1

⇒ Filled Area Primatives:- Filled area primitives are used to filling solid colors to area or image or polygon. filling the polygon means highlighting its pixel with different solid colors.

→ following are two filled are primitives

1) Seed fill Algo.

2) Scan line Algorithm.

1) Seed fill Algorithm:- In this seed file Algorithm, we select a starting point called seed inside the boundary of the polygon.

2) Scan Fill algorithm:- Scan file algorithm is an area filling Algorithm that fill colors by scanning horizontal times. these horizontal lines intersected the boundary of the polygon and fill colors b/w the intersection point.

→ For filling a given picture on object with colors we can do it in two ways in C programming.

① Using filling Algorithm such as Flood fill algo., Boundary fill Algorithm and Scan line Algorithm Polygon fill Algorithm.

② Using in built graphics functions such as Floodfill() setfillstyle() we can fill the object with colors directly without using any filling Algorithm.

If
liv

⇒ Filling algorithm:-

① **Solid fill:-** All the pixel inside the polygon's boundary are illuminated.

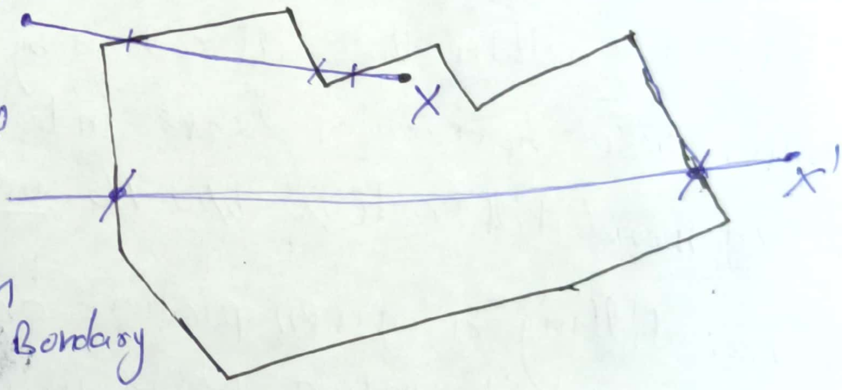
② **Pattern fill:-** The polygon is filled with an arbitrary predefined pattern.

⇒ Polygon Area Filling:-

Inside Test:-

→ **ODD_EVEN METHOD**

* Construct a line segment b/w the point in question & a point known to be outside polygon.



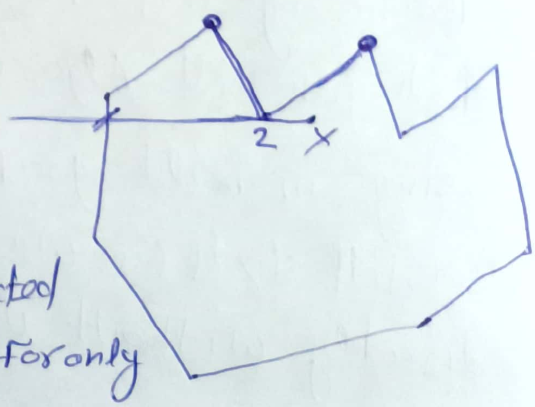
→ Count the intersection on line with polygon boundary

X
Inside → odd no. of Intersection ⇒ 3 (Inside)

X'
outside → Even no of Intersection ⇒ 2 (outside)

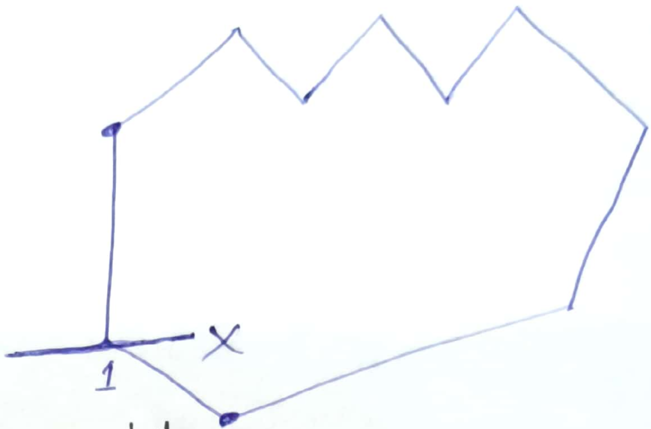
→ **If intersection point is vertex:**

* then look at other end points of line segment.



* If points are on same side of constructed line then EVEN no. of Intersection for only that intersection (2).

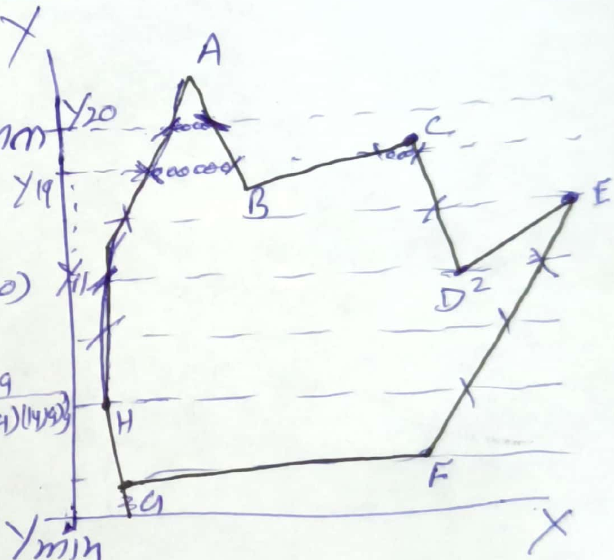
If Points are on different side of Constructed line' then ODD No of Intersection for only that Intersect.



⇒ Scan Line Polygon Fill Algorithm:-

steps:-

1) Locate the Intersection Points of the scan line with the Polygon Edges.
2) Paring Intersection Points. (6,20),(7,20) (Y_{20})
3) move down side as Per scan line Y_{19} & sort all Pairs. Y_{20} {(6,20)(7,20)}, {(5,19) (8,19)}{(12,19)(14,19)}
Y_{11} ⇒ {(3,11) (6,11) (16,11) (21,11)} → vertex cut.
4) All Pairs are sorted from Y_{max} to Y_{min}
5) Side Get sorted on Intersection Point Bases. Y_{20}-Y_{max},
6) Area filling starts now.



5 step

	Edge list
Y_{20}-max	A H (Active Edge)
	A B
	B C
	C D
	D E
	E F
	H G
	G F

Edge list

→ Coherence Property:-
* Relating Property of one scene to Another Part of scene
→ slope of Line Is 'm'

$$m = \frac{Y_2 - Y_1}{x_2 - X_1} \Rightarrow \frac{DY}{DX}$$ Here, $\Delta Y = 1$ Always

{ $\frac{20}{19}$ ⇒ 20-19 = 1, 19-18 = 1, 18-17 = 1 }

$$x_{k+1} = \frac{1}{m} + x_K . \; (x_2 = X_{K+1},\; X_K = X_1)$$

Unit - 3
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BCA-407

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⇒ Types of Transformations:-

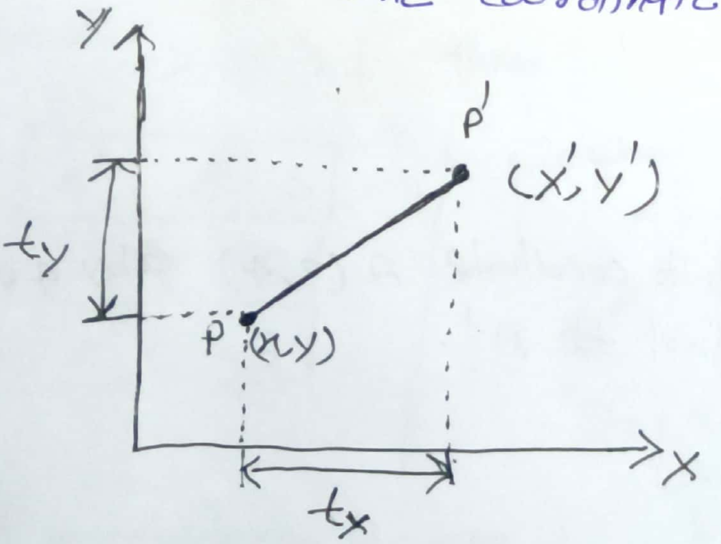
1) Translation → change in position
2) Scaling → change in size-shape
3) Rotating → movement along circular path
4) Reflection
5) Shearing

⇒ Transformation:-

Transformation means changes in orientation size & shape of the object. They are used to position the object to change the shape of object and even to change how something is viewed the basic geometrical transformation are:-

1) Translation:-

It is repositioning an object along straight line path from one coordinate location to another.



