UNIT-IV
PROBABILITY

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PROBABILITY

- **Probability denotes the possibility of the outcome of any random event. The meaning of this term is to check the extent to which any event is likely to happen.**
- For example, when we flip a coin in the air, what is the possibility of getting a head? The answer to this question is based on the number of possible outcomes. Here the possibility is either head or tail will be the outcome. So, the probability of a head to come as a result is 1/2.

PROBABILITY……………

- **P(E) = Number of Favourable Outcomes/Number of total outcomes**
- **P(E) = n(E)/n(S)**
- Here,
- n(E) = Number of event favourable to event E
- n(S) = Total number of outcomes

EXAMPLE

- **There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?**

SOLUTION

- Here,

n(E) = Number of event favourable to event E

= 2

n(S) = Total number of outcomes

=6

Therefore,

P(E) = n(E)/n(S)

= 2/6

= 1/3

EXAMPLE

- There is a container full of coloured bottles, red, blue, green and orange. Some of the bottles are picked out and displaced. Sumit did this 1000 times and got the following results:

  No. of blue bottles picked out: 300

  No. of red bottles: 200

  No. of green bottles: 450

  No. of orange bottles: 50

a) What is the probability that Sumit will pick a green bottle?

b)If there are 100 bottles in the container, how many of them are likely to be green?

SOLUTION

- **a) What is the probability that Sumit will pick a green bottle?**

Ans: For every 1000 bottles picked out, 450 are green.

Therefore, P(green) = 450/1000 = 0.45

- **b) If there are 100 bottles in the container, how many of them are likely to be green?**

Ans: The experiment implies that 450 out of 1000 bottles are green.

Therefore, out of 100 bottles, 45 are green.

Question 1: Find the probability of ‘getting 3 on rolling a die’.

Question 2: Draw a random card from a pack of cards. What is the probability that the card drawn is a face card?

SOLUTION 1

- **Solution:**
- Sample Space = S = {1, 2, 3, 4, 5, 6}
- Total number of outcomes = n(S) = 6
- Let A be the event of getting 3.
- Number of favourable outcomes = n(A) = 1
- i.e. A = {3}
- Probability, P(A) = n(A)/n(S) = 1/6
- Hence, P(getting 3 on rolling a die) = 1/6

SOLUTION 2

- A standard deck has 52 cards.
- Total number of outcomes = n(S) = 52
- Let E be the event of drawing a face card.
- Number of favourable events = n(E) = 4 x 3 = 12 (considered Jack, Queen and King only)
- Probability, P = Number of Favourable Outcomes/Total Number of Outcomes
- P(E) = n(E)/n(S)
- = 12/52
- = 3/13
- P(the card drawn is a face card) = 3/13

PROBLEM

Two coins are tossed 500 times, and we get:

Two heads: 105 times

One head: 275 times

No head: 120 times

Find the probability of each event to occur.

SOLUTION

Let us say the events of getting two heads, one head and no head by E_1, E_2 and E_3, respectively.

$P(E_1) = 105/500 = 0.21$

$P(E_2) = 275/500 = 0.55$

$P(E_3) = 120/500 = 0.24$

The Sum of probabilities of all elementary events of a random experiment is 1.

$P(E_1)+P(E_2)+P(E_3) = 0.21+0.55+0.24 = 1$

PROBLEM

- **A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of**

Distance(in km)	Less than 4000	4000 to 9000	9001 to 14000	More than 14000
Frequency	20	210	325	445

If a tyre is bought from this company, what is the probability that :

(i) it has to be substituted before 4000 km is covered?

(ii) it will last more than 9000 km?

(iii) it has to be replaced after 4000 km and 14000 km is covered by it?

SOLUTION

- (i) Total number of trials = 1000.
- The frequency of a tyre required to be replaced before covering 4000 km = 20
- So, $P(E_1) = 20/1000 = 0.02$
- (ii) The frequency that tyre will last more than 9000 km = 325 + 445 = 770
- So, $P(E_2) = 770/1000 = 0.77$
- (iii) The frequency that tyre requires replacement between 4000 km and 14000 km = 210 + 325 = 535.
- So, $P(E_3) = 535/1000 = 0.535$

Terms Used in Probability and Statistics

- There are various terms utilised in the probability and statistics concepts, Such as:

1. Random Experiment
2. Sample Space
3. Random variables
4. Expected Value
5. Independence
6. Variance
7. Mean

Random Experiment

- An experiment whose result cannot be predicted, until it is noticed is called a random experiment. For example, when we throw a dice randomly, the result is uncertain to us. We can get any output between 1 to 6. Hence, this experiment is random.

