3

Multiplication

Introduction

Multiplication is repeated addition. When we say 3 multiplied by 4, we simply say 3 + 3 + 3 + 3. The century-old method practiced all over the world has the unique pattern of multiplication that involves: Multiplicand x Multiplier = Product

```
    4 2 3 5 ———→ Multiplicand
    X   2 5 ———→ Multiplier
  ---------
  2 1 1 7 5
+ 8 4 7 0 X
-----------
1 0 5 8 7 5 ———→ Product
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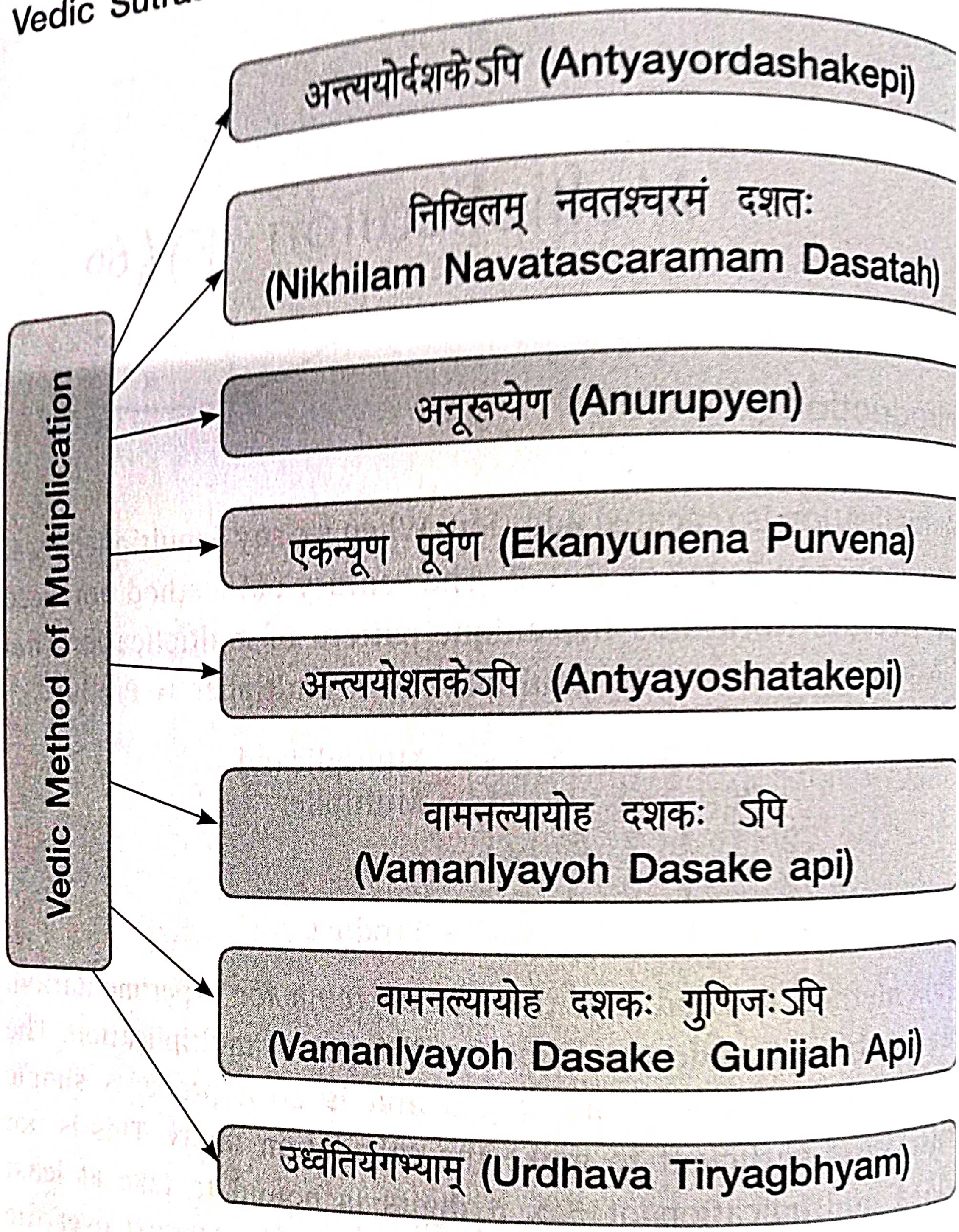
This method is good, but there is no room for experimentation. We are bound to do the same in all types of multiplication. The process involves multiple stages and is error prone. A simple mistake is sufficient to make the calculation faulty. This is not all; a multiplication of 5 x 5 digits is bound to take at least 3-4 minutes. The best advantage of the Vedic Method over the present day calculation is that Vedic mathematics has ample fruit in its basket. You have different choices and you can choose the best possible method in the best situation. There are many special sutras that help you to find the answer of a special type of multiplication even in seconds and the ***Urdhva Tiryagbhyam*** method helps you to encounter all types of multiplication. So friends, jump into the ocean of Vedic sutras and gather the pearls of your own choice

Vedic Sutras for Multiplication



Meaning of Vedic Sutra:

1. अन्त्ययोर्दशकेऽपि **(Antyayordashakepi)**: The Vedic sutra is applicable when the sum of unit digit at multiplicand and multiplier is 10 and the remaining digits in the multiplicand as well as in the multiplier are the same.
2. निखिलम् नवतश्चरमं दशतः **(Nikhilam Navatascaramam**

Dasatah): The literal meaning is – **All from 9 and last from ten**. This sutra works better when the numbers to be multiplied are very close to the base number. The base number should be the multiple of 10.

3. अनूरूप्येण **(Anurupyen)**: This Vedic sub-sutra literally means "**to proportionately**". This sub- sutra is applicable when either the multiplicand or multipliers is sufficiently very far from the power of 10.
4. एकन्यूण पूर्वेण **(Ekanyunena Purvena)**: It literally means "**one less than the previous**". This sutra has limited applications. It is used for multiplication wherein the multiplier digits consist entirely of nines. It comes up under three types.
   a. When the multiplicand and multipliers each consist of equal number of digits
   b. When the multiplier has more digits than the multiplicand
   c. When the multiplicand has more digits than the multipliers
5. अन्त्ययोशतकेऽपि **(Antyayoshatakepi)**: The literal meaning is – **the sum of unit and ten's digit at multiplicand and multiplier is 100**. This sutra is valid as long as the sum of the last two digits is 100 and the remaining digits in multiplicand and multiplier are the same.
6. वामनल्यायोह दशकःऽपि **(Vamanlyayoh Dasake Api)**: This sutra is not taken from the original Vedic Text available, but is equally valuable as far as speedy calculation is concerned. The literal meaning is – **Sum of left two digits is equal to 10**. This sutra is valid if the digit at the unit place in both multiplicand and multiplier is equal and the sum of the left two digits is 10.
7. वामनल्यायोह दशकः गुणिजःऽपि **(Vamanlyayoh Dasake Gunijah Api)**: The literal meaning of this sutra is – **Sum of left two digits other than unit is a multiple of ten and**

unit digits are the same. This is also not a Vedic Sutra and is not taken from the Original Vedic Text.

8. उर्ध्वतिर्यगभ्याम् **(Urdhava Tiryagbhyam)**: It is a general formulae applicable to all cases of multiplication. It is a process of vertical and cross-wise multiplication. This method has been further simplified and dealt with Dot and Cross method in this book. A better understanding of this formula will also help you in multiplying two numbers with the other formulae mentioned above.

निखिलम् नवतश्चरमं दशतः (Nikhilam Navatascaramam Dasatah)

This sutra works better when both the multiplicand and multiplier are very close to the base. The base should be in the form of 10^n, where n is a natural number.

Rule:

- Write the two numbers to be multiplied above and below at the right side of your notebook.
- Write the deviation of multiplicand and multiplier from the base and place them next to the digit to be multiplied.
- The final result will have two parts.

a) The left hand part will be obtained by cross operation of two numbers written diagonally.

b) The right side of the answer will be obtained by multiplying the deviations.

- The number of digits in the right hand part will be in accordance to the number of zeros in the base number. In simple words, if the base is 100, the right hand part will have two digits and if the base is 1000, the right hand part will have three digits.
- In case there is lesser number of digits in the right side, accommodate as many zeros before the right hand part so that the total number of digits in that part is equal to the number of zeros in the base.
- Here is the table that will guide you in deciding the number of digits to be placed on the right hand side.

Base	*Number of digits at the right side of vertical line.*	
10	1	0
100	2	00
1000	3	000
10000	4	0000
100000	5	00000
1000000	6	000000
10000000	7	0000000

Let us take a few examples to understand the modus operandi of the above Vedic Sutra.

Case 1: When both numbers are below the base.

Example 4: Multiply 8 by 7

Solution:

(a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{r} 8 \\ \underline{\times\ 7} \end{array}$$

(b) Both the numbers are closer to the base 10, so take Base = 10.

Deviation of 8 = 8 – 10 = – 2

Deviation of 7 = 7 – 10 = – 3

(c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{ll} 8 & -2 \\ \underline{\times\ 7} & \underline{-3} \end{array}$$

(d) Write the left hand digit by cross operation of any of two diagonal. Here 8 – 3 = 5 and 7 – 2 = 5

$$\begin{array}{ll} 8 & -2 \\ \underline{\times\ 7} & \underline{-3} \\ 5\ / & \end{array}$$

(e) The right hand digit will be the multiplication of the deviation. The product of deviation is (–2) x (–3) = 6.

