let Z_0 is the ~~un~~ input impedance of a transmission line of infinite length.

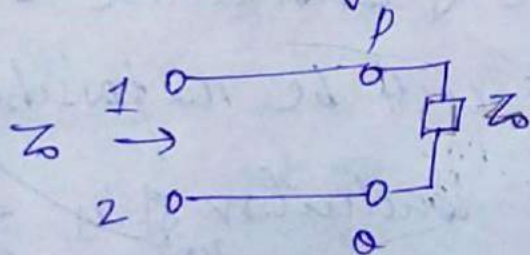


If a small section is removed from this line PQ, then



→ The remaining section ~~is~~ 3-4, is of infinite length, hence input impedance of 3-4 will be Z_0 .

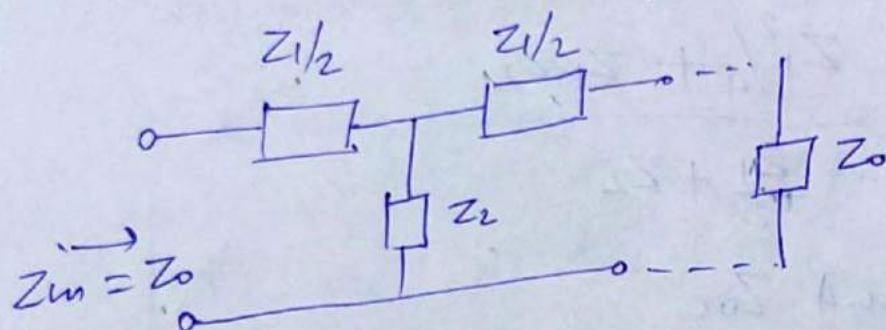
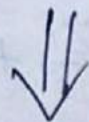
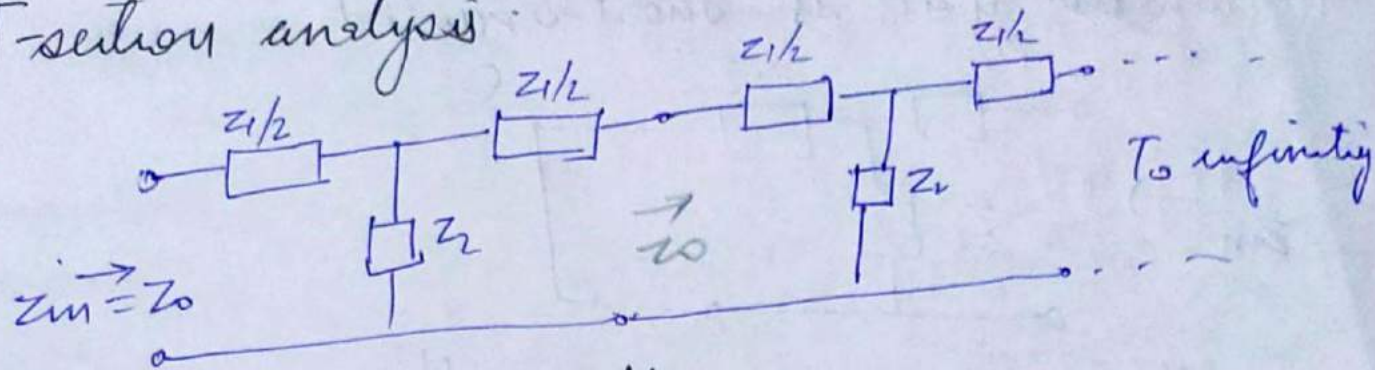
Thus we can say the impedance b/w PQ will be Z_0 .



→ Thus from this we can say that a transmission line of infinite length is ~~equal~~ equivalent to a transmission line terminated by its characteristic impedance (Z_0).

→ And so, if a ~~trans~~ transmission line is terminated by the characteristic impedance (Z_0), then there will be no reflection of signal.

