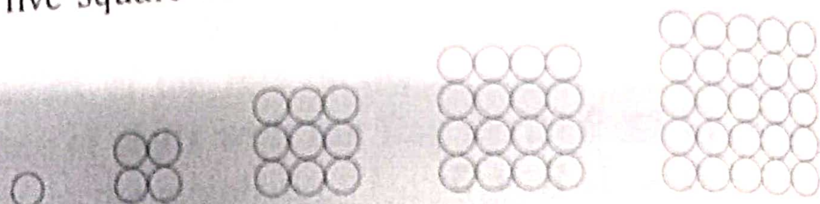
Square

Introduction

Squaring a number means multiplication of the number by itself. Mathematically, $a \times a = a^2$. Here is the geometrical representation of first five square numbers.



1   2 x 2   3 x 3   4 x 4   5 x 5

The school curriculum follows only two methods for finding the square of a number.

- By multiplying the number by itself through long multiplication process.
  Example: $(12)^2 = 12 \times 12 = 144$
- By Algebraic Expansion:
  Here, we generally use either of the two formulae–
  i) $(a+b)^2 = a^2 + 2ab + b^2$
  ii) $(a-b)^2 = a^2 - 2ab + b^2$

Example (a): $(13)^2 = (10 + 3)^2$
$= 10^2 + 2 \times 10 \times 3 + 3^2$
$= 100 + 60 + 9 = 169$
Example (b): $(96)^2 = (100 - 4)^2$
$= 100^2 - 2 \times 100 \times 4 + 4^2$
$= 10000 - 800 + 16$
$= 9216$

But in Vedic mathematics, there are around 5 methods to find the square of a number. Moreover, the Vedic method is 10 times faster than the traditional method. Some of these methods have limited application but there is a special method called the Duplex method which is said to be the gem of all methods.

Vedic Method

VEDIC SUTRAS FOR SQUARING

- एकाधिकेन पूर्वेण (EKADHIKENA PURVENA)
- यावदूनम तावदूनीकृत्य वर्गेण योजयेत् (YAVADUNAM TAVADUNI KRITYA VARGANEA YOJAYET)
- द्वंद्व योग (DWANDA YOGA)
- उर्ध्व तिर्यगभ्याम् (URDHVA TIRYAGBHYAM)

Meaning of Vedic Sutra

1. **Ekadhikena Purvena (एकाधिकेन पूर्वेण):** This Vedic method of finding a square has limited application and is valid only when the unit digit of any number is 5. The answer comes into two parts. The RHS part of the answer is the square of 5 and LHS part of the answer is the product of the remaining digit and its Ekadhikena. In case the number before 5 is bigger, the multiplication of the number and its Ekadhikena can be carried out by the long method of multiplication.

2. **Yavadunam Tavaduni kritya vargena yojayet (यावदूनम तावदूनी कृत्य वर्गेण योजयेत):** This Vedic sub-sutra is used for squaring numbers which are closer to the base (10^n). With a little practice, though, you can extend it to numbers which are farther from the base using the sub-base provided the sub-base is a multiple of 10^n.

   This Vedic-sutra simply says –

   (a) Find the extent or deficiency of a number to be squared with respect to its base. This extent or deficiency is termed here as the **deviation.**

   (b) Set up the square of the deviation at the end.

3. **Dwanda-Yoga or Duplex Mehod (द्वंद्व योग):** The two Vedic methods discussed above have limited application. So the question is – what will you do if the number that is to be squared does not satisfy either of the above condition?

   The Duplex combination is applicable in all the cases. The term Dwanda-Yoga is used in two different senses– The first one is by squaring and the second one is by cross multiplication.

   i) In the case of a single central digit a, the duplex is its square. i.e. $\mathbf{a^2}$

   ii) In the case of an even number of digits (say a and b) equidistant from the two ends, the duplex is taken as double the cross product of a and b (i.e.2ab)

4. **Urdhva–Tiryak (उर्ध्व तिर्यक):** Squaring a number means multiplying a number twice by itself. This method of squaring is nothing new but it is the same as that of the multiplication of two numbers in the Cross and Dot method discussed in the chapter on Multiplication. It is therefore expected from readers to do the squaring of any number by Urdhva-Tiryak Method, which is nothing but the multiplication of a single number twice.

Ekadhikena Purvena (एकाधिकेन पूर्वेण)

As discussed above, this Vedic sutra is applicable when the number to be squared has 5 as its unit digit. Mathematically,

$$(A\ 5)^2 = A \times (A+1) \ /\ 5^2$$

Here, the Ekadhikena of A is A+1. Let us understand the modus operandi of this method as discussed above, with the help of some examples.

Example 1: Find the square of 85

Solution: At the beginning, I said that the answer is split into two parts.

RHS = Square of 5 = 25
LHS = Remaining digit x Next digit
= 8 x 9 = 72

Hence, $(85)^2 = 7225$

Example 2: Find the square of 55

Solution:

RHS = Square of 5 = 25
LHS = Remaining digit x Next digit
= 5 x 6 = 30

Hence, $(55)^2 = 3025$

Example 3: Find the square of 125

Solution:

RHS = Square of 5 = 25
LHS = Remaining digit x Next digit
= 12 x13 = 156

Multiplication of 12 x 13 can be done by the Nikhilam method.

12 + 2
13 + 3
15 / 6

Moreover, multiplication in such a case can be done by the Urdhva-Tiryak method.

1 2
x 1 3

1 x 1 / 1 x 3 + 1 x 2 / 2 x 3

= 1 5 6

Hence, $(125)^2$ = 15625

Example 4: Find the square of 165

Solution:

RHS = Square of 5 = 25

LHS = Remaining digit x Next digit

= 16 x17 = 156

Multiplication of 12 x 13 can be done by the Nikhilam method.

16 + 6

17 + 7

23 / 42

= 272

Moreover, multiplication in such case can be done by the Urdhva-Tiryak method.

1 6

x 1 7

1 x 1 / 1 x 6 + 1 x 7 / 6 x 7

= 1 / 13 / 42

= 2 7 2

Hence, $(165)^2$ = 27225

Example 5: Find the square of 245

Solution:

RHS = Square of 5 = 25

LHS = Remaining digit x Next digit

= 24 x 25 = 600

(See special method of multiplication by 25 in *Multiplication through Observation*.)

Hence, $(245)^2$ = 60025

Yavadunam Tavaduni kritya vargena yojayet
(यावदूनम तावदूनी कृत्य वर्गेण योजयेत)

This sutra works better when the number to be squared is near the base 10, 100, 1000 ... or is the multiple of the base i.e. 20, 30, 40, --- 200, 300, 400--- etc. Let us divide the squaring concept through this Vedic sub-sutra in two parts.

Case 1: When the number is near the base 10, 100, 1000...... 10^n

The answer is arrived at in two parts.

LHS = Number + Deviation

(Deviation may be positive or negative, depending on the base)

RHS = Square of deviation

The RHS will contain the same number of digits as the number of zeros in the base. The excess digit if any will be carried over to LHS and the deficit digit, if any, will be filled up by putting the zeros to the left of the RHS.

Example 6: Find the square of 13

Solution: Number 13 is closer to base 10.

Deviation = 13 – 10 = 3.

$(13)^2 = 13 + 3 / 3^2$

$= 169$

Example 7: Find the square of 16

Solution: Number 16 is closer to base 10.

Deviation = 16 – 10 = 6.

$(16)^2 = 16 + 6 / 6^2$

$= 22 / 36$

$= 256$

Since Base = 10, the RHS will contain single digit.

Example 8: Find the square of 91?

Solution: Number 91 is closer to base 100.

$$\text{Deviation} = 91 - 100 = -9$$
$$(91)^2 = 91 - 9 \,/\, (-9)^2$$
$$= 82/\,81$$

Example 9: Find the square of 97

Solution: Number 97 is closer to base 100.

$$\text{Deviation} = 97 - 100 = -3$$
$$(97)^2 = 97 - 3 \,/\, (-3)^2$$
$$= 94/\,9$$

Since the Base =100, the RHS should have 2 digits, so one additional zero will be placed before 9.

$$(97)^2 = 9409$$

Case 2: When the base is not in the form of 10^n, but the multiple of 10.

If the number to be squared is near the base 20, 30, 40, ---- or 200, 300, 400, ---- or 2000, 3000, 4000, --- the Yavadunam Tavduni sub-sutra will work with a slight change.

The answer will be arrived at in two parts.

The RHS part of the answer will be the square of the deviation from the base. The LHS part of the answer should be written with utmost care. LHS = (Number to be squared + Deviation) x sub-base.

Example 10: Find the square of 32

Solution: Number 32 is closer to base 30.

$$\text{Deviation} = 32 - 30 = 2$$
$$30 = 3 \times 10$$
$$\text{Sub-base} = 3$$
$$\text{Actual base} = 10$$

$(32)^2 = (32 + 2) \times 3 / (2)^2$
$= 102/\ 4$
$= 1024$

Example 11: Find the square of 47

Solution: Number 47 is closer to base 50.

Deviation $= 47 - 50 = -3$
$50 = 5 \times 10$
Sub-base $= 5$
Actual base $= 10$

$(47)^2 = (47 - 3) \times 5 / (-3)^2$
$= 220/\ 9$
$= 2209$

Example 12: Find the square of 204

Solution: 204 is closer to base 200.

Deviation $= 204 - 200 = 4$
$200 = 2 \times 100$
Sub-base $= 2$
Actual base $= 100$

$(204)^2 = (204 + 4) \times 2 / (4)^2$
$= 416/\ 16$
$= 41616$

Example 13: Find the square of 482

Solution: 482 is closer to base 500.

Deviation $= 482 - 500 = -18$
$500 = 5 \times 100$
Sub-base $= 5$
Actual base $= 100$

$(482)^2 = (482 - 18) \times 5 / (-18)^2$
$= 5 \times 464/\ 324$
$= 232\ 0/3\ 24$
$= 232324$

Let us take the base 500 and find the square of 482 in another way.

