

6

Division

Introduction

Division is the reverse process of multiplication. If we say $12 \div 3 = 4$, it means $3 \times 4 = 12$. Division basically means repeated subtraction. Let us take an example: $12 \div 3 = ?$

$$1^{\text{st}} \text{ stage} = 12 - 3 = 9$$

$$2^{\text{nd}} \text{ stage} = 9 - 3 = 6$$

$$3^{\text{rd}} \text{ stage} = 6 - 3 = 3$$

$$4^{\text{th}} \text{ stage} = 3 - 3 = 0$$

Since at the 4^{th} stage, we get a remainder of 0, $12 \div 3 = 4$. The conventional method is lengthy and time-consuming as it is based on a hit and trial method. This method is effective as long as the divisor is a single digit, but when the divisor is a bigger number we are at loss because we keep multiplying the dividend by different numbers from 1 to 9. On the other hand, in the Vedic system, we get the quotient and remainder in one line.

Conventional Method

$$\begin{array}{r) } 54 \\ 43854 \left(812 \\ -432 \\ \hline 65 \\ -54 \\ \hline 114 \\ -108 \\ \hline 6 \end{array}$$

Rough area

$$54 \times 6 = 324$$

$$54 \times 7 = 378$$

$$54 \times 8 = 432$$

Vedic one-line Dhwajak method

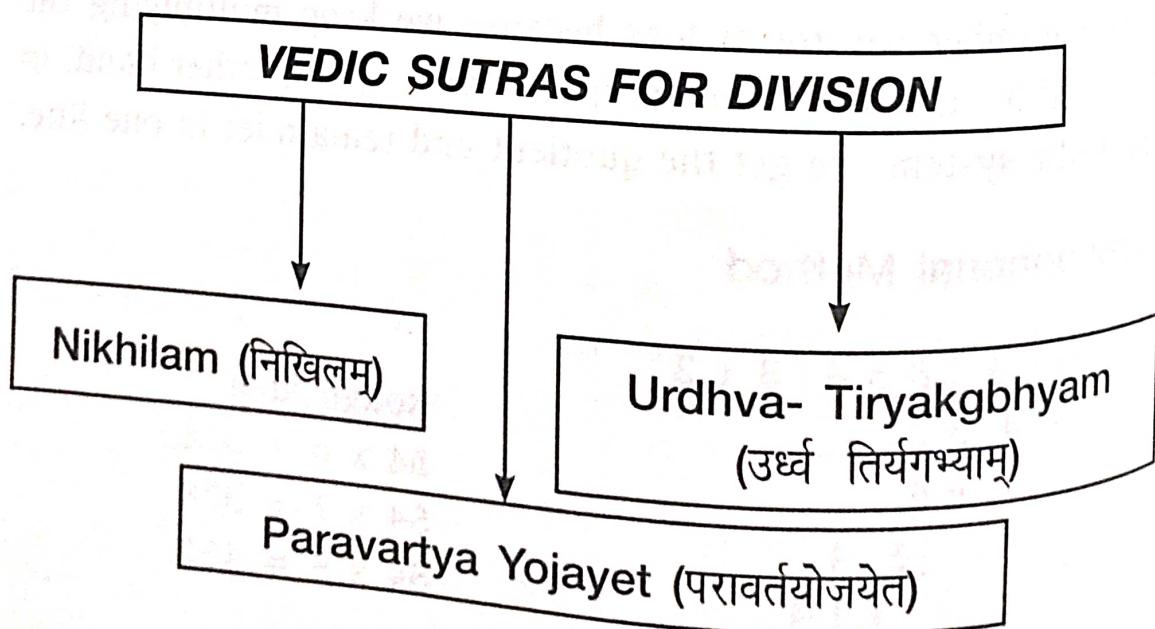
$$\begin{array}{r} 5^4 \mid 4 & 3 & 8 & 15 \\ \hline & 8 & 1 & 2 & 6 \end{array}$$

The conventional method is a fixed stereotypical method where there is no room for experiment, while there are various simple methods of division in Vedic mathematics, where readers have a choice of the best method fitted to that very situation.

In this chapter we shall deal with three Vedic sutras.

In the Nikhilam Method, there is no subtraction to be done at all. The second method, Paravartya Yojyet is based on simple algebraic division and the last method popularly known as Dhwajanka method of Division is based on the long established Vedic process of mathematical calculations. The Dhwajank method is popularly known as Urdhva Tiryagbhyam. Different from the Nikhilam and Paravartya methods, the Dhwajanka method is capable of immediate application to all cases and can be described as the "crowning gem of all" for the universality of its application.

Vedic Method for Division



Meaning of Vedic Sutra:

Nikhiam (निखिलम्): You must be acquainted with this sutra, Nikhilam Navataha Charmam Dastah. We have used this Vedic sutra in subtraction and multiplication earlier. The meaning of this sutra has been explained in Subtraction – "All from nine and last from 10". This sutra has limited application and is useful when every digit of the divisor is greater than 5. The best part of this sutra is that there is no subtraction process to be carried out at all.