Note:- (t_x, t_y) → translation vector or shift vector

→ Translation is rigid body translation that moves an object without deformation.

→ From the above figure, you can write that-

$$\boxed{\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}}$$

Addition

→ The matrix Representation will be -

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Note:-

$P = \frac{x}{y}$, $P' = \frac{x'}{y'}$, $T = \frac{t_x}{t_y}$

→ we can write that

$$\boxed{P' = P + T}$$

Q. ⇒ Translate a point (2,4) where T (-1,14) Find P'?

→ P = (2, 4)
→ T = (-1,14)

* $P' = P + T$
 $P' = 2 + (-1) \Rightarrow 1$

* $P' = P + T$
 $= 4 + 14 = 18$

$P' = (1, 18)$

Q:. Translate a Polygon with coordinates A (2,7), B (7,10) C (10,2) by 3 units in x direction and 4 unit in y direction

→ $A' = P + T$

$A' = (5, 11)$
$B' = (10, 14)$
$C' = (13, 6)$

ex Translate a Polygon with cordinate A (3,4) by 4 unit X and 5 unit y. find the A'.

⇒ $A' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

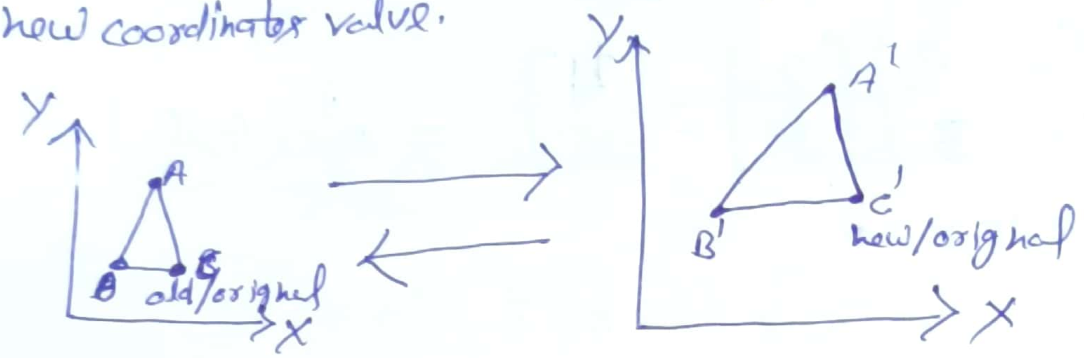
$A = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $T = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$A' = A + T \Rightarrow A' = \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$

$A' = (7, 9)$ Ae.

2) **Scaling:-** → A scaling transformation alters the size of an object.

→ Scaling of a polygon requires multiplying the coordinates value of each vertex by the scaling factor to get the new coordinates value.

→ 

→ Scaling function used multiplication.

→ S_x & S_y as known as scaling factor

$S_x, S_y > 1$ size ↑ increase (zooming)

$S_x, S_y < 1$ size ↓ Decrease

$S_x = S_y$ → uniform scaling (No change)

⇒ Now $S_x, S_y > 1$ then

$$\boxed{A' = A \cdot S}$$
$$\boxed{B' = B \cdot S}$$
$$\boxed{C' = C \cdot S}$$

We want

$A' = [x', y']$

$A = [x, y]_{1\times 2}$

*

$S = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}_{2\times 2}$

Note

$\begin{cases} x' = x \cdot S_x \\ y' = y \cdot S_y \end{cases}$

→ I matrix form, it can be Represented as

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{P' = S \cdot P}$$

$[x \cdot S_x + 0 \quad 0 + y \cdot S_y]$

$[x \cdot S_x \quad y \cdot S_y]$

$A = [\underbrace{x S_x}_{x'} \quad \underbrace{y S_y}_{y'}]$

Ex: Scale a polygon with coordinates A(2,5), B(7,10), C(10,2) by 2 units in x-direction & 3 units in y-direction

→ A(2,5) | $S_x = 2$ unit, $S_y = 3$ unit

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \end{bmatrix} = A' = (4,15)$$

$A' = (4,15)$

→ B(7,10)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 10 \end{bmatrix} = \begin{bmatrix} 14 \\ 30 \end{bmatrix} \Rightarrow B' = (14,30)$$

→ C(10,2)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 2 \end{bmatrix} = \begin{bmatrix} 20 \\ 6 \end{bmatrix} \Rightarrow C' = (20,6)$$

Ex: Translate a square ABCD A(0,0), B(3,0), C(3,3) & D(0,3) by 2 unit in both direction and to scale it by 1.5 unit in x direction and 0.5 unit in y directi[on]. Determine the resultant coordinates of polygon.

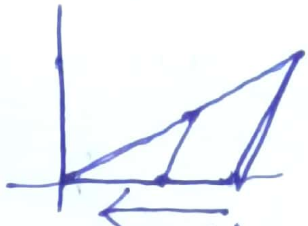
→ Translation – Both side 2 units
A'(2,2), B'(5,2) C'(5,5) & D'(2,5)

→ Scaling.

A''(2×1.5, 2×.5)
A'' = (3,1), B''(7.5,1) C''(7.5,2.5) D''(3,2.5)

Ex Magnify the triangle Δ with vertices (0,0), (1,1) (5,2) to twice its size while keeping (5,2) fixed.

