

Inferential Statistics

- ① Hypothesis Testing
- ② p-value
- ③ Confidence Interval
- ④ Significance Value

Z test
t test
Chi square test
Anova test (F-test)

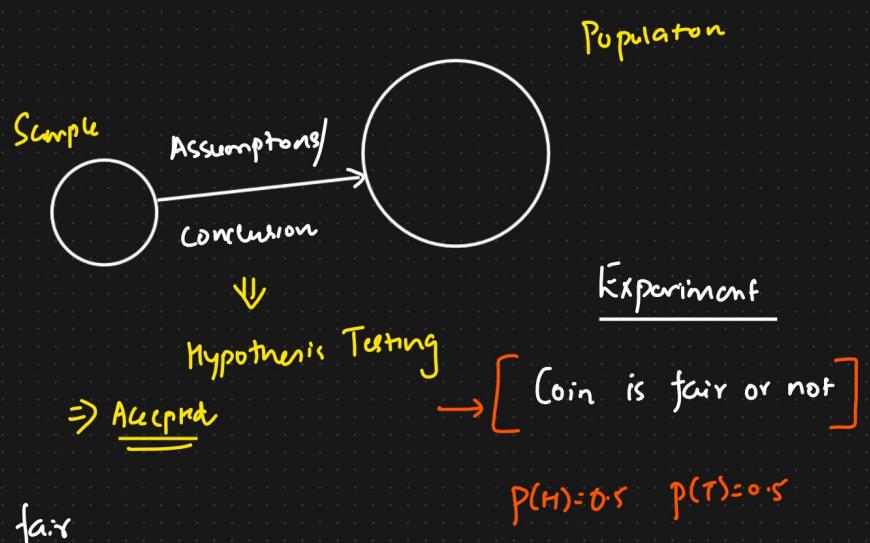
3 Distributions

- ① Bernoulli's
- ② Binomial
- ③ Power Law

Transformation

Inferential Stat

Steps of hypothesis Testing



- ① Null Hypothesis: Coin is fair \Rightarrow Accepted \rightarrow [Coin is fair or not]
- ② Alternate hypothesis: Coin is not fair

③ Perform Experiments

10 \rightarrow Null Hypothesis is Rejected
 \rightarrow Alternate Hypothesis is Accepted

10 times	75	60 40
100 times	70 30	80 20
50 times Head	\Rightarrow Fair	
60 times Head		

$$CI = [20-80]$$



Coin is fair



$C-I \Rightarrow$ Confidence Interval

70 times \Rightarrow Domain Expert



Confidence Interval

- ↳ We fail to Reject the Null Hypothesis [within C-I] \Rightarrow Conclusions
- ↳ We Reject the Null Hypothesis [outside C-I] \Rightarrow Conclusions