$$\begin{array}{ll} 8 & -2 \\ \underline{7} & \underline{-3} \\ 5\ / & 6 \end{array}$$

Example 5: Multiply 95 by 91

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{r} 95 \\ \underline{\times\ 91} \end{array}$$

b) Both the numbers are closer to the base 100, so take Base = 100.

Deviation of 95 = 95 – 100 = – 5

Deviation of 91 = 91 – 100 = – 9

c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{r} 95 - 5 \\ \underline{\times\ 91 - 9} \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals. Here 95 – 9 = 86 or 91 – 5 = 86 is written in the left hand part.

$$\begin{array}{r} 95 - 5 \\ \underline{\times\ 91 - 9} \\ 86\ / \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{r} 95 - 5 \\ \underline{\times\ 91 - 9} \\ 86\ /45 \end{array}$$

Case 2: When both the numbers are above the base.

Example 6: Multiply 15 by 11

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{r} 15 \\ \underline{\times\ 11} \end{array}$$

b) Both the numbers are closer to the base 10, so take Base = 10.

Deviation of 15 = 15 – 10 = 5

Deviation of 11 = 11 – 10 = 1

c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{r} 15 + 5 \\ \underline{\times\ 11 + 1} \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{r} 15 + 5 \\ \underline{\times\ 11 + 1} \\ 16\ / \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{lr} 15 & +5 \\ \underline{\times\ 11} & \underline{+1} \\ 16\ / & 5 \end{array}$$

Example 7: Multiply 105 by 104

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{l} 105 \\ \underline{\times\ 104} \end{array}$$

b) Both the numbers are closer to the base 100, so take Base = 100.

Deviation of 105 = 105 – 100 = +5

Deviation of 104 = 104 – 100 = + 4

c) Put the deviation at the right side along with the number to be multiplied. .

$$\begin{array}{lr} 105 & +5 \\ \underline{\times\ 104} & \underline{+4} \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{lr} 105 & +5 \\ \underline{\times\ 104} & \underline{+4} \\ 109\ / & \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{lr} 105 & +5 \\ \underline{104} & \underline{+4} \\ 109\ / & 20 \end{array}$$

Case 3: When one number is above the base and another is less than the base.

Example 8: Multiply 12 by 8

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{r} 12 \\ \times\ 8 \\ \hline \end{array}$$

b) Both the numbers are closer to the base 10, so take Base = 10.

Deviation of 12 = 12 − 10 = +2

Deviation of 8 = 8 − 10 = −2

c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{rr} 12 & +2 \\ \times\ 8 & -2 \\ \hline \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{rr} 12 & +2 \\ \times\ 8 & -2 \\ \hline 10\ / & \end{array}$$

e) The right hand digit will be the multiplication of the deviation. The product of (+2) x (−2) = −4 is written in the RHS.

$$\begin{array}{rr} 12 & +2 \\ \times\ 8 & -2 \\ \hline 10\ / & -4 \end{array}$$

e) When there is a minus (−) sign at the right hand product, use the Nilhilam formulae which states, "All from 9 and the last from 10." Hence subtract the right hand digit (−4) from 10 and left hand part will get diminished by 1. i.e. 10 − 1 = 9

$$\begin{array}{rr} 12 & +2 \\ \times\ 8 & -2 \\ \hline 10\ / & -4 \end{array}$$

= 9 / 10 − 4

= 9 / 6

Example 9: Multiply 122 by 98

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{r} 122 \\ \underline{\times\ 98} \end{array}$$

b) Both the numbers are closer to the base 100, so take Base = 100.

Deviation of 122 = 122 – 120 = +22

Deviation of 98 = 98 – 100 = –2

c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{rr} 122 & +\ 22 \\ \underline{\times\ 98} & \underline{-\ \ 2} \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{rr} 122 & +\ 22 \\ \underline{\times 98} & \underline{-\ \ 2} \\ 120\ / & \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{rr} 122 & +\ 22 \\ \underline{\times\ 98} & \underline{-\ \ 2} \\ 120\ / & -44 \end{array}$$

f) When there is a minus (–) sign at the right hand product, use the Nilhilam formulae which states, "All from 9 and the last from 10." Hence subtract the right hand digit (–44) from 100 and left hand part 120 will get diminished by 1. i.e. 120 – 1 =119

$$\begin{array}{rl} 122 & +\ 22 \\ \underline{\times\ 98} & \underline{-\ \ 2} \\ 119 & /\ 100 - 44 \\ = 119 & /\ 56 \end{array}$$

Case 4: Adjustment of right side digit of the product.

Two sub-cases may arise here:

a) When the number of digits on the right hand side is more than the permissible limit.

b) When the number of digits on the right hand side is less than the permissible limit.

Sub case (a): When the number of digits on the right hand side is more than the permissible limit.

Example 10: Multiply 16 by 15

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{r} 16 \\ \underline{\times\ 15} \end{array}$$

b) Both the numbers are closer to the base 10, so take Base = 10.

Deviation of 16 = 16 – 10 = +6

Deviation of 15 = 15 – 10 = +5

c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{rr} 16 & +6 \\ \underline{\times\ 15} & \underline{+5} \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{rr} 16 & +6 \\ \underline{\times\ 15} & \underline{+5} \\ 21\ / & \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{rr} 16 & +6 \\ \underline{\times\ 15} & \underline{+5} \\ 21\ / & 30 \end{array}$$

Here, the number of digit in RHS is two, which is more than the permissible number of digits in RHS (See Table 1). The number of permissible digits in RHS should be in accordance with the base number. Since, the base is 10, the number placed at the right side should be of one digit. In such a case, we transfer the extreme left digit of RHS to the LHS and add them.

16 + 6

x 15 + 5

21 / 30

+

= 240

Example 11: Multiply 13 by 18

Solution:

a) Put the multiplicand and multiplier as shown here.

13

x 18

b) Both the numbers are closer to the base 10, so take Base = 10.

Deviation of 13 = 13 – 10 = +3

Deviation of 18 = 18 – 10 = +8

c) Put the deviation at the right side along with the number to be multiplied.

13 +3

x 18 +8

d) Write the left hand digit by cross operation of any of the two diagonals.

13 +3

x 18 +8

21 /

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{l} 13 + 3 \\ \underline{18 + 8} \\ 21 \,/\, 24 \end{array}$$

Since, the base is 10, the number placed at the right side should be of one digit, so transfer the extreme left digit of RHS to the LHS and add them.

$$\begin{array}{l} 13 \;\; +3 \\ \underline{\times\, 18 \;\; +8} \\ 21 \,/\, 24 \\ \;\;\; + \\ = 234 \end{array}$$

Sub case (b): When the number of digits on the right hand side is less than the permissible limit.

Example 12: Multiply 96 by 98

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{l} 96 \\ \underline{\times\, 98} \end{array}$$

b) Both the numbers are closer to the base 100, so take Base = 100.

Deviation of 96 = 96 − 100 = − 4

Deviation of 98 = 98 − 100 = − 2

c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{l} 96 \;\; -4 \\ \underline{\times\, 98 \;\; -2} \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{l} 96 \;\; -4 \\ \underline{\times\, 98 \;\; -2} \\ 94 \,/ \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{ll} 96 & -4 \\ \underline{98} & \underline{-2} \\ 94 \ / \ 8 & \end{array}$$

Since, the base is 100, the number placed at the right side should consist of two digits. But there is a single digit in the RHS. In such a case, we place the zero to the left in RHS so that the total number of digits in RHS is equal to the permissible number of digits. See Table 1 for better understanding.

$$\begin{array}{ll} 96 & -4 \\ \underline{98} & \underline{-2} \\ 94 \ / \ 08 & \end{array}$$

Example 13: Multiply 989 by 995

Solution:

a) Put the multiplicand and multiplier as shown here.

$$\begin{array}{r} 989 \\ \underline{\times\ 995} \end{array}$$

b) Both the numbers are closer to the base 1000, so take Base = 1000.

Deviation of 989 = 989 – 1000 = –11

Deviation of 995 = 995 – 1000 = – 5

c) Put the deviation at the right side along with the number to be multiplied.

$$\begin{array}{ll} 989 & -11 \\ \underline{\times\ 995} & \underline{-5} \end{array}$$

d) Write the left hand digit by cross operation of any of the two diagonals.

$$\begin{array}{ll} 989 & -11 \\ \underline{\times\ 995} & \underline{-5} \\ 984 \ / & \end{array}$$

e) The right hand digit will be the multiplication of the deviation.

$$\begin{array}{ll} 989 & -11 \\ \underline{\times\ 995} & \underline{-5} \\ 984\ /\ 55 & \end{array}$$

Since the base is 1000, the number placed at the right side should consist of three digits. So in order to meet the requirement of permissible digit in RHS, we place the zero to the left in the RHS (Refer Table 1).

$$\begin{array}{ll} 989 & -11 \\ \underline{995} & \underline{-5} \\ 984\ /\ 055 & \end{array}$$

Till now, we have seen examples in which both the numbers were closer to the base. Now let us consider a case where the two numbers are nearer to a different base. Hey, are you worried? Don't panic, the problem will be solved in a similar fashion with a slight change in the LHS.

- Write the numbers with their respective deviations from the base as done earlier.
- Write the base of each number in a bracket and cancel an equal number of zeros in the bracket.
