Numerical Methods 2
Task-1 Report
14. Numerical Calculation of the integral $\int_a^b f(x)\, dx$. Use the n-point (n = 3,5,7,...,33) Gauss-Legendre rule.

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1 Introduction

Legendre-Gauss quadrature, often known as Gauss-Legendre quadrature, is a numerical approximation of a definite integral,

$$\int_{-1}^{1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i).$$

where,

- n is the number of sample points used,
- w_i are quadrature weights, and
- x_i are the roots of the nth Legendre polynomial.

2 Task

In this task problem, there is a limit on n for calculating the integral using the n-point Gauss-Legendre formula, i.e. the minimum is 3 and maximum is 33, and only odd numbers are used. So taking that into account the code has been written with the if-statements properly catering to the above mentioned limits. Thankfully, the coefficients and nodes, which were very important for calculating the integrals, for each different value of n was provided in a .dat file and they were very key in calculating the integrals.

The results for some of the functions using this method have been displayed in the Results section of this report.

3 Result

3.1 Integral test result for $\int_0^1 log(x)\, dx$

For this integral, we already know that the **value for this integral is -1.** Below are the values that were achieved for different point values including the absolute and relative errors.

Point	Integral	Absolute Error	Relative Error
3	-0.947672	0.052328	0.052328
5	-0.979001	0.020999	0.020999
7	-0.977739	0.022261	0.022261
9	-0.99299	0.00701	0.00701
11	-0.995219	0.004781	0.004781
13	-0.996532	0.003468	0.003468
15	-0.99737	0.00263	0.00263
17	-0.997937	0.002063	0.002063
19	-0.998339	0.001661	0.001661
21	-0.998634	0.001366	0.001366
23	-0.998856	0.001144	0.001144
25	-0.999029	0.000971	0.000971
27	-0.999165	0.000835	0.000835
29	-0.999274	0.000726	0.000726
31	-0.999364	0.000636	0.000636
33	-0.999437	0.000563	0.000563

As we can see, as the point value increases, the value for the integral gets closer to the right value of the integral, which is -1.

3.2 Integral test result for $\int_0^1 x + log^2(x)\, dx$

For this integral, we already know that **the value for this integral is 2.5.** Below are the values that were achieved for different point values.

Point	Integral	Absolute Error	Relative Error
3	2.04127	0.458733	0.183493
5	2.27688	0.223121	0.0892484
7	2.3662	0.1338	0.05352
9	2.41003	0.089968	0.0359872
11	2.43497	0.065027	0.0260108
13	2.4506	0.049401	0.0197604
15	2.46108	0.038922	0.0155688
17	2.46847	0.031532	0.0126128
19	2.47389	0.026113	0.0104452
21	2.47799	0.022013	0.0088052
23	2.48117	0.018832	0.0075328
25	2.48369	0.016311	0.0065244
27	2.48572	0.014276	0.0057104
29	2.48739	0.01261	0.005044
31	2.48877	0.011226	0.0044904
33	2.48994	0.010064	0.0040256

As we can see, as the point value increases, the value for the integral gets closer to the right value of the integral, which is 2.5.

3.3 Integral test result for $\int_{-1}^{2} x^3 - x^2\,dx$

For this integral, we already know that **the value for this integral is 0.75.** Below are the values that were achieved for different point values.

Point	Integral	Absolute Error	Relative Error
3	0.750000	0	0
5	0.750000	0	0
7	0.750000	0	0
9	0.750000	0	0
11	0.750000	0	0
13	0.750000	0	0
15	0.750000	0	0
17	0.750000	0	0
19	0.750000	0	0
21	0.750000	0	0
23	0.750000	0	0
25	0.750000	0	0
27	0.750000	0	0
29	0.750000	0	0
31	0.750000	0	0
33	0.750000	0	0

As we can see, the value of the integral is equal to the exact correct answer for all the points as listed in the table. Hence, this is a perfect solution with all the error values equal to 0.

3.4 Integral test result for $\int_{-3}^{5} e^{-x^2}\,dx$

For this integral, we already know that **the value for this integral is 1.772434.** Below are the values that were achieved for different point values.

Point	Integral	Absolute Error	Relative Error
3	1.335211	0.437224	0.24668
5	1.343808	0.428626	0.241829
7	1.675776	0.096658	0.054534
9	1.765344	0.00709	0.00400015
11	1.772757	0.000323	0.000182235
13	1.772533	9.9e-05	5.58554e-05
15	1.772443	9e-06	5.07776e-06
17	1.772435	1e-06	5.64196e-07
19	1.772434	0	0
21	1.772434	0	0
23	1.772434	0	0
25	1.772434	0	0
27	1.772434	0	0
29	1.772434	0	0
31	1.772434	0	0
33	1.772434	0	0

As we can see, even for such a complex integral, as the point value increases, the value for the integral gets equal to the correct value of the integral, which is 1.772434, after point number 19.

3.5 Integral test result for $\int_{0}^{2} 3 * log^2(x) + 4 * log(x)\,dx$

For this integral, we already know that **the value for this integral is 4.110129.** Below are the values that were achieved for different point values.

Point	Integral	Absolute Error	Relative Error
3	2.211599	1.89853	0.4619149
5	3.114059	0.99607	0.2423452
7	3.491084	0.619045	0.1506145
9	3.684712	0.425417	0.1035045
11	3.797979	0.31215	0.07594652
13	3.870314	0.239815	0.05834732
15	3.919511	0.190618	0.04637762
17	3.954596	0.155533	0.03784139
19	3.980558	0.129571	0.0315248
21	4.000346	0.109783	0.02671035
23	4.0158	0.094329	0.02295037
25	4.028115	0.082014	0.01995412
27	4.038099	0.07203	0.017525
29	4.046313	0.063816	0.01552652
31	4.053158	0.056971	0.01386112
33	4.058927	0.051202	0.01245752

As we can see, as the point value increases, the value for the integral gets closer to the right value of the integral, which is 4.110129.

4 Conclusion

As we can see from the results section of this report, the Gauss-Legendre method to calculate definite integrals is quite precise. However, we do notice that it performs quite poorly with logarithmic functions when compared to the usual quadratic, exponential, and/or trigonometric functions. It is sure that if provided with a complex logarithmic function, such as the one showed in 3.5, the absolute and relative error values will be present.

-x-x-x-x-x-x-x-x-x-x-x-x-x-x-END Of REPORTx-x-x-x-x-x-x-x-x-x-x-x-x-x-