② Person is Criminal or not {Murder Case}

① Null Hypothesis : Person is not Criminal

② Alternative Hypothesis : Person is Criminal

③ Evidence / Proof : DNA, finger print, weapons, eye witness, foot age



Judge \Rightarrow

Vaccines \Rightarrow Medical \Rightarrow critical

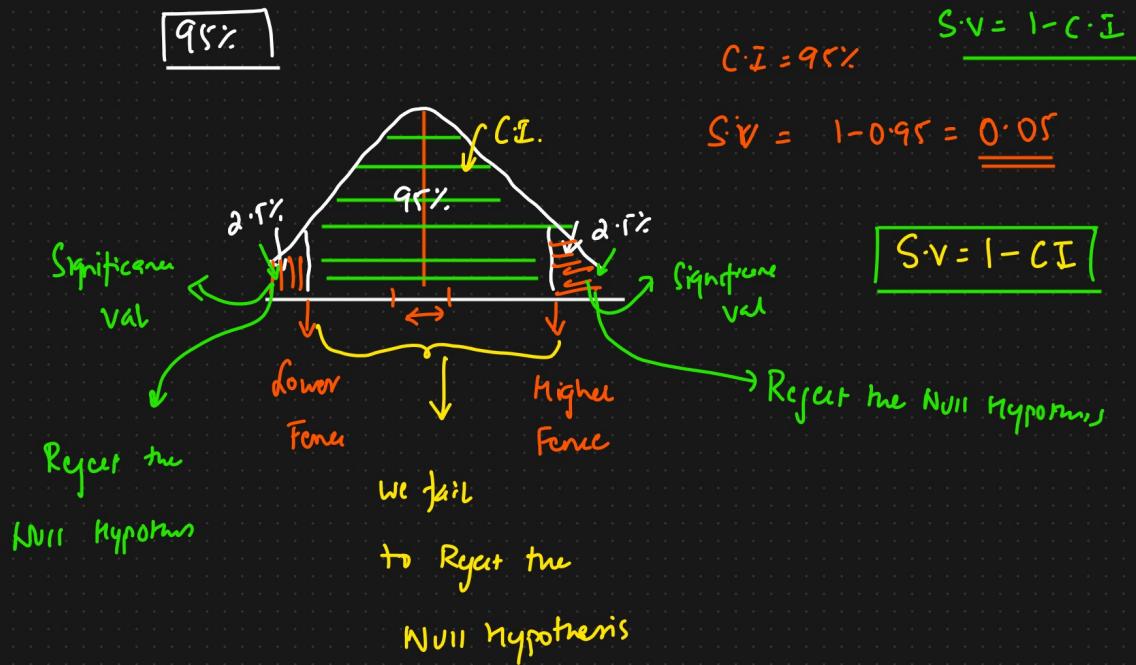
Conclusions

Confidence Interval : (CI)

\Rightarrow Domain Experiment

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Significance Value



Point Estimate : The value of any statistic that estimates the value of a parameter is called Point Estimate

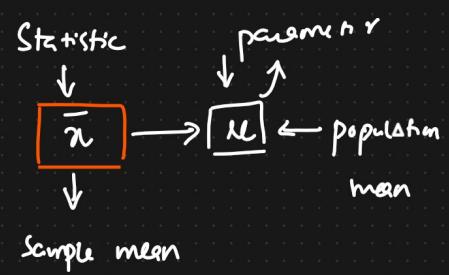
Point Estimate



$$\boxed{\bar{x}}$$

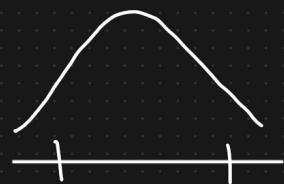
μ

$$\begin{cases} \bar{x} > \mu \\ \bar{x} \leq \mu \end{cases}$$



$$\boxed{\text{Point Estimate}} \pm \boxed{\text{Margin of Error}} = \boxed{\text{Parameter} \Rightarrow \text{population mean}}$$

Lower C.I :- Point Estimate - Margin of Error



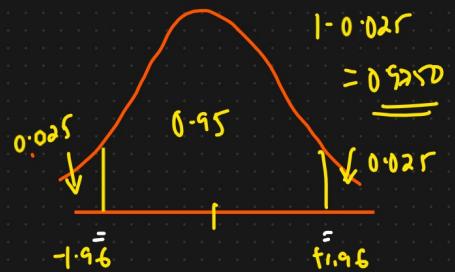
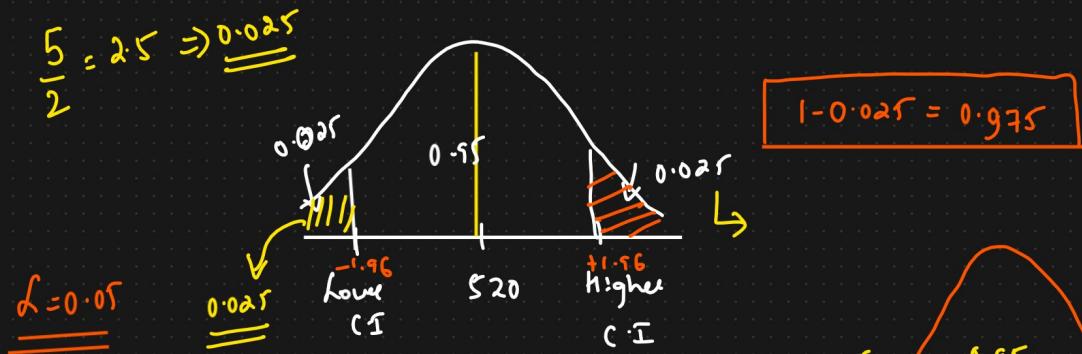
Higher C.I :- Point Estimate + Margin of Error

$$\text{Margin of Error} = Z_{\alpha/2} \left[\frac{\sigma}{\sqrt{n}} \right] \Rightarrow \text{Standard Error}$$

Population Std
 α = Significance Value.

