5.4 Head-on Elastic Collisions

In previous sections, you read about systems involving elastic and inelastic collisions. You applied the law of conservation of momentum and the law of conservation of kinetic energy to solve problems. You also read about the ideal cases of the perfectly elastic and perfectly inelastic collisions, and learned that in both cases momentum is conserved. You discovered that kinetic energy is conserved in the case of perfectly elastic collisions, but not inelastic collisions. In this section, you will examine in greater detail situations involving perfectly elastic collisions in one dimension. You will calculate the final velocities of two objects after an elastic collision in one dimension with equations for special cases. Once again, we consider near-perfectly elastic collisions in one dimension (**Figure 1**).

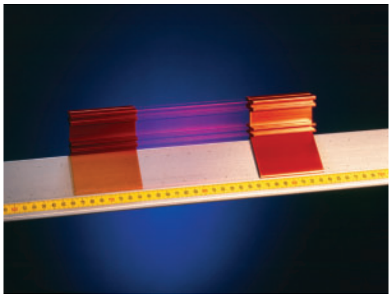

Figure 1 Gliders on an air track can undergo elastic collisions in one dimension.

Perfectly Elastic Head-on Collisions in One Dimension

head-on elastic collision an impact in which two objects approach each other from opposite directions; momentum and kinetic energy are conserved after the collision

In a one-dimensional **head-on elastic collision**, two objects approach each other from opposite directions and collide. In such collisions, both momentum and kinetic energy are conserved. You can derive expressions for the final velocities of two objects in a head-on collision in terms of the initial velocities and the objects' masses.

Suppose an object of mass m_1 travels with initial velocity v_{i_1} and collides head-on with an object of mass m_2 travelling at velocity v_{i_2}. If we assume a one-dimensional collision, we can omit the vector notation for velocities, and instead use positive and negative values to identify motion in one direction or the opposite direction. We begin the analysis with the conservation of momentum:

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2} \qquad \text{(Equation 1)}$$

Rewrite Equation 1 by bringing all the terms with m_1 to one side and all the terms with m_2 to the other side, and common factoring the m coefficients:

$$m_1v_{i_1} - m_1v_{f_1} = m_2v_{f_2} - m_2v_{i_2}$$

$$m_1(v_{i_1} - v_{f_1}) = m_2(v_{f_2} - v_{i_2}) \qquad \text{(Equation 2)}$$

Since this is an elastic collision, conservation of total kinetic energy can be applied:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

Multiply both sides of the equation by 2 to clear the fractions:

$$\cancel{2}\left(\frac{\cancel{1}}{\cancel{2}}m_1v_{i_1}^2 + \frac{\cancel{1}}{\cancel{2}}m_2v_{i_2}^2\right) = \cancel{2}\left(\frac{\cancel{1}}{\cancel{2}}m_1v_{f_1}^2 + \frac{\cancel{1}}{\cancel{2}}m_2v_{f_2}^2\right)$$

$$m_1v_{i_1}^2 + m_2v_{i_2}^2 = m_1v_{f_1}^2 + m_2v_{f_2}^2$$

Collect m_1 terms on the left side and m_2 terms on the right side and divide out the common factors.

$$m_1v_{i_1}^2 - m_1v_{f_1}^2 = m_2v_{f_2}^2 - m_2v_{i_2}^2$$

$$m_1(v_{i_1}^2 - v_{f_1}^2) = m_2(v_{f_2}^2 - v_{i_2}^2)$$

Factor both sides using the difference of squares:

$$m_1(v_{i_1} - v_{f_1})(v_{i_1} + v_{f_1}) = m_2(v_{f_2} - v_{i_2})(v_{f_2} + v_{i_2}) \qquad \text{(Equation 3)}$$