Paravartya Yojyet (परावर्त्योजयेत्): This sutra has an upper hand in comparison with the Nikhilam method. The basic meaning of this sutra is "Transpose and Apply." The well-known rule relating to transposition is to invariably change the sign with every change of side. This method like the Nikhilam method has limited application.

Urdhya-Tiryag bhyam (उर्ध्वतिर्यग्भ्यम्): This Vedic sutra is called the crowning gem of all due to its applicability in all types of division. The meaning is self-explanatory as this method is discussed well in detail in the chapter on Multiplication. This is also called the Dhwajank method of division. The word "dhwaj" means flag and "anka" means digit. Hence, the literal meaning of the word "Dhwajank" is to place a digit or two of the divisor at the top and division is then carried out with the remaining digits of the divisor.

The Nikhilam method

Working Rule

1. Take a base (in the power of 10) nearest to the divisor and write its complement below the original divisor, in the divisor column. Complement = Base – Divisor. The complement can be carried out easily by the Nikhilam

Navataha Charmam Dastah method as explained in the Subtraction chapter. Write the complement of the divisor below it.

2. Separate the extreme right digit of the dividend by drawing a slash equal to the number of digits in the divisor. This block is known as the remainder block and the left block is known as the quotient block.
3. The number of digits to be placed in the remainder column should be equal to the number of zeros in the base.
4. Carry down the first digit of divisor in the first column. This gives you the first digit of the quotient. Multiply the quotient digit by the complement, and place it in the dividend column; next to the first digit of the dividend.
5. Write mechanically the sum of the digits of the second column to get the second digit of the quotient.
6. Repeat the process until you get a number in the remainder column. If the remainder is greater than the divisor, continue the same process in the remainder block until the digit in the remainder column is less than that of the original divisor.

Case 1: When the remainder is less than the divisor.

Example 1: Divide 22 by 8

Solution:

Divisor	8)	2 / 2	
Complement	2	\downarrow	4
		<hr/>	(Quotient \times Complement)
		2 / 6	

Here the divisor is nearer to the base 10.
Complement = $10 - 8 = 2$

- Since the base has one zero, one digit will be separated by a slash for the remainder column. In the above example, the right-most 2 is separated by a slash.
- Carry down the first digit (2) of the dividend. This is the first digit of the quotient.

- Multiply the first digit of the quotient by the complement and place it in the dividend column, next to the first digit of the dividend. Here the first digit of the quotient is 2 and the complement is also 2 hence $2 \times 2 = 4$ is placed below 2 as shown above.
- Write the sum of the two digits of the remainder column. Since the remainder 6 is less than the divisor, we have completed the division.
- Hence, Quotient = 2 ; Remainder = 6

Example 2: Divide 213 by 9

Solution:

$$\begin{array}{r}
 \text{Divisor} \quad 9) \quad 2 \ 1/ \ 3 \\
 \text{Complement} \quad 1 \quad \downarrow \quad 2 \ \underline{3} \quad (\text{Quotient} \times \text{Complement}) \\
 \quad \quad \quad 2 \ 3 / 6
 \end{array}$$

Here, the divisor is nearer to the base 10.

$$\text{Complement} = 10 - 9 = 1$$

- Since the base has one zero, one digit will be separated by a slash for the remainder column. In the above example, the right-most 3 is separated by a slash.
- Carry down the first digit (2) of the dividend. This is the first digit of the quotient.
- Multiply the first digit of the quotient by the complement and place it in the dividend column; next to the first digit of the dividend. i.e. $2 \times 1 = 2$
- Write mechanically the sum of the digits of the second column to get the second digit of the quotient. Here $1 + 2 = 3$ is the second digit of the quotient.
- Multiply the second digit of the quotient by the complement and write it next to the second digit of the quotient. Place $3 \times 1 = 3$ in the remainder column.
- Write the sum of the two digits of the remainder column. Since the remainder 6 is less than the divisor, we have completed the division.
- Hence, Quotient = 23; Remainder = 6

Example 3: Divide 10025 by 88

Solution: Base = 100

$$\text{Complement} = 100 - 88 = 12$$

Column 1		Column 2 (Q)			Column 3 (R)	
Divisor	8 8	1	0	0	2	5
Complement =	1 2		1	2	-	-
					2	-
					3	6
		1	1	3	8	1

Hence,

$$\text{Quotient} = 113$$

$$\text{Remainder} = 81$$

Explanation:

1. Here Divisor = 88
Nearest base in power of 10 = 100
Complement = Base - Divisor
 $= 100 - 88 = 12$
2. Arrange the digits in the columns as shown above, separating quotient and remainder. Since there are two zeros in the base, the remainder column will have the two right-most digits of the dividend.
3. Carry down 1 of column 2 (the first digit of the dividend). This is the first digit of the quotient.
4. Multiply the first digit of the quotient with the complement and place it in the dividend column, next to the first digit of the dividend. $12 \times 1 = 12$ is placed below 0.

