Collisions

5.3

If you have ever been to an amusement park, chances are you have ridden in the bumper cars, where the objective is for you and your friends to crash into one another (**Figure 1**). In Section 5.2, you learned about two types of interactions within a system—collisions and explosions. In this section, you will learn more about different types of collisions, as well as what happens to the energy of systems when a collision occurs. How can you use what you have learned about linear momentum and kinetic energy to understand what happens when two bumper cars collide and bounce apart, and what happens when they collide and stay together?

Figure 1 Momentum and kinetic energy can help explain what happens to the directions and speeds of objects when they collide with one another.

Elastic and Inelastic Collisions

In Section 5.2, you learned how to analyze a system of two objects. The total momentum of the objects before a collision is equal to the total momentum of the objects after the collision. In this section, you will apply the law of conservation of momentum to analyze several different types of collisions. In general, a collision changes the velocities of the objects involved. The final velocities (those found just after the collision) are different from the initial velocities (from just before the collision). Since the kinetic energy of an object depends on its speed, the kinetic energy of the object also changes as a result of the collision. Collisions fall into two general types, depending on what happens to the total kinetic energy of the entire system: elastic and inelastic collisions.

Elastic Collisions

In an **elastic collision**, the system's kinetic energy is conserved. That is, the total kinetic energy of the two objects after the collision is equal to the total kinetic energy of the two objects before the collision. This is called **conservation of kinetic energy**. The term *elastic* can help you understand how and why a collision affects the kinetic energy. For example, an extremely elastic ball (such as a rubber ball) is compressed during a collision, and this compression stores energy in the ball just as energy is stored in a compressed spring. In an ideal rubber ball, all of this potential energy is turned back into kinetic energy when the ball decompresses (springs back) at the end of the collision.

elastic collision a collision in which momentum and kinetic energy are conserved

conservation of kinetic energy the total kinetic energy of two objects before a collision is equal to the total kinetic energy of the two objects after the collision

Figure 2 shows two rubber balls before and after an elastic collision.

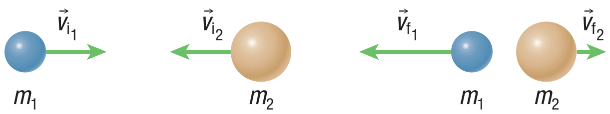

Figure 2 In elastic collisions, both momentum and kinetic energy are conserved.

Inelastic Collisions

inelastic collision a collision in which momentum is conserved, but kinetic energy is not conserved

In contrast to elastic collisions, **inelastic collisions** are collisions in which some kinetic energy is lost. The kinetic energy is transformed into other forms, such as thermal energy or sound energy. Consider a collision involving a ball composed of soft putty or clay. The ball will not spring back at the end of the collision. Energy is absorbed, causing the kinetic energy after the collision to be less than the kinetic energy before the collision. The collision, therefore, is inelastic.

In summary,

- In an elastic collision, both momentum and kinetic energy are conserved.
- In an inelastic collision, momentum is conserved, but kinetic energy is not conserved.

Note that in both types of collisions, momentum is conserved. The total energy is also conserved in both cases, even if the kinetic energy is not. WEB LINK

Mini Investigation

Newton's Cradle

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this investigation, you will explore the concept of conservation of momentum in collisions using a Newton's cradle (**Figure 3**).

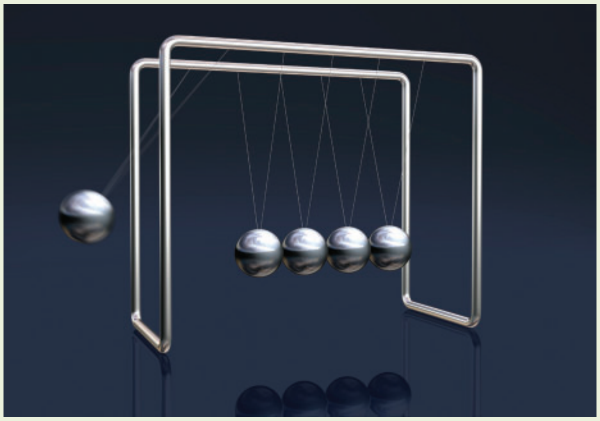

Figure 3

Equipment and Materials: Newton's cradle

1. Set up the Newton's cradle. Make sure all of the metal spheres are correctly aligned.
2. Pull back one of the spheres at the end and release. Observe how momentum is transferred from one end of the cradle to the other.
3. Explore what happens when you change the initial setup. For example, try changing the number of active spheres by holding up one of the end spheres during the investigation. You can also try moving one of the middle spheres out of the way of the collisions.