$\Rightarrow$



$\Delta \xrightarrow{\text{Scale}} \Delta \xrightarrow{\text{translate}} \Delta$

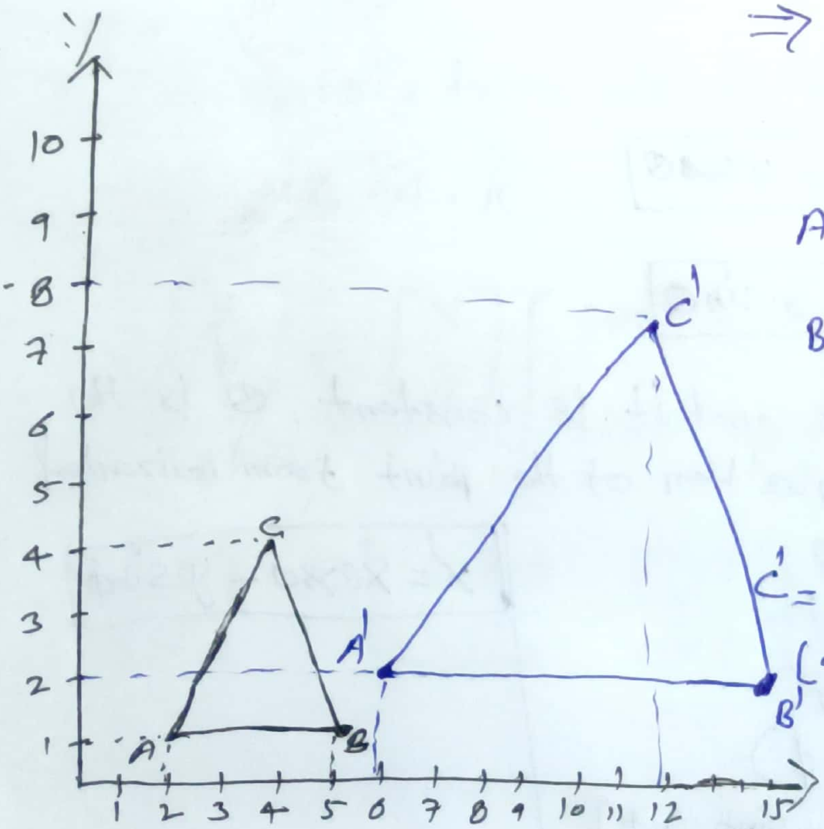
Scaling = $A'(0,0)$, $B'(2,2)$ $C'(10,4)$

Translate = $A''(-5,2)$ $B''(-3,0$ $C''(5,2)$

Translate –
[Increasing = +
Decreasing = –]

Ex old/original Point

A (2,1)	3 unit in x' → Sx
B (5,1)	2 unit in y' → Sy
C (4,4)	



$\Rightarrow A' = A \cdot S$

$= \underset{1\times 2}{[2 \;\; 1]} \underset{2\times 2}{\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}}$

$A' = \begin{array}{c} x \;\; y \\ [6 \;\; 2] \end{array}$

$B' = B \cdot S$

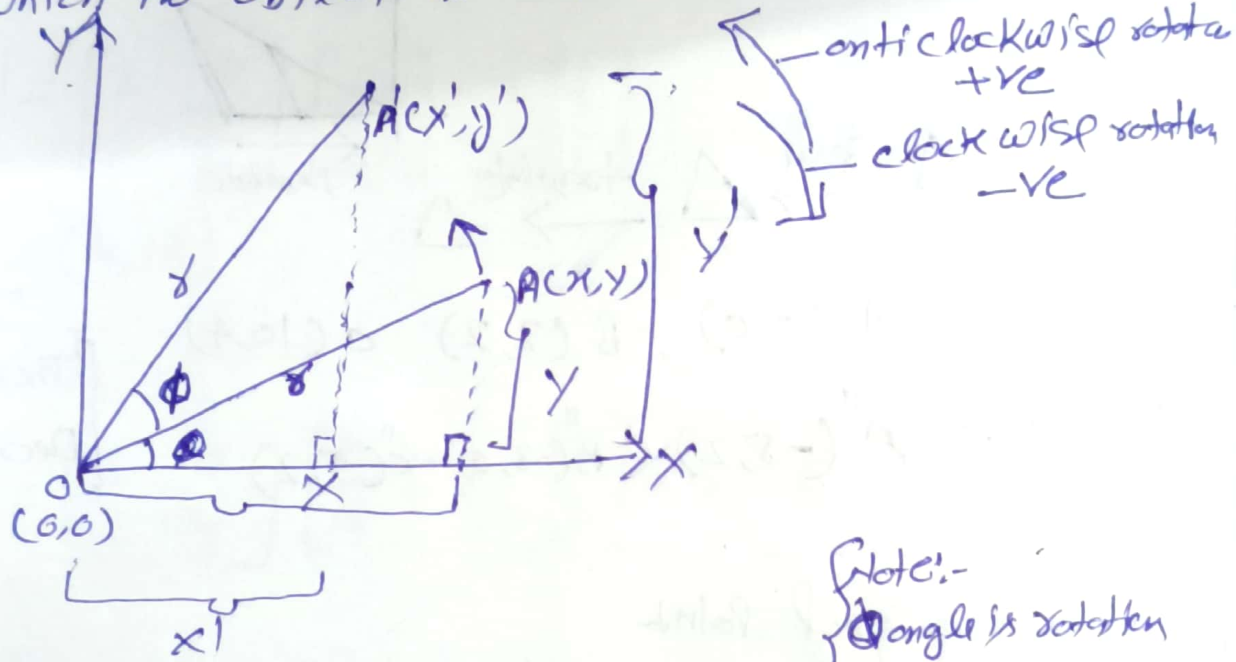
$= [5 \;\; 1] \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$B' = [15 \;\; 2]$

$C' = C \cdot S$

$[4, 4] \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} = C'(12, 8)$

3) **Rotation:-** A 2D rotation is applied to an object by repositioning it along a circle path in the X-Y plane. To generate a rotation, we specify a rotation point about which the object is to be rotate.



Note:-
θ angle is rotation angle
r is radius in circle

→ orignal cordinates are:.

$\cos\theta = \frac{x}{r} \Rightarrow \boxed{x = r\cos\theta}$

$\sin\theta = \frac{y}{r} \Rightarrow \boxed{y = r\sin\theta}$

Now Here 1 r is radius and it is constant θ is the ~~angle~~ orignal angle position of the point from horizontal φ is the rotation angle

$\boxed{x' = x\cos\phi - y\sin\phi}$

$\cos(\theta+\phi) = \frac{x'}{r}$

$x' = r\cos(\theta+\phi)$

$= r[\cos\theta\cos\phi - \sin\theta\sin\phi]$

$x' = \boxed{r\cos\theta}\cos\phi - \boxed{r\sin\theta}\sin\phi)$

→ $\sin(\theta+\phi) = \frac{y'}{r}$

$y' = r\sin(\theta+\phi)$

$= r[\sin\theta\cos\phi + \cos\theta\sin\phi]$

$= \boxed{r\sin\theta}\cos\phi + \boxed{r\cos\theta}\sin\phi$

$$\boxed{y' = x\sin\phi + y\cos\phi}$$

→ Anti clockwise ~~a~~ Rotation for negative value of ϕ.

$x = x\cos(-\phi) - y\sin(-\phi)$

$= x\cos(\phi) - y - \sin(\phi)$

$$\boxed{x = x\cos\phi + y\sin\phi}$$

$= y = x\sin(-\phi) + y\cos(-\phi)$

$$\boxed{y = -x\sin(\phi) + y\cos(\phi)}$$

→ In matrix from we can reprsent it as:.

$A' = A \cdot R$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$ → ~~Anti~~ clockwise Rotation equation

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$ → Anti clockwise Rotation

Ex A point (4,3) is rotated counter clockwise direction by the angle of 45°. Find the rotation matrix R and the resultant points.