Column 1	Column 2 (Q)	Column 3 (R)
8 8 C = 1 2 (Divisor)	1 0 0 1 2 ↓ ↓	2 5 - -

5. Carry down the sum of the circled digits; this will give you the second digit of the quotient. The second digit of quotient is $0 + 1 = 1$
6. Multiply the second digit of the quotient with the complement and place it in the dividend column, next to the second digit of the dividend. $12 \times 1 = 12$ is placed below the second zero.

Column 1	Column 2 (Q)	Column 3 (R)
8 8 C 1 2	1 0 0 1 2 1	2 5 - - 2

7. Carry down the sum of the second circled digits; this will give you the third digit of the quotient. The third digit of the quotient is $0 + 2 + 1 = 3$
8. Multiply the complement 12 with the third quotient (3) and write it below the fourth digit of the remainder. $12 \times 3 = 36$ is placed below 2 in the column 3.

Column 1	Column 2 (Q)	Column 3 (R)
8 8 C = 1 2	1 0 0 1 2 1 ↓	2 5 - - 2 3 6

9. Sum up the digits of Column 3 to get the remainder. The above process repeats until the digit thus obtained in the remainder column is less than the original divisor. Here the sum of the digits of Column 3 = 81 and $81 < 88$ (Divisor), hence Quotient = 113 and Remainder = 81

Example 4: Divide 1121134 by 8988

Solution:

Column 1	Column 2 (Q)	Column 3 (R)
Divisor 8 9 8 8	1 1 2	1 1 3 4
Complement = 1 0 1 2		
	↓	— — — —
		1

Explanation:

1. Here Divisor = 8988
Nearest base in power of 10 = 10000
Complement = Base - Divisor
 $= 10000 - 8988 = 1012$
2. Arrange the digits in the columns as shown above, separating quotient and remainder. Since there are four zeros in the base, the remainder column will contain 4 digits.
3. Carry 1 of column 2 (the first digit of the dividend) down. This is the first digit of the quotient.
4. Multiply the first digit of the quotient with the complement and place it in the dividend column, next to the first digit of the dividend. $1012 \times 1 = 1012$ is placed below 1.

	Column 1	Column 2 (Q)	Column 3 (R)
Divisor	8 9 8 8	1 1 2	1 1 3 4
Complement =	1 0 1 2	1 0	1 2
		↓ ↓	
		1 2	

5. Add the second digit of column 2; this will give you the second digit of the quotient. The second digit of the quotient is $1 + 1 = 2$
6. Multiply the second digit of the quotient with the complement and place it in the dividend column, next to the second digit of the dividend. $1012 \times 2 = 2024$ is placed below the second zero.

	Column 1	Column 2 (Q)	Column 3 (R)
Divisor	8 9 8 8	1 1 2	1 1 3 4
Complement =	1 0 1 2	1 0 2	1 2 0 2 4
		↓ ↓ ↓	
		1 2 4	

7. Carry down the sum of the third digit to the quotient column; this will give you the third digit of the quotient. The third digit of the quotient is $2 + 0 + 2 = 4$
8. Multiply the complement 1012 with the third quotient (4) and write it below 0 in the remainder column as shown below. $1012 \times 4 = 4048$.

	Column 1				Column 2 (Q)			Column 3 (R)			
Divisor	8	9	8	8	1	1	2	1	1	3	4
Complement =	1	0	1	2		1	0	1	2		
						2		0	2	4	
							2	4	0	4	8
								1	2	2	2
								6	6	2	2

9. Add the digits of Column 3 to get the remainder. The sum here is 6622, which is less than the divisor, hence the division process is complete.

10. Quotient = 124 and Remainder = 6622

Case 2: When the remainder is more than the divisor.

Example 5: Divide 10312 by 87?

Solution: Base = 100

$$\text{Complement} = 100 - 87 = 13$$

Number of digits to be placed in remainder column = 2
Follow the same method of calculation until you get the
Remainder > Dividend

	Column 2 (Q)	Column 3 (R)
8 7	1 0 3	1 2
C = 1 3		(13 X 1 = 13)
	1	(13 X 1 = 13)
	9 1	(13 X 7 = 91)
	1 1 7	13 3

Since the remainder $133 > 87$, add the complement 13 in the remainder column.

	Column 2 (Q)	Column 3 (R)
8 7	1 0 3	1 2
C = 1 3	1 3	- -
	1.	3
		9 1
	1 1 7	1 3 3
		+ 1 3
	1 1 7	1 4 6
	1 1 8	4 6

Since the base is 100, the number in the remainder column should be less than 100; therefore the left-most 1 of the remainder column will be transferred to the quotient column. Thus Quotient = 117 and Remainder = 46.

The importance of zero in the complement of divisor

While taking the complement from the base, we need to be very careful. If the base is 100, the complement should have two digits and if the base is taken as 1000, the complement will have three digits. Suppose the divisor is 9, then its complement from the base will be written as 1. For 99 and 999 as divisor, the complement from their respective bases 100 and 1000, will be written as 01 and 001. Let us take an example to understand the modus operandi.