A. What happened during Step 2? How did your results change when you modified the setup in Step 3? T/I A

B. Do the collisions appear to conserve momentum? Explain your answer. K/U C

C. Do the collisions appear to conserve kinetic energy? Explain your answer. K/U C

D. Does the device as a whole appear to conserve mechanical energy? If not, identify some reasons for the energy loss. K/U T/I C A

Perfectly Elastic Collisions and Perfectly Inelastic Collisions

A **perfectly elastic collision** is an idealized situation where friction and other external forces are negligible, and therefore momentum and kinetic energy are perfectly conserved. On the other hand, a **perfectly inelastic collision** is one in which the two objects in a collision stick together after the collision so that the objects have the same final velocity. Perfectly elastic and perfectly inelastic collisions occur in isolated systems in which the effects of friction and other external forces are negligible. Perfectly elastic and perfectly inelastic collisions are extremely rare in the world around us, and represent idealized cases. Most real collisions fall somewhere between these two extreme situations. However, it is useful to consider perfectly elastic and perfectly inelastic collisions as ideal examples of Newton's laws. As you do this, be mindful of external forces that may additionally affect the systems.

perfectly elastic collision an ideal collision in which external forces are minimized to the point where momentum and kinetic energy are perfectly conserved

perfectly inelastic collision an ideal collision in which two objects stick together perfectly so they have the same final velocity; in this situation, momentum is perfectly conserved, but kinetic energy is not conserved

Perfectly Elastic Collisions

By applying some basic assumptions, you can use collisions with billiard balls, bumper cars, and asteroid–planet systems as reasonable examples of perfectly elastic collisions.

In perfectly elastic collisions, both momentum and kinetic energy are conserved:

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

In one-dimensional collisions, each vector can point in only one of two ways. Designate directions in a way that is convenient for solving a particular problem. For example, you may choose to assign right as positive and left as negative. Then you can express the vectors using only their magnitudes, understanding that a negative value implies a left direction. In Tutorial 1, you will use these equations to explore perfectly elastic collisions in one dimension.

Tutorial 1 Perfectly Elastic Collisions in One Dimension

In the following Sample Problem, you will use conservation of momentum and kinetic energy to analyze a perfectly elastic collision.

Sample Problem 1: Analyzing Perfectly Elastic Collisions

Suppose you have two balls with different masses involved in a perfectly elastic collision. Ball 1, with mass $m_1 = 0.26$ kg travelling at a velocity $v_1 = 1.3$ m/s [right], collides head-on with stationary ball 2, which has a mass of $m_2 = 0.15$ kg. Determine the final velocities of both balls after the collision.

Given: $m_1 = 0.26$ kg; $\vec{v}_{i_1} = 1.3$ m/s [right]; $m_2 = 0.15$ kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$, $\vec{v}_{f_2}$

Analysis: The collision is perfectly elastic, which means that kinetic energy is conserved. Apply conservation of momentum and conservation of kinetic energy to construct and solve a linear-quadratic system of two equations in two unknowns. First use the conservation of momentum equation to express the final velocity of ball 1 in terms of the final velocity of ball 2. Then substitute the resulting equation into the conservation of kinetic energy equation to solve for the final velocity of ball 2. Use the result to solve for the final velocity of ball 1. This is a one-dimensional problem, so omit the vector notation for velocities, recognizing that positive values indicate motion to the right and negative values indicate motion to the left.

Solution: First use conservation of momentum to solve for $\vec{v}_{f_1}$ in terms of $\vec{v}_{f_2}$:

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$$

Isolate the term containing $\vec{v}_{f_1}$ on the left side of the equation. Since $v_{i_2} = 0$, the equation becomes

$$m_1v_{f_1} = m_1v_{i_1} - m_2v_{f_2}$$

Divide both sides by m_1:

$$v_{f_1} = v_{i_1} - \frac{m_2}{m_1}v_{f_2}$$

$$= 1.3\ \text{m/s} - \frac{0.15\ \text{kg}}{0.26\ \text{kg}}v_{f_2}$$

$$v_{f_1} = 1.3\ \text{m/s} - 0.58v_{f_2} \qquad \text{(Equation 1)}$$

The conservation of kinetic energy equation can be simplified by multiplying both sides of the equation by 2 and noting that $v_{i_2} = 0$:

$$\cancel{2}\left(\frac{1}{\cancel{2}}m_1v_{i_1}^2 + \frac{1}{\cancel{2}}m_2v_{i_2}^2\right) = \cancel{2}\left(\frac{1}{\cancel{2}}m_1v_{f_1}^2 + \frac{1}{\cancel{2}}m_2v_{f_2}^2\right)$$

$$m_1v_{i_1}^2 = m_1v_{f_1}^2 + m_2v_{f_2}^2 \qquad \text{(Equation 2)}$$

Substitute Equation 1 and the given values into Equation 2:

$$(0.26\ \text{kg})(1.3\ \text{m/s})^2 = (0.26\ \text{kg})(1.3\ \text{m/s} - 0.58v_{f_2})^2 + (0.15\ \text{kg})v_{f_2}^2$$