- The RHS will be calculated as done above by placing the product of deviations, and will have the number of digits equal to the number of zeros cancelled.
- In the LHS, write the sum of the cross product of the first diagonal and the deviation of second number.

Example 14: Multiply 107 by 1008

Solution: Here, the two numbers are of different base. 107 is closer to base 100 and 1008 is closer to base 1000. Hence, the respective deviations of the numbers are +7 (107 – 100) and +8 (1008 – 1000).

	Deviation	Base
107	+7	(100)
x 1008	+8	(1000)

- Cancel equal number of zeros of the different bases.

	Deviation	Base
107	+7	$(1\cancel{0}\cancel{0})$
x 1008	+8	$(10\cancel{0}\cancel{0})$

- LHS = 107 x 10 + 8 = 1078

  RHS = 7 x 8 = 56

- Hence, 1008 x 107 = 107856

This one liner multiplication technique is a perfect beauty of Vedic Mathematics. A few months ago, I had gone to Ahmedabad to attend a Vedic Mathematics seminar. While addressing the gathering of students and teachers, I wrote a 9 digit number on the blackboard and asked the audience to multiply the written digit with 9 times 9.

$$569876943 \times 999999999 = ?$$

I had even allowed them to use the calculator, but the calculator showed an error message. I asked the audience to multiply the numbers manually and in the mean time, I wrote 569876942430123057 on the blackboard. The audience took more than 5 minutes and I had taken less than 15 seconds to write down the answer. The audience was amazed to see that the result I had written in less than 15 seconds was absolutely correct.

The Vedic Sutra – Ekanyuena Purvena is simply an awesome method of multiplying two numbers but this has limited application. This sutra work only under three conditions:

1) When the number of digits in the multiplicand and number of 9s in multipliers is the same.
2) When the number of 9s in the multipliers are more than the number of digits in the multiplicand.
3) When there is less number of 9s in the multiplier than the number of digits in the multiplicand.

Now let us take each case one by one.

Case 1: When the number of digits in the multiplicand and the number of 9s in the multipliers is the same.

Rule:

- Subtract 1 from the multiplicand and write the result in LHS.

- Subtract the multiplicand by applying Nikhilam Navatascaramam Dasatah Vedic sutra and write the result in RHS.

Example 25: Multiply 6543 by 9999

Solution: Here, the number of digits in the multiplicand is equal to the number of 9s in the multipliers. As the rule suggests, the answer will have two parts.

LHS = Multiplicand – 1 = 6543 – 1 = 6542

RHS = Apply the Nikhilam method of subtraction and subtract the unit digit from 10 and the rest of the digits from 9. We get, 9 – 6 = 3, 9 – 5 = 4, 9 – 4 = 5 and, 10 – 3 = 7. Thus RHS will have 3457.

In order to simplify the calculation in RHS, we may subtract the result obtained in LHS from the multiplier.

RHS = 9999 – 6542 = 3457

Hence, 6543 x 9999 = 65423457

Example 26: Multiply 89654876 by 99999999

Solution:

LHS = 89654876 – 1 = 89654875

RHS = 99999999 – 89654875 = 10345124

Hence, 89654876 x 99999999 = 8965487510345124

Example 27: Multiply 83465087629 by 99999999999

Solution:

LHS = 83465087629 – 1 = 83465087628

RHS = 99999999999 – 83465087628 = 16534912371

Hence, 83465087629 x 99999999999 = 8346508762816534912371

Example 28: Multiply 45682 by 99999

Solution:

LHS = 45682 – 1 = 45681

RHS = 99999 – 45681 = 54318

Hence, 45682 x 99999 = 4568154318

With a little practice, you can write the RHS manually in no time. Subtract each digit of LHS from 9. If LHS = 23, then RHS will be 76.

Case 2: When the number of 9s in the multipliers is more than the number of digits in the multiplicand.

Rule: In case 2, the same procedure will be applied as in case 1.

Let us take a few examples.

Example 29: Multiply 456 by 9999

Solution:

LHS = 456 – 1 = 455

RHS = 9999 – 455 = 9544

Hence, 456 x 9999 = 4559544

Example 30: Multiply 56892 by 9999999

Solution:

LHS = 56892 – 1 = 56891

RHS = 9999999 – 56891 = 9943108

Hence, 56892 x 9999999 = 568919943108

Example 31: Multiply 13324 by 99999999

Solution:

LHS = 13324 – 1 = 13323

RHS = 99999999 – 13323 = 99986676

Hence, 13324 x 99999999 = 1332399986676

Case 3: When there is less number of 9s in the multiplier than the number of digits in the multiplicand.

This case is a little bit different from the last two cases discussed so far under Ekanyena Purvena. In order to get the result you have to –

a) Add as many zero as the numbers of 9s to the multiplicand.

b) Subtract the original multiplicand from the figure obtained in Step 1.

Example 32: Multiply 1564 by 99

Solution:

The multiplicand 1564 has 4 digits, whereas there are two 9s in the multiplier.

$$\begin{array}{r} 1\,5\,6\,4 \\ \underline{\times\,9\,9} \end{array}$$

1) Since there are two 9s, put two zeros at the end of 1564, making it 156400.

2) Subtract (1564 original multiplicand) from 156400 i.e.

$$\begin{array}{r} 1\,5\,6\,4\,0\,0 \\ \underline{-1\,5\,6\,4} \\ 1\,5\,4\,8\,3\,6 \end{array}$$

Hence, 1564 x 99 = 154836

Example 33: Multiply 783459 by 9999

Solution:

1) Since there are four 9s, put four zeros at the end of 783459, making it 7834590000

2) Subtract the original number 783459 from 7834590000.

$$\begin{array}{r} 7\,8\,3\,4\,5\,9\,0\,0\,0\,0 \\ \underline{-\,7\,8\,3\,4\,5\,9} \\ 7\,8\,3\,3\,8\,0\,6\,5\,4\,1 \end{array}$$

Hence, 783459 x 9999 = 7833806541

Example 34: Multiply 45678 by 999

Solution:

1) Since there are three 9s, put three zeros at the end of 45678, making it 45678000.

2) Subtract the original number from 45678000.

$$
\begin{array}{r}
45678000 \\
-\ 456778 \\
\hline
45632322
\end{array}
$$

Hence, 45678 x 999 = 45632322

This can be done by the Eknyuenane Purvena method effortlessly after a little practice. Let's see how it works. In this method:

a) Subtract 1 from the original number and place it in LHS.
b) Write as many digits from right to left equal to the number of 9s that of multiplicand in RHS. Suppose you have to multiply 147 by 99 so RHS part of answer should contain 2 digits 47 taken from right, equal to the number of 9 in the multiplier.
c) The remaining digits in the original number, after removing the digits from the right to left, placed in the RHS should be subtracted in the LHS.
d) Write the complement of the digits placed in the RHS by applying the Nikhilam sutra.

Example: Multiply 147 by 99

Solution: Since there are two 9s in the multiplier, two digits from right to left of the multiplicand will be placed in the RHS. In the LHS, subtract 1 from the original number.

LHS = 147 – 1 = 146

RHS = Complement of 47

Now subtract the remaining digits i.e. 1 from LHS and write the complement of 47 in RHS.

LHS = 146 – 1 = 145

RHS = 100 – 47 = 53

Hence, 147 x 99 = 14553

Example: Multiply 259648 by 9999.

Solution: Since there are four 9s in the multiplier, four digits from right to left of multiplicand i.e. 9648 will be placed in the RHS. In LHS, subtract 1 from the original number.

LHS = 259648 – 1 = 259647

RHS = Complement of 9648

Now subtract the remaining digits i.e. 25 from LHS and write the complement of 9648 in RHS.

LHS = 259647 – 25 = 259622

RHS = 10000 – 9648 =0352

Hence, 259648 x 9999 = 2596220352

Example: Multiply 52876 by 99

Solution: Since there are two 9s in the multiplier, two digits from right to left of the multiplicand i.e. 76 will be placed in the RHS. In LHS, subtract 1 from the original number.

LHS = 52876 – 1 = 52875

RHS = Complement of 76

Now subtract the remaining digits i.e. 528 from LHS and write the complement of 76 in RHS.

LHS = 52875 – 528 = 52347

RHS = 100 – 76 = 24

Hence, 52876 x 99 = 5234724

Urdhva Tiryagbhyam (उर्ध्वतिर्यगभ्याम्)

So far we have discussed seven Vedic sutras, but all of them have limited applicability. We shall now proceed to deal with a general formula of multiplication which is applicable in all the cases. This sutra is widely known as **Urdhva Tiryag** sutra, which means "**Vertically and Cross-wise**". Once you get mastery over this method you can multiply a 5 digit multiplication of any number in 15 seconds. Initially this method will seem tough to work out, but believe me; I have seen the change in calculating power of students after learning this sutra. The best feature of this method is:

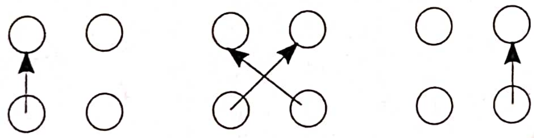
a) With practice, you can multiply any digits of number and obtain the result in one line.

b) You are free to multiply from both ends due to the flexibility of Vedic Sutra..