Example 6: Divide 11199171 by 99979

Solution: Base = 100000

Complement = 00021

Number of digits to be placed in Remainder column = 5

	Column 2 (Q)	Column 3 (R)
9 9 9 7 9	1 1 1	9 9 1 7 1
0 0 0 2 1	0 0	0 2 1 - -
	0	0 0 2 1 -
		0 0 0 2 1
	1 1 1	1 0 1 5 0 2
		+ 0 0 0 2 1
	1 1 1	1 0 1 5 2 3

Hence, Quotient = 112

Remainder = 0 1 5 2 3

↳ *Paravartya Yojyet* (परावर्त्योजयेत्) is another sutra.

Paravartya Yojyet (परावर्त्योजयेत्)

The literal meaning of this sutra is **Transpose and Apply**. There is a very small difference between the Nikhilam and Paravartya Yojyet methods. This sutra works effectively when the first digit of the divisor is 1. First, find the complement from the base as done in Nikhilam. This complement will be revised by Paravartya sutra by writing each digit of the complement with a changed sign separately. For example, if the divisor is 87, the nearest base = 100 and complement = $100 - 87 = 13$ and revised complement = $-1 - 3$. This revised complement will now be the basis of division.

Example 7: Divide 1358 by 113

Solution: Base = 100

$$\text{Complement} = 113 - 100 = 13$$

Complement with changed sign for each digit = $-1 - 3$

Column 1	Column 2(Q)	Column 3(R)
1 1 3	1 3	5 8
1 3	-1	-3
-1 -3		-2
	1 2	0 2

Explanation:-

1. Base = 100

$$\text{Complement} = 113 - 100 = 13$$

Writing each digits of complement with the changed sign
= -1 -3.

Since the number of zeros in the base is 2, the number of digits in the remainder column is 2.

2. Carry 1 (the first digit of Column 2) down; this will give you the first digit of the quotient.

Column 1	Column 2(Q)	Column 3(R)
1 1 3	1 3	5 8
1 3		
-1 -3		
	1	

3. Multiply the first digit of the quotient with the revised complement and write it next to the first digit of the dividend.

$$-1 -3 \times 1 = -1 -3$$

Now, add each digit of Column 2 and Column 3 to get the Quotient and Remainder. Here, the sum of the digits of the second column is $3 - 1 = 2$. Now multiply the revised complement with the second digit of the quotient and write it next to the second digit of the quotient.

$$-1 -3 \times 2 = -2 -6$$

Add the digits of column 3 to get the remainder.

	Column 1	Column 2(Q)	Column 3(R)
Complement =	1 1 3	1 3 -1	5 -3 -2 -6
Revised Complement =	-1 -3	+ 1 2	0 2

Example 8: Divide 239479 by 11213

Solution: Base = 10000

$$\text{Complement} = 11213 - 10000 = 1213$$

Complement with changed sign for each digit = -1 -2 -1 -3
Since the base has 4 zeros, arrange the digits in the quotient and remainder columns accordingly.

	Column 1	Column 2 (Q)	Column 3 (R)
Divisor	1 1 2 1 3	2 3 9 4 7 9	
Complement = 1 2 1 3		-2 4 -2 -6 -0	(Revised complement x 2)
Revised Complement = -1 -2 -1 -3		-1 -2 -1 -3	(Revised complement x 1)
		2 1 4 0 0 6	

Quotient = 21

Remainder = 4 0 0 6

Urdhva – Tiryagbhyam (उर्ध्वं तिर्यग्भ्याम्)

The division method studied so far has limited application and is useful when the divisor is near the base. The division method we are going to study now is a mix of the Vedic sutra, Urdhva Tiryagbhyam and sub-sutra, Dhwajanka. This is also known as straight division because we get the answer as a quotient and

remainder in one line. Thus we save time and space. In straight division; divisor is bifurcated as operator and Dhwajank so that we go on dividing not by the whole divisor, but by the small operator and go on modifying the sub dividend portions using the dhwajank and quotient by the Urdhva Tiryak sutra.

Rules:

- Irrespective of the number of digits in the divisor, our divisor is of one digit or two digits, depending upon the situation. The remaining digits of the divisor are placed at the top of that number. For example – If we are to divide $x \ y \ z \ p \ q \ r$ by $a \ b \ c$, then we shall write

$$\begin{array}{c|cccccc} a^{bc} & x & y & z & p & q & r \\ \hline \end{array}$$

Quotient / Remainder

Here our new divisor is a and bc is called the flag (Dhwajank) digit.

- The number of digits on the right side of the slash will depend on the number of flag digits on the top.
- Divide the first digit of the dividend by the divisor and write the quotient in the quotient column and the remainder is placed before the next dividend digit to give gross dividend.
- Subtract the product of the flagged digit and the first quotient digit from the gross dividend to get the net dividend.
- Now this net dividend is to be divided by the divisor. The net dividend should always be positive. In case you obtain a negative net dividend, make an adjustment by reducing the quotient and recalculating the remainder thereafter.
- The process of getting gross dividend and net dividend is to be repeated till you reach the desired result.
- The main problematic area in this straight division is subtraction from the gross dividend the product of the

quotient and flagged digits. The steps written here will help you in this regard.

