MATH 1040/1070 Lecture and Learning Activity 2.1
DMS

Section: ___ Group Number: _______ **Score: ______/10**

Names of Group Members PRESENT: Key LA-15

Credit is only given for group work to those present on all days L&LA is worked in class and who are also present the day it is turned in.

Chapter 2 is the beginning of Calculus.

Calculus has three BIG ideas: limits, derivative, and integrals. Derivative have limits as part of their definitions. Integrals are based on derivatives. So we start with **LIMITS.**

Calculus is often called the mathematics of change. In preparation for our discussion of limits and derivatives we will first look at the "tangent problem" and two types of change: average velocity and instantaneous velocity.

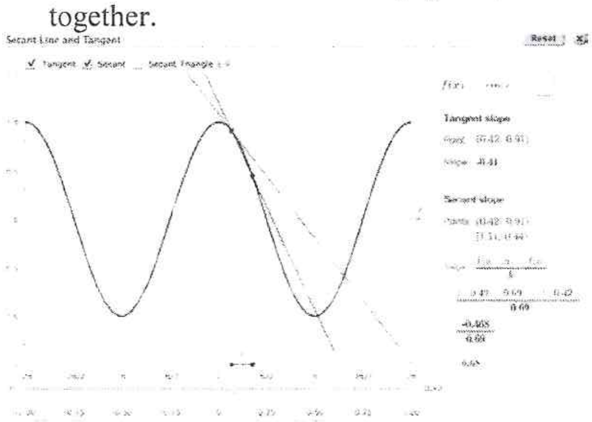
One of the big questions in Calculus is how to find the slope of the tangent line for a given function at a certain point.

In this section we see how limits arise when we attempt to find the tangent to a curve or the velocity of an object.

The Tangent Problem

The word *tangent* is derived from the Latin word *tangens*, which means "touching." Thus a tangent to a curve is a line that touches the curve. In other words, a tangent line should have the same direction as the curve at the point of contact. How can this idea be made precise?

In the electronic text in WebAssign there is a video lecture on page 82, several video examples, a Wolfram Demonstration on page 83, and a TEC visual on page 83. Let's look at the Visual together.



So what we are supposed to realize is that we cannot directly find the slope of the tangent. But we can estimate it with the slope of a secant line. If we let the two points get closer and closer together and can see a pattern to the slope of the secants, the result of the pattern is the limit and is the slope of the tangent.

Don't worry we will learn the Calculus in the next few weeks to write this down precisely.

1. (2.1:#2)

2. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute.

t (min)	36	38	40	42	44
Heartbeats	2530	2661	2806	2948	3080

The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of t.

(a) $t = 36$ and $t = 42$ (b) $t = 38$ and $t = 42$
(c) $t = 40$ and $t = 42$ (d) $t = 42$ and $t = 44$

What are your conclusions?

(a) $(36, 2530), (42, 2948)$

$$m_a = \frac{2948-2530}{42-36} = \frac{418}{6} = 66$$
$$= 69.67$$

(b) $(38, 2661), (42, 2948)$

$$m_b = \frac{2948-2661}{42-38} = \frac{287}{4}$$
$$= 71.75$$

(c) $(40, 2806), (42, 2948)$

$$m_c = \frac{2948-2806}{42-40} = \frac{142}{2}$$
$$= 71$$

(d) $(42, 2948), (44, 3080)$

$$m_d = \frac{3080-2948}{44-42}$$
$$= \frac{132}{2} = 66$$

Heart rate decreasing from 71 beats/min to 66 beats/min

2. (2.1:#4)

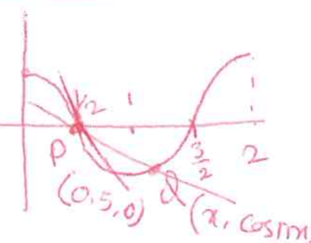
4. The point $P(0.5, 0)$ lies on the curve $y = \cos \pi x$.

(a) If Q is the point $(x, \cos \pi x)$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:

(i) 0 (ii) 0.4 (iii) 0.49 (iv) 0.499
(v) 1 (vi) 0.6 (vii) 0.51 (viii) 0.501

(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.

$$m = \frac{\cos \pi x - 0}{x - \frac{1}{2}} = \frac{\cos \pi x}{x - \frac{1}{2}}$$
$$m = \frac{2\cos \pi x}{2x-1}$$



(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 0)$.