Divide Equation 3 by Equation 2:

$$\frac{\cancel{m_1(v_{i_1} - v_{f_1})}(v_{i_1} + v_{f_1})}{\cancel{m_1(v_{i_1} - v_{f_1})}} = \frac{\cancel{m_2(v_{f_2} - v_{i_2})}(v_{f_2} + v_{i_2})}{\cancel{m_2(v_{f_2} - v_{i_2})}}$$

$$v_{i_1} + v_{f_1} = v_{f_2} + v_{i_2} \qquad \text{(Equation 4)}$$

Rearranging Equation 4 to isolate v_{f_2} on the left gives

$$v_{f_2} = v_{i_1} + v_{f_1} - v_{i_2} \qquad \text{(Equation 5)}$$

Substitute Equation 5 into Equation 1 to express v_{f_1} in terms of the masses and their initial speeds:

$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2(v_{i_1} + v_{f_1} - v_{i_2})$$
$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{i_1} + m_2 v_{f_1} - m_2 v_{i_2}$$

Collect the v_{f_1} terms on the right side of the equation and terms involving initial velocities on the left side, then collect like terms and divide out common m coefficients:

$$m_1 v_{i_1} - m_2 v_{i_1} + m_2 v_{i_2} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{f_1}$$
$$(m_1 - m_2)v_{i_1} + 2m_2 v_{i_2} = (m_1 + m_2)v_{f_1}$$

Divide both sides by $m_1 + m_2$ to isolate v_{f_1}:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

This equation expresses the final velocity of the first object in terms of the masses and initial velocities of the two objects. Similarly, we can rearrange Equation 4 to isolate v_{f_1} on the left:

$$v_{f_1} = v_{f_2} + v_{i_2} - v_{i_1} \qquad \text{(Equation 6)}$$

To derive a similar equation for v_{f_2}, follow the above steps for v_{f_1}, starting with substituting Equation 6 into Equation 1, and ending by dividing both sides by $m_1 + m_2$ and isolating v_{f_2} on the left side:

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

This equation expresses the final velocity of the second object in terms of the masses and initial velocities of the two objects. Note that the equation for $\vec{v}_{f_2}$ is the same as the equation for $\vec{v}_{f_1}$ if you interchange all the 1 and 2 subscripts. It is important to note that these equations hold true only for perfectly elastic collisions in one dimension.

In some cases, one of the objects is initially at rest. For instance, if v_2 is initially zero, the equations above simplify to

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$$
$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

In Tutorial 1, you will use these velocity relationships as an alternative way of analyzing some head-on elastic collisions. As you do Tutorial 1, compare these methods to the methods used in Section 5.3. WEB LINK

Tutorial 1 Analyzing Head-on Elastic Collisions

In these Sample Problems, you will use the relative velocity relationships derived above to solve problems related to head-on elastic collisions in one dimension.

Sample Problem 1: Head-on Elastic Collision with One Object at Rest in One Dimension

Consider an elastic head-on collision between two balls of different masses, as shown in **Figure 2**. The mass of ball 1 is 1.2 kg, and its velocity is 7.2 m/s [W]. The mass of ball 2 is 3.6 kg, and ball 2 is initially at rest. Determine the final velocity of each ball after the collision.

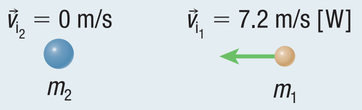

Figure 2

Given: $m_1 = 1.2$ kg; $\vec{v}_{i_1} = 7.2$ m/s [W]; $m_2 = 3.6$ kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Since one of the objects is initially at rest in the head-on elastic collision, use the simplified equations:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution: Let the negative x-direction represent west.

For ball 1,

$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&= \left(\frac{1.2\text{ kg} - 3.6\text{ kg}}{1.2\text{ kg} + 3.6\text{ kg}}\right)(-7.2\text{ m/s}) \\
&= 3.6\text{ m/s} \\
\vec{v}_{f_1} &= 3.6\text{ m/s [E]}
\end{aligned}$$

For ball 2,

$$\begin{aligned}
\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&= \left(\frac{2(1.2\text{ kg})}{1.2\text{ kg} + 3.6\text{ kg}}\right)(-7.2\text{ m/s}) \\
&= -3.6\text{ m/s} \\
\vec{v}_{f_2} &= 3.6\text{ m/s [W]}
\end{aligned}$$

Statement: The final velocity of ball 1 is 3.6 m/s [E]. The final velocity of ball 2 is 3.6 m/s [W].