A further simplification can be understood by the **Dot and Stick Method..**

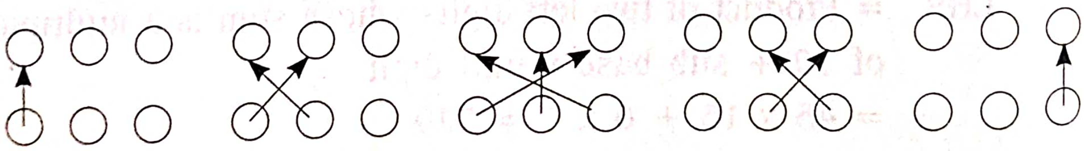
Dot and Stick Method

Multiplication of 2 digits number



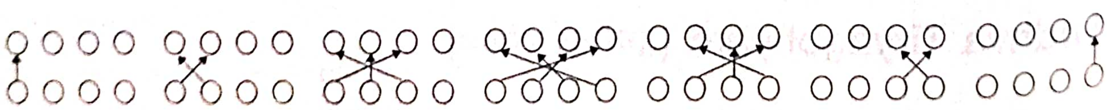
(1) (2) (1)

Multiplication of 3 digits number

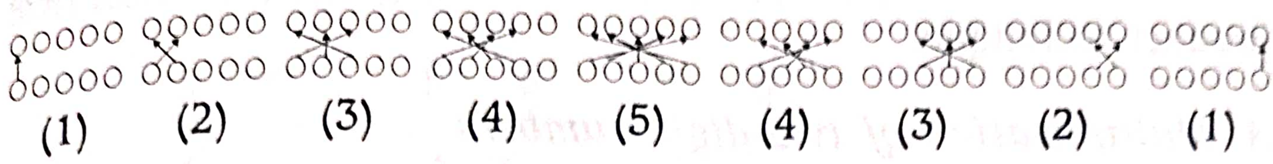


(1) (2) (3) (2) (1)

Multiplication of 4 digits number



(1) (2) (3) (4) (3) (2) (1)



The above pictorial representation may be extended upon need. The dot and stick method may bring some amount of discomfort initially, but the more you practice, the more comfortable you will feel with it. At the very beginning you might get puzzled over the multiplication by dot and cross as shown above pictorially, but here is an interesting technique to learn the dot and cross method. See this unique pattern of multiplication:

11 X 11 = 121

111 X 111 = 12321

1111 X 1111 = 1234321

11111 X 11111 = 123454321

Looking upon the multiplication of 11 x 11, 111 x 111 ...and the dot multiplication shown pictorially, you may find a very special clue. Did you notice the clue?

If you are multiplying a two-digit number, see the product of 11 X 11 i.e. 121 and now notice the number written below the dots of 2 digit multiplication.

Did you find any similarity?

Yes, the same 121 is written there. This process can be summed up in three points.

- First multiply vertically the number placed at unit digits.
- Find the sum of the cross product of the two figures.
- Finally, multiply vertically the remaining digits.

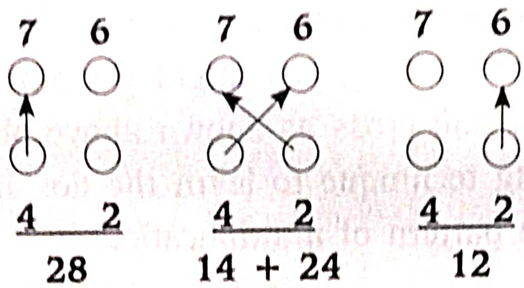
Similarly, if you are multiplying a three-digit number, look at the product of 111 X111 = 12321, this is the same number written at the bottom of the dot multiplication.

Let us take a few examples to clarify the dot and cross product technique.

A. Multiplication of two-digit numbers

Example 46: Multiply 76 by 42

Solution:



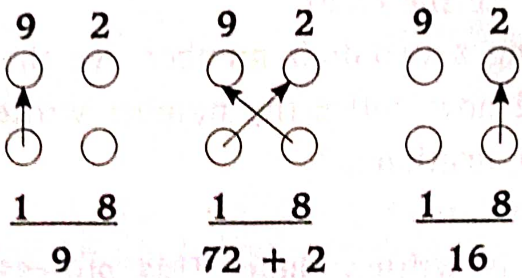
Arranging the number and adding them from right to left, taking only one digit at a time, we get the final result.

= 28 | 38 | 12

= 3192

Example 47: 92 X 18 = ?

Solution: Arranging the number on the dots.



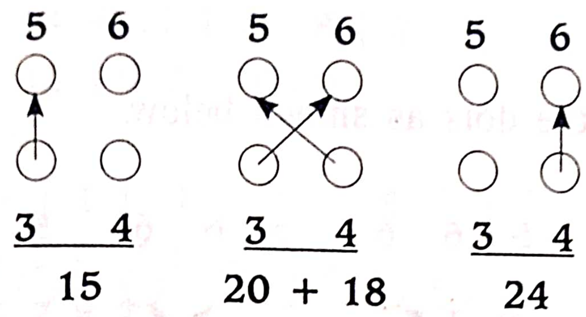
Arranging the number and adding them from right to left, taking only one digit at a time, we get the final result.

= 9 | 74 | 16

= 1656

Example 48: Multiply 56 by 34

Solution: Arranging the number on the dots.



Arranging the number and adding them from right to left, taking only one digit at a time, we get the final result.

= 15 | 38 | 24

= 1904

Once the concept is clear, the whole process can be done mentally in one line.

Example 49: Multiply 77 by 39.

Solution: I do hope the concept of dot and cross technique is clearly understood by you. Here is the one liner method. The sum of the cross multiplication of dots in the second stage has to be done mentally.

$$\begin{array}{lcr} & 7 & 7 \\ \times & 3 & 9 \\ \hline \end{array}$$

= 21 | 84 | 63

= 3003

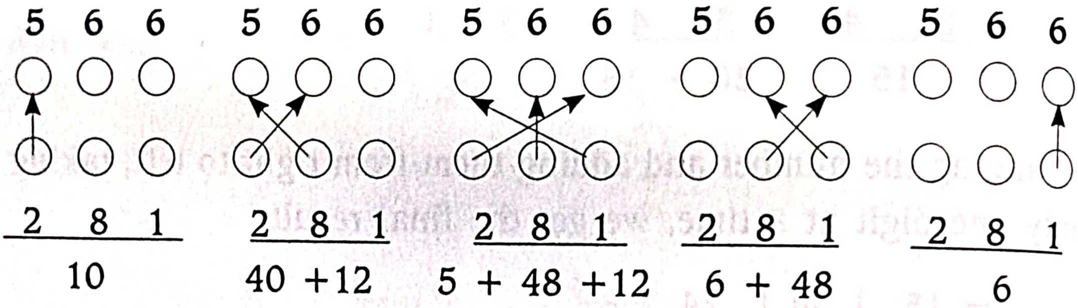
In the 1st vertical separator 7 x 9 = 63 is written, in the second vertical separator the sum of the cross product of 7 x 9 and 3 x 7 is written directly i.e. 7 x 9 + 3 x 7 = 6 3 + 2 1 = 8 4. In the third vertical separator, 7 X 3 = 21 is placed. As told earlier, take only one digit in each separator and add the remaining digit to the next digit separator as shown, in the direction of the arrow.

B. Multiplication of three- digit numbers

Example 50: Multiply 566 by 281

Solution:

Arrange the number on the dots as shown below.



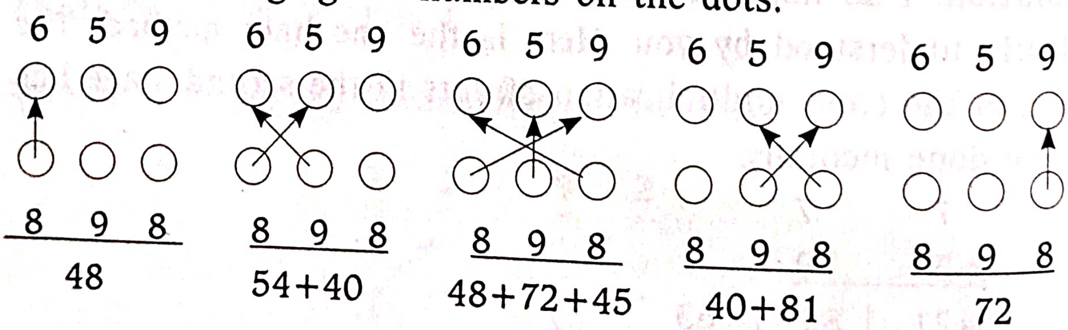
Arrange each product with vertical separator as shown below.

10 | 5 2 | 6 5 | 5 4 | 6

= 159046

Example 51: Multiply 659 by 898

Solution: Arranging the numbers on the dots.



Arranging the product with vertical separator as shown below:

= 4 8 | 9 4 | 1 6 5 | 1 2 1 | 7 2

= 48 | 94 | 165 | 121 + 7 | 2

= 48 | 94 | 165 | 12 8 | 2

$= 48 \mid 94 \mid 165 + 12 \mid 8 \mid 2$

$= 48 \mid 94 \mid \underline{17}\ \ 7 \mid 8 \mid 2$

$= 48 \mid 94 + 17 \mid 7 \mid 8 \mid 2$

$= 4\,8 \mid \underline{11}\ 1 \mid 7 \mid 8 \mid 2$

$= 4\,8 + 1\,1 \mid 1 \mid 7 \mid 8 \mid 2$

$= 5\,9 \mid 1 \mid 7 \mid 8 \mid 2$

$= 5\,9\,1\,7\,8\,2$

Hence $6\,5\,9 \times 8\,9\,8 = 5\,9\,1\,7\,8\,2$

The above steps are written for the sake of readers to understand the concept more vividly, though it is unnecessary to write all these steps. Keep only one thing in your mind that after one stage of operation is over, keep a single digit in each block and move the remaining to the next. Readers are expected to do these operations involving the addition of two or three number mentally.