- When there is a single flagged digit, we subtract the product of the last quotient digit and the flagged digit at each step.
- When there are two flag digits, in the first step product of the first flagged digit and the first quotient digit is subtracted and from the second step onwards, the cross product of the two flagged digits and the last two quotient digits is subtracted.
- When there are three flagged digits: – first subtract the product of the first flagged digit and the first quotient digit. In the second stage, the cross product of the first two flagged digits and first two quotient digits will be subtracted from the gross dividend. In the third stage, the cross product of the three flagged digits and the three quotients will be subtracted. After the third stage, the subtraction of the cross product of the three flagged digits and the three quotients will be continued.
- Let us take some examples to understand the modus operandi more clearly.

Example 9: Divide 1764 by 42

Solution:

Step 1: Take 4 as the main divisor and 2 as the flag digit. Arrange the divisor, flag digit, quotient and digit for remainder, if any, according to the rule explained above. Since we have taken only one digit as the flagged digit, the unit digit will be put in the remainder column.

4^2	1	7	6	4
	Quotient			Remainder

Step 2: Divide 17 by 4

First digit of quotient = 4

Remainder = 1

Put the quotient 4 in quotient column and the remainder 1 will be place before 6 in the dividend column.

4^2	1	7	1	6	4
		42			

Gross Dividend = 16

Net Dividend = Gross Dividend - First digit of quotient

\times Flag digit

$$= 16 - 2 \times 4 = 8$$

Step 3: Divide 8 by 4

Quotient = 2

Remainder = 0

4^2	1	7	1	6	0	4

Now our calculation has moved in remainder part. Here

Gross Dividend = 4

$$\text{Net Dividend} = 4 - 2 \times 2 = 0$$

Since, we are in the remainder part of division, our division process is completed.

Hence $1764 \div 42, Q = 42 R = 0$

Example 10: Divide 387 by 32

Solution:

Step 1: Here, we have a double-digit divisor, so 3 is taken as the main divisor for the whole operation and 2 is the flag digit (Dhwajank). Since the flag digit is a single digit, so is the remainder. Hence the remainder digit is 1

3^2	3	8	7

Step 2: Divide 3 by 3. Quotient = 1 and Remainder = 0
 Put the quotient in the quotient column and remainder 0 before 8 in the dividend column.

$$\begin{array}{c|cc|c} 3^2 & 3 & 0 & 8 \\ \hline & 1 & & \end{array}$$

Gross Dividend = 08

$$\begin{aligned} \text{Net Dividend} &= 08 - \text{first digit of quotient} \times \text{flag digit} \\ &= 08 - 1 \times 2 = 6 \end{aligned}$$

Step 3: Divide 6 by 3

Quotient = 2 and Remainder = 0.

0 is now placed before 7 in the dividend column, making it 07, the next dividend.

$$\begin{array}{c|cc|c} 3^2 & 3 & 0 & 8 \\ \hline & 1 & 2 & \end{array}$$

Gross Dividend = 07

$$\begin{aligned} \text{Net Dividend} &= 07 - \text{second digit of quotient} \times \text{flag digit} \\ &= 07 - 2 \times 2 = 3 \end{aligned}$$

Since, our calculation has moved inare in remainder side, we do not do the division.

$$\begin{array}{c|cc|c} 3^2 & 3 & 0 & 8 \\ \hline & 1 & 2 & 3 \end{array}$$

Hence, $387 \div 32$, Quotient = 12, Remainder = 3

Example 11: Divide 38982 by 73

Solution:

The first step in the Dhvajanka method is to make the column as shown below.

Step 1: Out of the divisor 73, we put down only the first digit i.e. 7 in the divisor- column and put the other digit i.e. 3 on the top of the flag.

Here main divisor = 7

Dhvajanka = 3 (one digit)

$$\begin{array}{r|rrrr|r} 7^3 & 3 & 8 & 9 & 8 & 2 \end{array}$$

Hence, a line is drawn vertically from the right, leaving one digit at the end in the remainder column as shown above. Now the entire division is to be carried out by 7.

Step 2: As the first digit from the left of dividend (3) is less than 7, we take 38 as our first dividend. Divide 38 by 7

$$Q_1 = 5 \quad R_1 = 3$$

Q_1 = The first quotient

R_1 = The first remainder.

Put the quotient below the horizontal line and prefix the remainder 3 below the digit 9.

$$\begin{array}{r|rrrr|r} 7^3 & 3 & 8 & \underline{3} & 8 & 2 \\ & & & 5 & & \end{array}$$

Step 3:

New gross dividend = 39

Subtract the product of dhvajanka 3 and the first quotient

(5) i.e. $3 \times 5 = 15$ from 39.