Sample Problem 2: Head-on Elastic Collision with Both Objects Moving in One Dimension

In a bumper car ride, bumper car 1 has a total mass of 350 kg and is initially moving at 4.0 m/s [E]. In a head-on completely elastic collision, bumper car 1 hits bumper car 2. The total mass of bumper car 2 is 250 kg, and it is moving at 2.0 m/s [W]. Calculate the final velocity of each bumper car immediately after the collision.

Given: Let east be positive and west be negative; $m_1 = 350$ kg; $\vec{v}_{i_1} = 4.0$ m/s [E] $= +4.0$ m/s; $m_2 = 250$ kg; $\vec{v}_{i_2} = 2.0$ m/s [W] $= -2.0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis:
$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution:
$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2} \\
&= \left(\frac{350\text{ kg} - 250\text{ kg}}{350\text{ kg} + 250\text{ kg}}\right)(4.0\text{ m/s}) \\
&\quad + \left(\frac{2(250\text{ kg})}{350\text{ kg} + 250\text{ kg}}\right)(-2.0\text{ m/s}) \\
&= -1.0\text{ m/s} \\
\vec{v}_{f_1} &= 1.0\text{ m/s [W]} \\
\vec{v}_{f_2} &= \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&= \left(\frac{250\text{ kg} - 350\text{ kg}}{350\text{ kg} + 250\text{ kg}}\right)(-2.0\text{ m/s}) \\
&\quad + \left(\frac{2(350\text{ kg})}{350\text{ kg} + 250\text{ kg}}\right)(4.0\text{ m/s}) \\
&= 5.0\text{ m/s} \\
\vec{v}_{f_2} &= 5.0\text{ m/s [E]}
\end{aligned}$$

Statement: The final velocity of bumper car 1 is 1.0 m/s [W]. The final velocity of bumper car 2 is 5.0 m/s [E].

Practice

1. A ball of mass 80.0 g is moving at 7.0 m/s [W] when it undergoes a head-on elastic collision with a stationary ball of mass 60.0 g. Assume the collision is one-dimensional. Calculate the velocity of each ball after the collision. T/I [ans: 1.0 m/s [W]; 8.0 m/s [W]]
2. Cart 1 has a mass of 1.5 kg and is moving on a track at 36.5 cm/s [E] toward cart 2. The mass of cart 2 is 5 kg, and it is moving toward cart 1 at 42.8 cm/s [W]. The carts collide. The collision is cushioned by a Hooke's law spring, making it an elastic head-on collision. Calculate the final velocity of each cart after the collision. T/I [ans: cart 1: 90 cm/s [W]; cart 2: 6 cm/s [W]]

Special Cases

Using these new equations for head-on elastic collisions in one dimension, special cases of collisions, such as objects of equal mass, produce some interesting results.

CASE 1: OBJECTS HAVE THE SAME MASS

The first case we consider is when the objects that are colliding have the same mass, so let $m_1 = m_2 = m$.

$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m - m}{m + m}\right)\vec{v}_{i_1} + \left(\frac{2m}{m + m}\right)\vec{v}_{i_2} \\
&= \left(\frac{0}{2m}\right)\vec{v}_{i_1} + \left(\frac{2m}{2m}\right)\vec{v}_{i_2} \\
&= \left(\frac{2m}{2m}\right)\vec{v}_{i_2} \\
\vec{v}_{f_1} &= \vec{v}_{i_2} \\
\vec{v}_{f_2} &= \left(\frac{m - m}{m + m}\right)\vec{v}_{i_2} + \left(\frac{2m}{m + m}\right)\vec{v}_{i_1} \\
&= \left(\frac{0}{2m}\right)\vec{v}_{i_2} + \left(\frac{2m}{2m}\right)\vec{v}_{i_1} \\
&= \left(\frac{2m}{2m}\right)\vec{v}_{i_1} \\
\vec{v}_{f_2} &= \vec{v}_{i_1}
\end{aligned}$$

In other words, when two objects with the same mass undergo a head-on elastic collision in one dimension, they exchange velocities almost as if they pass through each other.