$500 = 1000/2$

Hence, Base = 1000 and sub-base = ½

Deviation $= 482 - 500 = -18$

$(482)^2 = (482 - 18) \times ½ \,/\, (-18)^2$

$= 232 \,/\, 324$

$= 232324$

Example 14: Find the square of 709

Solution: 709 is closer to base 700.

Deviation $= 709 - 700 = 9$

$700 = 7 \times 100$

Sub-base = 7

Actual base = 100

$(709)^2 = (709 + 9) \times 7 \,/\, (9)^2$

$= 7 \times 718/\, 81$

$= 502681$

Example 15: Find the square of 8989

Solution: 8989 is closer to base 9000.

Deviation $= 8989 - 9000 = -11$

$9000 = 9 \times 1000$

Sub-base = 9

Actual base = 1000

$(8989)^2 = (8989 - 11) \times 9 \,/\, (-11)^2$

$= 9 \times 8978/\, 121$

$= 80802 \,/121$

$= 80802121$

Duplex or Dwanda yoga (द्वंद्व योग)

The Dwandwa-Yoga or Duplex method of squaring is one of the best squaring methods in Vedic Mathematics. By using this sutra, we can find the square of any number, of any length, with comfort and ease in one line. After a little practice, you can find the square of any number mentally. This is unique in the sense that it has universal application. Let us denote the duplex of a number by D.

- Duplex of 1 digit number = Square of that number

  $D(a) = a^2$

  Duplex of 2 = 2^2 = 4 Duplex of 6 = 6^2 = 36

- Duplex of 2 digit number = 2 X (Product of digits)

  D(ab) = 2ab

  Duplex of 24 = 2 x (2 x 4) = 16

  Duplex of 76 = 2 x (7 x 6) = 84

- Duplex of 3 digit number = 2 x (1st digit x 3rd digit) + (square of middle digit)

  $D(abc) = 2ac + b^2$

  Duplex of 126 = 2 x (1 x 6) + 2^2 = 16

  Duplex of 478 = 2 x (4 x 8) + 7^2 = 113

- Duplex of 4 digit number = 2 x (1st digit x 4th digit) + 2 x (2nd digit x 3rd digit).

  D(abcd) = 2ad + 2bc

  Duplex of 2468 = 2 x (2 x 8) + 2 x (4 x 6) = 80

  Duplex of 4567 = 2 x (4 x 7) + 2 x (5 x 6) = 116

- Duplex of 5 digit number = 2 x (1st digit x 5th digit) + 2 x (2nd digit x 4th digit) + (middle digit)^2

  $D(abcde) = 2ae + 2bd + c^2$

  Duplex of 16289 = 2 x (1 x 9) + 2 x (6 x 8) + 2^2 = 118

  Duplex of 50406 = 2 x (5 x 6) + 2x (0 x 0) + 4^2 = 76

- Duplex of 6 digit number = 2 x (1st digit x 6th digit) + 2 x (2nd digit x 5th digit) +2 x (3rd digit x 4th digit)

  D(abcdef) = 2af +2be +2cd

Duplex of 320416 = 2 x (3 x 6) + 2 x (2 x 1) + 2 x (0 x 4) = 40

Duplex of 125673 = 2 x (1 x 3) + 2 x (2 x 7) + 2 x (5 x 6) = 94

- Duplex of 7 digit number = 2 x (1^{st} x 7^{th} digit) + 2 x (2^{nd} x 6^{th} digit) + 2 x (3^{rd} x 5^{th} digit) + (4^{th} digit)^2

  D (abcdefg) = 2ag +2bf + 2ce + d^2

  Duplex of 2356214 = 2 x (2 x 4) + 2 x (3 x 1) + 2 x (5 x 2) + 6^2 =78

  Duplex of 1025962 = 2 x (1 x 2) + 2 x (0 x 6) + 2 x (2 x 9) + 5^2 =65

Once you learn to find the duplex of a number, you need to write the number in groups. The following pattern will help you in grouping the numbers.

$(11)^2 = 1\ 2\ 1$

$(1\ 1\ 1)^2 = 1\ 2\ 3\ 2\ 1$

$(1\ 1\ 1\ 1)^2 = 1\ 2\ 3\ 4\ 3\ 2\ 1$

$(1\ 1\ 1\ 1\ 1)^2 = 1\ 2\ 3\ 4\ 5\ 4\ 3\ 2\ 1$

$(1\ 1\ 1\ 1\ 1\ 1)^2 = 1\ 2\ 3\ 4\ 5\ 6\ 5\ 4\ 3\ 2\ 1$

$(1\ 1\ 1\ 1\ 1\ 1\ 1)^2 = 1\ 2\ 3\ 4\ 5\ 6\ 7\ 6\ 5\ 4\ 3\ 2\ 1$

Grouping of a number

The grouping of $(24)^2$ will follow the pattern of 1 2 1 of $(11)^2$
The groups for **24** are–

$$\underbrace{D(2)}_{1\text{ digit}} \qquad \underbrace{D(24)}_{2\text{ digit}} \qquad \underbrace{D(4)}_{1\text{ digit}}$$

The grouping of $(245)^2$ will follow the pattern of 1 2 3 2 1 of $(111)^2$.
The groups of numbers for **245** are-

$$\underbrace{D(2)}_{1\text{ digit}} \qquad \underbrace{D(24)}_{2\text{ digit}} \qquad \underbrace{D(245)}_{3\text{ digit}} \qquad \underbrace{D(45)}_{2\text{ digit}} \qquad \underbrace{D(5)}_{1\text{ digit}}$$

The grouping of $(2456)^2$ will follow the pattern of 1 2 34 3 2 1 of $(1111)^2$.

The groups of numbers for **2456** are–

$$\underbrace{D(2)}_{1\ \text{digit}}\quad \underbrace{D(24)}_{2\ \text{digit}}\quad \underbrace{D(245)}_{3\ \text{digit}}\quad \underbrace{D(2456)}_{4\ \text{digit}}\quad \underbrace{D(456)}_{3\ \text{digit}}\quad \underbrace{D(56)}_{2\ \text{digit}}\quad \underbrace{D(6)}_{1\ \text{digit}}$$

	Summary of the Duplex Method
1	$D(a) = a^2$
2	$D(ab) = 2ab$
3	$D(abc) = 2ac + b^2$
4	$D(abcd) = 2ad + 2bc$
5	$D(abcde) = 2ae + 2bd + c^2$
6	$D(abcdef) = 2af + 2be + 2cd$
7	$D(abcdefg) = 2ag + 2bf + 2ce + d^2$

How does the Duplex Method work?

- Form the groups of numbers to be squared as shown above
- Write the duplex of each group
- Once the duplex value for each group is written, add the figures from right to left, keeping only one digit in each separator

Example 16: Find the square of 32

Solution: The groups for 32 are

$$= \underbrace{D(3)}_{3^2} \quad | \underbrace{D(32)}_{2 \times 3 \times 2} | \quad \underbrace{D(2)}_{2^2}$$

$= 9 \mid 1\,2 \mid 4$

+

$= 1024$

Example 17: Find the square of 49

Solution: The groups for 49 are

$$= \underbrace{4^2}_{D(4)} \;|\; \underbrace{2 \times 4 \times 9}_{D(49)} \;|\; \underbrace{9^2}_{D(9)}$$

$= 16 \mid 7\,2 \mid 8\,1$

$= 16 / 80 / 1$

$= 2401$

Example 18: Find the square of 465

Solution: The groups of numbers for 465 are–

$$= \underbrace{4^2}_{D(4)} \quad \underbrace{2 \times 4 \times 6}_{D(46)} \quad \underbrace{2 \times 4 \times 5 + 6^2}_{D(465)} \quad \underbrace{2 \times 6 \times 5}_{D(65)} \quad \underbrace{5^2}_{D(5)}$$

$= 16 \mid 4\,8 \mid 7\,6 \mid 6\,0 \mid 2\,5$

$= 20 / {}_1 5 /{}_1 2/2/5$

$= 216225$

Example 19: Find the square of 687

Solution: The groups of numbers for 687 are–

D(6) / D(68) / D(687) / D(87) / D(7)

$= 6^2 / 2 \times 6 \times 8 / 2 \times 6 \times 7 + 8^2 / 2 \times 8 \times 7 / 7^2$

$= 36 / {}_9 6 / {}_{14} 8 / {}_{11} 2 / {}_4 9$

$= 45 / {}_2 0 / {}_1 9/6/9$

$= 471969$

Example 20: Find the square of 8254

Solution: The groups of numbers for 8254 are

8, 82, 825, 8254, 254, 54 and 4.

Duplex of 8 $= 8^2 = 64$

Duplex of 82 $= 2 \times 8 \times 2 = 32$

Duplex of 825 = 2 x 8 x 5 + 2^2 = 84

Duplex of 8254 = 2 x 8 x 4 + 2 x 2 x 5 = 84

Duplex of 254 = 2 x 2 x 4 + 3^2 = 41

Duplex of 54 = 2 x 5 x 4 = 40

Duplex of 4 = 4^2 = 16

Arrange the value of duplex as follows–

$$64 \mid {}_{3}2 \mid {}_{8}4 \mid {}_{8}4 \mid {}_{4}1 \mid {}_{4}0 \mid {}_{1}6$$

$$= 6\ 8\ 1\ 2\ 8\ 5\ 1\ 6$$

Hence $(8254)^2$ = 68128516

Example 21: Find the square of 4856

Solution: The groups for 4856 are:

4, 48, 485, 4856, 856, 56, and 6

Duplex of 4 = 4^2 = 16

Duplex of 48 = 2 x 4 x 8 = 64

Duplex of 485 = 2 x 5 x 4 + 8^2 =104

Duplex of 4856 = 2 x 4 x 6 + 2 x 8 x 5 =128

Duplex of 856 = 2 x 8 x 6 + 5^2 =121

Duplex of 56 = 2 x 5 x 6 = 60

Duplex of 6 = 6^2 = 36

Arrange the duplex of each number in digit separator.

$$= 16 / {}_{6}4 / {}_{10}4 / {}_{12}8 / {}_{12}1 / {}_{6}0 / {}_{3}6$$

= 23580736

Example 22: Find the square of 45612

Solution: The groups for 45612 are:

4, 45, 456, 4561 , 45612, 5612, 612, 12 and 2

Duplex of 4 = 4^2 = 16

Duplex of 45 = 2 x 4 x 5 = 40

Duplex of 456 = 2 x 4 x 6 + 5^2 =73

Duplex of 4561 = 2 x 4 x 1 + 2 x 5 x 6 = 68

Duplex of 45612 = 2 x 4 x 2 + 2 x 5 x 1 + 6^2 =62

Duplex of 5612 = 2x 5 x 2 + 2 x 6 x 1 = 32

Duplex of 612 = 2 x 6 x 2 + 1^2 = 25

Duplex of 12 = 2 x 1 x 2 = 4

Duplex of 2 = 2^2 = 4

Arrange the duplex of each number in digit separator.