Net dividend = $39 - 3 \times 5 = 24$

Divide 24 by 7

$$Q_2 = 3 \quad R_2 = 3$$

Q_2 = The second quotient

R_2 = The second Remainder

Prefix the remainder 3 below 8 above the horizontal line

$$\begin{array}{r|rrrr|r} 7^3 & 3 & 8 & \underline{3} & 9 & \underline{3} & 8 & 2 \\ & & & & & & & \end{array}$$

Step 4:

New gross dividend = 38

Net Dividend = Gross Dividend - product of 2nd quotient

and dhvajanka

$$= 38 - 3 \times 3 = 29$$

Net dividend = 29

Divide it by 7

$$Q_3 = 4 \quad R_3 = 1$$

Q_3 = The third digit of quotient

R_3 = The third digit of remainder

Place 4 and 1 as discussed above.

$$\begin{array}{r} 7^3 \\ \hline 3 & 8 & 3 & 9 & 3 & 8 & | & 1 & 2 \\ & 5 & 3 & 4 & & & & & \\ \hline & & & & & & & & 0 \end{array}$$

Step 5:

New Gross dividend = 12

$$\begin{array}{r} 7^3 \\ \hline 3 & 8 & 3 & 9 & 3 & 8 & | & 1 & 2 \\ & 5 & 3 & 4 & & & & & 0 \\ \hline & & & & & & & & \end{array}$$

Net Dividend = Gross Dividend - product of dhvajanka and third quotient

$$= 12 - 3 \times 4 = 0$$

Since, we are in remainder part so we stop division process.

Hence, Quotient = 534 and Remainder = 0

Example 12: Divide 72 38 761 by 524?

Solution: If we take two digits as dhvajanka, then the remainder column will certainly have two digits. See the arrangement below.

$$\begin{array}{r} 5^{24} \\ \hline 7 & 2 & 3 & 8 & 7 & | & 6 & 1 \\ & & & & & & & \\ \hline & & & & & & & \end{array}$$

Step 1: Divide 7 by 5

$$Q_1 = 1 \text{ and } R_1 = 2$$

$$\begin{array}{r} 5^{24} \\ \hline 7 & 2 & 3 & 8 & 7 & | & 6 & 1 \\ & 1 & & & & & & \\ \hline & & & & & & & \end{array}$$

Step 2: Gross dividend = 22

Net dividend = Gross Dividend - Product of the first quotient and the first flag digit

$$= 22 - 1 \times 2 = 20$$

Step 3: Divide 20 by 5

$$Q_2 = 4 \text{ and } R_2 = 0$$

Since $R_2 = 0$, we have to take the quotient below 4 i.e. New quotient = $4 - 1 = 3$. This is because we cannot consider the remainder zero in the middle of the division. The same process will be applicable in case the net divisor in the middle is either zero or negative.

Hence, Revised Quotient (Q_2) = 3 and $R_2 = 5$

$$\begin{array}{r} 5^{24} | & 7 & 2 & 5 & 3 & 8 & 7 & | & 6 & 1 \\ & 1 & 3 \end{array}$$

Step 4: Gross dividend = 53

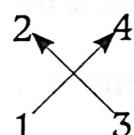
Net dividend = $53 - (\text{sum of cross product of two flag digits and two quotients})$

$$\begin{aligned} &= 53 - (2 \times 3 + 1 \times 4) \\ &= 43 \end{aligned}$$

Divide 43 by 5

$$Q_3 = 8 \text{ and } R_3 = 3$$

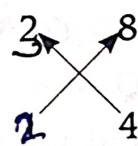
$$\begin{array}{r} 5^{24} | & 7 & 2 & 5 & 3 & 8 & 7 & | & 6 & 1 \\ & 1 & 3 & 8 \end{array}$$



Step 5: Gross dividend = 38

Net dividend = $38 - (\text{sum of cross product of } 38 \text{ and } 24)$

$$\begin{aligned} &= 38 - (3 \times 4 + 2 \times 8) \\ &= 38 - 28 = 10 \end{aligned}$$



Divide 10 by 5

$$Q_4 = 2 \text{ and } R_4 = 0$$

Since $R_4 = 0$ so we have to take $Q_4 = 1$ (discussed above)

$$\text{For } Q_4 = 1 \text{ and } R_4 = 5$$

$$\begin{array}{r} 5^{24} | & 7 & 2 & 5 & 3 & 8 & 5 & 7 & | & 6 & 1 \\ & 1 & 3 & 8 & 1 \end{array}$$

Net dividend = 29

Divide it by 7

$$Q_3 = 4 \quad R_3 = 1$$

Q_3 = The third digit of quotient

R_3 = The third digit of remainder

Place 4 and 1 as discussed above.

$$\begin{array}{r} 7^3 \\ \hline 3 & 8 & 3 & 9 & 3 & 8 & | & 1 & 2 \\ & 5 & 3 & 4 & & & & & \\ \hline & & & & & & & & \end{array}$$

Step 5:

New Gross dividend = 12

$$\begin{array}{r} 7^3 \\ \hline 3 & 8 & 3 & 9 & 3 & 8 & | & 1 & 2 \\ & 5 & 3 & 4 & & & & & 0 \\ \hline & & & & & & & & \end{array}$$

Net Dividend = Gross Dividend - product of dhvajanka and third quotient

$$= 12 - 3 \times 4 = 0$$

Since, we are in remainder part so we stop division process.