Expand and simplify both sides of the equation:

$$0.439\ \text{kg}\cdot\text{m}^2/\text{s}^2 = (0.26\ \text{kg})(1.69\ \text{m}^2/\text{s}^2 - 1.51\ \text{m/s}\ v_{f_2} + 0.34v_{f_2}^2) + (0.15\ \text{kg})v_{f_2}^2$$

$$0.439\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 0.439\ \text{kg}\cdot\text{m}^2/\text{s}^2 - 0.39\ \text{kg}\cdot\text{m/s}\ v_{f_2} + 0.088\ \text{kg}\ v_{f_2}^2 + (0.15\ \text{kg})v_{f_2}^2$$

$$0.439\ \cancel{\text{kg}}\cdot\text{m}^2/\text{s}^2 = 0.439\ \cancel{\text{kg}}\cdot\text{m}^2/\text{s}^2 - 0.39\ \cancel{\text{kg}}\cdot\text{m/s}\ v_{f_2} + 0.24\ \cancel{\text{kg}}\ v_{f_2}^2$$

$$0 = -0.39\ \text{m/s}\ v_{f_2} + 0.24v_{f_2}^2$$

Express the quadratic equation in standard form:

$$0 = -0.39\ \text{m/s}\ v_{f_2} + 0.24v_{f_2}^2$$

Solve by common factoring:

$$0 = (-0.39\ \text{m/s} + 0.24v_{f_2})v_{f_2}$$

The factor of v_{f_2} means the equation has a solution $v_{f_2} = 0$ m/s. This solution describes the system before the collision. The equation has a second solution describing the system after the collision:

$$0.24v_{f_2} - 0.39\ \text{m/s} = 0$$

$$v_{f_2} = \frac{0.39\ \text{m/s}}{0.24}$$

$$v_{f_2} = 1.63\ \text{m/s (one extra digit carried)}$$

Substitute this value into Equation 1 and calculate the final velocity of ball 1:

$$v_{f_1} = 1.3\ \text{m/s} - 0.58v_{f_2}$$

$$= 1.3\ \text{m/s} - (0.58)(1.63\ \text{m/s})$$

$$v_{f_1} = 0.35\ \text{m/s}$$

Statement: The second ball has a final velocity of 1.6 m/s [right] and the first ball has a final velocity of 0.35 m/s [right].

Practice

1. Two balls collide in a perfectly elastic collision. Ball 1 has mass 3.5 kg and is initially travelling at a velocity of 5.4 m/s [right]. It collides head-on with stationary ball 2 with mass 4.8 kg. Determine the final velocity of ball 2. T/I [ans: 4.6 m/s [right]]
2. A curling stone with initial speed v_1 collides head-on with a second, stationary stone of identical mass m. Calculate the final speeds of the two curling stones. K/U T/I A [ans: $v_{f_1} = 0$; $v_{f_2} = v_1$]

The analysis and solution used above apply to the special case of a perfectly elastic collision where one body is initially at rest. A more general case of perfectly elastic collisions involves two bodies that are already in motion before the collision. Collisions of this type will be explored in greater depth later in this chapter.

Perfectly Inelastic Collisions

The simplest type of inelastic collision is a perfectly inelastic collision, in which two objects stick together after the collision so that the objects have the same final velocity. If the colliding objects bounce, it is not a perfectly inelastic collision. A good example of a perfectly inelastic collision is one in which two cars lock bumpers. **Figure 4** on the next page shows two cars coasting on a straight road in one dimension with velocities $\vec{v}_{i_1}$ and $\vec{v}_{i_2}$. The cars collide and lock bumpers, and they have the same velocity $\vec{v}_f$ after the collision. The cars are moving in a horizontal direction, x, and if they are coasting, there are no external forces on the cars in this direction. Therefore, the total momentum along x is conserved.

