5.2 Conservation of Momentum in One Dimension

Figure 1 Curling requires a firm understanding of momentum and impulse to control the movement of the stones.

Success in the sport of curling relies on momentum and impulse. A player must accelerate a curling stone to a precise velocity to collide with an opponent's stone so that both end up in the desired location (**Figure 1**). The same is true in billiards. You must not only control where the target ball will go, but also where the cue ball will travel. Likewise, a football player receiving a pass, a tennis player delivering a serve, and a lacrosse player attempting a pass must all be in control of momentum and impulse.

In this section, you will examine what happens when two bodies interact such that momentum is exchanged. You will also examine a new aspect of momentum by considering what happens when an impulse is generated from within a single object, such as a rocket taking off, to create the motion of two independent masses, each with its own momentum.

The Law of Conservation of Momentum

collision the impact of one body with another

You may recall that the concept of a system plays an important role in discussions of energy and the law of conservation of energy. You can also apply ideas about momentum and impulse to the motion of a system of objects by examining a **collision**, where two or more objects come together. When two objects collide, they create an associated collision force. In the case of two hockey pucks or two billiard balls, this collision force is a normal (contact) force.

Consider a system of two colliding objects, as shown in **Figure 2**. WEB LINK

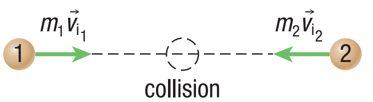

Figure 2 When two objects collide, the total momentum just after the collision is equal to the total momentum just before the collision.

Let $\vec{F}_{21}$ be the force exerted by object 2 on object 1 and $\vec{F}_{12}$ be the force exerted by object 1 on object 2. These forces are an action–reaction pair, so according to Newton's third law, they must be equal in magnitude and opposite in direction: $\vec{F}_{21} = -\vec{F}_{12}$. If this collision takes place over a time interval Δt, the impulse of object 1 is $\vec{F}_{21}\Delta t$. Since the impulse equals the change in momentum of object 1,

$$\vec{F}_{21}\Delta t = \Delta\vec{p}_1$$

and

$$\Delta\vec{p}_1 = \vec{p}_{f_1} - \vec{p}_{i_1}$$

Substituting gives

$$\vec{F}_{21}\Delta t = \vec{p}_{f_1} - \vec{p}_{i_1}$$

Here, $\vec{p}_{i_1}$ is the initial momentum of object 1 just before the collision and $\vec{p}_{f_1}$ is its final momentum just after the collision. Likewise, the impulse imparted to object 2 is $\vec{F}_{12}\Delta t$, which equals the change in the momentum of object 2:

$$\vec{F}_{12}\Delta t = \Delta\vec{p}_2$$

$$\Delta\vec{p}_2 = \vec{p}_{f_2} - \vec{p}_{i_2}$$

$$\vec{F}_{12}\Delta t = \vec{p}_{f_2} - \vec{p}_{i_2}$$

Since $\vec{F}_{21} = -\vec{F}_{12}$ and the interaction times Δt are the same, the impulse imparted to object 1 is equal in magnitude but opposite in sign to the impulse imparted to object 2:

$$\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$$
$$m_1\vec{a}_1 = -m_2\vec{a}_2$$
$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2$$
$$m_1(\vec{v}_{f_1} - \vec{v}_{i_1}) = -m_2(\vec{v}_{f_2} - \vec{v}_{i_2})$$
$$m_1\vec{v}_{f_1} - m_1\vec{v}_{i_1} = -m_2\vec{v}_{f_2} + m_2\vec{v}_{i_2}$$

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

This equation summarizes the **law of conservation of momentum** for two colliding objects:

Law of Conservation of Momentum

When two objects collide in an isolated system, the collision does not change the *total* momentum of the two objects. Whatever momentum is lost by one object in the collision is gained by the other. The total momentum of the system is conserved.

The law of conservation of momentum applies not only to isolated systems of two objects, but also to complex systems. For example, you can easily see how the momentum of the cue ball in a billiards game is transferred to the other balls in the system during the opening break. Momentum is always conserved, whether the colliding objects bounce off one another, as with billiard balls, or remain together, as in the case of a football player who catches a pass.