= 16 / $_4$0 / $_7$3 / $_6$8 / $_6$2 / $_3$2 / $_2$5 / 4 / 4

= 2080454544

Proceeding in the same fashion, readers can find the square of any number of their choice by using the Duplex Method. All the methods discussed above have their own beauty and it is now the readers who have to decide which method is the best according to a particular situation. The squaring of a number by the Urdhva-Tiryak sutra is the simple multiplication of a number twice; therefore it is left upto the readers to decide which method they want to adopt.

8

Square Root

Introduction

In mathematics, a **square root** of a number x is a number r such that $r^2 = x$, or, in other words, a number r whose square (the result of multiplying the number by itself, or $r \times r$) is x. For example, 4 is a square root of 16 because $4^2 = 16$. Mathematically, if $x^2 = y$ then $x = y^{1/2} = \pm \sqrt{y}$

If you square 2, you get 4, and if you "take the square root of 4", you get 2; if you square 3, you get 9, and if you "take the square root of 9", you get 3.

$$2^2 = 4, \text{ so } \sqrt{4} = 2$$
$$3^2 = 9, \text{ so } \sqrt{9} = 3$$

Generally, extracting the square root of a number is considered a tedious job. We do have two sets of methods taught in our present day classroom:

a) Factor Method
b) Long Division Method.

Both the methods are lengthy and time consuming. The Vedic sutra helps us to find the square root of an exact square, merely by observation. Before we move to the Vedic Methodology to find the square root of a number, let us understand the following fundamental rule.

- A perfect square ends in 0, 1, 4, 5, 6 and 9
- A number is not a perfect square if it ends with 2, 3,7 or 8

- If the given number has n digits then its square root will have n/2 digits if n is even else (n + 1)/ 2 digits if n is odd

Let us study the following square root table

Table 1

N	N^2	Last digit of N^2	Digit sum of square number
1	1	1	1
2	4	4	4
3	9	9	9
4	16	6	7
5	25	5	7
6	36	6	9
7	49	9	4
8	64	4	1
9	81	1	9
10	100	0	1

From the above table we conclude that –

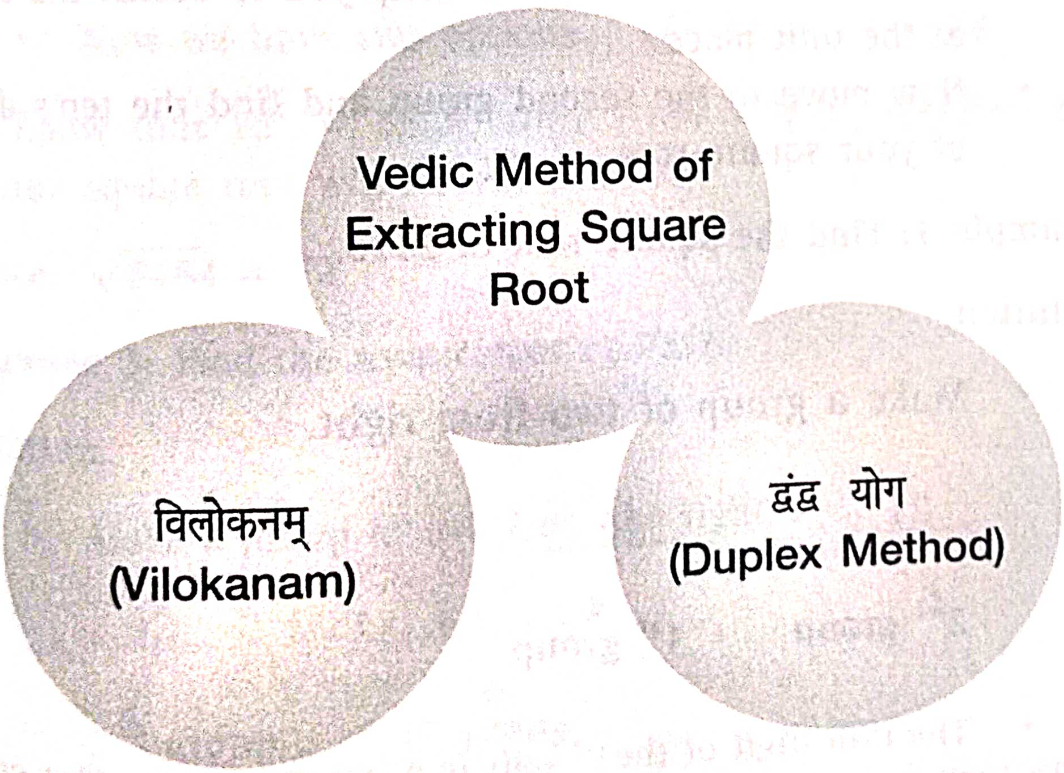
1. A Complete square ending in 1 must have either 1 or 9 as the last digit of the square root.
2. A square ending in 4 must have 2 or 8 as the last digit of the square root.
3. A square ending in 6 must have 4 or 6 as the last digit of the square root.
4. A square ending in 5 will have 5 as the last digit of the square root.
5. A square ending in 9 must have 3 or 7 as the last digit of the square root.
6. A square ending in 00 will have 0 as the last digit of the square root.

7. Apart from the above, let us look at the following nearest square root table that will help us to find the square root of a number instantly.

Table 2

Number	Nearest Square Root	Number	Nearest Square Root
1–3	1	4–8	2
9–15	3	16–24	4
25–35	5	36–48	6
49–63	7	64–80	8
81–99	9		

Vedic Method



Meaning of Vedic Sutra:

1. विलोकनम् (Vilokanam): It means, "by mere observation". This Vedic sutra will help you to find the square root

of 3–4 digits in 2–3 seconds, merely by observing the above two tables. The first table will help you to find the unit digit of the exact square root, whereas Table 2 will help you to find the ten's digit.

2. द्वंद्व-योग (Duplex method): A detailed description of this method can be found in this book itself in the chapter on Squares. This Vedic sutra is applicable to all, whether the given number is a perfect square root or not.

Exact square root of 3–4 digits by Vilokanam (विलोकनम्) method

Rule:

- Make a group of two starting from the right
- Look at the unit digit of the number and observe your answer in Table 1. This will help you to decide the digit at the unit place
- Now move to the second group and find the ten's digit of your square root

Example 1: Find the square root of 2116.

Solution:

- Make a group of two from right.

$$\underset{\swarrow}{\underline{\;21\;}} \qquad \underset{\searrow}{\underline{\;16\;}}$$

2^{nd} **group** $\qquad$ 1^{st} **group**

- The unit digit of the 1^{st} pair is 6, so the square root ends in 4 or 6 (see Table 1)
- Now look at the second group. Since $16 < 21 < 25$, the digit at the ten's place = 4 (see Table 2)
- Now we have two options $\sqrt{2116} = 44$ or 46

We know that $45^2 = 2025$, since $2116 > 2025$, therefore the desired square root will be more than 45.

Hence, $\sqrt{2116} = 46$

Example 2: Find the square root of 5184.

Solution:

- Make a group of two from the right.

$$\underset{\text{2nd group}}{\underline{51}} \qquad \underset{\text{1st group}}{\underline{84}}$$

- The unit digit of the 1st pair is 4, so the square root ends in 2 or 8 (see Table 1)
- Now look at the second group. Since $49 < 51 < 84$, the digit at the ten's place = 7 (see Table 2)
- Now we have two options $\sqrt{5184} = 72$ or 78

We know that $75^2 = 5625$, since $5184 < 5625$, therefore the desired square root will be less than 75

Hence, $\sqrt{5184} = 72$

Example 3: Find the square root of 9216.

Solution:

- Make a group of two from the right.

$$\underset{\text{2nd group}}{\underline{92}} \qquad \underset{\text{1st group}}{\underline{16}}$$

- The unit digit of 1st pair is 6, so the square root ends in 4 or 6 (see Table 1)
- Now look at the second group. Since $81 < 92 < 100$, the digit at the ten's place = 9 (see Table 2).
- Now we have two options $\sqrt{9216} = 94$ or 96

We know that $95^2 = 9025$, since $9216 > 9025$, therefore the desired square root will be more than 95.

Hence, $\sqrt{9216} = 96$.

Example 4: Find the square root of 676.

Solution:

- Make a group of two from the right.

$$\underbrace{\overline{06}}_{2^{nd}\ \text{group}} \qquad \underbrace{\overline{76}}_{1^{st}\ \text{group}}$$

- The unit digit of 1^{st} pair is 6, so the square root ends in 4 or 6 (see Table 1)
- Now look at the second group. Since $4 < 6 < 9$, the digit at the ten's place = 2 (see Table 2)
- Now we have two options $\sqrt{676} = 24$ or 26

We know that $25^2 = 625$, since $676 > 625$ therefore the desired square root will be more than 25

Hence, $\sqrt{676} = 26$.

Square Root of 5–6 digits by the Vilokanam Method

Rule:

- Make a group of two digits starting from the right. Here we will have three groups. Denote the left digit by L, the middle digit by M and the right digit by R
- The first (L) and third (R) group will give us the hundred's place digit and the unit place digit. These two can be written only through observation, with the help of Table 1 and Table 2.
- Subtract L^2 from the 1^{st} pair and carry down the next digit from the dividend, as done in simple division.