Example 52: Multiply 247 by 989.

Solution: The whole operation of dot and cross method is done here in one line.

$$
\begin{array}{r}
2\,4\,7 \\
\times\ 9\,8\,9 \\
\hline
{}_{1}8 \mid {}_{5}2 \mid {}_{11}3 \mid {}_{9}2 \mid {}_{6}3
\end{array}
$$

$$
\begin{array}{cccccc}
 & 8 & 2 & 3 & 2 & 3 \\
+\ 1 & 5 & 11 & 9 & 6 & \\
\hline
2 & 4 & 4 & 2 & 8 & 3
\end{array}
$$

In the very beginning, I had mentioned the fact that once the multiplication of every digit is completed, you have to add the digits from right to left, taking only one digit in each separator. Here the extra digits, leaving the unit digit in each separator, have been written in sub-script so that further addition becomes easy.

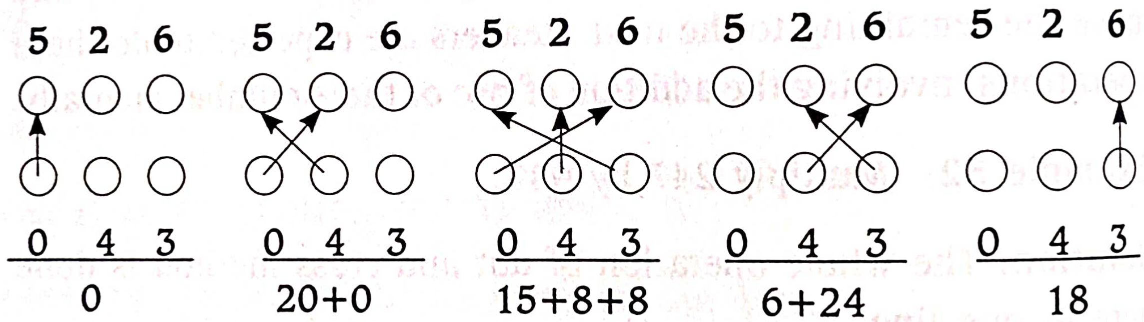
Example 53: Multiply 467 by 598.

Solution:

$$\begin{array}{rrrrrr} & & 4 & 6 & 7 & \\ & \times & 5 & 9 & 8 & \\ \hline {}_{2}0 \mid & {}_{6}6 \mid & {}_{12}1 \mid & {}_{11}1 \mid & {}_{5}6 & \\ & 0 & 6 & 1 & 1 & 6 \\ + & 2 & 6 & 12 & 11 & 5 \\ \hline & 2 & 7 & 9 & 2 & 6\ \ 6 \end{array}$$

Example 54: Multiply 526 by 43

Solution: This is a 3 x 2 digit multiplication so put a zero in front of 43, making it 043 and now apply the above 3 x 3 operation technique. Arrange the numbers on the dots as shown below.



0 | 20 | 23 | 30 | 18

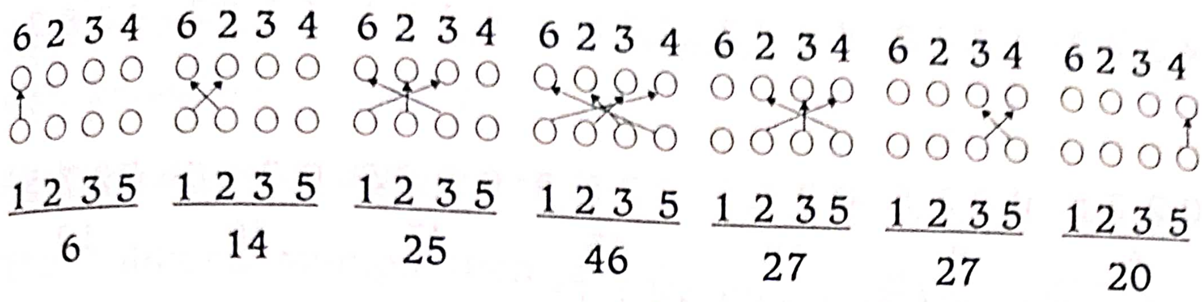
= 22618

C. Multiplication of 4 digits number

The first example here is done in detail with each operation illustrated clearly, but the next example onwards is done in a single line with the hope that readers are well acquainted with the process of multiplication.

Example 55: Multiply 6234 by 1235

Solution: Arranging the numbers on the dots.



Arrange the numbers in vertical separators.

$$6 \mid 14 \mid 25 \mid 46 \mid 27 \mid 27 \mid 20$$

$= 7698990$

Example 56: Multiply 8989 with 8892.

Solution:

$$\begin{array}{r} 8\;\;9\;\;8\;\;9 \\ \times\, 8\;\;8\;\;9\;\;2 \\ \hline \end{array}$$

$$64 \mid 136 \mid 208 \mid 233 \mid 162 \mid 97 \mid 18$$

$$\begin{array}{cccccccc} = 6 & 4 & 6 & 8 & 3 & 2 & 7 & 8 \\ + & 13 & 20 & 23 & 16 & 9 & 1 & \\ \hline 7 & 9 & 9 & 3 & 0 & 1 & 8 & 8 \end{array}$$

Example 57: Multiply 2134 by 3261

Solution:

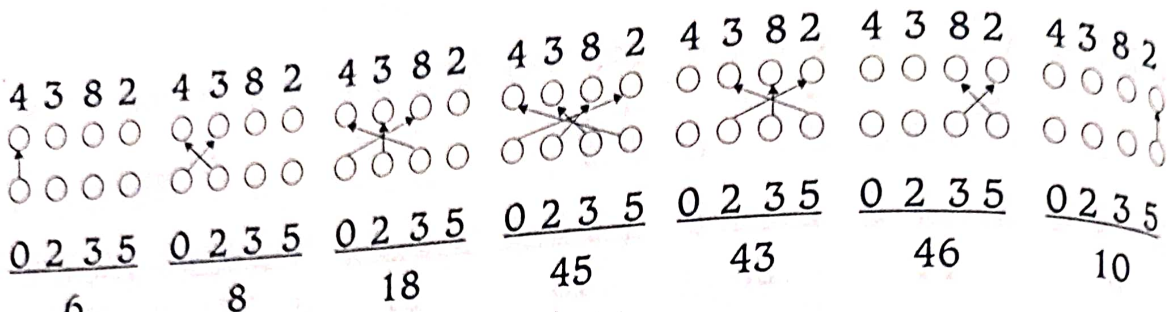
$$\begin{array}{r} 2\;1\;3\;4 \\ \times\, 3\;2\;6\;1 \\ \hline \end{array}$$

$$6 \mid 7 \mid {}_{2}3 \mid {}_{2}6 \mid {}_{2}7 \mid 4$$

$= 6958974$

Example 58: Multiply 4382 by 235

Solution: This is a 4 x 3 digit multiplication. The 4 x 4 method as explained above can be used here, provided we put a zero in front of 235, making it 0235. Arrangement of dots in this case is shown here.

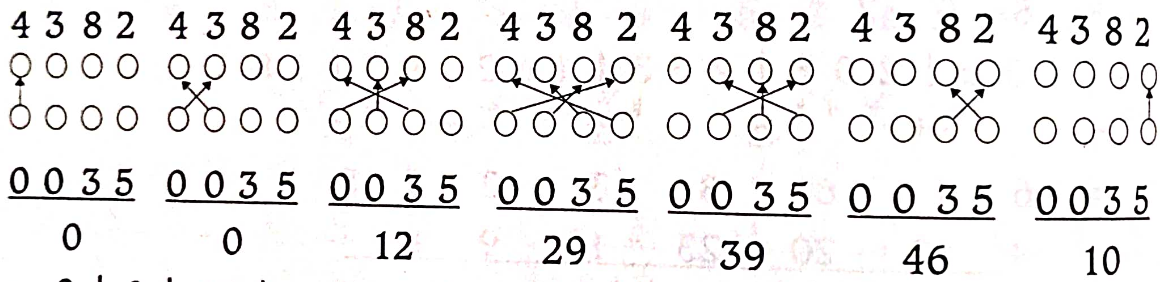


= 0 | 8 | 18 | 45 | 43 | 46 | 10
+ + + + +

= 1029770

Example 59: Multiply 4382 by 35

Solution: This is a 4 x 2 digit multiplication, so two zeros have been placed behind 35 to make the calculation by 4 x 4 techniques as described above. Arrangement of dots is shown here for the convenience of readers.



