Phase Modulation:

Assume carrier s/g before modulation $c(t) = A_c \cdot \cos 2\pi f_c t$

carrier s/g after modulation $S_{pm}(t) = A_c \cdot \cos[2\pi f_c t + \phi(t)]$

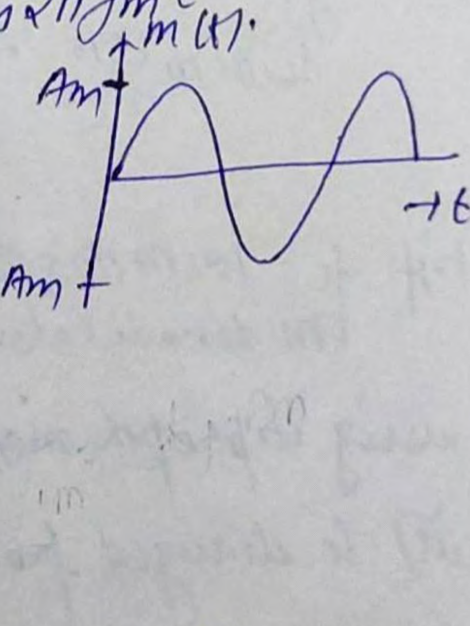
where $\phi = \overset{\text{radians/volt}}{K_p} \cdot m(t)$. $\phi(t) \rightarrow$ phase deviation or phase shift

$\Rightarrow$ $$\boxed{S_{pm}(t) = A_c \cdot \cos[2\pi f_c t + K_p \cdot m(t)]}$$

general expression of PM s/g.

$$\boxed{f_i = f_c + K_p \cdot m(t).}$$ $\rightarrow$ Freq deviation

Let $m(t) = A_m \cdot \cos 2\pi f_m t$ or $A_m \sin 2\pi f_m t$



max. phase deviation

$$\Delta\phi = |Max[\phi(t)]|$$
$$= |Max[K_p \cdot m(t)]|$$

$$\boxed{\Delta\phi = K_p \cdot A_m}$$ radi

Single tone PM.

$S_{pm}(t) = A_c \cdot \cos[2\pi f_c t + K_p \cdot m(t)]$.

let $m(t) = A_m \cdot \cos 2\pi f_m t$

$S_{pm}(t) = A_c \cdot \cos[2\pi f_c t + K_p \cdot A_m \cdot \cos 2\pi f_m t]$.

let $\boxed{K_p \cdot A_m = \beta}$ β = modulation index of PM.

For phase modulation –

$$\beta = \Delta\phi. \quad (\beta \text{ for PM only})$$

For frequency modulation

$$\beta = \Delta f / f_m. \quad (\beta \text{ for FM only}).$$

$$\Rightarrow \boxed{S_{PM}(t) = A_c \cdot \cos[2\pi f_c t + \beta \cdot \cos 2\pi f_m t]} \rightarrow PM$$

↳ single tone PM expression.

FM →

$$S_{FM}(t) = A_c \cdot \cos[2\pi f_c t + \beta \cdot \sin 2\pi f_m t] \rightarrow FM$$

Note → simply on the basis of sine or cosine function, it cannot be said that if it is a PM wave or FM wave.

* if an ~~AM~~ s/g given $s(t) = A_c \cdot \cos(2\pi f_c t + \beta \cdot \cos 2\pi f_m t)$.

if $m(t) = A_m \cdot \cos 2\pi f_m t$ → then $s(t)$ is PM.

if $m(t) = A_m \cdot \sin 2\pi f_m t$ → then $s(t)$ is FM.

* For same msg s/g given, the single tone PM and FM expressions will be same, except 90° phase shift at msg-frequency component.