- Compare the new dividend by 2 L M. Put different value of M in 2LM and select the best possible digit, so that 2 L M $\leq$ new dividend.
- In order to avoid confusion over the choice of the number, use the Casting out Nines rule.
- Exact square root = L M R

Example 1: Find the square root of 692224.

Solution:

Make a group of two digits, starting from the right.

$$\begin{array}{ccc} \underline{69} & \underline{22} & \underline{24} \\ \downarrow & \downarrow & \downarrow \\ L & M & R \end{array}$$

- Since the unit digit of the given number ends with 4, therefore the square root ends in 2 or 8 (Table 1).
- 69 in the left group lies between $8^2 < 64 < 9^2$, hence, L = 8 (Table 2).
- Now subtract L^2 from the given number and carry down the next digit from the dividend as shown here –

$$\begin{array}{cccl} 69 & 22 & 24 & \\ \underline{-\,8^2} & \downarrow & & \longrightarrow \text{carry down the next digit from the dividend} \\ 5 & 2 & & \longrightarrow \text{new dividend} \end{array}$$

- Compare the new dividend by 2 L M = 2 x 8 x M = 16 M. Put different values of M.
  For M = 3, 16 x 3 = 48 < 52
  and For M = 4 , 64 > 52. Hence, M = 3
- Now we are left with two options:
  $$\sqrt{69\ 22\ 24} = 8\ 3\ 2 \text{ or } 8\ 3\ 8$$
- Apply the Casting out Nines method to overcome the confusion in selecting the correct answer.

Digit sum of 69 22 24	Digit sum of $(832)^2$	Digit sum of $(838)^2$
$6 + 9 + 2 + 2 + 2 + 4 = 7$	7	1

Hence, $\sqrt{69\ 22\ 24} = 832$

Example 1: Find the square root of 103041.

Solution:

- Make a group of two digits, starting from the right.

$$\underset{\downarrow\atop L}{\underline{10}} \quad \underset{\downarrow\atop M}{\underline{30}} \quad \underset{\downarrow\atop R}{\underline{41}}$$

- Since the unit digit of the given number ends with 1, the square root ends in 1 or 9 (Table 1)
- 10 in the left group lies between $3^2 < 10 < 4^2$, hence, L = 3 (Table 2)
- Now subtract L^2 from the given number and carry down the next digit from the dividend as shown here –

$$\begin{array}{lll} 10 & 30 & 41 \\ \underline{-\ 3^2} & \downarrow & \\ 1 & 3 & \end{array}$$

$\longrightarrow$ carry down the next digit from the dividend

3 $\longrightarrow$ new dividend

- Compare the new dividend by 2 L M = 2 x 3 x M = 6 M. Put different values of M.
  For M = 2, 6 x 2 = 12 < 13
  and For M = 3 , 18 > 13. Hence, M = 2
- Now we are left with two options:–
  $\sqrt{10\ 30\ 41} = 321$ or 329
- Apply the Casting out Nines method to overcome the confusion in selecting the correct answer.

Digit sum of 10 30 41	Digit sum of $(321)^2$	Digit sum of $(329)^2$
0	0	7

Hence, $\sqrt{10\ 30\ 41} = 321$

द्वंद्व-योग विधि (Duplex Method)

Rule:

- The given number is first arranged in two-digit groups from right to left, and a single digit if any left over at the left hand end, is counted as a simple group by itself.
- The number of digits in the square root will be the same as the number of digit-groups in the given number itself including a single digit if any such there is.
- If a square root contains n digits, the square must consist of 2n or 2n–1 digits.
- And conversely, if the given number has n digits, the square root will contain n| 2 or (n+1)|2 digits.
- Group the number by placing a bar and put them in between the horizontal and vertical line bar, as shown in the given examples.

Working procedure of the Duplex Method

The working of the Duplex method is as simple as straight division. Let us take a few examples to understand the modus operandi of this method.

Example: Find the square root of 529 by using the Duplex method.

Solution:

- Group the number $\overline{5}\ \overline{29}$ by placing a bar over it.
- Put a horizontal and vertical line as shown below.

	5	29
Q		

- Since $2^2 < 5 < 3^2$, the first digit of the square root in the quotient column is 2. Double the quotient and set this down as divisor

4	5	29
Q	2	

Remainder $= 5 - 2^2 = 1$

- Put this remainder below the next dividend digit. Hence the next gross dividend = 12

4	5	$_1 29$
Q	2	3

- Divide 12 by 4 and put the quotient 3 in the quotient column.

$$12 \div 4 = 3$$

- Next dividend = 9
  Subtract the square of the quotient from the next dividend.
  Net dividend $= 9 - 3^2 = 0$
  Since no more digits are left, the square root of 529 is 23.

Example: Find the square root of 4225

Solution:

Group the number $\overline{42}\ \overline{25}$ by placing a bar over it.
Put a horizontal and vertical line as shown below.

	42	25
Q		

- Since $6^2 < 12 < 7^2$, the first digit of the square root in the quotient column is 6. Double the quotient and set this down as divisor.

12	42	25
Q	6	

Remainder $= 42 - 6^2 = 6$

- Put this remainder below the next dividend digit. Hence the next gross dividend = 62.

12	42	$_{6}25$
Q	6	

- Divide 62 by 12 and put the quotient 5 in the quotient column and the remainder 2 below 5.

12	42	$_{6}2 \quad _{2}5$
Q	6	5

- Next dividend = 25 Subtract the square of the quotient from the next dividend.
  Net dividend $= 25 - 5^2 = 0$
  Since no more digits are left, the square root of 4225 is 65.

Example: Find the square root of 20736

Solution:

- Group the number 2 07 36, by placing a bar over it.
- Put horizontal and vertical line as shown below.

	2	07 36
Q		

- Since $1^2 < 2 < 2^2$, the first digit of the square root in the quotient column is 1. Double the quotient and set this down as divisor.

2	2	07 36
Q	1	4

Remainder $= 2 - 1^2 = 1$.

- Put this remainder below the next dividend digit. Hence the next gross dividend = 10

2	2	$_1 07$	36
Q	1		

- Divide 10 by 2
  $10 \div 2 = 5$, $Q = 5$ and $R = 0$

Since the remainder cannot be taken as 0 until the whole operation is completed, we need to take the quotient as less than 5. This is because we cannot consider the remainder zero in the middle of division.

Hence the **Revised Quotient** = 4 and **Revised Remainder** = 2

2	2	$_1 0\ _2 7$	36
Q	1	4	

- Next dividend = 27
  Subtract the square of quotient from the next dividend.
  Net dividend = $27 - 4^2 = 11$
  Divide 11 by 2 and write the quotient (Q = 5) and remainder (R = 1) at their respective places.

2	2	$_1 0\ _2 7$	$_1 36$
Q	1	4	5

- Gross dividend = 13
  Net dividend = 13 – (Duplex of 45)
  $= 13 - 2 \times 4 \times 5$
  $= 13 - 40 = -27 < 0$
  (Since Net dividend is less than zero, we can't take the quotient (Q= 5) as taken above. Now for $11 \div 2$, **Revised** Q = 4 and **Revised R** = 3.)

2	2	$_1 0\ _2 7$	$_3 36$
Q	1	4	4

- Gross dividend = 33

  Net dividend = 33 – (Duplex of 44)

  = 33 – 2 x 4 x 4

  = 33 – 32

  = 1

  Divide 1 by 2 and put the quotient Q = 0 and remainder R = 1 in its proper place.

2	2	${}_{1}0\ {}_{2}7\ \ {}_{1}3\ {}_{1}6$
Q	1	4 4.0

- Next gross dividend = 16
- Net dividend = 16 – duplex of 4

  = 16 – 4^2

  = 0

  Since no more digits are left, the square root of 20736 is 144

Example: Find the square root of 25747576

Solution:

10	25	${}_{7}4\ {}_{4}5\ {}_{5}4\ {}_{5}7\ {}_{1}6$
Q	5	0 7 4.000

A step by step illustration has been done here for your convenient.

- Number of digits in the square root = 4
- The first digit of the square root = 5
- Remainder = 25 – 5^2 = 0, place it below 7.
- Next Dividend = 07 and Divisor = 2 x 5 = 10
- For 07 ÷10 , Q = 0 and R = 7, place it at proper place.
- Next dividend = 74 and corrected dividend

  = 74 – 0^2 = 74
- For 74 ÷10 , Q = 7 and R = 4
- Next dividend = 45 and corrected dividend

  = 45 – 2 x 0 x7 = 45
- For 45÷10 , Q= 4 and R = 5

- Since we have so far got 4 digits in the quotient column, the perfect square root is obtained. The next digit in the quotient column will give the remainder, if any.
- Remainder= $54 - 2 \times 0 \times 4 - 7^2 = 5$
- Next dividend = 57 and remainder
  $= 57 - 2 \times 4 \times 7 = 1$
- Next dividend = 16 and the last remainder
  $= 16 - 4^2 = 0$
- I think you have understood the Duplex method, therefore the next two examples given below are without much detail, though a brief description is provided here for your convenience.

Example: Find the square root of 45 31 98 24

Solution:

12	45	${}_{9}3\ {}_{9}1\ {}_{6}9\ {}_{3}8\ {}_{1}2\ 4$
Q	6	7 3 2 .000

A step by step illustration is done here for your convenient.

- Number of digits in the square root = 4
- The first digit of square root = 6
- Remainder = $45 - 6^2 = 9$, place it below 3.
- Next Dividend = 93 and Divisor = $2 \times 6 = 12$
- For $93 \div 12$, Q = 7 and R = 9, place it at the proper place.
- Next dividend = 91 and corrected dividend
  $= 91 - 7^2 = 42$
- For $42 \div 12$, Q = 3 and R = 6
- Next dividend = 69 and corrected dividend
  $= 69 - 2 \times 7 \times 3 = 27$
- For $27 \div 12$, Q= 2 and R = 3
- Since we have so far got 4 digits in the quotient column, the perfect square root is obtained. The next digit in the quotient column will give the remainder if any.