Sample Space

- A sample space is the set of all possible results or outcomes of a random experiment. Suppose, if we have thrown a dice, randomly, then the sample space for this experiment will be all possible outcomes of throwing a dice, such as;
- Sample Space = { 1,2,3,4,5,6}

Random Variables

- **The variables which denote the possible outcomes of a random experiment are called random variables. They are of two types:**

1. Discrete Random Variables

   (variables take only those distinct values which are countable.)

2. Continuous Random Variables

   (could take an infinite number of possible values.)

EVENTS

- **In Probability, the set of outcomes of an experiment is called events.**
- There are different types of events such as:

1. Independent events,
2. Dependent events,
3. mutually exclusive events

and so on.

Independent Event

- **When the probability of occurrence of one event has no impact on the probability of another event, then both the events are termed as independent of each other.**
- For example, if you flip a coin and at the same time you throw a dice, the probability of getting a ‘head’ is independent of the probability of getting a 6 in dice.

Probability of Independent Events

- Consider an example of rolling a die. If A is the event 'the number appearing is odd' and B be the event 'the number appearing is a multiple of 3', then

  $P(A) = 3/6 = 1/2$ and $P(B) = 2/6 = 1/3$

  Also A and B is the event 'the number appearing is odd and a multiple of 3' so that

  $P(A \cap B) = 1/6$

  $P(A|B) = P(A \cap B)/ P(B)$

  =1613=12

  $P(A) = P(A|B) = 1/2$, which implies that the occurrence of event B has not affected the probability of occurrence of the event A .

  If A and B are independent events, then $P(A|B) = P(A)$

  Using the Multiplication rule of probability, $P(A \cap B) = P(B) .P(A|B)$

  $P(A \cap B) = P(B) .P(A)$

What are Mutually Exclusive Events?

- Two events A and B are said to be mutually exclusive events if they cannot occur at the same time. Mutually exclusive events never have an outcome in common.

ADDITION LAW OF PROBABILITY

Statement: If A and B are two events associated with an experiment; then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Proof. Let m_1, m_2, and m be the number of favourable outcomes to the events A, B and $A \cap B$ respectively. **Total outcomes** of the experiment be n.

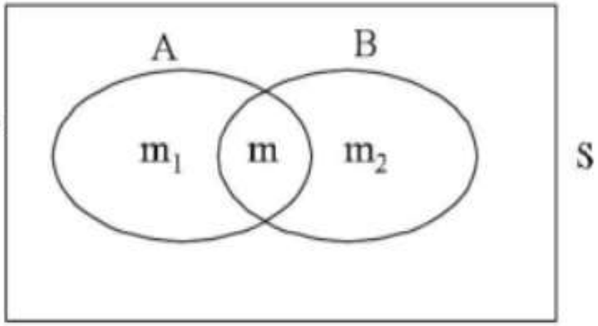
$$P(A) = \frac{m_1}{n},\ P(B) = \frac{m_2}{n},\ P(A \cap B) = \frac{m}{n}$$

The favourable outcomes to the event A only $= m_1 - m$

The favourable outcomes to the event B only $= m_2 - m$

The favourable outcomes to the event $A \cap B$ only $= m$.

The favourable outcomes to the events A or B or both i.e.,



$$A \cup B = m_1 - m) + (m_2 - m) + m$$

$$= m_1 + m_2 - m$$

So, $$P(A \cup B) = \frac{m_1 + m_2 - m}{n}$$

$$= \frac{m_1}{n} + \frac{m_2}{n} - \frac{m}{n}$$

$$= P(A) + P(B) - P(A \cap B)$$

Note: If Both the event are mutually exclusive: then P(**A ∪ B) =** P (A or B) = P(A) +P(B)

MULTIPLICATION LAW OF PROBABILITY

Statement: If there are two independent events the respective probabilities of which are known, then the probability that both will happen is the product of the probabilities of their happening respectively.

$$P(AB) = P(A) \times P(B)$$

Proof. Suppose A and B are two independent events. Let A happen in m_1 ways and fail in n_1 ways.

$$\therefore \qquad P(A) = \frac{m_1}{m_1+n_1}$$

Also let B happen in m_2 ways and fail in n_2 ways.

$$\therefore \qquad P(B) = \frac{m_2}{m_2+n_2}$$

Now there are four possibilities

A and B both may happen, then the number of ways $= m_1 . m_2$.
A may happen and B may fail, then the number of ways $= m_1 . n_2$
A may fail and B may happen, then the number of ways $= n_1 . m_2$
A and B both may fail, then the number of ways $= n_1 . n_2$

Thus, the total number of ways $= m_1m_2 + m_1n_2 + n_1m_2 + n_1n_2 = (m_1+n_1)(m_2+n_2)$

Hence the probabilities of the happening of both A and B

$$P(AB) = \frac{m_1m_2}{(m_1+n_1)(m_2+n_2)} = \frac{m_1}{m_1+n_1} . \frac{m_2}{m_2+n_2} = P(A) . P(B)$$