→ $A' = A \cdot R$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos 45° & \sin 45° \\ \sin 45° & \cos 45° \end{bmatrix}$$

$$= \underset{2\times1}{\begin{bmatrix} 4 \\ 3 \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}$$

$$= \underset{2\times1}{\begin{bmatrix} \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}} \\ \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}} \end{bmatrix}}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{7}{\sqrt{2}} \end{bmatrix}$$

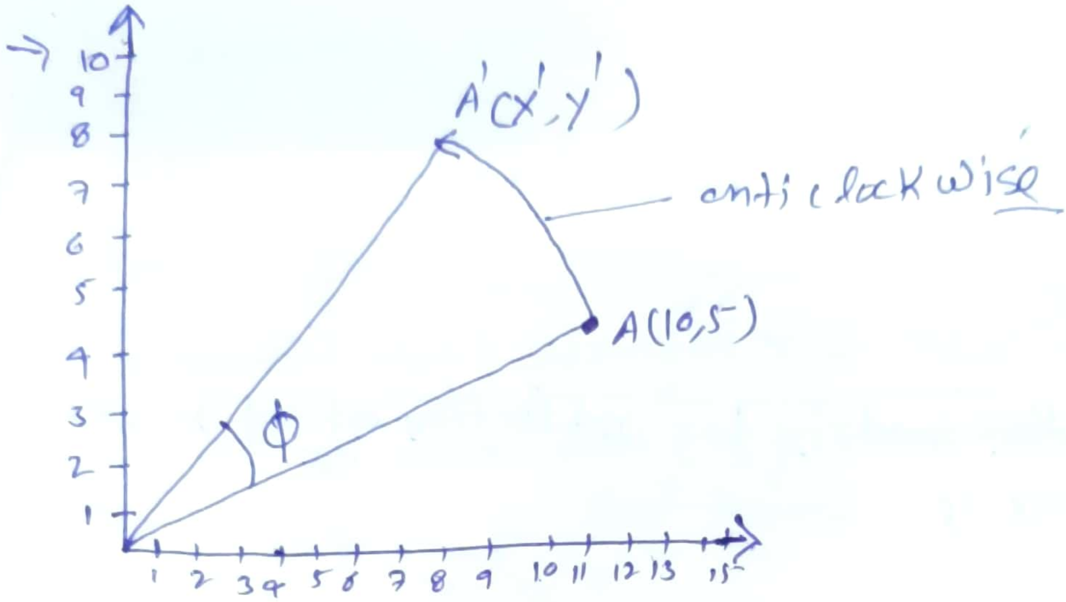
Ex:- $\phi = 45°$

$A(10,5)$

Anticlock
→ clock wise Rotation

$A' = A \cdot R$

$$\rightarrow \begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x, & y \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$



→ $$[x', y'] = [10 \;\; 5] \begin{bmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{bmatrix}$$

$$= \overset{\text{row}}{[10 \;\; 5]} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}_{\text{col}}$$

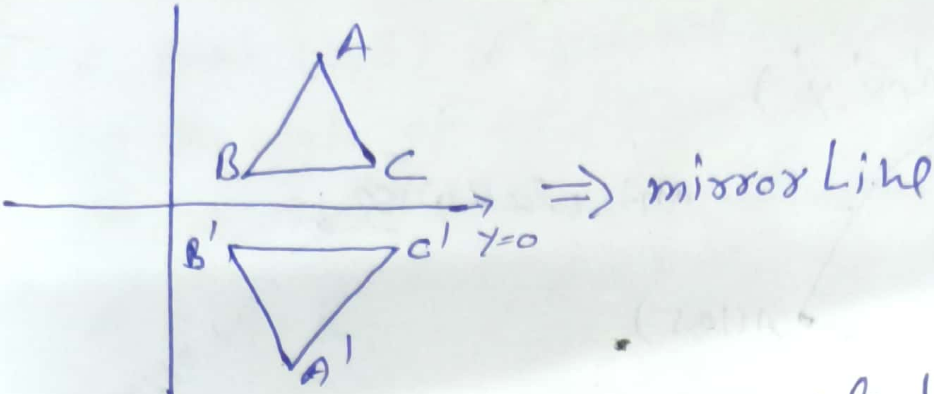
$$= \begin{bmatrix} \frac{10}{\sqrt{2}} - \frac{5}{\sqrt{2}} & \frac{10}{\sqrt{2}} + \frac{5}{\sqrt{2}} \end{bmatrix}_{1 \times 2}$$

$$[x', y'] = \begin{bmatrix} \frac{5}{\sqrt{2}} & \frac{15}{\sqrt{2}} \end{bmatrix}_{1 \times 2} \quad \underline{\text{Ans}}$$

⇒ **Reflection:-** A reflection is a transformation that produce a mirror image of an object.

a) **Reflection about x-axis:-** x-coordinates is not change and sign of y-coordinate is change.

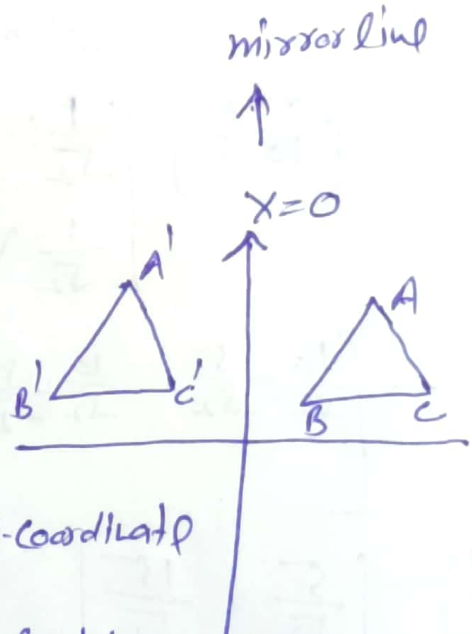
→ If we selected point (x, y) in the x-axis we get (x, −y)



→ The transformation matrix for reflection about x-axis or y = 0 axis is

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \end{array}$$



b) Reflection about y-axis:-

$x' = -x$

$y' = y$

→ X-coordinate is change and y-coordinate is not change

→ The transformation matrix for reflection about y-axis or x = 0 ~~as~~ axis is

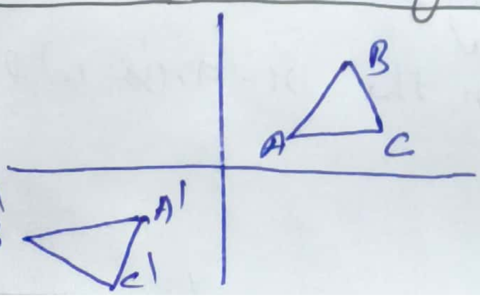
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

c) Reflection about origin:-

x & y coordinate are both change

$x' = -x$

$y' = -y$



→ The transformation matrix for reflection about the origin is

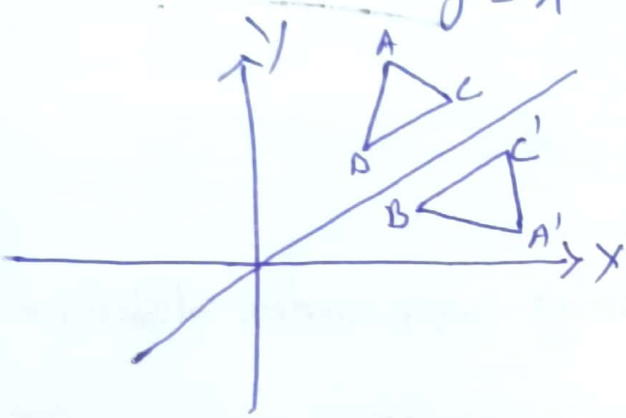
$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

d) Reflection about straight line $y = x$

→ If we reflect point (x, y) about the line $y = x$, then we get (y, x) that means

$$x' = y$$
$$y' = x$$

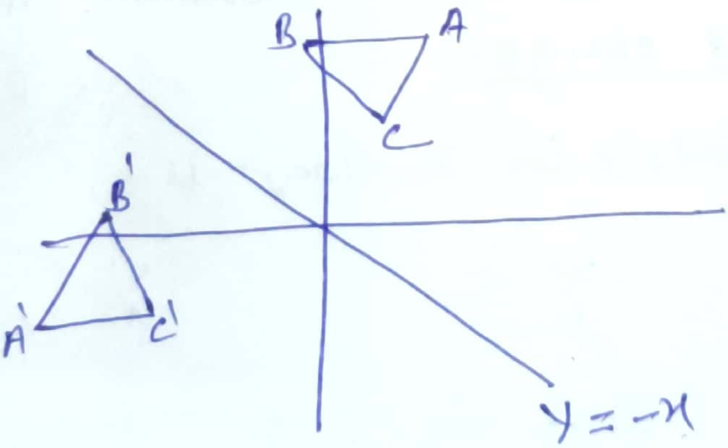
Transformatrix = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$



e) Reflection about the straight line:

If we reflect point (x, y) in the line $y = -x$ if we reflect point (x,y) in the line $y = -x$ then we get $(-y, -x)$ that means $x' = -y$

$y' = -x$



→ The transformation matrix

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

Ex Determine the transformation matrix of triangle A(4,1) B(5,2) & C(4,3) about the line x = 0 and determine the resultant coordinates.