- Remainder= 38 – 2 x 7 x 2 – 3^2 = 1
- Next dividend = 12 and remainder
  = 12 – 2 x 3 x 2 =0
- Next dividend = 4 and the last remainder
  = 4 – 2^2 = 0

Example: Find the square root of 52443907 up to 1 decimal place.

Solution:

14	52	${}_{3}4\ {}_{6}4\ {}_{4}3\ {}_{13}9\ {}_{7}0\ 7$
Q	7	2 4 1. 8

A step by step illustration is done here for your convenient.

- Number of digits in the square root = 4
- The first digit of square root = 7
- Remainder = 52 – 7^2 = 3, place it below 4.
- Next Dividend = 34 and Divisor = 2 x 7 = 14
- For 34 ÷14, Q = 2 and R = 6, place it at proper place.
- Next dividend = 64 and corrected dividend
  = 64 – 2^2 = 60
- For 60 ÷14, Q = 4 and R = 4
- Next dividend = 43 and corrected dividend
  = 43 – 2 x 2 x 4 = 27
- For 27 ÷14 , Q= 1 and R = 13
- Since we have so far got 4 digits in the quotient column, the perfect square root is obtained. The next digit in the quotient column will give the digit after decimal.
- Remainder= 139 – 2 x 2 x 1 – 4^2 = 119
- Next dividend = 119 and Quotient = 119÷14 = 8 and Remainder = 119 –112= 7

9

Square Root of Irrational Number

Introduction

In the previous chapter we learnt the square root of a perfect number, but suppose you are in a situation where you need to find the square root of an irrational number? Now the big question is – how will you extract the square root of such a number? You may find yourself in such a situation while solving the problem of surds in arithmetic, mensuration or trigonometry. The conventional method of finding the square root of such a number is possible only through Long Division method and this method is time consuming and error prone in this case. Here is a method that helps you to find the square root of the irrational by mere observation, and with a little practice; you will be able to find the square root of such numbers mentally.

Vedic Method

There is no such reference of this method in the Vedic mathematics book written by Bharti Krishna Tirthaji Maharaj, but I have used the theory of Differential Calculus and the Vedic sutra, Vilokanam (विलोकनम्) in extracting the square root of such a number. The best part of this method is that we are free to choose the number of digits after the decimal. Generally, in competitive examinations, we use the technique of approximation in finding the square root of such numbers and take a maximum of two digits after the decimal place.

Square root of Irrational Number =

$$\sqrt{\text{Nearest Perfect square}} + \frac{\text{Deviation from Irrational number}}{2 \times \sqrt{\text{Nearest Perfect square}}}$$

Let us take few examples to understand the modus operandi of extracting the square root of a number that is not a perfect square.

Example 1: Find the square root of 79

Solution: Perfect square approaching 79 is 81.

$$\text{Deviation} = 79 - 81 = -2$$

$$\sqrt{79} = \sqrt{81} - \frac{2}{2 \times \sqrt{81}}$$

$$= 9 - {}^{1}/_{9} = 9 - 0.999 = 8.001$$

Example 2: Find the square root of 174

Solution: Perfect square approaching 174 is 169

$$\text{Deviation} = 174 - 169 = 5$$

$$\sqrt{174} = \sqrt{169} + \frac{5}{2 \times \sqrt{169}}$$

$$= 13 + {}^{5}/_{26}$$

$$= 13.\ 192$$

Example 3: Find the square root of 474

Solution: Perfect square approaching 474 is 484.

$$\text{Deviation} = 474 - 484 = -10$$

$$\sqrt{474} = \sqrt{484} - \frac{10}{2 \times \sqrt{484}}$$

$$= 22 - {}^{5}/_{22}$$

$$= 22 - 0.227 = 21.773$$

Example 4: Find the square root of 187

Solution: Perfect square approaching 187 is 196.

Deviation = 187 − 196 = − 9

$$\sqrt{187} = \sqrt{196} - \frac{9}{2 \times \sqrt{196}}$$

$= 14 - {}^{9}/_{28}$

$= 14 - 0.32$

$= 13.67$

Example 5: Find the square root of 28

Solution: Perfect square approaching 28 is 25.

Deviation = 28 − 25 = 3

$$\sqrt{28} = \sqrt{25} + \frac{3}{2 \times \sqrt{25}}$$

$= 5 + 3/_{10}$

$= 5.3$

Example 6: Find the square root of 34

Solution: Perfect square approaching 34 is 36.

Deviation = 34 − 36 = −2

$$\sqrt{34} = \sqrt{36} - \frac{2}{2 \times \sqrt{36}}$$

$= 6 - {}^{1}/_{6}$

$= 6 - 0.166 = 5.844$

Example 7: Find the square root of 204

Solution: Perfect square approaching 204 is 196.

Deviation = 204 − 196 = 8

$$\sqrt{204} = \sqrt{196} + \frac{8}{2 \times \sqrt{196}}$$

$= 14 + {}^{8}/_{28}$

$= 14 + 0.285$

$= 14.\ 285$

10

Cube

Introduction

When a number is multiplied by itself three times, the number so obtained is called the cube of that number. In general, a x a x a = a^3. Here is the cube of the first ten numbers.

Number	1	2	3	4	5	6	7	8	9	10
Cube	1	8	27	64	125	216	343	512	729	1000

Cubes of large numbers are not useful as far as the syllabus of CBSE, ICSE are concerned, though many questions based on cubing a number are asked in competitive examinations. In arithmetic too, while solving the problem of mensuration, we can't escape the tiring process of cubing the numbers. Cubing a large number is a tough task, as the traditional method only allows us to multiply the particular number three times or use the formula for binomial expansion. There is no room to experiment in the traditional method.

Traditional method of cubing:

$(988)^3 = 988 \times 988 \times 988$

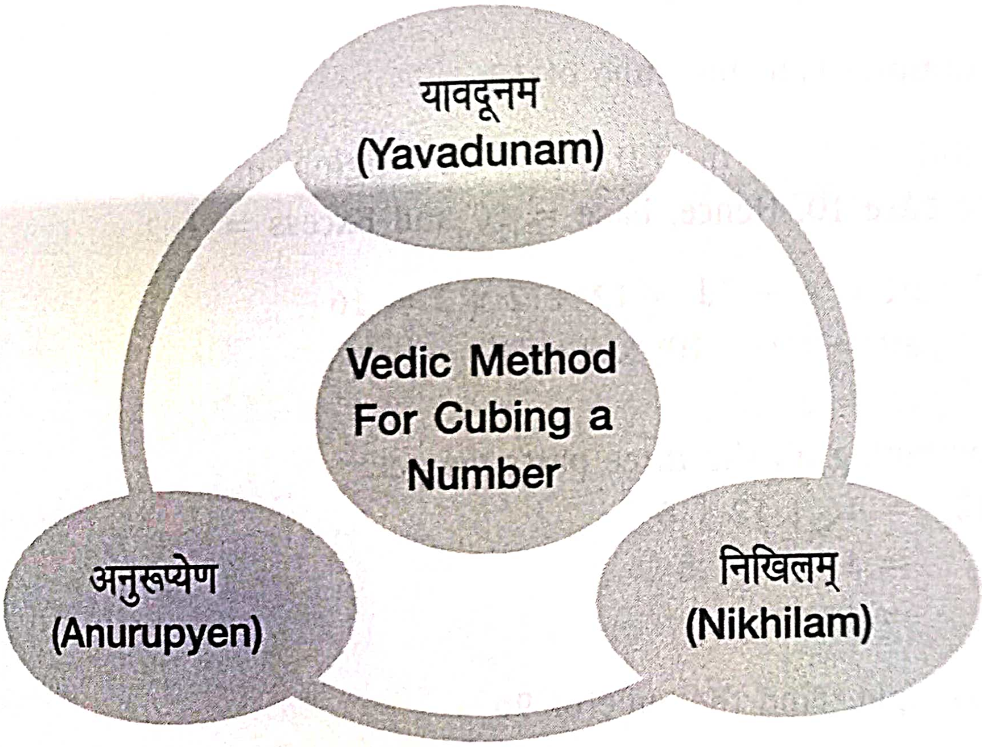
$$
\begin{array}{r}
988 \\
\times 988 \\
\hline
7904 \\
7904\times \\
8892\times\times \\
\hline
976144 \\
\times 988 \\
\hline
7809152 \\
7809152\times \\
8785296\times\times \\
\hline
964430272
\end{array}
$$

There is another traditional method which can be termed as better than the above method. The binomial expansion of $(a + b)^3$ and $(a - b)^3$ will reduce calculation time a bit but this method is still not suitable as far as its application in competitive examinations is concerned.

- $(988)^3 = (1000 - 12)^3 = (1000)^3 - 3 (1000)^2 \times 12 + 3 \times 1000 \times (12)^2 - (12)^3$
  $= 1000000000 - 36000000 + 432000 - 1728$
  $= 964430272$
  [Note: Applying the binomial expansion
  $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$]
- $(108)^3 = (100 + 8)^3 = (100)^3 + 3 \times (100)^2 \times 8 + 3 \times 100 \times (8)^2 + (8)^3$
  $= 1000000 + 240000 + 19200 + 512$
  $= 1259712$
  [Applying binomial expansion
  $(a-b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

Vedic mathematics presents several very interesting methods to find the cube of any number in a few seconds. Unlike the traditional method, it is easy, interesting and short. It reduces the time by 1/10th and that's why the Vedic method is far superior to the traditional method of finding cubes.

Vedic Sutra for cubing a number



1. Yavadunam (यावदूनम)

This formula works better when the number to be cubed is near the base. The base should be in the form of 10^n, where n is a natural number. This formula has limited application.

Working Rule

- Check whether the number is near the base 10, 100, 1000... or not
- Find the excess or deficit number from the base.
- The whole operation is to be performed in three parts. In the 1st part, add twice the excess/ deficit to the original number.
- 2nd part = New Excess (Number obtained in 1st part – base) x original excess/deficit
- 3rd part = cube of excess.

Mathematically, if a = original number and d = deviation (Excess/ Deficit from the Base) then the whole operation can be summed up as —

$a^3 = a+2d \ / \ [\ (a + 2d\ ) - \text{base}\ ] \times d \ / \ d^3$

let us take some example.