Hence, Quotient = 534 and Remainder = 0

Example 12: Divide 72 38 761 by 524?

Solution: If we take two digits as dhvajanka, then the remainder column will certainly have two digits. See the arrangement below.

$$\begin{array}{r} 5^{24} \\ \hline 7 & 2 & 3 & 8 & 7 & | & 6 & 1 \end{array}$$

Step 1: Divide 7 by 5

$$Q_1 = 1 \text{ and } R_1 = 2$$

$$\begin{array}{r} 5^{24} \\ \hline 7 & 2 & 3 & 8 & 7 & | & 6 & 1 \\ & 1 & & & & & & \end{array}$$

Step 2: Gross dividend = 22

Net dividend = Gross Dividend - Product of the first quotient and the first flag digit

$$= 22 - 1 \times 2 = 20$$

Step 3: Divide 20 by 5

$$Q_2 = 4 \text{ and } R_2 = 0$$

Since $R_2 = 0$, we have to take the quotient below 4 i.e. New quotient = $4 - 1 = 3$. This is because we cannot consider the remainder zero in the middle of the division. The same process will be applicable in case the net divisor in the middle is either zero or negative.

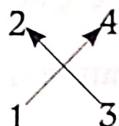
Hence, Revised Quotient (Q_2) = 3 and $R_2 = 5$

$$\begin{array}{r} 5^{24} | 7 & 2 & 5 & 3 & 8 & 7 \\ & 1 & 3 & & & & \\ \hline & 6 & 1 & & & & \end{array}$$

Step 4: Gross dividend = 53

Net dividend = $53 - (\text{sum of cross product of two flag digits and two quotients})$

$$\begin{aligned} &= 53 - (2 \times 3 + 1 \times 4) \\ &= 43 \end{aligned}$$



Divide 43 by 5

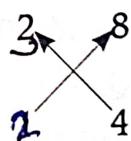
$$Q_3 = 8 \text{ and } R_3 = 3$$

$$\begin{array}{r} 5^{24} | 7 & 2 & 5 & 3 & 8 & 7 \\ & 1 & 3 & 8 & & & \\ \hline & 6 & 1 & & & & \end{array}$$

Step 5: Gross dividend = 38

Net dividend = $38 - (\text{sum of cross product of 38 and 24})$

$$\begin{aligned} &= 38 - (3 \times 4 + 2 \times 8) \\ &= 38 - 28 = 10 \end{aligned}$$



Divide 10 by 5

$$Q_4 = 2 \text{ and } R_4 = 0$$

Since $R_4 = 0$ so we have to take $Q_4 = 1$ (discussed above)

For $Q_4 = 1$ and $R_4 = 5$

$$\begin{array}{r} 5^{24} | 7 & 2 & 5 & 3 & 8 & 5 & 7 \\ & 1 & 3 & 8 & 1 & & & \\ \hline & 6 & 1 & & & & & \end{array}$$

Step 6: Gross dividend = 57

Net dividend = $57 - (\text{sum of cross product of } 24 \text{ and } 81)$

$$= 57 - (2 \times 1 + 4 \times 8)$$

$$= 57 - 34$$

$$= 23$$

Divide 23 by 5

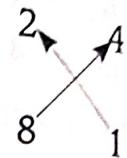
$$Q_5 = 4 \text{ and } R_5 = 3$$

Remainder = $361 - (\text{sum of cross product of } 24 \text{ and } 14) \times 10$
 $- (\text{last flag digit} \times \text{last quotient})$

$$= 361 - (8 + 4) \times 10 - (4 \times 4)$$

$$= 361 - 120 - 16 = 225$$

$$\begin{array}{c|ccccc|cc} 5^{24} & 7 & 2 & 3 & 8 & 7 & 6 & 1 \\ \hline & 1 & 3 & 8 & 1 & 4 & 2 & 25 \end{array}$$



$$\text{Hence Quotient} = 13814 \text{ and } R = 225$$

Example 13: Divide 9862145 by 20132

Solution: Arrange the dividend, divisor and remainder digit as discussed in the above examples.

$$\begin{array}{c|ccccc|cc} 20^{132} & 9 & 8 & 6 & 2 & | & 1 & 4 & 5 \\ \hline & & & & & | & & & \end{array}$$

Step 1: For convenience, we have taken the dividend of two digits and flag digit of three digits.