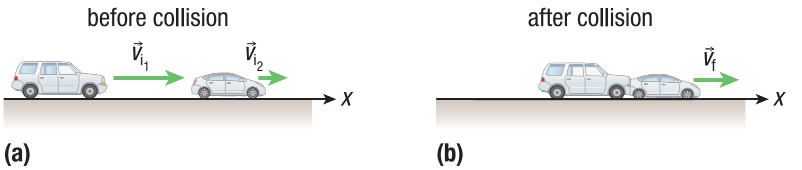

Figure 4 In an inelastic collision, only momentum is conserved. (a) Velocities just before a one-dimensional collision. (b) After the collision the cars travel with $\vec{v}_f$.

The condition for conservation of momentum along x is

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

In this case, there is only one unknown, the final velocity $\vec{v}_f$, since the two velocities are the same when the two objects stick together. Solving the above equation for $\vec{v}_f$ gives

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$$

This equation relates the final velocity of two objects in a perfectly inelastic collision to their masses and their initial velocities. You will apply this equation in Tutorial 2.

UNIT TASK BOOKMARK

You can apply what you learn about collisions to the Unit Task on page 270.

Tutorial 2 Perfectly Inelastic Collisions in One Dimension

In these Sample Problems, you will calculate the final velocities of two vehicles after a one-dimensional perfectly inelastic collision.

Sample Problem 1: Applying the Conservation of Momentum to a One-Dimensional Perfectly Inelastic Collision

Two cars, a sports utility vehicle (SUV) of mass 2500 kg and a compact model of mass 1200 kg, are coasting at a constant velocity along a straight road. Their initial velocities are 40.0 m/s [W] and 10.0 m/s [W], respectively, so the SUV is catching up to the compact car. When they collide, the cars lock bumpers. Assume no external forces exist along the direction of travel, and the total momentum along x is conserved. Determine the velocity of the cars just after the collision.

Given: $m_1 = 2500$ kg; $\vec{v}_{i_1} = 40.0$ m/s [W]; $m_2 = 1200$ kg; $\vec{v}_{i_2} = 10.0$ m/s [W]

Required: $\vec{v}_{f_1}$

Analysis: The cars stick together after the collision, so this is an example of a perfectly inelastic collision. In this system, both vehicles are moving in the same direction (west).

Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$ to calculate the final velocity.

Solution:

$$\vec{v}_f = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$$

$$= \frac{(2500\text{ kg})(40.0\text{ m/s [W]}) + (1200\text{ kg})(10.0\text{ m/s [W]})}{2500\text{ kg} + 1200\text{ kg}}$$

$$= \frac{(100\,000\ \cancel{\text{kg}}\cdot\text{m/s [W]}) + (12\,000\ \cancel{\text{kg}}\cdot\text{m/s [W]})}{3700\ \cancel{\text{kg}}}$$

$$\vec{v}_f = 3.0 \times 10^1\text{ m/s [W]}$$

Statement: The final velocity of the cars is 3.0×10^1 m/s [W].

Sample Problem 2: Applying Conservation of Momentum and Energy in a One-Dimensional Perfectly Inelastic Collision

A child with a mass of 22 kg runs at a horizontal velocity of 4.2 m/s [forward] and jumps onto a stationary rope swing of mass 2.6 kg. The child "sticks" on the rope swing and swings forward.

(a) Determine the horizontal velocity of the child plus the swing just after impact.

(b) How high do the child and swing rise?

Solution

(a) **Given:** Let m_1 and v_{i_1} represent the mass and initial velocity of the child, and m_2 and v_{i_2} represent the mass and initial velocity of the swing. $m_1 = 22$ kg; $\vec{v}_{i_1} = 4.2$ m/s [forward]; $m_2 = 2.6$ kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_f$

Analysis: Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$ to determine the final velocity just after the child collides with the swing.

Solution:

$$\begin{aligned}\vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}\\ &= \frac{(22\text{ kg})(4.2\text{ m/s}) + (2.6\text{ kg})(0\text{ m/s})}{22\text{ kg} + 2.6\text{ kg}}\\ &= \frac{(92.4\ \cancel{\text{kg}}\cdot\text{m/s}) + (0\ \cancel{\text{kg}}\cdot\text{m/s})}{24.6\ \cancel{\text{kg}}}\\ \vec{v}_f &= 3.76\text{ m/s [forward] (one extra digit carried)}\end{aligned}$$

Statement: The child and rope swing have a final velocity of 3.8 m/s [forward] just after the collision.

(b) **Given:** $m_1 = 22$ kg; $m_2 = 2.6$ kg; $\vec{v}_f = 3.76$ m/s

Required: Δy

Analysis: Conservation of energy requires that the initial kinetic energy of the child and swing transform to gravitational potential energy at the highest point of the swing: $E_k = mg\Delta y$.