CASE 2: A LIGHTER OBJECT COLLIDING WITH A MUCH HEAVIER, STATIONARY OBJECT

Our second case deals with situations in which the mass of one of the objects is much greater than the mass of the other object, and the heavier object is stationary. For example, if object 2 is stationary and has a much greater mass, then since m_2 is much greater than m_1, you can consider m_1 to be approximately zero, or negligible. So

$$\begin{aligned}
\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&\approx \left(\frac{0 - m_2}{0 + m_2}\right)\vec{v}_{i_1} \\
\vec{v}_{f_1} &\approx -\vec{v}_{i_1} \\
\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\
&\approx \left(\frac{2(0)}{0 + m_2}\right)\vec{v}_{i_1} \\
\vec{v}_{f_2} &\approx 0
\end{aligned}$$

Investigation 5.4.1

Head-on Elastic Collisions (page 259)

Now that you have an understanding of how head-on elastic collisions work, perform Investigation 5.4.1 to study these types of collisions in greater detail.

In other words, if an object collides with a stationary, much heavier object, the velocity of the light object is reversed, and the heavier object stays at rest. To put this scenario into perspective, consider a collision between a table tennis ball and a stationary transport truck: the transport truck will not move and the table tennis ball will bounce back with the same speed. You will explore more special cases in the questions at the end of Section 5.4.

Conservation of Mechanical Energy

You have discovered what happens to momentum in head-on elastic collisions. What do you suppose happens to the conservation of total mechanical energy during elastic collisions? One of the two gliders in **Figure 3** has been fitted with a spring bumper. When the two gliders collide head-on in an elastic collision, the bounce is not immediate. If you viewed the collision in slow motion, you would see the bumper compress initially and then spring back to its original shape. During the compression, some of the kinetic energy of the moving gliders is converted into elastic potential energy. This potential energy is converted back into kinetic energy during the rebound.

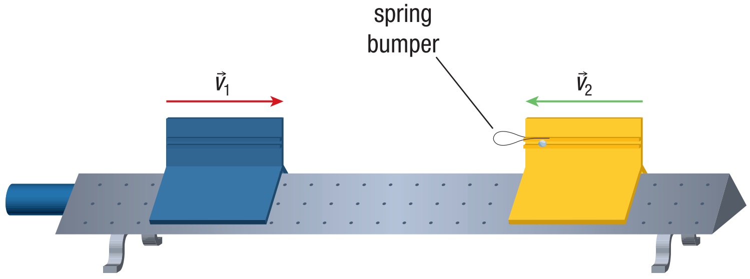

Figure 3 When the two gliders collide, the duration of the collision is greater than it would be without the spring bumper on one of the gliders.

If the compression of the spring bumper during the collision is x, then the law of conservation of energy states:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2 + \frac{1}{2}kx^2$$

This equation and the graph in **Figure 4** both show that as the spring compresses, the elastic energy increases and the total kinetic energy of the two carts decreases. The total mechanical energy, however, stays constant. As the compression decreases, the elastic energy decreases and the total kinetic energy increases. The total mechanical energy still remains constant.

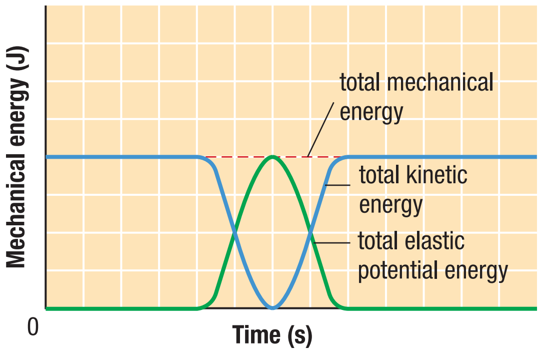

Figure 4 In this graph of total mechanical energy versus time, you can see how the total mechanical energy, the total kinetic energy, and the total elastic potential energy relate to each other throughout the collision.