Divide 98 by 20

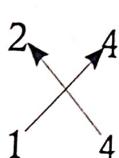
$$Q = 4, R = 18$$

Put the quotient below the horizontal line and prefix the remainder 18 below the digit 6.

$$\begin{array}{c|ccccc|cc} 20^{132} & 9 & 8 & 18 & 6 & 2 & | & 1 & 4 & 5 \\ \hline & & & & & & | & & & \end{array}$$

$$\text{Gross Dividend} = 186$$

$$\begin{aligned} \text{Net Dividend} &= 186 - 1 \times 4 \\ &= 182 \end{aligned}$$



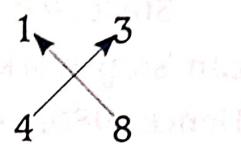
Step 2: Divide 182 by 20

$$Q = 8, R = 22$$

Put the quotient below the horizontal line and prefix the remainder 22 below the digit 2.

$$\begin{array}{r} 20 \overline{)132} \\ \underline{-16} \\ 18 \\ \underline{-16} \\ 2 \end{array}$$

$$\text{Gross Dividend} = 222$$



$$\begin{aligned} \text{Net Dividend} &= 222 - \text{Cross product of } 13 \text{ and } 48 \\ &= 202 \end{aligned}$$

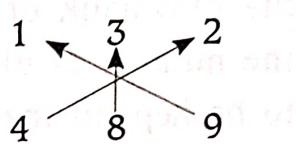
Step 3: Divide 202 by 20

$$Q = 9, R = 22$$

Put the quotient below the horizontal line and prefix the remainder 22 below the digit 1 in the remainder column.

$$\begin{array}{r} 20 \overline{)132} \\ \underline{-16} \\ 18 \\ \underline{-16} \\ 2 \end{array}$$

$$\text{Gross Dividend} = 221$$



$$\begin{aligned} \text{Net Dividend} &= 221 - \text{Cross product of } 132 \text{ and } 489 \\ &= 221 - 41 \quad 1 \quad 3 \quad 2 \end{aligned}$$

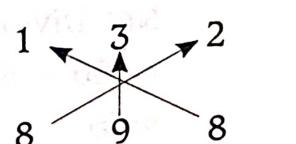
Step 4: Divide 180 by 20

$$Q = 8, R = 20$$

Put the quotient below the horizontal line and prefix the remainder 20 below the digit 4 in the remainder column.

$$\begin{array}{r} 20 \overline{)132} \\ \underline{-16} \\ 18 \\ \underline{-16} \\ 2 \end{array}$$

$$\text{Gross Dividend} = 204$$



$$\begin{aligned} \text{Net Dividend} &= 204 - \text{Cross product of } 132 \text{ and } 898 \\ &= 204 - 51 \quad 8 \quad 9 \quad 8 \\ &= 153 \end{aligned}$$

Step 5: Divide 153 by 20

$$Q = 7, R = 13$$

Put the quotient below the horizontal line and prefix the remainder 13 below the digit 5 in the remainder column.

20^{132}	9	8	₁₈ 6	₂₂ 2	₂₂ 1	₂₀ 4	₁₃ 5
	4	8	9		8	7	

Since, we are left with two digits in the remainder column, we can stop working.

$$\text{Hence, } 9862145 \div 20132 = 489.87$$

Example 14: Divide 5362968527 by 9213649875

Solution: At first look, this sum seems horrible as the number of digits in the divisor is 10. The conventional method will take much of your time in the hit and trial method of multiplication, but the Vedic method of division makes this sum child's play. For convenience, we will take the actual divisor of one digit and the dhwajank of the rest of the 9 digits. As discussed earlier, the number of digits with the flag decides the number of digits to be kept in the remainder.

$9^{213649875}$	5	3	6	2	9	6	8	5	2	7

Step 1: Here, $5 < 9$, so we put a decimal at the beginning.

Now divide 53 by 9

$$Q = 5, R = 8$$

$$\text{Gross Dividend} = 86$$

Net Dividend = 86 - first flagged digit \times quotient

$$= 86 - 5 \times 2$$

$$= 76$$

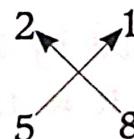
$9^{213649875}$	5	3	₈ 6	2	9	6	8	5	2	7

Step 2: Divide 76 by 9

$$Q = 8, R = 4$$

$$\text{Gross Dividend} = 42$$

Net Dividend = 42 - cross product of 21 and 58



$$= 42 - 21$$

$$= 21$$

9	213649875	5	3	8	6	4	2	9	6	8	5	2	7
		0.5	8										

Step 3: Divide 21 by 9

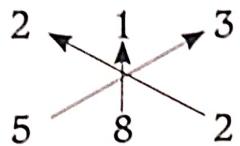
$$Q = 2, R = 3$$

$$\text{Gross Dividend} = 39$$

$$\text{Net Dividend} = 39 - \text{cross product of } 213 \text{ and } 582$$

$$= 39 - 27$$

$$= 12$$



9	213649875	5	3	8	6	4	2	3	9	6	8	5	2	7
		0.5	8	2										

Step 4: Divide 12 by 9

$$Q = 1, R = 3$$

9	213649875	5	3	8	6	4	2	3	9	3	6	8	5	2	7
		0.5	8	2	1										

Since we have reached upto 4 decimal places, there is no need to move any further.