Solution:

$$\begin{aligned}E_k &= mg\Delta y\\ \frac{1}{2}\cancel{(m_1 + m_2)}(\vec{v}_f)^2 &= \cancel{(m_1 + m_2)}g\Delta y\\ \Delta y &= \frac{(\vec{v}_f)^2}{2g}\\ &= \frac{(3.76\text{ m/s})^2}{2(9.8\text{ m/s}^2)}\\ &= \frac{(3.76)^2\text{ m}^{\cancel{2}}/\cancel{\text{s}^2}}{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}^2})}\\ \Delta y &= 0.72\text{ m}\end{aligned}$$

Statement: The child and rope swing will rise to 0.72 m above the initial height.

Practice

1. A child rolls a 4.0 kg ball with a speed of 6.0 m/s toward a 2.0 kg ball that is stationary. The two balls stick together after the collision. What is their velocity immediately after the perfectly inelastic collision? K/U A [ans: 4.0 m/s [forward]]
2. In a scene in an action film, a car with a mass of 2200 kg, travelling at 60.0 km/h [E], collides with a car of mass 1300 kg that is travelling at 30.0 km/h [E], and the two cars lock bumpers. T/I A
   (a) Calculate the velocity of the vehicles immediately after the perfectly inelastic collision. [ans: 14 m/s [E]]
   (b) Calculate the total momentum of the two cars before and after the collision. [ans: 4.8×10^4 kg·m/s]
   (c) Determine the decrease in kinetic energy during the collision. [ans: 2.8×10^4 J]
3. A 66 kg snowboarder slides down a hill 25 m high and has a perfectly inelastic collision with an initially stationary 72 kg skier at the bottom of the hill. Friction is negligible. Calculate the speed of each person after the collision. K/U A [ans: 11 m/s]

5.3 Review

Summary

- In an isolated system, the total momentum of the system is conserved for all elastic, inelastic, perfectly elastic, and perfectly inelastic collisions:

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

- In a perfectly elastic collision, total kinetic energy is conserved:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

- In an inelastic collision, the total kinetic energy is not conserved, although total energy is always conserved.

Questions

1. Two boxes are on an icy, frictionless, horizontal surface. You push one box, which collides with the other. If the boxes stick together after the collision, which of the following is conserved in the collision? Explain your reasoning. K/U
   (a) momentum
   (b) kinetic energy
2. An 85 kg skateboarder takes a running jump onto his skateboard, which has a mass of 8.0 kg and is initially at rest. After he lands on the skateboard, the speed of the board plus the skateboarder is 3.0 m/s. Determine the speed of the skateboarder just before he landed on the skateboard. T/I
3. A student puts two dynamics carts with a speed bumper between them on a track and presses them together. The total mass of the carts is 3.0 kg. Once the student releases them, the carts spring apart and roll away from each other. One cart has a mass of 2.0 kg and a final velocity of 2.5 m/s [S]. Calculate the final velocity of the other cart. T/I
4. Two people are riding inner tubes on an ice-covered (frictionless) lake. The first person has a mass of 85 kg and is travelling with a speed of 6.5 m/s. He collides head-on with the second person with a mass of 120 kg who is initially at rest. They bounce apart after the perfectly elastic collision. The final velocity of the first person is 1.1 m/s in the opposite direction to his initial direction. K/U T/I C A
   (a) Are momentum and kinetic energy conserved for this system? Explain your answer.
   (b) Determine the final velocity of the second person.
5. Two skaters are studying collisions on an ice-covered (frictionless) lake. Skater 1 has a mass of 95 kg and is initially travelling with a speed of 5.0 m/s, and skater 2 has a mass of 130 kg and is initially at rest. Skater 1 then collides with skater 2, and they lock arms and travel away together. K/U T/I
   (a) Does the system undergo an elastic collision or an inelastic collision? Explain your answer.
   (b) Solve for the final velocity of the two skaters.
6. Two cars of equal mass (1250 kg) collide head-on in a perfectly inelastic collision. Just before the collision, one car is travelling with a velocity of 12 m/s [E] and the other at 12 m/s [W]. Determine the velocity of each car after the collision. A T/I
7. A moving object collides with a stationary object. If the collision is perfectly elastic, is it possible for both objects to be at rest after the collision? Explain your answer. K/U C A
8. A truck of mass 1.3×10^4 kg, travelling at 9.0×10^1 km/h [N], collides with a car of mass 1.1×10^3 kg, travelling at 3.0×10^1 km/h [N]. The collision is perfectly inelastic. T/I A
   (a) Calculate the magnitude and direction of the velocity of the vehicles immediately after the collision.
   (b) Calculate the total kinetic energy before and after the collision described in (a).
   (c) Determine the decrease in kinetic energy during the collision.