To determine the maximum compression of the spring during the collision, use the fact that when the two gliders collide they have the same velocity at that point. If they did not have the same velocity at maximum compression, then one would be catching up to the other or pulling away from the other. Therefore, at maximum compression (closest approach), the two objects must have the same velocity, v_f. The equation above then reduces to the following:

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}(m_1 + m_2)v_f^2 + \frac{1}{2}kx^2$$

In Tutorial 2, you will apply the conservation of mechanical energy to problems involving the physics of spring carts.

Tutorial 2 Applying Conservation of Mechanical Energy

In the following Sample Problem, you will apply the conservation of mechanical energy to solve collision problems.

Sample Problem 1: Two-Cart Spring System

Dynamics cart 1 has a mass of 1.8 kg and is moving with a velocity of 4.0 m/s [right] along a frictionless track. Dynamics cart 2 has a mass of 2.2 kg and is moving at 6.0 m/s [left]. The carts collide in a head-on elastic collision cushioned by a spring with spring constant $k = 8.0 \times 10^4$ N/m (**Figure 5**).

(a) Determine the compression of the spring, in centimetres, during the collision when cart 2 is moving at 4.0 m/s [left].

(b) Calculate the maximum compression of the spring, in centimetres.

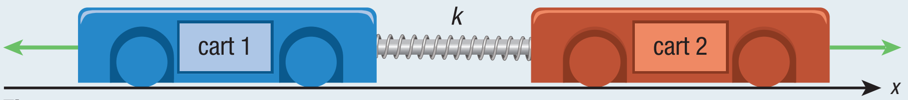

Figure 5

Solution

(a) **Given:** $m_1 = 1.8$ kg; $\vec{v}_{i_1} = 4.0$ m/s [right]; $m_2 = 2.2$ kg; $\vec{v}_{i_2} = 6.0$ m/s [left]; $\vec{v}_{f_2} = 4.0$ m/s [left]; $k = 8.0 \times 10^4$ N/m

Required: x

Analysis: Use the conservation of momentum to determine the velocity of cart 1 during the collision, when cart 2 is moving 4.0 m/s [left]. Then apply the conservation of mechanical energy to determine the compression of the spring at this particular moment during the collision. Consider right to be positive and left to be negative, and omit the vector notation.

Solution: Begin with the conservation of momentum equation.

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$$

Rearrange this equation to express the final velocity of cart 1 in terms of the other given values.

$$m_1v_{i_1} + m_2v_{i_2} - m_2v_{f_2} = m_1v_{f_1}$$

$$\frac{m_1v_{i_1} + m_2v_{i_2} - m_2v_{f_2}}{m_1} = v_{f_1}$$

Substitute the given values and solve.

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2} - m_2 v_{f_2}}{m_1}$$

$$= \frac{(1.8\ \text{kg})(4.0\ \text{m/s}) + (2.2\ \text{kg})(-6.0\ \text{m/s}) - (2.2\ \text{kg})(-4.0\ \text{m/s})}{1.8\ \text{kg}}$$

$$v_{f_1} = 1.56\ \text{m/s (one extra digit carried)}$$

Cart 1 is moving 1.6 m/s [right] when cart 2 is moving 4.0 m/s [left].

Now use the conservation of mechanical energy to determine the compression of the spring, x.

$$\frac{1}{2}m_1 v_{i_1}^2 + \frac{1}{2}m_2 v_{i_2}^2 = \frac{1}{2}m_1 v_{f_1}^2 + \frac{1}{2}m_2 v_{f_2}^2 + \frac{1}{2}kx^2$$

Multiply both sides of the equation by 2 to clear the fractions, and then isolate the term containing x on one side of the equation.

$$\cancel{2}\left(\frac{\cancel{1}}{\cancel{2}}m_1 v_i^2 + \frac{\cancel{1}}{\cancel{2}}m_2 v_i^2 - \frac{\cancel{1}}{\cancel{2}}m_1 v_f^2 - \frac{\cancel{1}}{\cancel{2}}m_2 v_f^2\right) = \cancel{2}\left(\frac{\cancel{1}}{\cancel{2}}kx\right)$$

Divide both sides by k and then take the square root of both sides.