T-section analysis



Solving
$$Z_{in} = z_{1/2} + z_2 \parallel (z_{1/2} + z_0)$$

$$Z_{in} = \frac{z_1}{2} + \frac{z_2 \left(\frac{z_1}{2} + z_0 \right)}{z_2 + \frac{z_1}{2} + z_0}$$

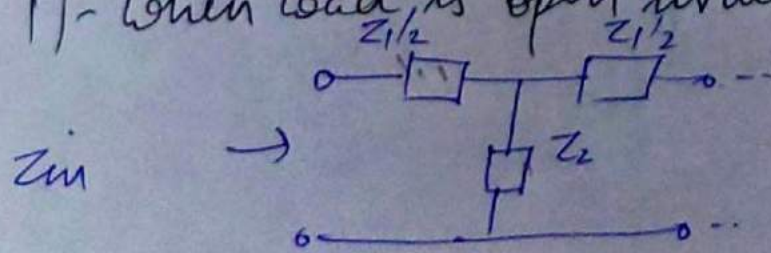
For transmission line of infinite length
 $Z_{in} = z_0$

thus
$$z_0 = \frac{z_1}{2} + \frac{\frac{z_1 z_2}{2} + z_0 z_2}{\frac{z_1}{2} + z_2 + z_0}$$

on solving
$$z_0^2 = \frac{z_1^2}{4} + z_1 z_2$$

or
$$z_0 = \sqrt{\frac{z_1^2}{4} + z_1 z_2}$$
 ✓

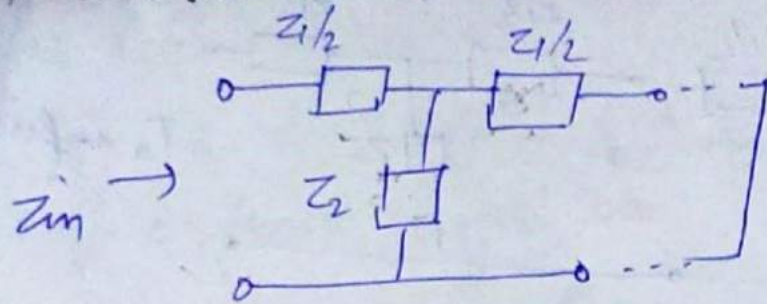
ii) - When load is open circuited



then

$$Z_{in} = Z_{oc} = \frac{z_1}{2} + z_2$$

iii) when load is short-circuited



then $z_{in} = z_{sc} = \frac{z_1}{2} + z_2 \parallel \frac{z_1}{2}$

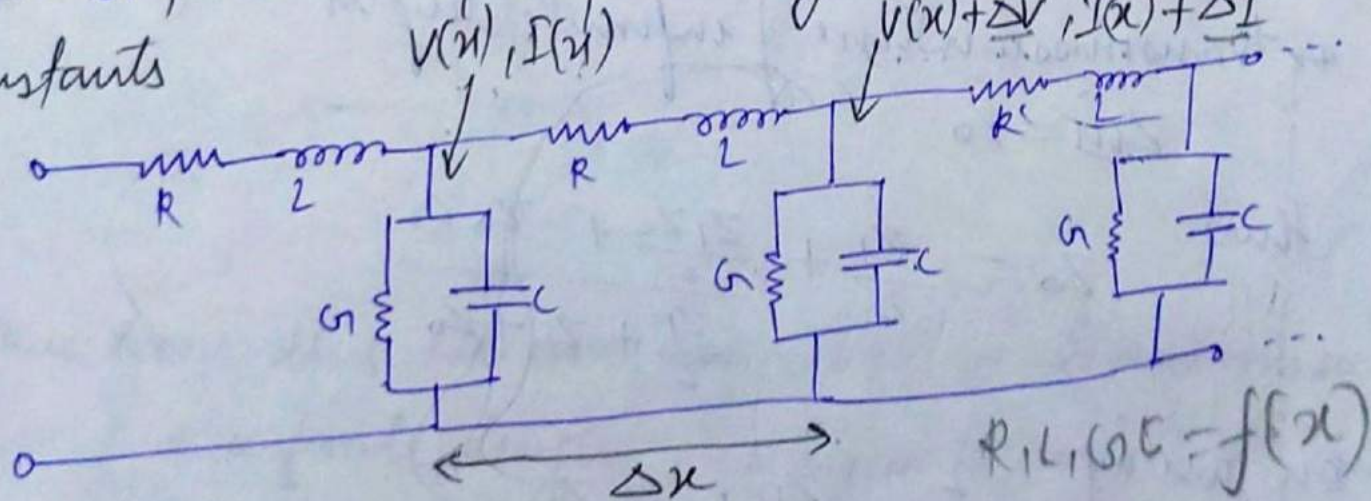
$$z_{sc} = \frac{\frac{z_1^2}{4} + z_1 z_2}{\frac{z_1}{2} + z_2}$$