- Q) On the quant test of CAT Exam, a sample of 25 test takers has a mean of 520 with a population standard deviation of 100. Construct a 95% C.I about the mean?

$$\text{Ans) } n=25 \quad \bar{x}=520 \quad \sigma=100 \quad C.I = 95\% \quad S.V = 1-C.I = 0.05$$



Lower C.I = Point Estimate - Margin of Error

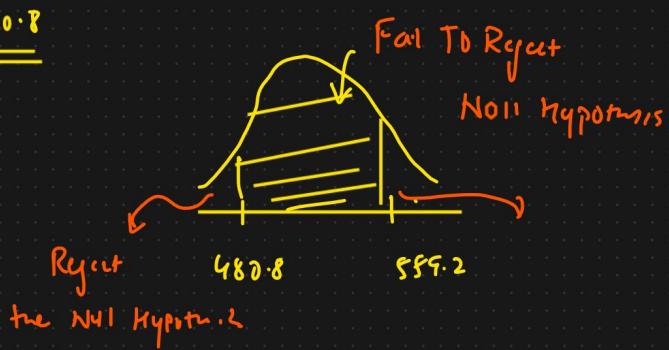
$$= 520 - Z_{0.05/2} \frac{\sigma}{\sqrt{n}}$$

$$= 520 - Z_{0.025} \frac{100}{\sqrt{25}}$$

$$Z_{0.05/2} \Rightarrow \boxed{Z_{0.025}}$$

$$S.V = 1 - 0.95 = 0.05$$

$$= 520 - 1.96 \times 20 = \underline{\underline{480.8}}$$

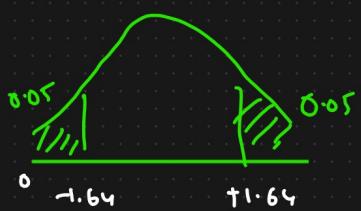


$$\text{Higher CI} = 520 + 1.96 \times 20 = \underline{\underline{559.2}}$$

Reject the Null Hypothesis

① $\bar{x} = 480 \quad \sigma = 85 \quad n=25 \quad C.I = 90\% \quad \text{Significance}$

$$= 1 - 0.90 = \boxed{0.10}$$



$$\text{Lower CI} = 480 - Z_{0.10/2} \left[\frac{85}{5} \right]$$

$$\text{Higher CI} = 480 + Z_{0.10/2} \left[17 \right]$$

$$= 480 - Z_{0.05} \left[\frac{85}{5} \right]$$

$$= 480 + 27.8$$

$$= 480 - 1.64 \left[17 \right]$$

$$= 507.8$$

$$= 480 - 27.8 = 452.12$$

$$\left[452.12 \leftrightarrow 507.8 \right].$$

② On the Quant test of CAT exam, a sample of 25 test takers has a mean of 520, with a sample standard deviation of 80.

Construct 95% CI about the mean?

Ans) $\bar{x} = 520 \quad s = 80 \quad C.I = 95\% \quad S.V = 1 - 0.95 = 0.05 \quad \underline{\underline{}}$
 $n=25$

$$\bar{x} \pm t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right).$$

t test



$$\text{Degree of freedom} = \boxed{n-1} = 25-1 = \boxed{24}$$

19 20 21

$$\text{Lower C.I} = 520 - t_{0.05/2} \left(\frac{\frac{16}{80}}{81} \right) =$$

$$= 520 - 2.064 \times 16$$

$$\text{Lower C.I} = 486.976$$

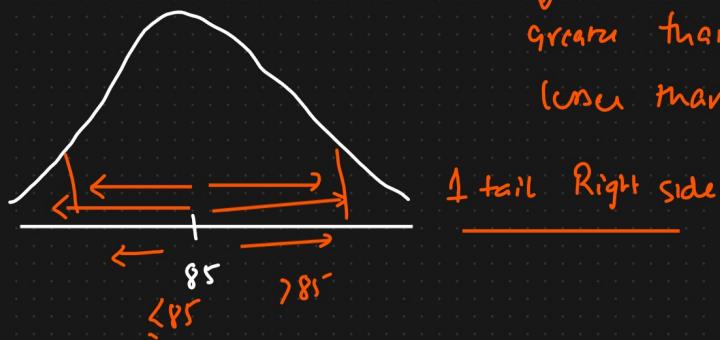
$$\text{Higher C.I} = 553.024$$



① 1 Tail and 2 Tail Test

- ① Colleges in Town A has 85% placement rate. A new College was recently opened and it was found that a sample of 150 students had a placement rate of $\underline{88\%}$ with a standard deviation of $\underline{4\%}$. Does this college has a different placement rate with 95% C.I?

Two Tail.



↓
greater than 85%.

less than 85%.

1 tail Right side

- ① Z-test }
② t-test }.