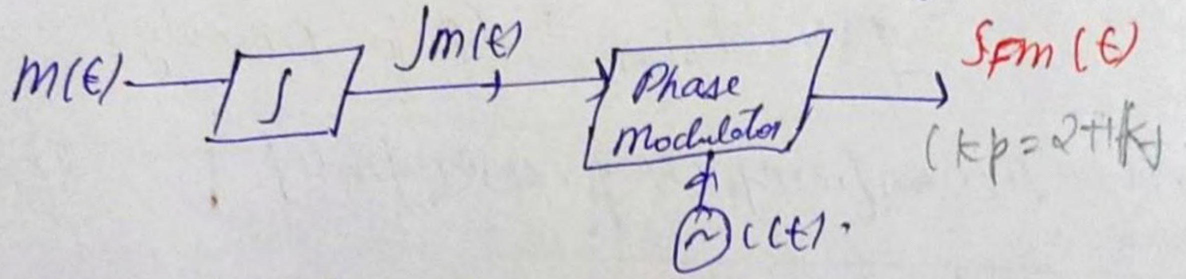
Generation of PM for ~~AM~~ FM or vice versa.

$$S_{PM} = A_c \cos[2\pi f_c t + k_p \cdot m(t)]$$

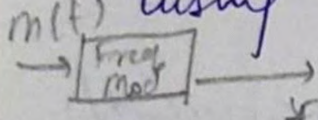
$$S_{FM} = A_c \cdot \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\, dt\right]$$

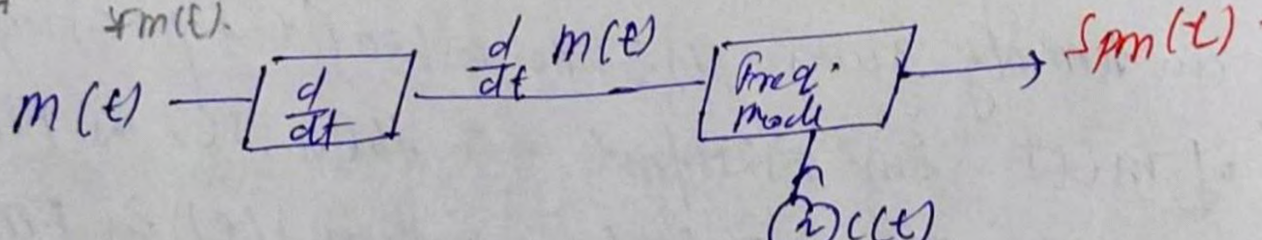
~~at~~ ~~via~~ $m(t) \rightarrow$ Phase Modulator $\rightarrow S_{PM}(t)$.

(1) - if give $k_p = 2\pi k_f$ ✓
and $m(t)$ as $\int m(t)$ then so by
using phase modulator we can generate FM.



(2) - if g we give $2\pi k_f = ~~\beta~~\, k_p$
and $\beta\, m(t)$ as $\frac{d}{dt} m(t)$, then by
using frequency modulator we can generator PM.





*
$$\boxed{\begin{aligned} FM[m(t)] &= PM\left[\int m(t)\right] \\ PM[m(t)] &= FM\left[\frac{d}{dt} m(t)\right] \end{aligned}}$$

* - if given $S_{FM}(t) = A_c \cdot \cos[\quad]$
$\downarrow$
~~pm~~ $FM[m(t)]$

$PM(x(t))$

$x(t) = \int m(t)$

$S_{PM}(t) = A_c \cos[\quad]$.
$\downarrow$
PM of $m(t) = ~~x~~$ FM of $\left(\frac{d}{d}\, m(t)\right)$