multiplying z_{sc} and z_{oc}

$$z_0 z_{sc} \cdot z_{oc} = \frac{z_1^2}{4} + z_1 z_2 = z_0^2$$

$$\Rightarrow \boxed{z_0 = \sqrt{z_{sc} \cdot z_{oc}}}$$

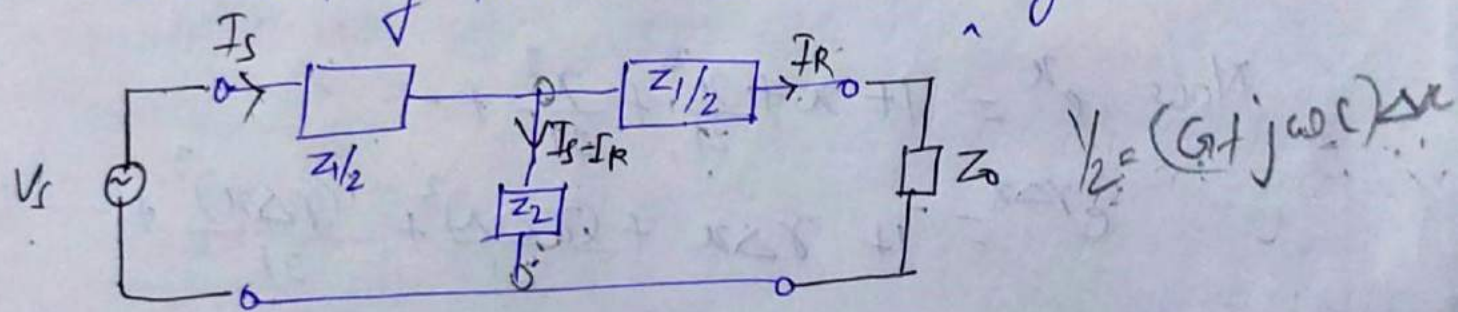
~~If a~~ Relation b/w primary and secondary constants



→ Since primary constants are distributed elements, so the net value of these elements will change with distance. or to say these elements will be a function of distance.

For a small distance Δx , the voltage and current will change by ΔV and ΔI , due to change in the primary constants (R, L, G, C) in Δx length. So let the primary constant in the small length Δx will be $R\Delta x, L\Delta x, G\Delta x, C\Delta x$.

For a T-section of transmission line of length Δx given as -



where $\frac{Z_1}{2} = \frac{1}{2}(R + j\omega L)\Delta x$, $Z_2 = (G + j\omega C)\Delta x$

$$Z_0 = \sqrt{\frac{Z_1^2}{4} + Z_1 Z_2}$$

$$Z_0 = \sqrt{\frac{(R + j\omega L)^2 (\Delta x)^2}{4} + \frac{(R + j\omega L) \Delta x (G + j\omega C) \Delta x}{1}}$$

For a very small section of transmission line $\Delta x \rightarrow 0$
then

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Propagation constant

$$e^{\gamma} = \frac{I_s}{I_R}$$

$$e^{\gamma} = \frac{Z_1/2 + Z_0 + Z_2}{Z_2}$$

$$= 1 + \frac{Z_1}{2Z_2} + \frac{Z_0}{Z_2}$$

→ For Δx

$$e^{\gamma \Delta x} = 1 + \frac{(R + j\omega L) \Delta x \times (G + j\omega C) \Delta x}{2} + \sqrt{\frac{R + j\omega L}{G + j\omega C}} (G + j\omega C) \Delta x$$

$$= 1 + \frac{(R + j\omega L)(G + j\omega C)(\Delta x)^2}{2} + \sqrt{(R + j\omega L)(G + j\omega C)} (\Delta x)$$

$$e^{\gamma \Delta x} = 1 + \sqrt{(R + j\omega L)(G + j\omega C)} \Delta x + \frac{(R + j\omega L)(G + j\omega C)(\Delta x)^2}{2}$$

Now $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$

$$e^{\gamma \Delta x} = 1 + \gamma \Delta x + \frac{(\gamma \Delta x)^2}{2!} + \frac{(\gamma \Delta x)^3}{3!} + \dots \infty$$

For $\Delta x \rightarrow 0$, high order terms can be neglected.

So $e^{\gamma \Delta x} = 1 + \gamma \Delta x + \frac{(\gamma \Delta x)^2}{2!}$

∴ Thus

$$\boxed{\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}} \quad \checkmark$$

Also

$$\gamma = \alpha + j\beta \rightarrow \begin{matrix} \text{attenuation} \\ \text{phase constant} \end{matrix}$$