Example: Find the cube of 12

Solution: Here, in $(12)^3$; a = 12 (original number) is nearer to the base 10. Hence, base = 10 and Excess = + 2.

1^{st} part = $a + 2d = 12 + 2 \times 2 = 16$
2^{nd} part = $(16 - 10) \times 2 = 12$
3^{rd} part = 2^3
Combining all the three parts we get,
$(12)^3 = 16 \mid 12 \mid 8$
$= 17 \mid 2 \mid 8$
$= 1728$

Example: Find the cube of 96

Solution: Here, in $(96)^3$; a = 96 (original number) is nearer to the base 100. Hence, base = 100 and deficit = – 4

1^{st} part = $a + 2d = 96 - 2 \times 4 = 88$
2^{nd} part = $(88 - 100) \times -4 = 48$
3^{rd} part = $(-4)^3 = -64$
Combining all three parts we get,
$(96)^3 = 8\,8 \mid 4\,8 \mid -64$
$= 88 \mid 47 + 1 \mid -64$
$= 88 \mid 47 \mid 100 - 64$
$= 88 \mid 47 \mid 36$
(3^{rd} part is negative, so add 100 to the negative part to make it 100 – 64 = 36. Subtract 1 from the previous part, thus making 48 to 47.)

Example: Find the cube of 105

Solution: a = 105, Base = 100. Excess = 5

1^{st} part = a + 2d = 105 + 2 x 5 = 115
2^{nd} part = (115 – 105) x 5 = 50
3^{rd} part = 5^3 = 125
Combining all three parts, we get,

$(105)^3$ = 115 | 7 5 | 1 25
= 115| 75 + 1 | 25
= 115 | 76 | 25

(The number of digits in each part depends on the number of zeros in the base digit. If the base is taken as 100, the number of digits permissible in each part is 2.)

2. Anurupyen (अनुरूप्येण):

Anurupyen Vedic sutra is based on the concept of Geometric Progression. In a geometric series, every next number is the multiple of some constant ratio called **r**, or the common ratio.
If a = first term, r = common ratio, then the n^{th} term (t_n) = $a\,r^{n-1}$.
If a, b, c are in Geometric series then,
Second term/ first term = third term/ second term = r

b / a = c / b

or, b^2 = ac

Here are a few examples of geometric series.

i) 2, 8, 32 ...

ii) 5, 25, 125, 625,...

In the first example (i), a = first term and r = common ratio = 8/2 = 4

$T_n = 2\,(4)^{n-1} = 2.\,2^{2(n-1)}$

In the second example, a = 5 and r = 5, hence $T_n = 5\,(5)^{n-1}$

Rule:

- First, take the cube of the first digit (a) and multiply it with the common ratio (b/a), in a row of 4 figures

- Double the second and the third number and put it down under the second and third numbers. Finally add up the two rows

This can be further clarified with the help of this table

	1^{st}	2^{nd}	3^{rd}	4^{th}
$(ab)^3 =$	a^3	a^2b	ab^2	b^3
	+	$2a^2b$	$2ab^2$	
	a^3	$3a^2\ b$	$3\ a\ b^2$	b^3

In the binomial expansion we know,

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.

The Anurupyen method is just an extension of the above expansion. This can be simplified again if we take the help of geometric progression.

$$
\begin{array}{lcccc}
(ab)^3 = & a & ar & ar^2 & ar^3 \\
 & + & 2ar & 2ar^2 & \\
\hline
 & a & 3\ ar & 3ar^2 & a\ r^3
\end{array}
\qquad \text{(Where } r = b/a)
$$

Let us take a few examples to understand the basic modus operandi.

Example: Find the cube of 12?

Solution: $(12)^3 = ?$

Here, say **a** = 1 and **b** = 2

r = common ratio = b/a = 2/ 1 = 2

Hence the table arrangement will be as follows:–

$$
\begin{array}{lcccc}
(12)^3 = & 1 & 2 & 4 & 8 \\
 & + & 4 & 8 & \\
\hline
 & 1 & 7 & 2 & 8
\end{array}
$$

Important points:

1. *If you start with the cube of the first digit and multiply with the geometric ratio up to the next three numbers*

the 4^{th} *number of the series will be the cube of the* 2^{nd} *digit.*

2. *Addition should be done from right to left, keeping only a single digit at a time and the remaining digit will be carried-over to the next column, and so on.*

Example: Find the cube of 15

Solution: $(15)^3 = ?$

Here say **a** = 1 and **b** = 5

r = common ratio = b/a = 5 / 1 = 5

Hence the table arrangement will be as follows

$$
\begin{array}{lllll}
(15)^3 = & 1 & 5 & 25 & 125 \\
 & + & 10 & 50 & \\
\hline
 & 1 & \underline{1}\,5 & \underline{7}\,5 & \underline{1\,2}\,5
\end{array}
$$

$= 3\ 3\ 75$

The excess digit, leaving the unit digit from each column (from right to left) is transferred to the next column. In the above example, the excess digit is underlined.

Example: Find the cube of 19

Solution: $(19)^3 = ?$

Here say **a** = 1 and **b** = 9

r = common ratio = b/a = 9/ 1 = 9

Hence the table arrangement will be as follows:-

$$
\begin{array}{llll}
1 & 9 & 81 & 729 \\
 & 18 & 162 & \\
\hline
1 & \underline{2}\,7 & \underline{24}\,3 & \underline{72}\,9
\end{array}
$$

$= 6\ 8\ 5\ 9$

Example: Find the cube of 32

Solution: $(32)^3 = ?$

Here, say **a** = 3 and **b** = 2

r = common ratio = b/a = 2/3

Hence the first line of the table arrangement will be as follows:-

$(32)^3 = 3^3\ (27)\quad 27 \times 2/3\ (18)\quad 18 \times 2/3\ (12)\quad 12 \times 2/3\ (8)$

=	27	18	12	8
		36	24	
=	2 7	<u>5</u> 4	<u>3</u> 6	8
=	3 2 7 6 8			

Example: Find the cube of 46

Solution: $(46)^3 = ?$

Here say **a** = 4 and **b** = 6

r = common ratio = b/a = 6 / 4

Hence ,the table arrangement will be as follows:-

$(46)^3 = 4^3\ (=64)\quad 64 \times 6/4\ (=96)\quad 96 \times 6/4\ (=144)\quad 144 \times 8/9\ (=512)$

	64	96	144	216
		192	288	
	64	288	432	216
=	64	288	432	<u>21</u>6
=	64	288	<u>45</u>3	6
=	64	<u>33</u>3	3	6

= 97336

Example: Find the cube of 105

Solution: $(105)^3 = ?$

Here a = 10 and b = 5, hence common ratio = 5 / 10 = 1/ 2

$(105)^3 =$	1000	500	250	125
	+	1000	500	
	1000	1500	750	125

Simple addition as done in the above example will give you wrong answer. Now the big question is what next?-

The answer is very simple. If the number is between 100 – 999, put 1, 2, and 3 zeros after each digit as shown here-

$$1000000 + 150000 + 7500 + 125 = 1157625$$

In case the number is above 1000, you need to put 2, 4 and 6 zeros after each digit. Let us find the cube of a number above 1000.

Example: Find the cube of 1001

Solution: $(1001)^3 = ?$

Here, a = 10 and b =01, hence common ratio = 01 / 10 = 1/ 10

$(1001)^3 =$	1000	100	10	1
	+	200	20	
	1000	300	30	1

Since the number is above 1000, start putting 6, 4 and 2 zeros from the extreme left before adding. Once the process of putting zeros get complete, you can simply add to get the result.

$(1001)^3 = 1000000000 + 3000000 + 3000 + 1 = 1003003001$

3. Nikhilam (निखिलम्) Vedic Sutra

Rule

The modus operandi of the Nikhilam Sutra can be described in the following steps:-

- First, take the deviation of the number to be cubed from its base. The base should be the multiple of 10. If the base is 10 , 100,1000------ then sub-base = 1; on the other hand, if the base = 40, then the sub- base = 4, because 40 = 4 x 10,

- The whole cubing process then involves 3 steps.

  A) (Number to be cubed + 2 x deviation from the base) x $(\text{sub-base})^2$

  B) $\{3 \times (\text{deviation})^2\} \times$ sub-base

  C) $(\text{Deviation})^3$

- If there is no sub-base, then the calculation becomes very easy.

Let us take some example to understand the modus-operandi of this method.

Example: Find the cube of 25 using Nikhilam Sutra

Solution: 25 is nearer to the base 20 (2 x 10), hence–

$$\text{Deviation} = 25 - 10 = 5,\ \text{sub- base} = 2$$

$$= (25 + 2 \times 5) \times 2^2 \mid (3 \times 5^2) \times 2 \mid 5^3$$

$$= 140 \mid 1\underline{5}0 \mid \underline{12}5$$

$$= 14\underline{0} \mid \underline{16}2 \mid 5$$

$$= 15625$$

Example: Find the cube of 58 by using Nikhilam Sutra

Solution: 58 = 5 x 10 + 8

base = 10 sub- base = 10 and excess = 8

$$\text{Hence } (58)^3 = [\underbrace{58 + 2 \times (8)}_{1^{st}\text{ term}}] \times 5^2 \mid \underbrace{3 \times (8)^2 \times 5}_{2^{nd}\text{ term}} \mid \underbrace{(8)^3}_{3^{rd}\text{ term}}$$

$$= 1850 \mid 960 \mid 512$$

$$= 1850 \mid \underline{10}11 \mid 2$$

$$= 1951 \mid 1 \mid 2$$

Hence $(58)^3 = 195112$

Example: Find the cube of 98 by using Nikhilam Sutra?