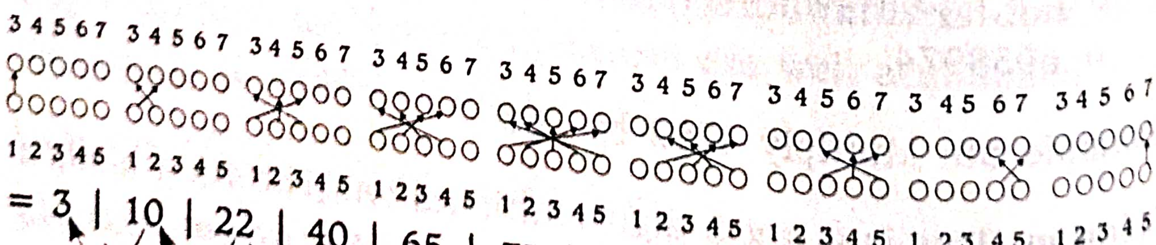
= 0 | 0 | 12 | 29 | 39 | 46 | 10
+ + + + +

=153370

Multiplication of 5 digits

Example 60: Multiply 34567 by 12345

Solution:



= 3 | 10 | 22 | 40 | 65 | 72 | 70 | 58 | 35
+ + + + + + + +

= 426729615

Readers can extend the multiplication of 6 or 7 digits by their own by dot and cross method.

Multiplication of 3 numbers by Vedic method

The traditional multiplication can't do the multiplication of three numbers in one go, but Vedic Mathematics can, in no time, do the multiplication of three-digit numbers closer to the base and sub-base with ease. For the convenient of readers, the modus operandi of the whole process is illustrated with the help of some examples.

Case 1: When the numbers to be multiplied are nearer to the power of 10.

Example 61: Multiply 1 2 x 1 3 x 1 5

Solution:

Step 1: Write the deviation of each number from its base. Place it against each number. For the above multiplication, the working base = 10, and their deviations from the base are 2, 3 and 5 respectively.

Number	Deviation from the Base
12	+ 2
13	+ 3
15	+ 5

Step 2: Add all the deviations to the working base.

10 + 2 + 3+ 5 = 20

Step 3: Make all possible permutations of the deviation, taking two at a time and add them. For example, the possible permutation of abc is ab, bc and ac.

2 x 3 + 2 x 5 + 3 x 5 = 31

Step 4: Multiply the deviations

2 x 3 x 5 = 30

Step 5: Arrange the result obtained in above steps as shown here.

20 | 31 | 30

Adding the result from right to left in the direction of the arrows we can find the result.

$$20 \mid 31 \mid 30$$

$$= 2340$$

Example 62: Multiply 105 x 104 x 109

Solution: The working base = 100

Step 1: Find the deviation of each number from its base. Write it against each number.

Number	Deviation
105	+ 5
104	+ 4
109	+ 9

Step 2: Find the sum of base and deviations.

Base + deviations = 100 + 5 + 4 + 9 = 118

Step 3: Multiply the deviations in pairs of two and sum up the results so obtained

5 x 4 + 4 x 9 + 5 x 9 = 101

Step 4: Multiply the deviations.

5 x 4 x 9 = 180

Step 5: Arrange the result of all the above steps in a vertical separator and add them up, from right to left as done previously. But never forget to keep two digits in each separator as constant because the base taken here is 100.

$$118 \mid 101 \mid 180$$

$$= 1190280$$

Example 63: 989 x 995 x 1012 = ?

Solution: The working base for the above number = 1000

Step 1: Find the deviation of each number from its base. Write it against each number.

Number	Deviation
989	– 11
995	– 5
1012	+ 12

Step 2: Find the sum of base and deviations.

Base+ deviations = 1000 – 11 – 5 + 12 = 996

Step 3: Multiply the deviations in pair of two and sum up the result so obtained.

$(-11) \times (-5) + 5 \times (-12) + (-11) \times (+12)$

$= 55 - 60 - 132 = -137$

Step 4: Multiply the deviations

$-11 \times -5 \times 12 = 660$

Step 5: Arrange the result of all the above steps in a vertical separator.

$= 996 \mid (-137) \mid 660$

$= 995 \mid 1000 - 137 \mid 660$

$= 995 \mid 863 \mid 660$

$= 995863660$

Case 2: When the working base is 50, 500, 5000...

The working rule of case 2 is same as that of case 1, except for the following:

- Write the last result in column.
- Keep the right hand figure of column 3 intact and divide the figure in the 2nd column by ½ and the figure in the 1st column by ¼.
- Add the digits of each column as shown in the example.

Example 64: Multiply 54 x 56 x 51

Solution:

Step 1: Write the deviation of each number from its base. Place it against each number.

Number	Deviation
54	+ 4
56	+ 6
51	+ 1

Step 2: Find the sum of base and deviations.

$50 + 4 + 6 + 1 = 61$

Step 3: Multiply the deviations in pairs of two and sum up the results so obtained

$4 \times 6 + 4 \times 1 + 6 \times 1 = 34$

Step 4: Multiply the deviations

$4 \times 6 \times 1 = 24$

Arrange the result obtained above in a vertical separator.

61 | 34 | 24

Make three columns and write the result accordingly, as shown here.

Column 1	Column 2	Column 3
61	34	24
= 61 x ¼	34 x ½	24
= 15 + ¼	17	24
= 15	+ 25 + 17	24
= 15 42 24		

Here the theoretical base = 100 and working base = 50 = 100|2, so ½ = 50 and ¼ = 25 is taken.

Example 65: Multiply 54 x 48 x 61

Solution: The working base = 50 = 100 | 2 and Theoretical base = 100

Step 1: Find the deviation of each number from its base. Write it against each number.

Number	Deviation
54	+4
48	–2
61	+11

Step 2: Find the sum of base and deviations.

Base + deviations = 50+ 4 – 2 + 11 = 63

Step 3: Multiply the deviations in pairs of two and sum up the results so obtained

$4 \times (-2) + 4 \times 11 + 11 \times (-2) = 14$

Step 4: Multiply the deviations.

$4 \times (-2) \times 11 = -88$

The final result is 63 | 14 | – 88

Write the result in columns.

Column 1	Column 2	Column 3
63 x ¼	14 x ½	–88
= 15 + ¾	7	–88
= 15	+ 75 + 7	–88
= 15	82	–88
= 15	81	100 –88
= 15	81	12

Hence, 54 x 56 x 61 = 158112

Multiplication of four numbers by Vedic Method

If you are asked to multiply four numbers the first thing that will strike you is to multiply the first two numbers and then multiply the product with the third number and finally the product with the fourth number. Don't you think that even multiplication of 4 two-digit numbers will take 5 minutes of your time? On the contrary, multiplication of four numbers near the same base or sub-base can be done through the Vedic method quite easily and that too in a few seconds. Are you ready to ride on the chariot of Vedic Sutra?

The answer of such multiplication will consist of 4 parts.

Rule:

- First, write the deviation of numbers from its base against the number.

Suppose you have to multiply a x b x c x d if the deviation of these numbers from its base is d_1 d_2 d_3 and d_4, then we will write–

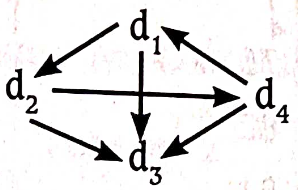
a d_1

b d_2

c d_3

d d_4

- 1^{st} part = Sum of any number and other three deviations.
  $= a + (d_2 + d_3 + d_4)$ or $b + (d_1 + d_3 + d_4)$,or $c + (d_1 + d_2 + d_4)$ or $d + (d_2 + d_3 + d_4)$
- 2^{nd} part = Sum of the product of two deviations



$= d_1d_2 + d_1d_3 + d_1d_4 + d_2d_3 + d_2d_4 + d_3d_4$

- 3^{rd} part = Sum of the product of three numbers at a time
  $= d_1d_2d_3 + d_1d_2d_4 + d_1d_3d_4 + d_2d_3d_4$
- 4^{th} part = Product of all deviations
  $= d_1d_2d_3d_4$

Example 66: Multiply 102 x 103 x 104 x 105

Solution: Here all the numbers are near the base 100 and their deviations are respectively 2, 3, 4 and 5.

102 + 2

103 + 3

104 + 4

105 + 5

- 1^{st} part = Sum of any number and other three deviations.
  = 102 + 3 + 4 + 5= 114

- 2^{nd} part = Sum of the product of two deviations
  $= 2 \times 3 + 2 \times 4 + 2 \times 5 + 3 \times 4 + 3 \times 5 + 4 \times 5 = 71$

- 3^{rd} part = Sum of the product of three numbers at a time
  $= 2 \times 3 \times 4 + 2 \times 3 \times 5 + 2 \times 4 \times 5 + 3 \times 4 \times 5$
  $= 154$

- 4^{th} part = Product of all deviations
  $= 2 \times 3 \times 4 \times 5 = 120$
  Hence, $102 \times 103 \times 104 \times 105 = 114 \mid 71 \mid 154 \mid 120$

  $= 114 \mid 72 \mid 55 \mid 20 = 114725520$

Since the base is 100, each part will contain a maximum of two digits and the excess digits will be transferred to the next part.

Example 67: Multiply $995 \times 996 \times 997 \times 998$

Solution: Here all numbers are near the base 100 and their deviations are respectively 2, 3, 4 and 5.

995	−5
996	−4
997	−3
998	−2

- 1^{st} part = Sum of any number and other three deviations.