→ $A' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$A' = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$A' = \begin{bmatrix} -4 \\ 1 \end{bmatrix} \Rightarrow (-4, 1)$

$B' = (-5, 2)$ & $C' = (-4, 3)$ A

⇒ **Shear Transformation:-** The shearing transformation distorts the shape of object.

→ There are 2 types of shearing transformation
1) X - ~~shear~~ shearing
2) Y - shearing

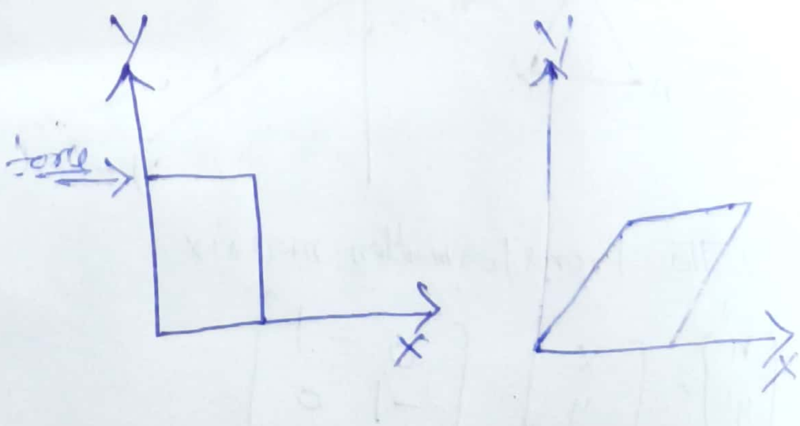
1) **X - Shearing:-** ~~the proc~~ In this y-coordinate remain unchange but x is change.

→ The transformation matrix for x-shear is

$\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$[x + yb \quad y]$

$[x, y] = [x + yb \quad y]$



2) **Y-shearing:-** In this y coordinate change but x coordinate remain unchanged.

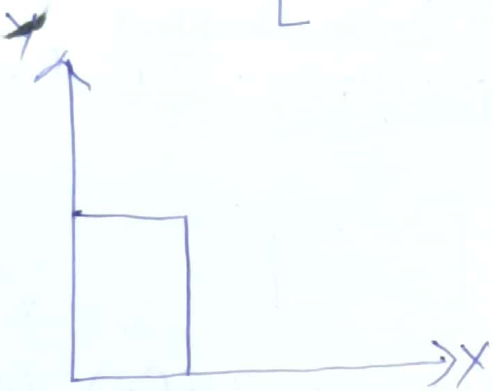
→ The transformation matrix for y-axis is $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

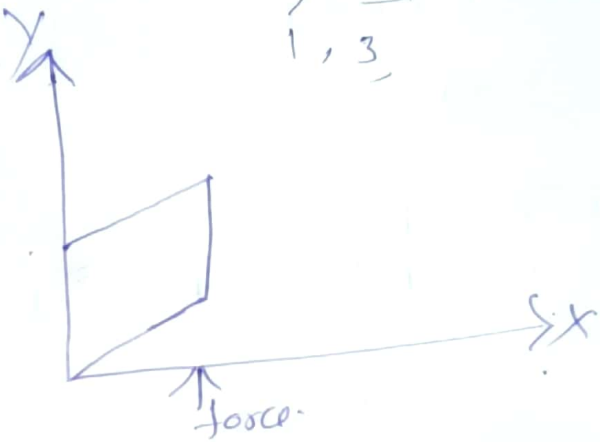
$$[x\ y] = [x\ y]\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

$$= [x,\ x.a + y]$$

1, 1×2+1

1, 3

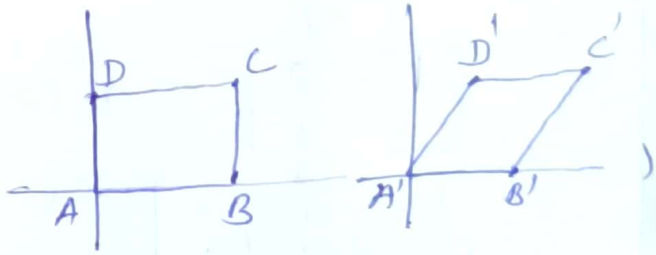




Ex:- Shear a polygon A(0,0), B(1,0) C(1,1) D(0,1) by shearing vector $sh_x = 2$ and determine the new coordinate

→ A(0,0) B(1,0) C(1,1) D(0,1)

A'(0,0) B(1,0) C'(3,1) (2,1)



→ matrix

A' = [

Ex → A polygon with vertices A(0,0) B(1,0) C(1,1) D(0,1) perform the following transformation and conclude the result of sharing.

a) x-sharing with a=2 followed by y-sharing with b=3

b) simultaneous x and y sharing with a=2 and b=3.

a) $A' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$A' = (0,0)$

$(1,0)$

$x, \; x.2+y$

$1, \; 2+1$

$B' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$B' = (1,0)$

$C' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

$C' = (3,1)$

$D' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$D' = (2,1)$

now

→ y-sharing.

$\begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} \Rightarrow A''(0,0)$

$\cancel{B''(1,3)}$ $A' \begin{bmatrix} x'' \\ y'' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\cancel{C'(3,10)}$ $A''(0,0)$

$B'' = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$B' = (1,3)$

$C'' = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3 \\ 9+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \end{bmatrix}$

$C'' = (3,10)$

$D'' = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 6+1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 7 \end{bmatrix}$

$D'' = (2,7)$

→ b) Simaltennious

$a=2, \; b=3$

$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

$\Rightarrow A' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$A' = (0,0)$

$B' = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $(1,3)$

$B' = (1,3)$

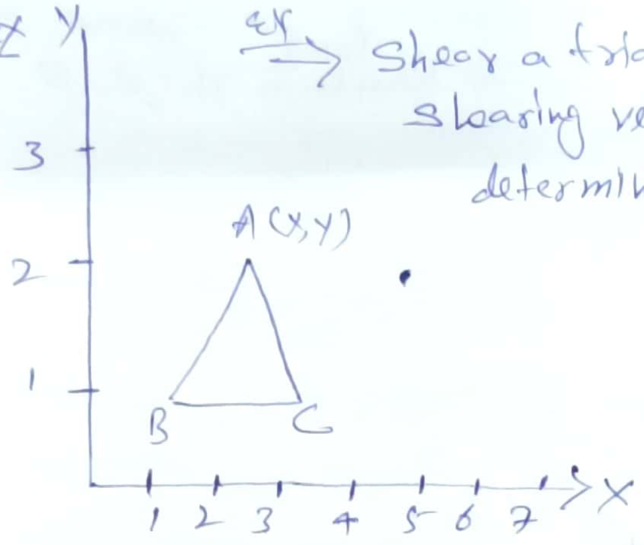
$C' = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1+2 \\ 3+1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$C' = (3,4)$

$D' = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ 19.