Solution: 98 is nearer to the base 100

$$\text{Deviation} = 98-100 = -2$$

$$\text{Hence } (98)^3 = \underbrace{98 + 2 \times (-2)}_{1^{st}\text{ term}} \mid \underbrace{3 \times (-2)^2}_{2^{nd}\text{ term}} \mid \underbrace{(-2)^3}_{3^{rd}\text{ term}}$$

$$= 94 \mid 12 \mid -8$$

$$= 94 \mid 11 \mid 100-8$$

$$= 94 \mid 11 \mid 92$$

Hence $(98)^3 = 941192$

Example: Find the cube of 104 by using Nikhilam Sutra

Solution: Working base = 100

$$\text{Deviation} = 104 - 100 = 4$$

$$(104)^3 = 104 + 4 \times 2 \mid 3 \times 4^2 \mid 4^3$$

$$= 112 \mid 48 \mid 64$$

$$= 1124864$$

(Since the base =100, there should be 2 digits in each digit separator)

Example: Find the cube of 997 by using Nikhilam Sutra

Solution: Working base = 1000

$$\text{Deviation} = 997 - 1000 = -3$$

$$(997)^3 = \underbrace{997 + (-3) \times 2}_{1^{st}\text{ part}} \mid \underbrace{3 \times (-3)^2}_{2^{nd}\text{ part}} \mid \underbrace{(-3)^3}_{3^{rd}\text{ part}}$$

$$= 991 \mid 27 \mid -27$$

$$= 991 \mid 026 \mid 1000 - 27$$

$$= 991\ 026\ 973$$

Since the base is 1000, there should be 3 digits in each digit separator. Therefore, 26 in the second part should be changed to 026. In the 3rd part, there is a negative sign, so subtract it from the base 1000. Hence in the 3rd part, we will have, 1000 – 27 = 973.This change will be adjusted by reducing 1 from the 2nd part. Thus, 27 in the second part now become 27 – 1 = 26.

$$(997)^3 = 991026973$$

11

Cube Root

Introduction

The cube of a number is that number raised to the power 3. Thus, the cube of **a** is $\mathbf{a^3}$.

Suppose $a^3 = x$

$$\Rightarrow a = x^{1/3}$$

The present curriculum taught in our school uses only Factorization of a number and thus is a cumbersome technique. Moreover, it is a time consuming process and thus not fit for the competitive examinations. Suppose you are asked to find the cube root of a number having 10 digits, the present method taught in our school curriculum will take you 5–10 minutes, but if you are familiar with the Vedic method, it will take only 20–25 seconds to extract the cube root of even 8–9 digits. There are around 4 or 5 Vedic methods written in the original work of Sri Swami Krisna Tirthaji Maharaj.

Let us see one example.

Example: Find the cube root of 830584

Solution:

2	592704
2	296352
2	148176
2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Hence, $592704 = \underbrace{2 \times 2 \times 2} \times \underbrace{2 \times 2 \times 2} \times \underbrace{3 \times 3 \times 3} \times \underbrace{7 \times 7 \times 7}$

$= 2 \times 2 \times 3 \times 7$

$= 84$

The Vedic method will instantly give you the answer 84 by mere observation.

The great Indian Astronomer and Mathematician Aryabhatta has also mentioned in his book, *Ganita Pada,* a method to extract the cube root of any number, but the method is too complex to understand. The fifth sloka of **Aryabhatta's** book **Ganita-Pada** reads as follows.

अघनाद भजेद द्वितीयात त्रिगुणेन घनस्य मूलवर्गेण ।
वर्ग स्त्रिपूर्व गुणितः शोध्यः प्रथमाद धनस्य घनात् ।।

Before Aryabhatta's, there was no proper method to extract the cube root of any number. All the mathematicians after him followed his method with some modifications here or there. This elegant method described by Aryabhatta has more symmetry with the Vedic Method used by Swami Bharathi Krishna Tirtathji Maharaj

Before we move further, look at the following table carefully. This table will help you to determine the unit digit of a cube root.

Table 1

If a cube ends in	The unit digit of a cube root will be
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

From the above table, we can conclude the following facts:-

1, 4, 5, 6, 9, and 0 repeat themselves in the cube ending.

2, 3, 7 and 8 have an inter–play of complements from 10.

The Left digit of a cube root having more than 7 digits, or the ten's digit of a cube root having less than 7 digits, can be extracted with the help of the following table.

Table 2

Left-most pair of the cube root	Nearest cube root
1 – 7	1
8– 26	2
27 – 63	3

64 – 124	4
125 – 215	5
216 – 342	6
343 – 511	7
512 – 728	8
729 – 999	9

There is another very important table that will help you to find the cube in ambiguous cases. The details will follow the next table.

Table 3

Number	Cube	Beejank of cube
1	1	1
2	8	8
3	27	0
4	64	1
5	125	8
6	216	0
7	343	1
8	512	8
9	729	0

The beejank (बीजांक) of a number is determined through the Casting out Nines (नवशेष) method described in the book. The Vedic method described here is effective as long as the number whose cube root is to be extracted is a perfect cube.

Now the bigger question is – ***How will you determine that the given number is a perfect cube or not?***

The Vedic method has the answer of your question. Find the digit sum of that number by using the Casting out Nines method. If the digit sum of that number is found to be 0, 1 or 8 then the given number is a perfect cube.

Example: Is 1729 a perfect cube?

Solution: The digit sum of 1729 is 1 + 7 + 2 + 9 = 1

Hence, it is a perfect cube

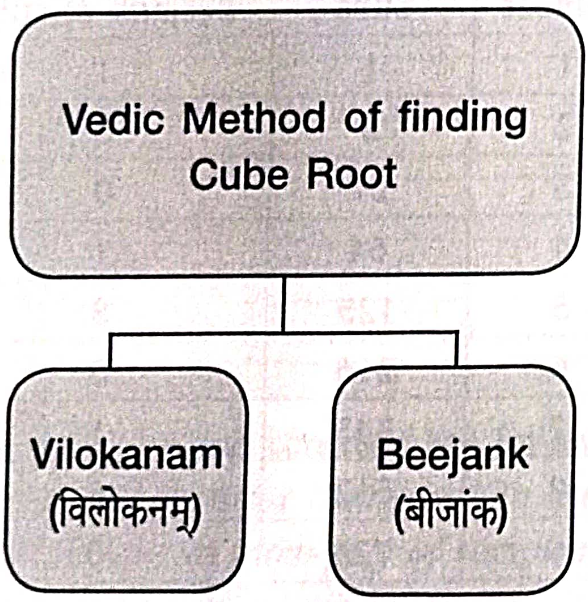
Example: Is 9528127 a perfect cube?

Solution: The digit sum of 9528127 is

9 +5 +2 + 8 +1 +2 + 7 = 7

Hence, it is not a perfect cube.

Vedic Method



Meaning of Vedic Sutra:

1. **Vilokanam (विलोकनम्):** The literal meaning of this sutra is – by inspection or observation. This sutra helps you to determine the unit digit and ten's digit of a perfect cube root in a few seconds, if the number whose cube root is to be extracted is less than 7 digits. Moreover, if the number whose cube root is to be extracted is more than 7 digits, this sutra will help you to find the L(Left digit) and R(Right digit) of the cube root.

2. **Beejank method** (बीजांक विधि): Many a time, while extracting the cube root of a number having more than 7 digits, you will get two results for M (Middle term) and will be little confused in deciding which one is the suitable value for M and the Beejank method will be there to help you out. The beejank of a number is extracted using the Casting out Nines method.

Let us now start with the Vedic method of finding cube root. For the convenience of readers, this chapter has been divided into two segments.

Case 1: Cube Root of a number having less than 7 digits
Case 2: Cube Root of a number having more than 7 digits, but less than 10 digits
Case 3: Cube Root of a number greater than 7 digits, but ending with even numbers.

General rule to follow in all the three cases:

1. Form the groups of three digits from right to left. The number of groups thus formed will give you an idea of the number of digits in its cube root.
   i. A number having 4, 5 or 6 digits will have a cube root of 2 digits number.
   ii. A number having 7, 8 or 9 digits will have a cube root of 3 digits number.
   iii. A number having 10, 11 or 12 digits will have a cube root of 4 digit numbers.

   Example: a) $\overline{3}\ \overline{4\ 3\ 1}\ \overline{3\ 3\ 1}$ has three groups, hence its cube root will contain 3 digits.
   b) $\overline{3\ 9}\ \overline{3\ 0\ 4}$ has two groups, hence its cube root will have 2 digits.
2. Find the digit of the unit place by observing the ght-most pair and the left digit of the cube root by observing the left-most pair. Table 1 will help you to determine the

unit digit of cube root and Table 2 will give you the Left digit of the cube root.

Case 1: Cube Root of a number having less than 7 digits.

The Vedic method of extracting the cube root of a number having less than 7 digits is done by the Vilokanam (विलोकनम्) method. Form the group of three digits from right to left as discussed above and find the unit digit and ten's digit with the help of Table 1 and Table 2.

Example 1: Find the cube root of 3375

Solution: The Vedic method as described above will have the following steps.

1. Place the bar over the number making the group $\overline{3}\ \overline{375}$
2. Since the bar is placed on two numbers, the cube root of this number contains two digits.
3. The first bar falls on unit digit 5, and with the help of Table 1 we can say that the unit digit of the cube root is 5.
4. The next bar falls on 3, and looking upon the Table 2 we see that the left digit of the cube root will be 1.

Hence $\sqrt[3]{3375} = 15.$

Example 2: Find the cube root of 97336

Solution:

1. Place the bar over the number from right to left. $\overline{9\ 7}\ \overline{3\ 3\ 6}$
2. The cube ends with 6 hence the unit digit of cube root is 6 (see Table 1 for the reference)
3. The ten's digit of the cube root is 4, because $4^3 < 97 < 5^3$

Hence, $\sqrt[3]{9\ 7\ 3\ 3\ 6} = 4\ 6$

Example 3: Find the cube root of 941192

Solution:

1. Place the bar over the number from left to right, leaving two digits at a time.
   $\overline{9\ 4\ 1}\ \overline{1\ 9\ 2}$
2. Since the bar is placed on two numbers, the cube root will contain only two digits.
3. Since the unit digit of this number is 2, the unit digit of cube root is 8.
4. For the ten's digit, take the least of
   $9^3 < 941 < 10^3$
   Ten's digit = 9

   Hence, $\sqrt[3]{941192} = 98$

Case 2: Cube root of a number having more than 7 digits but less than 10 digits.