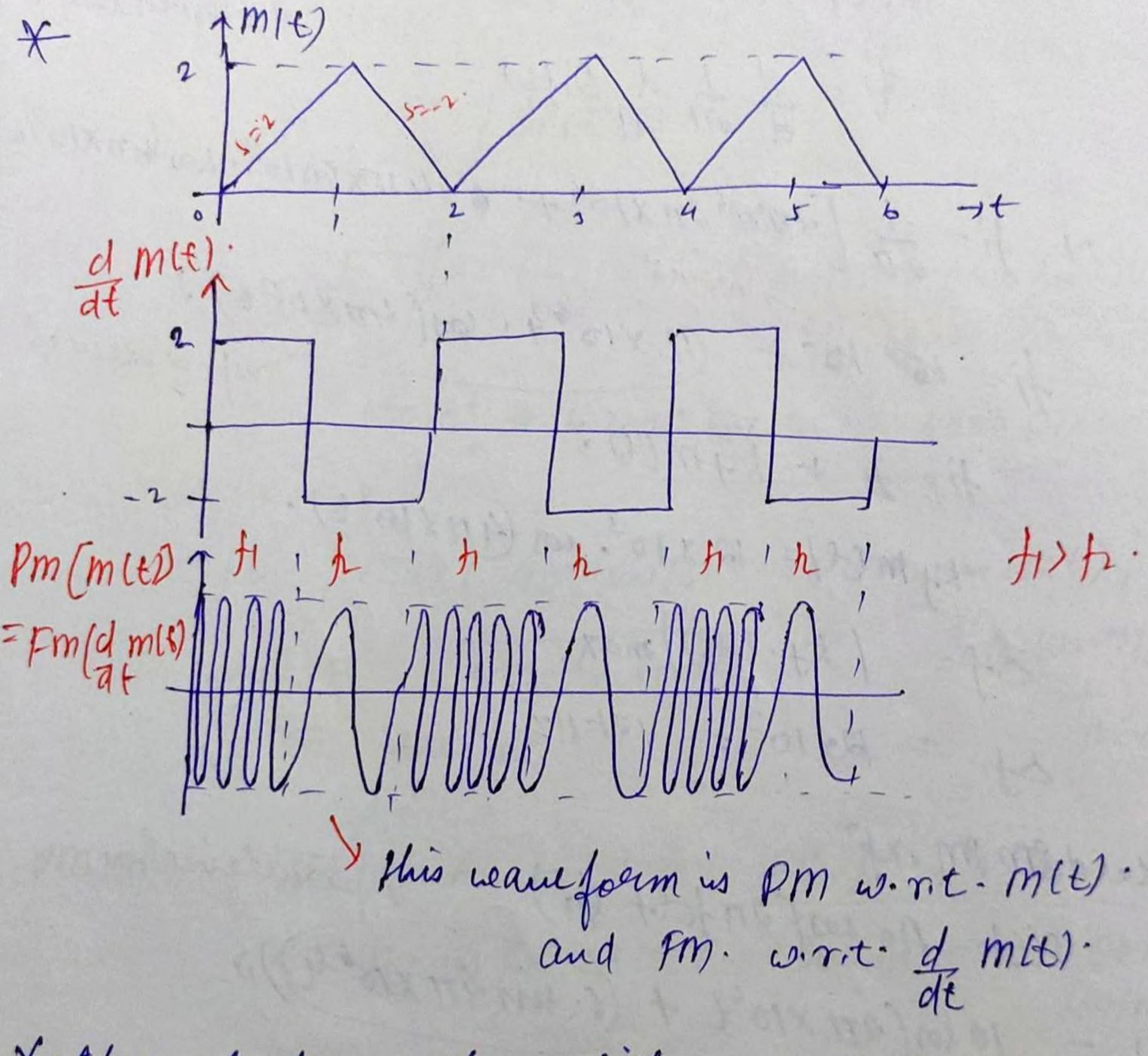
$3 = 12KHz$

* From application point of view, there is no difference between PM and FM coz.

$$PM(x(t)) = FM(y(t))$$

when $x(t) = \int y(t)$.