Investigation 5.2.1

Conservation of Momentum in One Dimension (page 258)
Now that you understand conservation of momentum, perform Investigation 5.2.1 to see how a collision between two objects in one dimension affects the momentum of each object.

Interactions within a System

You have explored the momentum of an object as the product of its mass, m, and its velocity, $\vec{v}$. You have also examined how the application of a force over a specified period of time, an impulse, can change the momentum of an object. You have seen how to derive the equation describing the conservation of motion in an isolated system. In the remainder of this section, you will read more about the concepts of momentum and impulse within an isolated system. Two general categories of interactions exist within a system: collisions and explosions.

COLLISIONS

Momentum is conserved in a system when two or more objects come together in a collision. Examples of a collision are a cue ball hitting another billiard ball, a car rear-ending another car, and one lacrosse player delivering a body check to another.

EXPLOSIONS

Momentum is also conserved in systems when an object or a collection of objects breaks apart in an **explosion**. Fireworks provide vivid images of explosions that give us a feel for the masses and velocities of the many individual objects involved. Accounting for the masses and velocities of all the elements is a challenging task.

explosion a situation in which a single object or group of objects breaks apart

Other examples of explosions may be less obvious. A force used to send an arrow flying affects the momentum of the arrow and the momentum of the bow and archer. Similarly, a squid gains momentum by ejecting water that possesses its own momentum. The same principle is used to put spacecraft into orbit. The following Tutorial illustrates how the law of conservation of momentum can be used to predict the outcome of a collision or explosion. CAREER LINK

Tutorial 1 Applying the Law of Conservation of Momentum

The Sample Problems in this Tutorial apply the law of conservation of momentum to problems involving collisions or explosions in one dimension.

Sample Problem 1: Collision Analysis

A hockey player of mass 97 kg skating with a velocity of 9.2 m/s [S] collides head-on with a defence player of mass 105 kg travelling with a velocity of 6.5 m/s [N]. An instant after impact, the two skate together in the same direction. Calculate the final velocity of the two hockey players.

Given: $m_1 = 97$ kg; $\vec{v}_1 = 9.2$ m/s [S]; $m_2 = 105$ kg; $\vec{v}_2 = 6.5$ m/s [N]

Required: $\vec{v}_f$

Analysis: The players stick together after the collision, so they can be treated as a single object having mass $m_1 + m_2$, velocity v_f, and momentum p_f.

$$\vec{p}_f = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{\vec{p}_f}{(m_1 + m_2)}$$

By conservation of momentum,

$$\vec{p}_f = \vec{p}_1 + \vec{p}_2$$

First determine the total momentum before the collision and then use this result to calculate the velocity of the two players after the collision.

Solution:
$$\begin{aligned}
\vec{p}_f &= \vec{p}_1 + \vec{p}_2 \\
&= (97\text{ kg})(9.2\text{ m/s [S]}) + (105\text{ kg})(6.5\text{ m/s [N]}) \\
&= (97\text{ kg})(9.2\text{ m/s [S]}) - (105\text{ kg})(6.5\text{ m/s [S]}) \\
&= 892\text{ kg·m/s [S]} - 682\text{ kg·m/s [S]} \\
\vec{p}_f &= 210\text{ kg·m/s [S]} \\
\vec{v}_f &= \frac{\vec{p}_f}{(m_1 + m_2)} \\
&= \frac{210\ \cancel{\text{kg}}\text{·m/s [S]}}{(202\ \cancel{\text{kg}})} \\
\vec{v}_f &= 1.0\text{ m/s [S]}
\end{aligned}$$

Statement: After the collision, the two skaters will be travelling at 1.0 m/s [S].

Sample Problem 2: Explosion Analysis

In a science fiction novel, a large asteroid is approaching Earth. Scientists decide to use explosive devices to blow the asteroid into two equal halves before impact with Earth. The asteroid has a mass of 2.4×10^9 kg. For each half to safely miss Earth, an explosion must cause each to travel a minimum of 8.0×10^6 m at a right angle away from Earth within 24 h (**Figure 3**). Assume this is a one-dimensional problem. The magnitude of the impulse applied to each fragment is the same, but the halves are directed in opposite directions.