The cube root of a number having more than 7 digits through the Vedic method needs a little practice and patience initially. A little practice and understanding of the fundamental concepts will help you to extract the cube root of much bigger number in less than 10 seconds. As discussed above, the cube root of more than 7 digit numbers will involve three digits, the unit digit (R) and the digit at the hundred's place (L) can be obtained quite easily with the help of Vilokanam (विलोकनम्) method, whereas the middle digit (M) will be determined with the help of the Beejank (बीजांक या नवशेष) method.

Rule:

1. Denote the left digit by L, the middle digit by M and the right digit by R.
2. Subtract R^3 from the number and cancel the last zero.
3. The Middle digit of the cube root is obtained by $3R^2M$. Place a different value of M so that we may reach to the

unit digit of the number obtained in the previous step. In case you obtain more than one value of M, check by theBeejank method (Refer Table 3) which value of M is best suited in this case.

4. The cube root of such a number = L M R.

Example 4: Find the cube root of 76765625

Solution:

1. Group the number from right to left by placing the bar.

$$\sqrt[3]{7\;6\;\overline{7}\;6\;5\;\overline{6}\;2\;5}$$

2. The unit digit of the cube root is 5 (Refer Table 1). Hence R = 5.
3. Moreover $4^3 < 76 < 5^3$, so L = 4 (Refer Table 2)
4. Subtract R^3 from the number and forget the last digit (0) obtained after subtraction.

$$\begin{array}{r} 7\;6\;7\;6\;5\;6\;2\;5 \\ -1\;2\;5 \\ \hline 7\;6\;7\;6\;5\;5\;0\;\cancel{0} \end{array}$$

5. Middle digit of the cube root is obtained by $3\ R^2\ M$.
$3\ R^2\ M. = 3 \times 5^2 \times M = 75\ M$
6. Now, the bigger question is what value of M should be put in 75 M, so that the unit digit of the result obtained is equal to the unit digit of 767550 obtained in step 3. We find here 4 options – they are 2, 4, 6 and 8. Hence, M = 2, 4, 6 or 8. M = 8 can be ruled out as 75 x 8 = 600 > 550. Still, we are left with 3 options.
7. This situation can be handled by applying the Beejank method.

Beejank of 76765625 = 8
Beejank of $(425)^3$ = 8
Beejank of $(445)^3$ = 1
Beejank of $(465)^3$ = 0

This clearly shows that our answer is 425.

Example 5: Find the cube root of 84604519

Solution:

1. Group the number by placing the bar as shown below. Since there are three groups,the cube root will have three digits.

$$\sqrt[3]{8\ 4\ 6\ 0\ 4\ 5\ 1\ 9}$$

2. The unit digit of the cube root is 9. Hence R = 9 (Refer Table 1)
3. Moreover $4^3 < 84 < 5^3$, so L = 4 (Refer Table 2)
4. Subtract R^3 from the number and forget the last digit (0) obtained after subtraction.

$$\begin{array}{r} 8\ 4\ 6\ 0\ 4\ 5\ 1\ 9 \\ -7\ 2\ 9 \\ \hline 8\ 4\ 6\ 0\ 3\ 7\ 9\ \cancel{0} \end{array}$$

5. Middle digit of the cube root is obtained by 3 R^2 M.
R^2 M. = 3 x 9^2 x M = 243M
6. The unit digit of the number obtained in step 5 is 9 so put M = 3 to obtain the Middle digit.
7. We have thus got
L = 4 M = 3 and R = 9

$$\sqrt[3]{8\ 4\ 6\ 0\ 4\ 5\ 1\ 9} = 4\ 3\ 9$$

Example 6: Find the cube root of 279726264

Solution:

1. Placing the bar over the number we find that there are three groups of numbers, so the cube root will have three digits.

$$\sqrt[3]{2\ 7\ 9\ 7\ 2\ 6\ 2\ 6\ 4}$$

2. From the Table 1, we can say that the unit digit of the cube root is 4. Hence R = 4.

3. Moreover, $6^3 < 33 < 7^3$, so L = 6 (Refer Table 2)
4. Subtract R^3 from the number and cancel out the last zero.

$$\begin{array}{r} 2\ 7\ 9\ 7\ 2\ 6\ 2\ 6\ 4 \\ -\ 6\ 4 \\ \hline 2\ 7\ 9\ 7\ 2\ 6\ 2\ 0\ \cancel{0} \end{array}$$

5. Middle digit of the cube root is obtained by $3\ R^2\ M$. Here, $3\ R^2\ M. = 3 \times 4^2 \times M = 48\ M.$
6. We are now looking for a suitable value of M so that the unit digit of 48 M becomes equal to the unit digit of the number obtained in step 4. Thus M = 5.
7. We have thus got
L = 6 M = 5 and R = 4
$\sqrt[3]{2\ 7\ 9\ 7\ 2\ 6\ 2\ 6\ 4} = 6\ 5\ 4.$

Case 3: Cube Root of a number greater than 7 digits, but ending with even number.

Many a time, while extracting the cube root of a number having its unit digit, even you may get two values of M and it becomes difficult to ascertain the exact value of M. In such a case, you may reach to the exact answer by two ways.

- Divide the number whose cube root has to be extracted by 8, until an odd cube emanates
- Use the Beejank (बीजांक) method to reach the exact value of M

Let us take some examples to understand this case.

Example 7: Find the cube root of 1906624

Solution: Since this is an even number, to avoid the ambiguous case in extracting the cube root, we need to divide the number by 8 until an odd cube emanates.

$$\begin{array}{r|l} 8 & 1\,9\,0\,6\,6\,2\,4 \\ \hline 8 & 2\,3\,8\,3\,2\,8 \\ \hline & 2\,9\,7\,9\,1 \end{array}$$

The cube root of 29761 can be extracted merely by the Vilokanam (विलोकनम्) method using Table 1 and Table 2. You may refer to case1.

Group the number

$$\overline{2\,9}\ \overline{7\,9\,1}$$

Since there are two groups, the cube root of 29791 contains a two digit number.

Unit digit = 1

Ten's digit = 3

Hence, the cube root of 29791 = 31

We have,

$$\sqrt[3]{1906624} = \sqrt[3]{8 \times 8 \times 29791}$$

$$= 2 \times 2 \times 31 = 124$$

Example 8: Find the cube root of 51478848?

Solution: Since this is an even number, to avoid the ambiguous case in extracting the cube root, we need to divide the number by 8 until an odd cube emanates.

$$\begin{array}{r|l} 8 & 51478848 \\ \hline 8 & 6434856 \\ \hline & 804357 \end{array}$$

The cube root of 804357 can be extracted merely by the Vilokanam (विलोकनम्) method using Table 1 and Table 2. You may refer to case 1.

Group the number

$$\overline{804}\ \overline{357}$$

Since there are two groups, the cube root of 804357 contains a two digit number.

Unit digit = 3 (Refer Table 1)

Since, $9^3 < 804 < 10^3$

Hence, Ten's digit = 9 (Refer Table 2)

We have,

$$\sqrt[3]{51478848} = \sqrt[3]{8 \times 8 \times 804357}$$

$$= 2 \times 2 \times 93 = 372$$

Let us extract the cube root by the ***Beejank method***.

Here the method is slightly changed. In case 2, we have subtracted R^3 from the number and neglected the last zero but here L^3 is subtracted. You may consider either of these cases.

1. Group the number from right to left by placing the bar.

   $\sqrt[3]{5\ \overline{1\ 4\ 7}\ \overline{8\ 8\ 4\ 8}}$

2. The unit digit of the cube root is 8 (Refer Table 1). Hence R = 2.
3. More over $3^3 < 51 < 4^3$, so L = 3 (Refer Table 2)
4. Subtract L^3 from the number.

$$\begin{array}{l} 5\ 1\ 4\ 7\ 8\ 8\ 4\ 8 \\ \underline{-\ 2\ 7\qquad\qquad\qquad} \\ \ 1\ 4\ 4 \end{array}$$

5. Middle digit of the cube root is obtained by $3\ R^2\ M$.

   $3\ R^2\ M. = 3 \times 2^2 \times M = 12\ M$

6. Now, the bigger question is what value of M should be put in 12 M so that the unit digit ends with 4. We find here 2 options they are 2, and 7.
7. This situation can be handled by applying the Beejank method

   Beejank of 51478848 =0

   Beejank of $(322)^3 = 1$

   Beejank of $(372)^3 = 0$

This clearly shows that our answer is 372.

Example 9: Find the cube root of 7738893352

Solution: Since this is an even number, to avoid the ambiguous case in extracting the cube root, we need to divide the number by 8 until an odd cube emanates.

$$\begin{array}{r|l} 8 & 7738893352 \\ \hline & 967361669 \end{array}$$

The cube root of 967361669 can be extracted by the ***Beejank method***.

1. Group the number from right to left by placing the bar

   $\sqrt[3]{\overline{967}\ \overline{361}\ \overline{669}}$

2. The unit digit of the cube root is 9 (Refer Table 1). Hence $R = 9$
3. Moreover $9^3 < 967 < 10^3$, so $L = 9$ (Refer Table 2)
4. Subtract R^3 from the number and neglect the last zero.

$$\begin{array}{r} 9\,6\,7\,3\,6\,1\,6\,6\,9 \\ -\ 7\,2\,9 \\ \hline 9\,6\,7\,3\,6\,0\,9\,4\,\cancel{0} \end{array}$$

5. Middle digit of the cube root is obtained by $3\ R^2\ M$.

   $3\ R^2\ M. = 3 \times 9^2 \times M = 243\ M$

6. Now, the bigger question is what value of M should be put in 243 M, so that the unit digit ends with 4.

This clearly shows that $M = 8$

Hence,

$$\sqrt[3]{967\ 361\ 669} \qquad = 9\ 8\ 9$$

Therefore,

$$\sqrt[3]{7738893352} = \sqrt[3]{8 \times 967361669} \quad = 2 \times 989 = 1978$$