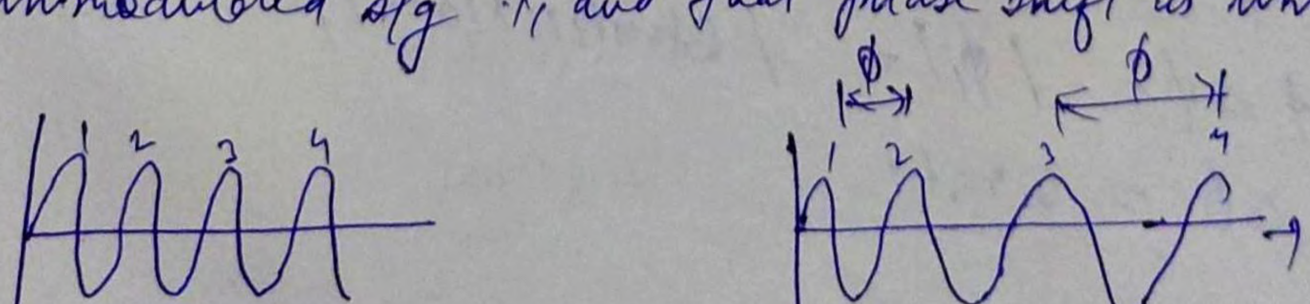
*



this waveform is PM w.r.t. $m(t)$ and FM w.r.t. $\frac{d}{dt} m(t)$.

* If a s/g has a phase shift w.r.t. ~~to~~ the ~~[illegible]~~ unmodulated s/g x_1, and that phase shift is time-varying

*



this is valid PM, as ϕ is changing with time.

maximum phase deviation of FM s/g · (only if m(t) → sinusoidal)

$$S_{FM} = A_c \cdot \cos\left[2\pi f_c t + 2\pi k_f \int m(t) \cdot dt\right]$$

$$[\Delta\phi]_{FM} = \left| \max\left[2\pi k_f \int m(t) \cdot dt\right] \right|$$

$$[\Delta\phi_{PM}] = \left| \max\left[k_p \cdot m(t)\right] \right|$$

for $m(t) = A_m \cdot \cos 2\pi f_m t$

$$[\Delta\phi]_{FM} = \left| \max\left(2\pi k_f \frac{A_m \sin 2\pi f_m t}{2\pi f_m}\right) \right|$$

$$\boxed{\Delta\phi_{FM} \Rightarrow \frac{k_f \cdot A_m}{f_m} = \beta} \checkmark = \frac{(\Delta f)_{FM}}{f_m}$$

$$\boxed{\Delta\phi_{PM} = k_p \cdot A_m}$$

Maximum frequency deviation of PM signal (only if $m(t)$ - sinusoidal)

$$S_{pm} = A_c \cdot \cos[\underbrace{2\pi f_c t + k_p \cdot m(t)}_{\phi_i}]$$

$$S_{pm}(t) = A_c \cdot \cos[\phi_i(t)]$$

$$f_i = \frac{1}{2\pi} \frac{d}{dt} [\phi_i].$$

$$= \frac{1}{2\pi} \left[2\pi f_c + k_p \cdot \frac{d}{dt} m(t)\right]$$

** $\boxed{f_i = f_c + \frac{k_p}{2\pi} \frac{d}{dt} m(t)}$ for PM.

$$[\Delta f]_{pm} = \left|\max\left[\frac{k_p}{2\pi} \frac{d}{dt} m(t)\right]\right| ; [\Delta f]_{FM} = [k_f \cdot M(t)]_{max}$$

$\cancel{[\Delta f]_{pm}}$ =

Bandwidth of PM sig - (only if $m(t) \to$ sinusoidal.

$$[BW]_{pm} = [\beta_{pm} + 1] 2f_m = 2[\Delta f_{pm} + f_m].$$

By Carson's rule

$$[BW_{FM}] = [\beta_{FM} + 1] 2f_m = 2[\Delta f_{FM} + f_m]$$

Let $m(t) = A_m \cdot \cos 2\pi f_m t$

$$[\Delta f]_{pm} = \left|\max\left[\frac{k_p}{2\pi} \frac{d}{dt} m(t)\right]\right|$$

=) $\Delta\phi_{pm} = \frac{(\Delta f)_{pm}}{f_m}$ $= \left|\max\left[\frac{k_p}{2\pi} \cdot A_m \cdot 2\pi f_m \cdot \sin 2\pi f_m t\right]\right|$

$$\boxed{[\Delta f]_{pm} = k_p \cdot A_m \cdot f_m = \beta_{pm} \cdot f_m} = \Delta\phi \cdot f_m$$