  $= 995 - 4 - 3 - 2 = 986$

- 2^{nd} part = Sum of the product of two deviations
  $= (-5) \times (-4) + (-5) \times (-3) + (-5) \times (-2) + (-4) \times (-3) + (-4) \times (-2) + (-3) \times (-2)$
  $= 20 + 15 + 10 + 12 + 8 + 6 = 71$

- 3^{rd} part = Sum of the product of three numbers at a time
  $= (-5) \times (-4) \times (-3) + (-5) \times (-4) \times (-2) + (-4) \times (-3) \times (-2) + (-5) \times (-3) \times (-2)$
  $= -60 - 40 - 24 - 30$
  $= -154$

- 4^{th} part = Product of all deviations
  $= -2 \times -3 \times -4 \times -5 = 120$

Hence, 995 x 996 x 997 x 998 = 986 | 71 | −154 | 120

Here, the base is taken as 1000, so each part will consist of three digits. In the 2^{nd} part, we have a two-digit number, so put a zero in front of it, making it 071. In the 3^{rd} part, we have a negative number, so subtract 1 from the 2^{nd} part and the negative number of part 3 from the base 1000. Our answer will now look like–

$$
\begin{aligned}
995 \times 996 \times 997 \times 998 &= 986 \mid 071 \mid -154 \mid 120 \\
&= 986 \mid 070 \mid 1000 - 154 \mid 120 \\
&= 986 \mid 070 \mid 846 \mid 120 \\
&= 986070846120
\end{aligned}
$$

Case 2: When the base is the multiple of 100, 1000... etc.

Suppose the four numbers to be multiplied are closer to 200, then we shall first write 200 = 2 x 100. Here Base = 100 and sub- base = 2. In such a case, the above process will be applied with a slight change. In the 1^{st} part, multiply the result with $(\text{sub-base})^3$, 2^{nd} part with $(\text{sub-base})^2$ and 3^{rd} part will be multiplied by the sub-base. The number of zeros in the base will decide the number of digits to be kept in each part while writing the final answer.

Example 68: Multiply 506 x 507 x 508 x 509

Solution:

Here all the numbers to be multiplied are near to 500 = 5 x 100, hence Base = 100 and sub-base = 5. Deviations of the number from 500 are 6, 7, 8 and 9 respectively

506	+ 6
507	+ 7
508	+ 8
509	+ 9

- 1st part = (Sum of any number and other three deviations) x (Sub-base)^3
  $= (506+7+8+9) \times 5^3 = 530 \times 125 = 66250$
  (Refer **Multiplication through observation** for multiplying a number by 125)

- 2nd part = (Sum of product of two deviations) x (Sub-base)^2
  $= (6 \times 7 + 6 \times 8 + 6 \times 9 + 7 \times 8 + 7 \times 9 + 8 \times 9) \times 5^2 = 335 \times 25 = 8375$
  (Refer **Multiplication through observation** for multiplying a number by 25)

- 3rd part = (Sum of product of three numbers at a time) x (Sub-base)
  $= (6 \times 7 \times 8 + 6 \times 7 \times 9 + 6 \times 8 \times 9 + 7 \times 8 \times 9) \times 5 = 1650 \times 5 = 8250$

- 4th part = Product of all deviations
  $= 6 \times 7 \times 8 \times 9 = 3024$

Hence, $506 \times 507 \times 508 \times 509 = 66250 \mid {}_{83}75 \mid {}_{82}50 \mid {}_{30}24$

Since the base = 100, each part will contain maximum two digits and the rest will be transferred to the next part. The excess digit is written in subscript.

$506 \times 507 \times 508 \times 509 = 66250 \mid {}_{83}75 \mid {}_{82}50 \mid {}_{30}24$
$= 66334578024$

4

Multiplication through Observation

Introduction

The best part of Vedic maths is that you can do calculations in a few seconds and sometimes, orally. In this chapter we shall learn how to do quick and flawless calculations and that too by mere observation. Once the techniques for such special cases are mastered, you will feel enthusiastic enough to do some more tricks of your own.

Mental Multiplication

A: Multiplication by 11

Multiplication of any number with 11 can be done orally in a single line. Once the technique for multiplication of a number with 11 is mastered, it can be further extended for a number such as 22, 33, 44 etc by simply splitting the multiplicand as 11 x 2, 11 x 3 or 11 x 4. In mensuration, you need to calculate the volume and surface area of three-dimensional objects such as cylinder, sphere, cone, pyramid, frustum etc and there you need to multiply the number by 22 (π =22/7). This method will help you immensely here.

Rule:

- Place the number to be multiplied by 11 in a bracket and put zeros on either side.

- Start adding the two numbers at a time from right to left. If the sum of two numbers in any case exceeds 10, the digit at the tenth place shall be carried over to the next sum, as is usually done in simple addition.

Example 1: Multiply 3251 by 11

Solution:

Place the number in a bracket and put zeros on either side.

0 (3 2 5 1) 0

Add the digit from the right to left as shown above.

0+ 3 | 3+2 | 2 + 5 | 5 + 1 | 1 + 0

= 3 5 7 6 1

Hence 3251 x 11 = 35761

Example 2: Multiply 4876254 by 11

Solution:

Place the number in a bracket and put zeros on either side.

0 (4 8 7 6 2 5 4) 0

Add the digit from the right to left as shown above.

= 0+4| 4+8 | 8+7 | 7+6 | 6+2 | 2+5 | 5+4 | 4+0

= 4 | 12 | 15 | 13 | 8 | 7 | 9 | 4

= 4 | 12 | 16 | 3 | 8 | 7 | 9 | 4

= 4 | 13 | 6 | 3 | 8 | 7 | 9 | 4

= 5 3 6 3 8 7 9 4

Hence, 4876254 x 11 = 53638794

Example: Multiply 384 by 11

Solution: Place the number in a bracket and put zeros on either side.

0 (3 8 4) 0

Add the digit from the right to left as shown above.

0+3 | 3+8 | 8+4 | 4+0

= 3 | 11 | 12 | 4

= 3 | 12 | 2 | 4

= 4 2 2 4

Hence, 384 x 11 = 4224 .

B: Multiplication by 111

Rule:- Multiplication with 111 involves the same pattern with a slight difference.

- Place the number to be multiplied inside a bracket and two zeros on either sides of the bracket.
- Add from right to left, taking the sum of three digits at a time.
- If the sum of the digits exceeds 10, the digit at the ten's place will be carried over to the next sum.

Example: Multiply 34 by 111

Solution: Place the number in a bracket and put 2 zeros on either side.

0 0 (3 4) 0 0

Keep adding three digits from the right at a time as shown above.

= 0 + 0 + 3 | 0 + 3 + 4 | 3 + 4 +0 | 4 + 0 + 0

= 3774

Example 2: Multiply 497 by 111

Solution: Place the number in a bracket and put zeros on either side.

0 0 (4 9 7) 0 0

Keep adding from right to left, taking the sum of three digits at a time.

0 0 (4 9 7) 0 0

= 0+0+4 | 0+4+9 | 4+9+7 | 9+7+0 | 7+0+0

= 4 | 13 | 20 | 16 | 7

= 4 | 13 | 21 | 6 | 7

= 4 | 15 | 1 | 6 | 7

= 5 5 1 6 7

C: Multiplication with 1111

Rule: Multiplication with 1111 involves the same sort of operations as discussed above.

- Place the number to be multiplied inside a bracket and three zeros on either side of the bracket.
- Add from right to left, taking the sum of four digits at a time.
- If the sum of the digits exceeds 10, the digit at the ten's place will be carried over to the next sum.

Example: Multiply 2172 by 1111

Solution: Place the number in a bracket and put 3 zeros on either side.

0 0 0 (2 1 7 2) 0 0 0

Keep adding from right to left, taking 4 digits at a time.

Step 1: 0 0 0 (2 1 7 2) 0 0 0

0 + 0 + 0 + 2 = 2

Step 2: 0 0 0 (2 1 7 2) 0 0 0

0 + 0 + 2 + 7 = 9

Step 3: 0 0 0 (2 1 7 2) 0 0 0

0 + 2 + 7 + 1 = 10

Step 4 : 0 0 0 (2 1 7 2) 0 0 0

2 + 7 + 1 + 2 = 12

Step 5: 0 0 0 (2 1 7 2) 0 0 0

0 + 2 + 1 + 7 = 10

Step 6: 0 0 0 (2 1 7 2) 0 0 0

0 + 0 + 2 + 1 = 3

Step 7: 0 0 0 (2 1 7 2) 0 0 0

0 + 0 + 0 + 2 = 2

Arranging all the steps in one line, we get—

= 2 | 3 | 10 | 12 | 10 | 9 | 2

= 2| 3 | 10 | 13 | 0 | 9 | 2

= 2 | 3 | 11 | 13| 0 | 9 | 2

= 2 | 4 | 1 | 3| 0 | 9 | 2

Hence, 2172 x 1111 = 2413092

D: Multiplication by 25

Rule: This is a very simple technique. When you are multiplying a number by 25, put two zeros (00) to the right of the multiplicand and divide it by 4.