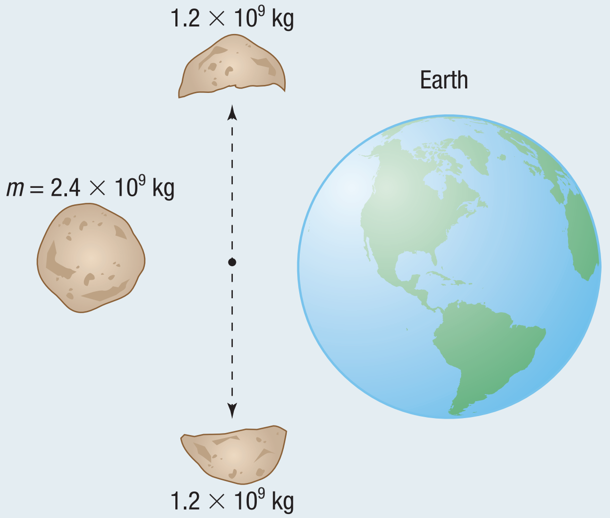

Figure 3

(a) Calculate the momentum of each part of the asteroid after the explosion.

(b) Determine the impulse delivered to each part by the explosion.

Solution

(a) **Given:** $m = 2.4 \times 10^9$ kg; $v = 8.0 \times 10^6$ m/24 h

Required: $\vec{p}$

Analysis: $\vec{p} = m\vec{v}$. Since the total mass of the asteroid is 2.4×10^9 kg, the mass of each half is 1.2×10^9 kg. Use this mass to calculate the momentum. First, convert the speed to metres per second.

$$\frac{8.0 \times 10^6\text{ m}}{24\text{ h}} = \frac{8.0 \times 10^6\text{ m}}{24\ \cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}}$$
$$= 92.6\text{ m/s (one extra digit carried)}$$

Solution:
$$\begin{aligned}
\vec{p} &= m\vec{v} \\
&= (1.2 \times 10^9\text{ kg})(92.6\text{ m/s}) \\
\vec{p} &= 1.111 \times 10^{11}\text{ kg·m/s (two extra digits carried)}
\end{aligned}$$

Statement: The momentum of each part of the asteroid is 1.1×10^{11} kg·m/s.

(b) **Given:** $m = 2.4 \times 10^9$ kg; $v = 8.0 \times 10^6$ m/24 h = 92.6 m/s; $\vec{p} = 1.111 \times 10^{11}$ kg·m/s

Required: $\Delta\vec{p}$

Analysis: The original momentum of each half of the asteroid is zero. So the change in momentum for each half is

$$\Delta\vec{p} = 1.1 \times 10^{11} \text{ kg·m/s}$$

Solution: $\Delta\vec{p} = 1.1 \times 10^{11}$ kg·m/s

$$\Delta\vec{p} = 1.1 \times 10^{11} \text{ N·s}$$

The impulse for the other fragment is 1.1×10^{11} N·s in the opposite direction.

Statement: The explosion will deliver an impulse of magnitude 1.1×10^{11} N·s to each fragment in the directions shown in Figure 3.

Practice

1. A 1350 kg car travelling at 72 km/h [S] collides with a slow-moving car of mass 1650 kg, also initially travelling south. After the collision, the velocity of the two cars together is 24 km/h [S]. Determine the initial velocity at which the second car was travelling. T/I [ans: 15 km/h [N]]
2. After shooting a 28 g arrow with an initial velocity of 92 m/s [forward], an archer standing on a frictionless surface travels in the opposite direction at a speed of 0.039 m/s. Calculate the combined mass of the archer and the bow. T/I A [ans: 6.6×10^1 kg]

You can demonstrate conservation of momentum experimentally by creating systems that minimize the influence of external forces, such as friction. For example, air pucks interacting on a cushion of air near motion sensors provide a means for carefully measuring the initial and final velocities of two objects after a collision.

Keep in mind that while momentum is always conserved in an isolated system, real-life systems are subject to many outside influences, such as friction and other complicated forces, which can make detecting conservation of momentum difficult. For example, if you jump on Earth, it does move, even if you cannot sense the movement. Momentum is conserved when you jump, but because of the huge mass of Earth, the change in its velocity is quite small.