$D' = (2,1)$

ex



ex → Shear a triangle A(2,2) B(1,1) C(3,1) shearing vector $sh_x = 1$ & $sh_y = 1$ then determine the new coordinate.

A(2,2)
B(1,1)
C(3,1)
$sh_x = 1$
$sh_y = 1$

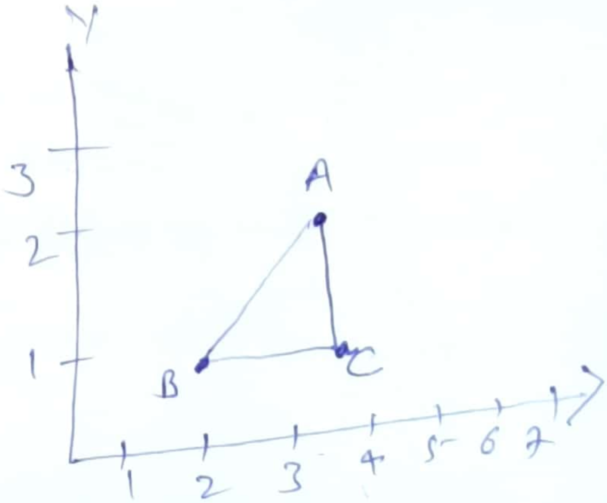
→ Shearing

→ In x axis →

→ $x' = x + sh_x * y$

$y' = y$

$A' = \begin{cases} x' = 2 + 1 \times 2 = 4 \\ y' = 2 \end{cases} \Rightarrow (4,2)$



In y axis →

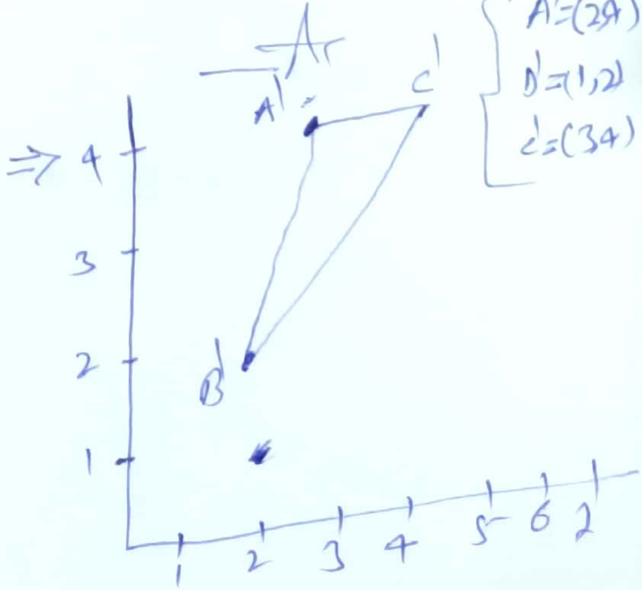
$x' = x$

$y' = y + sh_y * x$

$B' \begin{cases} x' = 1 + 1 \times 2 = 2 \\ y' = 1 \end{cases} \Rightarrow (2,1)$

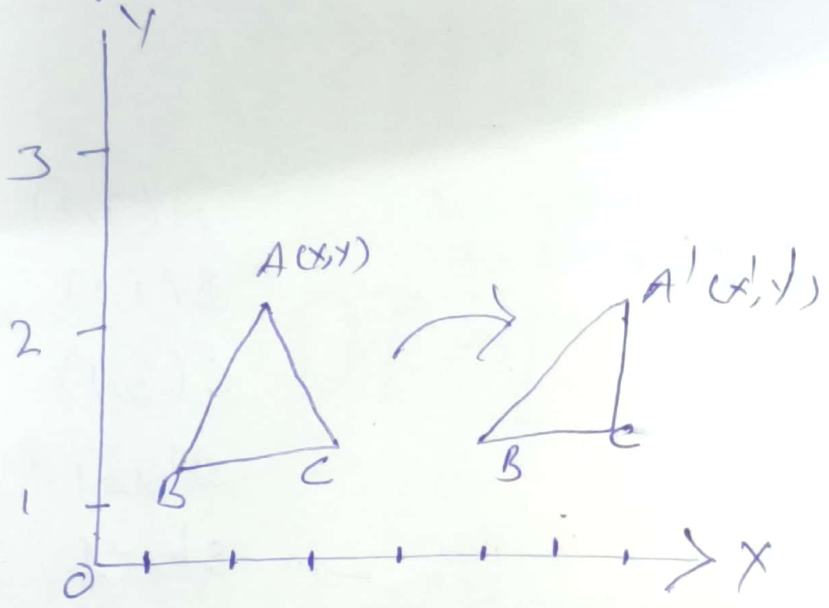
$C' \begin{cases} x' = 3 + 1 \times 1 = 4 \\ y' = 1 \end{cases} \Rightarrow (4,1)$

⇒



$\begin{cases} A' = (2,4) \\ B' = (1,2) \\ C' = (3,4) \end{cases}$

→ 2D-Shearing :→ It is to simply ~~transform~~ tring change to shape changing to an object.



Shearing :-

→ In x-axis

→ $x' = x + Shx * y$

→ $y' = y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & Shx \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

→ In y-axis

→ $x' = x$

→ $y' = y + Shy * x$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Shy & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Note

shx }
shy } ⇒ Shearing Parameters

⇒ Homogenous coordinates' system:-

→ Translation
→ Rotatation
→ Scaling.

* many times we may require to perform translation, rotations and scaling to fit the picture componets into their proper positions to produce a sequence of transformation with above equations, such as translation followed by rotation and then scaling, we must calculate the transformed coordinates one step at a time.

* In order to combine sequence of transformations we have to eliminate the matrix addition associated with translation. to achive this we have represent matrix m_1 as 3×3 matrix instead of 2×2 introducing an additional dummy coordinate w. Here, points are specified by the three numbers instead of two. this coordinate system is called homogeneous coordinate system.

→ Homogeneous coordinate system is represented by a triplet $(x, y, 1)$

1) **Homogeneous coordinates for translation**:- The homogenous coordinate for translate are given

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$$[x' \ y' \ 1] = [x, y, 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$$

$$\boxed{= [x+t_x \quad y+t_y \quad 1]}$$

2) Homogeneous coordinates for Rotation :-

The homogenous coordinates for rotation are given

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{= [x\cos\theta - y\sin\theta \quad x\sin\theta + y\cos\theta \quad 1]}$$

3) Homogeneous coordinates for Scaling :

The homogeneous coordinates for scaling are given as

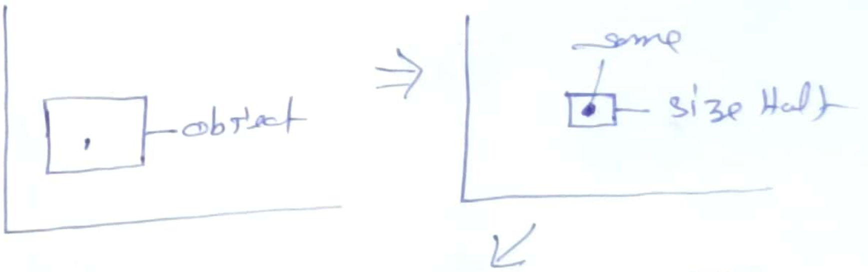
$$S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

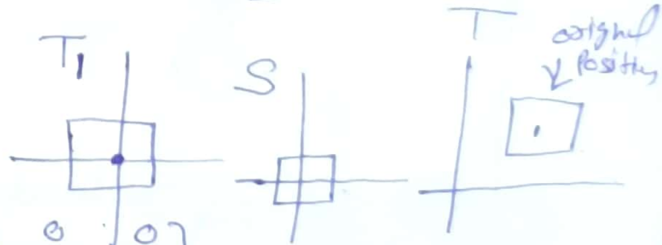
$$[x' \ y' \ 1] = [x \ y \ 1] \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{= \quad x S_x \quad y . S_y \quad 1}$$

Ex Find the Transformation matrix that transform the square ABCD to Half its size with the centre remaining At the same position
A (1,1) B (3,1) C(3,3) D (1,3) & centre At (2,2)
Also find Resultant coordinates of square.