Example 1: Multiply 16 by 25

Solution: Put two zeros to the right of 16, i.e. 1600
Divide it by 4 = 1600 / 4 = 400
25 x 16 = 400

Example 2: Multiply 98 by 25

Solution: Put two zeros to the right making it 9800
Divide it by 4, 9800 / 4 = 2450
98 x 25 = 2450

Example 3: Multiply 428764 by 25

Solution: Put two zeros to the right of 428764, i.e. 42876400
Divide it by 4 = 42876400 / 4 = 10719100
428764 x 25 = 10719100

Example 4: Multiply 82456 by 25

Solution: Put two zeros to the right of 82456, i.e. 8245600
Divide it by 4 = 8245600 / 4 = 2061400
82456 x 25 = 2061400

E: Multiplication by 125

Rule: Put three zeros to the right of the multiplicand and divide it by 8.

Example 1: Multiply 624 by 125

Solution: Place 3 zeros after 624, making it 624000.
Divide it by 8= 624000/8 = 78000
Hence, 624 x 125 = 78000

Example 2: Multiply 48 by 125

Solution: Place 3 zeros after 48, making it 48000
Divide it by 8= 48000/8 = 6000
Hence, 48 x 125 = 6000

Example 3: Multiply 24376 by 125

Solution: Place 3 zeros after 24376, making it 24376000
Divide it by 8= 24376000/8 = 3047000
Hence, 24376 x 125 = 3047000

F: Multiplying with 625

Rule: Put 4 zeros (0000) to the right of the number and divide it by 16.

Example 1: Multiply 428 by 625

Solution: Place 4 zeros after 428, making it 4280000
Divide it by 16 = 4280000/16 = 267500
Hence, 428 x 625 = 267500

Example 2: Multiply 246284 by 625

Solution: Place 4 zeros after 246284, making it 2462840000
Divide it by 16 = 2462840000/16 = 153927500
Hence, 246284 x 625 = 153927500

Example 3: Multiply 144 by 625

Solution: Place 4 zeros after 144, making it 1440000
Divide it by 16 =1440000/16 = 90000
Hence, 144 x 625 = 90000

G: Multiply by 5

Rule: Put one zero to the right of the number and divide it by 2.

Example 1: Multiply 42 by 5

Solution: Place 1 zero after 42, making it 420

Divide it by 2 = 420/2 = 210
Hence, 42 x 5 = 210

Example 2: Multiply 5986 by 5

Solution: Place 1 zero after 5986, making it 59860
Divide it by 2 = 59860/2 = 29930
Hence, 5986 x 625 = 29930

H: Multiply by 50

Rule: Put two zeros to the right of the number and divide it by 2.

Example 1: Multiply 47 by 50

Solution: Place 2 zero after 47, making it 4700
Divide it by 2 = 4700/2 = 2350
Hence, 47 x 50 = 2350

Example 2: Multiply 62876 by 50

Solution: Place 2 zeros after 62876, making it 6287600
Divide it by 2 = 6287600/2 = 3143800
Hence, 62876 x 50 = 3143800

I: When the sum of the unit's place digit is 10 and the rest of the digits are the same

Rule:

- Multiply the unit digit whose sum is 10 and place it on the right side.
- Increase the multiplicand by 1 and then multiply it with the original multiplier. Put this result to the left.

Example: Multiply 46 by 44

Solution: Here the sum of the unit digit is 10 and the digit at the ten's place in the multiplier and multiplicand are also same.

$$\begin{array}{r} 46 \\ \times\ 44 \\ \hline \end{array}$$

Multiply the encircled digit and write it to the right side.

$$\begin{array}{r} 4\,6 \\ \times\ 4\,4 \\ \hline /\ 24 \end{array}$$

Increase the multiplicand by 1 and then multiply it with the original multiplier.

$$(4+1) = \begin{array}{r} 5\,6 \\ \times\ 4\,4 \\ \hline 20/\ 24 \end{array}$$

Hence, 46 x 44 = 2024

Example: Multiply 113 by 117

Solution: Here the sum of the unit digit is 10 and the digit at the ten's place in the multiplier and multiplicand are also same.

$$\begin{array}{r} 11\ 3 \\ \times\ 11\ 7 \\ \hline \end{array}$$

Multiply the encircled digit and write it to the right side.

$$\begin{array}{r} 11\ 3 \\ \times\ 11\ 7 \\ \hline /\ 21 \end{array}$$

Increase the multiplicand by 1 and then multiply it with the original multiplier.

$$(11+1) = \begin{array}{r} 12\ 3 \\ \times\ 11\ 7 \\ \hline 132/\ 21 \end{array}$$

Hence, 113 x 117 = 13221
(Multiplication of 12 and 11 can be done as discussed in part A in this chapter itself.)

Example: Multiply 168 by162

Solution: Here the sum of the unit digit is 10 and the digit at the ten's place in the multiplier and multiplicand are also same.

```
  16 8
x 16 2
------
```

Multiply the encircled digit and write it to the right side.

```
  16 8
x 16 2
------
   /16
```

Increase the multiplicand by 1 and then multiply it with the original multiplier.

```
(16+1) =   17  8
         x 16  2
         -------
         272/ 16
```

Hence, 168 x 162 = 27216

(The multiplication of 17 x 16 can be done through the Vedic sutra Urdhyagtiryagbhyam)

J: When the sum of the last two digits is 100 and the rest of the digits at the hundred's places are the same).

This method is applicable when the sum of the unit's and ten's place digits of the multiplicand and multiplier is 100 and hundred's place digit is the same.

```
  H T O
  1 0 3
x 1 9 7
-------
```

Example: Multiply 103 by 197

Rule:

- Multiply the number (unit and ten's place digit) of the multiplicand and multiplier whose sum is 100. The product should be of 4 digits. If the product in RHS is less than four digits, place zero at the extreme left.

HTO

1/03

X1\97

/0291

(Since 03 x 97 = 297 contains only 3 digits, one zero has been placed at the extreme left, making it 0297. As explained above, the right side of the product should contain 4 digits.)

- Increase the multiplicand (digit at hundred's place) by 1 and multiply it with the number placed at the hundred's place in the multiplier.

HTO

(1 + 1)= 2/03

X 1\97

2 /0291

Hence, 103 x 197 = 20291

Example: Multiply 425 by 475

Solution:

Multiply the number (unit and ten's place digit) of the multiplicand and the multiplier whose sum is 100.

HTO

4/25

X4\75

/1875

(Here the sum of the encircled number is 100 (25 +75 =100). For multiplication of 25 x 75, see Rule D.

- Increase the multiplicand (digit at hundred's place) by 1 and multiply it with the number placed at the hundred's place in the multiplier.

HTO

(4 + 1)= 5/2 5

X 4\7 5

2 0 /1875

Hence, 425 x 475 =201875

Example: Multiply 211 by 289

Solution:

Multiply the number (unit and ten's place digit) of the multiplicand and the multiplier whose sum is 100.

HTO

2/1 1

X 4\8 9

/0979

[Here the sum of the encircled numbers is 100 (25 +75 =100). For multiplication of 25 x 75, see Rule A.]

- Increase the multiplicand (digit at the hundred's place) by 1 and multiply it with the number placed at the hundred's place in the multiplier.

HTO

(2 + 1) = 3/1 1

X 2\8 9

6 /0979

Hence, 2 1 1 x 2 8 9 = 60979

K: Multiply any number by 51

Rule:

- Place the multiplicand on the right side. The right side should contain only two digits. If the number of digits at the right side is more than 2, the left- most digit will be carried to the next column on the left side.
- On the left, place half of the multiplicand.
- In case the half of the multiplicand is fractional, add 50 to the digit placed on the right side. This case will occur if the **digit of the multiplicand is odd.**

Example: Multiply 42 by 51

Solution:

$$\begin{array}{r} 42 \\ \underline{\times\ 51} \end{array}$$

- Place 42 at the right side

$$\begin{array}{r} 42 \\ \underline{\times\ 51} \\ /42 \end{array}$$

- Place half of the multiplicand to the left, i.e. 42/2 = 21

$$\begin{array}{r} 42 \\ \underline{\times\ 51} \\ 21\ /42 \end{array}$$

Hence, 42 x 51 = 2142

Example: Multiply 124 by 51

Solution:

$$\begin{array}{r} 124 \\ \underline{\times\ 51} \end{array}$$

- Place 124 at the right side

$$\begin{array}{r} 124 \\ \underline{\times\ 51} \\ /124 \end{array}$$

- Place half of the multiplicand to the left, i.e. 124/2 = 62. The excess digit from the right side will get transferred to the left side.

$$\begin{array}{r} 124 \\ \underline{\times\ 51} \\ 62/124 \end{array}$$

= 63/24

Hence, 124 x 51 = 6324

Example: Multiply 41 by 51

Solution:

$$\begin{array}{r} 41 \\ \underline{\times\ 51} \end{array}$$

- Place 41 at the right side

41
x 51
/41

- Place half of the multiplicand to the left, i.e. 41/2 = 20½.

41
x 51
20 ½ /41

= 20 / 50 + 41= 91

Since, the left most part is fractional, add 50 to the right side and remove the fractional part from the left.

Hence, 41 x 51 = 2091

Example: Multiply 23 by 51

Solution: 23
x 51

- Place 23 at the right side

23
x 51
/23

- Place half of the multiplicand to the left, i.e. 23/2 = 11½.

23
x 51
11½/23

= 11 / 50 + 23= 73

Since, the left most part is fractional, add 50 to the right side and remove the fractional part from the left.

Hence, 23 x 51 = 1173