$$\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - m_1 v_{f_1}^2 - m_2 v_{f_2}^2}{k} = \frac{\cancel{k}x^2}{\cancel{k}}$$

$$\sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - m_1 v_{f_1}^2 - m_2 v_{f_2}^2}{k}} = x$$

Substitute the known values to determine the compression of the spring when cart 2 is moving 4.0 m/s [left].

$$x = \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - m_1 v_{f_1}^2 - m_2 v_{f_2}^2}{k}}$$

$$x = \sqrt{\frac{(1.8\ \text{kg})(4.0\ \text{m/s})^2 + (2.2\ \text{kg})(-6.0\ \text{m/s})^2 - (1.8\ \text{kg})(1.56\ \text{m/s})^2 - (2.2\ \text{kg})(4.0\ \text{m/s})^2}{8.0 \times 10^4\ \text{N/m}}}$$

$$x = 2.9 \times 10^{-2}\ \text{m}$$

Statement: The compression of the spring is 2.9 cm during the collision, when cart 2 is moving 4.0 m/s [left].

(b) **Given:** $m_1 = 1.8$ kg; $\vec{v}_{i_1} = 4.0$ m/s [right]; $m_2 = 2.2$ kg; $\vec{v}_{i_2} = 6.0$ m/s [left]; $k = 8.0 \times 10^4$ N/m

Required: x

Analysis: At the beginning of the collision, as the carts come together and the spring is being compressed, cart 1 is moving faster than cart 2. Toward the end of the collision, as the carts separate and the spring is being released, cart 2 will be moving faster than cart 1. At the point of maximum compression of the spring, the two carts will have the same velocity, v_f. Use the conservation of momentum equation to determine this velocity. Then apply the conservation of mechanical energy to calculate the maximum compression of the spring.

Solution: $m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_f + m_2 v_f$

Factor out the common factor v_f.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2)v_f$$

Divide both sides by $m_1 + m_2$ to isolate v_f.

$$\frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2} = v_f$$

Substitute the given values to calculate the velocity of both carts at maximum compression.

$$v_f = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

$$= \frac{(1.8\text{ kg})(+4.0\text{ m/s}) + (2.2\text{ kg})(-6.0\text{ m/s})}{1.8\text{ kg} + 2.2\text{ kg}}$$

$$v_f = -1.5\text{ m/s}$$

Now use the law of conservation of mechanical energy to determine the maximum compression of the spring. Clear the fractions first, and then isolate x.

$$\cancel{2}\left(\frac{1}{\cancel{2}} m_1 v_{i_1}^2 + \frac{1}{\cancel{2}} m_2 v_{i_2}^2\right) = \cancel{2}\left(\frac{1}{\cancel{2}}(m_1 + m_2) v_f^2 + \frac{1}{\cancel{2}} kx^2\right)$$

$$m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2) v_f^2 = kx^2$$

$$\sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2) v_f^2}{k}} = x$$

Substitute the known values and solve for the maximum compression of the spring.

$$x = \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2) v_f^2}{k}}$$

$$= \sqrt{\frac{(1.8\text{ kg})(4.0\text{ m/s})^2 + (2.2\text{ kg})(-6.0\text{ m/s})^2 - (1.8\text{ kg} + 2.2\text{ kg})(-1.5\text{ m/s})^2}{8.0 \times 10^4\text{ N/m}}}$$

$$x = 3.5 \times 10^{-2}\text{ m}$$

Statement: The maximum compression of the spring is 3.5 cm.

Practice

1. A 1.2 kg glider moving at 3.0 m/s [right] undergoes an elastic head-on collision with a glider of equal mass moving at 3.0 m/s [left]. The collision is cushioned by a spring whose spring constant, k, is 6.0×10^4 N/m. T/I A
   (a) Determine the compression in the spring when the second glider is moving at 1.5 m/s [right]. [ans: 1.6 cm]
   (b) Calculate the maximum compression of the spring. [ans: 1.9 cm]
2. A student designs a new amusement park ride that involves a type of bumper car that has a spring on the front to cushion collisions. To test the bumper, the student attaches one spring to the front of a single car. In a collision, car 1 with total mass 4.4×10^2 kg is moving at 3.0 m/s [E] toward car 2 with total mass 4.0×10^2 kg moving at 3.3 m/s [W]. During the collision, the spring compresses a maximum of 44 cm. Determine the spring constant. T/I [ans: 4.3×10^4 N/m]

UNIT TASK BOOKMARK

You can apply what you learn about head-on elastic collisions and conservation of mechanical energy to the Unit Task on page 270.