→ 1) Translate the square so that its centre coincide with the origin.
2) Scale the square with respect to the origin
3) Translate the square back to the original position





→ $T_1 \cdot S \cdot T =$

$$T_1 \cdot S \cdot T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Now:

$$\begin{bmatrix} A' \\ B' \\ C' \\ D' \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} x & y & w \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ 3 & 3 & 1 \\ 1 & 3 & 1 \end{bmatrix} \end{array} \cdot \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & .5 & 0 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1.5 & 1.5 & 1 \\ 2.5 & 1.5 & 1 \\ 2.5 & 2.5 & 1 \\ 1.5 & 2.5 & 1 \end{bmatrix}$$

⇒ Composite of 2D Transformation:-

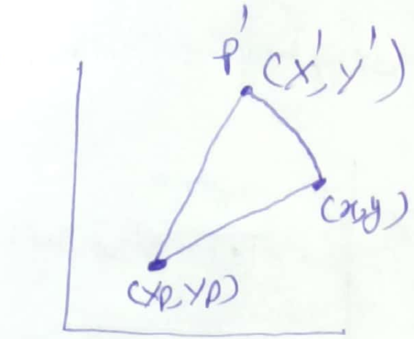
2) 1:- Rotation About an Arbitrary point:- to Rotate an object on arbitrary point (x_p, y_p) we have to carry out three steps.

1) Translate Arbitrary point (x_p, y_p) to the origin. ⟶ $t_x = -x_p$, $t_y = -y_p$

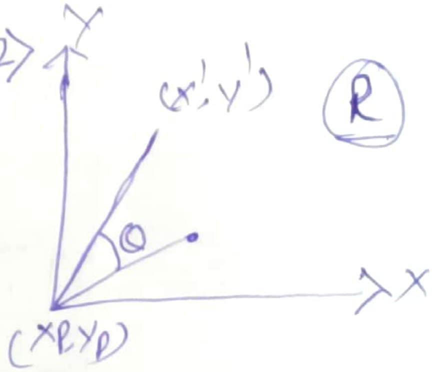
3) Rotate translate the center of rotation back. ⟶ $t_x = x_p$, $t_y = y_p$

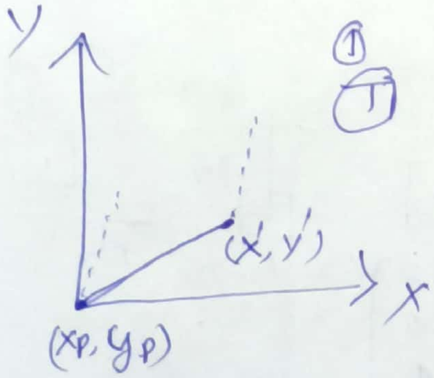
2) Rotate it about the origin. (R)

1>



2>





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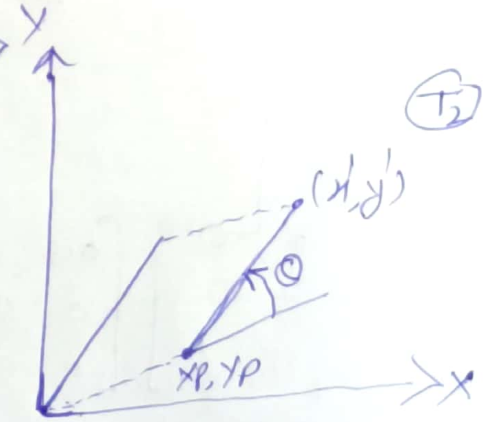


⟶ Represent matrix form_

① $T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_p & -x_y & 1 \end{bmatrix}$

Counter clockwise

~~Are back position~~

② $R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

③ $T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_p & y_p & 1 \end{bmatrix}$

→ Therefor the overall transformation matrix for a counter clockwise Rotation by an angle θ about the point (x_P, y_P) is given as

$$T_1 R T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_P & -y_P & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_P & y_P & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_P\cos\theta + y_P\sin\theta & -x_P\sin\theta - y_P\cos\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_P & y_P & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_P\cos\theta + y_P\sin\theta + x_P & -x_P\sin\theta - y_P\cos\theta + y_P & 1 \end{bmatrix}$$

Ex ⇒ Perform a counterclockwise (Anticlockwise) 45° rotation of triangle A(2,3) B(5,5) & C(4,3) about point (1,1).

→ $x_P = 1$ $\quad \theta = 45°$

$y_P = 1$

$$T_1 \cdot R \cdot T_2 = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_P\cos\theta + y_P\sin\theta + x_P & -x_P\sin\theta - y_P\cos\theta + y_P & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -\frac{1}{\sqrt{2}} + 1/\sqrt{2} + 1 & -1/\sqrt{2} - 1/\sqrt{2} + 1 & 1 \end{bmatrix}$$

$-1 \times \frac{1}{\sqrt{2}} + 1 \cdot \frac{1}{\sqrt{2}} + 1$

Put value $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 1$

$x_P = 1$ $\quad = 1$

$y_P = 1$ ⟶

$$T_1 \cdot R \cdot T_2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & -\frac{2}{\sqrt{2}}+1 & 1 \end{bmatrix}$$

→ How used point $P' = P \cdot T$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 2 & 3 & 1 \\ 5 & 5 & 1 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1 & -\frac{2}{\sqrt{2}}+1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} 2 \times \frac{1}{\sqrt{2}} \; 3\left(-\frac{1}{\sqrt{2}}\right)+1 & 2 \cdot \frac{1}{\sqrt{2}} + 3\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{2}{\sqrt{2}}+1\right) & 1 \\ 5\left(\frac{1}{\sqrt{2}}\right)+5\left(\frac{-1}{\sqrt{2}}\right)+1 & 5\left(\frac{1}{\sqrt{2}}\right)+5\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{2}{\sqrt{2}}+1\right) & 1 \\ 4\left(\frac{1}{\sqrt{2}}\right)+3\left(\frac{-1}{\sqrt{2}}\right)+1 & 4\left(\frac{1}{\sqrt{2}}\right)+3\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{2}{\sqrt{3}}+1\right) & 1 \end{bmatrix}$$

$$\to \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2}+1, & \frac{3}{\sqrt{2}}+1 & 1 \\ 1 & \frac{8}{\sqrt{2}}+1 & 1 \\ 1/\sqrt{2}+1 & \frac{5}{\sqrt{2}}+1 & 1 \end{bmatrix}$$

$\Rightarrow A' = \left(-1/\sqrt{2}+1, \; 3/\sqrt{2}+1\right)$

$B' = \left(1, \; \frac{8}{\sqrt{2}}+1\right)$

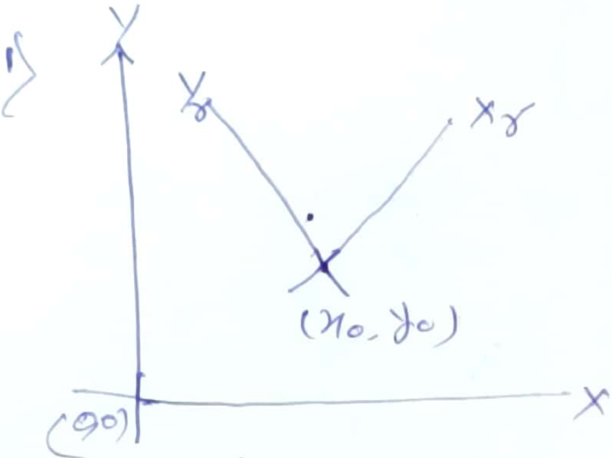
$C' \left(1/\sqrt{2}+1, \; 5/\sqrt{2}+1\right)$