UNIT TASK BOOKMARK

You can apply what you have learned about conservation of momentum to the Unit Task on page 270.

Rocket Propulsion

Momentum is conserved when a rocket engine burns fuel and expels a continuous stream of gases at an extremely high velocity (**Figure 4**). In this explosion, the expanding gases act against the rocket, propelling the rocket forward. In the vacuum of deep space, where gravity is negligible, it is possible to achieve a nearly perfectly isolated system, free from the influences of friction and gravity, providing ideal conditions for the study of momentum. The study of rocket propulsion is complicated, however, because the mass of the rocket changes continuously as the rocket burns fuel. For this reason, a detailed study of rocket propulsion requires knowledge of calculus, so we will not discuss it here. However, it may be something you wish to explore if you have already studied calculus or plan to do so in the future. CAREER LINK

Figure 4 The momentum of combusted gases ejected from the rocket is balanced by the forward momentum of the rocket.

5.2 Review

Summary

- For any interaction involving a system that experiences no external forces, the total momentum before the interaction is equal to the total momentum after the interaction.
- During an interaction between two objects in a system that experiences no external forces, the change in momentum of one object is equal in magnitude but opposite in direction to the change in momentum of the other object: $m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$.
- Interactions within a system can be categorized as collisions, in which two or more objects come together, or explosions, in which a single object or collection of objects separates.

Questions

1. Identify the conditions required for the total momentum of a system to be conserved. K/U
2. A 55 kg student stands on a 4.6 kg surfboard moving at 2.0 m/s [E]. The student then walks with a velocity of 1.9 m/s [E] relative to the surfboard. Determine the resulting velocity of the surfboard, relative to the water. Neglect friction. T/I
3. Two stationary hockey players push each other so that they move in opposite directions. One player has a mass of 35.6 kg and a speed of 2.42 m/s. What is the mass of the other player if her speed is 3.25 m/s? Neglect friction. T/I
4. A baseball pitcher with a mass of 80 kg is initially standing at rest on extremely slippery artificial turf. He then throws a baseball with a mass of 0.14 kg with a horizontal velocity of 50 m/s. Determine the recoil velocity of the pitcher. T/I
5. Consider a collision in one dimension that involves two objects of masses 4.5 kg and 6.2 kg. The larger mass is initially at rest, and the smaller mass has an initial velocity of 16 m/s [E]. The final velocity of the larger object is 10.0 m/s [E]. Calculate the final velocity of the smaller object after the collision. T/I
6. Two objects of masses m and $3m$ undergo a collision in one dimension. The lighter object is moving at three times the speed of the heavier object. Describe what happens to their speeds after the collision. Explain your reasoning. Assume that the lighter mass is moving to the right. K/U T/I C A
7. An object of mass $m_1 = 2.5$ kg has a one-dimensional collision with another object of mass $m_2 = 7.5$ kg, as shown in **Figure 5**. Their initial speeds along x are $v_1 = +6.0$ m/s and $v_2 = -15$ m/s. The two objects stick together after the collision. Calculate the velocity after the collision. T/I

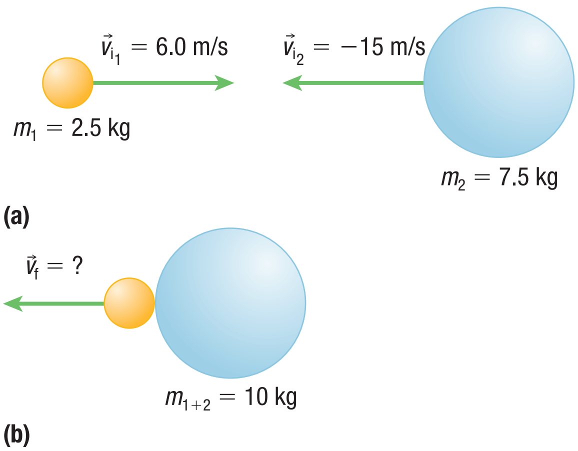

Figure 5

8. An astronaut on a spacewalk outside the International Space Station (ISS) has a safety equipment failure that leaves her floating in space just out of reach of the station airlock. Fortunately, she is still holding a tool bag. Explain how she can use the tool bag and conservation of momentum to return safely to the ISS. K/U T/I A