5.4 Review

Summary

- In a perfectly elastic head-on collision in one dimension, momentum and kinetic energy are conserved.
- Using the law of conservation of momentum and the law of conservation of kinetic energy, we can derive equations to determine the final velocities of two objects in a perfectly elastic head-on collision in one dimension: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$ and $\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$.
- In cases where $\vec{v}_2$ is initially zero, $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$ and $\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$.
- In cases where the masses of the colliding objects are identical, $\vec{v}_{f_1} = \vec{v}_{i_2}$ and $\vec{v}_{f_2} = \vec{v}_{i_1}$.
- In cases in which one mass is significantly larger than the other mass, and the larger mass is stationary, $\vec{v}_{f_1} \approx -\vec{v}_{i_1}$ and $\vec{v}_{f_2} \approx 0$.
- During a head-on collision in one dimension, the kinetic energy of the moving masses is converted into elastic potential energy, and then back into kinetic energy during the rebound. Total mechanical energy is conserved throughout the collision.

Questions

1. Is it possible for two moving masses to undergo an elastic head-on collision and both be at rest immediately after the collision? Is it possible for an inelastic collision? Explain your reasoning. K/U
2. In curling, you will often see one curling stone hit another and come to rest while the stationary stone moves away from the one-dimensional collision. Explain how this can happen. K/U
3. The particles in **Figure 6** undergo an elastic collision in one dimension. Particle 1 has mass 1.5 g and particle 2 has mass 3.5 g. Their velocities before the collision are $\vec{v}_{i_1} = 12$ m/s [right] and $\vec{v}_{i_2} = 7.5$ m/s [left]. Determine the velocity of the two particles after the collision. K/U T/I A

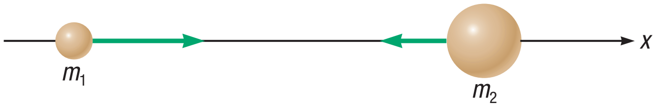

Figure 6

4. Two chunks of space debris collide head-on in an elastic collision. One piece of debris has a mass of 2.67 kg. The other chunk has a mass of 5.83 kg. After the collision, both chunks move in the direction of the second chunk's initial velocity with speeds of 185 m/s for the smaller chunk and 172 m/s for the larger. What are the initial velocities of the two chunks? K/U T/I
5. Dynamics cart 1 has a mass of 0.84 kg and is initially moving at 4.2 m/s [right]. Cart 1 undergoes an elastic head-on collision with dynamics cart 2. The mass of cart 2 is 0.48 kg, and cart 2 is initially moving at 2.4 m/s [left]. The collision is cushioned by a spring with spring constant 8.0×10^3 N/m. T/I
   (a) Calculate the final velocity of each cart after they completely separate.
   (b) Determine the compression of the spring during the collision at the moment when cart 1 is moving at 3.0 m/s [right].
   (c) Determine the maximum compression of the spring.
6. Ball 1 has a mass of 2.0 kg and is suspended with a 3.0 m rope from a post so that the ball is stationary. Ball 2 has a mass of 4.0 kg and is tied to another rope. The second rope also measures 3.0 m but is held at a 60.0° angle, as shown in **Figure 7**. When ball 2 is released, it collides, head-on, with ball 1 in an elastic collision. T/I
   (a) Calculate the speed of each ball immediately after the first collision.
   (b) Calculate the maximum height of each ball after the first collision.

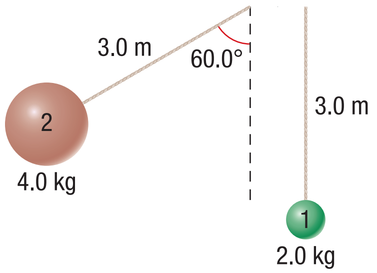

Figure 7