A

⇒ Transformation between coordinate systems:-

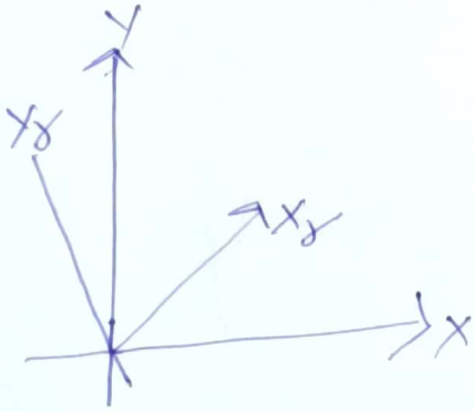
1) * Translation
2) * Rotation about an origin
3) * Scaling with respect to origin
4) * mirror reflection about an axis.

1)



1) $T(-X_0, -Y_0)$

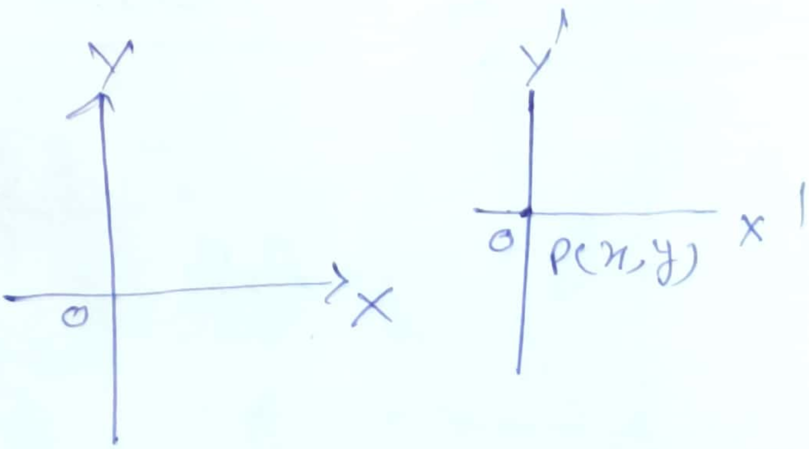
2)



2) $R(\theta)$

$$\boxed{R_m = T(-x_0, y_0)\, R(\theta)}$$

→ Translation:-



$$x' = x - t_x$$
$$y' = y - t_y$$

→ Rotation about on origin:-

$$\overline{R_\theta} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

→ Scaling with respect to origin:-

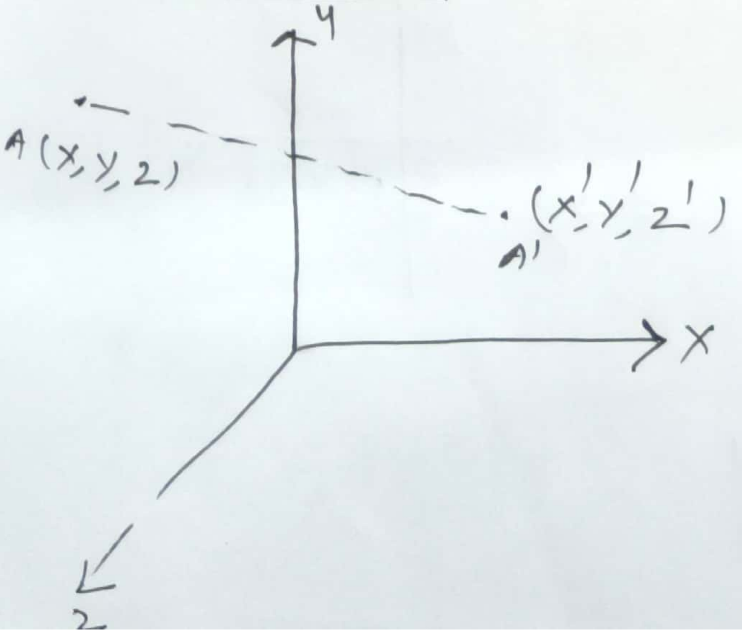
$$\overline{S_{S_x S_y}} = \begin{bmatrix} \frac{1}{S_x} & 0 \\ 0 & \frac{1}{S_y} \end{bmatrix}$$

→ mirror reflection about the ~~x~~ on -axis.

x axis $\quad \overline{m_x} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \begin{bmatrix} x' = -x \\ y' = y \end{bmatrix}$

y axis $\quad \overline{m_y} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} x' = x \\ y' = -y \end{bmatrix}$

⇒ Introduction of 3-D Transformation:-



translation

$$\Rightarrow x' = x + T_x$$
$$y' = y + T_y$$
$$z' = z + T_z$$

→

→ Representation matrix – 3D Translation

→ moving an object →

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x + T_x \\ y + T_y \\ z + T_z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

2) Scaling :-

$$x' = x \cdot S_x$$
$$y' = y \cdot S_y$$
$$z' = z \cdot S_z$$

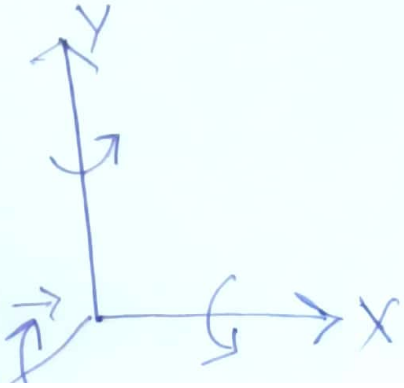
matrix form →

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3) 3-D Rotation :-

① Angle Z-axis

$$z' = z$$
$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$



→ Representation matrix – 3D – Translation

→ moving an object →

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} x + T_x \\ y + T_y \\ z + T_z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix}$$

2) Scaling :-

$x' = x \cdot S_x$

$y' = y \cdot S_y$

$z' = z \cdot S_z$

matrix form →

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

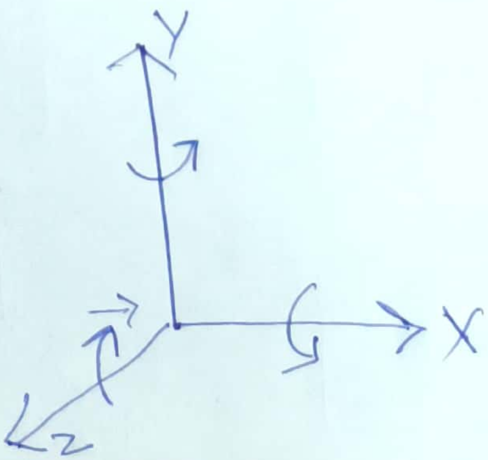
3) 3-D Rotation :-

① Angle Z-axis

$z' = z$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$



⇒ matrix form - 3D - Rotation:-

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ii> x-axis

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

⇒ matrix form

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta, & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

iii> angle y-axis

$y' = y$

$z' = z\cos\theta - x\sin\theta$

$x' = z\sin\theta + x\cos\theta$

⇒ matrix form

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

→ 3-D Reflection:

i> in xy plane

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

ii> yz plane

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

iii> zx plane

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$