① A factory has a machine that fills 80ml of Baby medicine in a bottle. An employee believes the average amount of baby medicine is not 80ml. Using 40 samples, he measures the average amount dispersed by the machine to be 78ml with a standard deviation of 2.5.

(a) State Null & Alternate Hypotheses

(b) At 95% C.I, is there enough evidence to support Machine is working properly or not

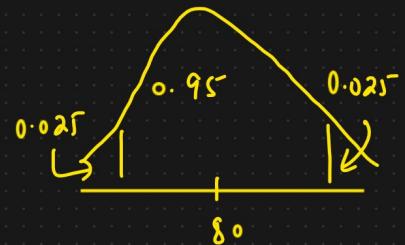
Step 1

Ans) Null Hypothesis $\mu = 80$ \rightarrow

$$\mu = 80 \text{ ml} \quad n = 40 \quad \bar{x} = 78 \quad s = 2.5$$

Alternate Hypothesis $\mu \neq 80$ \rightarrow

Step 2 $\therefore C.I = 0.95 \quad S.V(\alpha) = 1 - 0.95 = 0.05$



Step 3 \therefore

① $n > 30$ or population std $\} \rightarrow Z \text{ test}$

$$n = 40$$

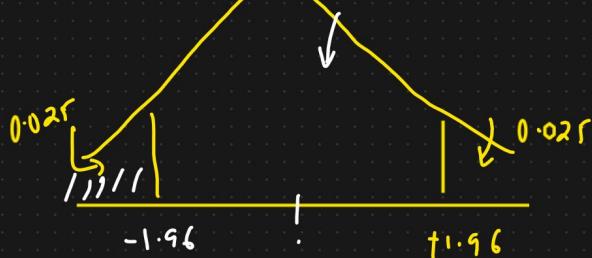
$$s = 2.5$$

② $n < 30$ and sample std $\} \rightarrow t \text{ test}$

Z test

Let's perform the Experiment

Decision Boundary



$$1 - 0.025 = 0.975$$

④ Calculate test statistics (Z-test)

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \rightarrow \text{Standard Error}$$
$$= \frac{78 - 80}{\frac{2.5}{\sqrt{40}}} = -5.05$$

⑤ Conclusions

Decision Rule : If $Z = -5.05$ is less than -1.96 or greater than $+1.96$, Reject the Null Hypothesis with 95% CI

Reject the Null Hypothesis } There is some fault in the
machine.

⑥ A complaint was registered, the boys in a Government School are underfed. Average weight of the boys of age 10 is 32 kgs with $S.D = 9$ kgs. A sample of 25 boys were selected from the Government School and the average weight was found to be 29.5 kgs? With CI = 95%. Check if it is True or False.

Ans) Conditions for Z-test

$$n=25 \quad \mu=32 \quad \sigma=9 \quad \bar{x}=29.5$$

① We know the population sd. OR

② We do not know the population sd but our sample is large $n > 30$

Conditions For T test

- ① We do not know the population std.
- ② Our sample size is small $n < 30$
- ③ Sample std is given.

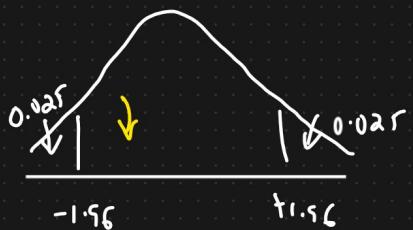
Step 1

① $H_0 : \mu = 32$

$H_1 : \mu \neq 32$

② C.I = 0.95 $\alpha = 1 - 0.95 = 0.05$

Z test



$$Z\text{-score} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{29.5 - 32}{9 / \sqrt{25}} = -1.39$$

σ / \sqrt{n}

Conclusion $= -1.39 > -1.96$ Accept the Null Hypothesis 95% C.I

We fail to Reject the Null Hypothesis

The Boys are fed well.

① A factory manufactures cars with a warranty of 5 years ^{or more} on the engine and transmission. An engineer believes that the engine or transmission will malfunction in less than 5 years. He tests a sample of 40 cars and finds the average time to be 4.8 years with a standard deviation of 0.50. ① State the null & alternate hypothesis

② At a 2% significance level, is there enough evidence to support the idea that the warranty should be revised?

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Step 1: $H_0 : \mu \geq 5$

$H_1 : \mu < 5$

Step 2: $\alpha = 0.02$ $C.I = 0.98$

② In the population the average IQ is 100 with a standard deviation of 15. A team of scientists wants to test a new medication to see if it has a tre or -ve effect, or no effect at all.

A sample of 30 participants who have taken the medication has a mean of 140. Did the medication affect intelligence? $\{ 95\% \}$