(d) Sketch the curve, two of the secant lines, and the tangent line.

(a)

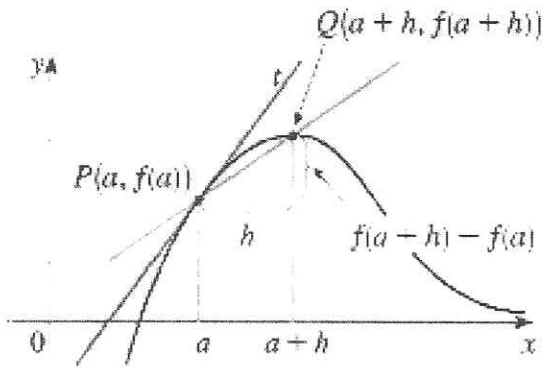
x	m
0	-2
0.4	-3.090170
0.49	-3.141076
0.499	-3.141587
1	-2
0.6	-3.090170
0.51	-3.141076
0.501	-3.141587

(b) We guess the value of the slope of the tangent line at $P(0.5, 0) = -\pi$
$= -3.141592$

(c) Point-slope form:

$$y - 0 = -\pi\left(x - \tfrac{1}{2}\right)$$
$$\Rightarrow \boxed{y = \frac{\pi}{2} - \pi x}$$

If we consider a velocity application, the question of how to find the slope of the tangent is the same as the question of how to find instantaneous velocity – velocity at a specific instant.
If you think of the ends of the time interval as two points on the graph of $s(t)$, then the average velocity is also the slope of the secant line between those two points.



$$m_{PQ} = \frac{f(a+h) - f(a)}{h}$$

$$v_{avg} = \frac{\text{change in position}}{\text{change in time}} = \frac{\text{displacement}}{\text{elapsed time}}$$

and

$$\boxed{v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

3. (2.1:#6)

6. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$. $y(1) = 10 - 1.86 = 8.14$

(a) Find the average velocity over the given time intervals:

(i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$

(iv) $[1, 1.01]$ (v) $[1, 1.001]$

(b) Estimate the instantaneous velocity when $t = 1$.

(a) $P(1, 8.14)$, $Q(1+h, y(1+h))$

$$V_{avg} = \frac{y(1+h) - y(1)}{h}$$

$$= \frac{8.14 + 6.28h - 1.86h^2 - 8.14}{h}$$

$$= 6.28 - 1.86h$$

(i) $h = 1$, $V_{avg} = 6.28 - 1.86 = 4.42$ m/s

(ii) $h = 0.5$, $V_{avg} = 6.28 - \frac{1}{2}(1.86) = 5.35$ m/s

(iii) $h = 0.1$, $V_{avg} = 6.28 - 0.186 = 6.094$ m/s

(iv) $h = 0.01$, $V_{avg} = 6.28 - 0.0186 = 6.2614$ m/s

$y(1+h) = 10(1+h) - 1.86(1+h)^2$

$= 10 + 10h - 1.86(1 + 2h + h^2)$

$= 10 + 10h - 1.86 - 3.72h - 1.86h^2$

$= 8.14 + 6.28h - 1.86h^2$

(v) $h = 0.001$, $V_{avg} = 6.28 - 0.00186$

$= 6.27814$ m/s

(b) Instantaneous velocity

$V \approx 6.28$ m/s

4. The position of an object moving along a line is given by the function $s(t) = -4.9t^2 + 30t + 20$.
Find the average velocity of the object over the following intervals.

$S(0) = -4.9\times0 + 30\times0 + 20 = 20$

a. $[0,3]$

$S(3) = -4.9\times9 + 90 + 20$
$= -44.1 + 110 = 65.9$

$$V_{avg} = \frac{S(3)-S(0)}{3-0} = \frac{65.9-20}{3}$$
$$= \frac{45.9}{3} = 15.3$$

b. $[0,2]$

$S(2) = -4.9\times4 + 60 + 20 = 60.4$

$$V_{avg} = \frac{S(2)-S(0)}{2-0} = \frac{60.4-20}{2} = 20.2$$

c. $[0,1]$

$S(1) = -4.9 + 30 + 20$
$= 45.1$

$$V_{avg} = \frac{S(1)-S(0)}{1-0} = \frac{45.1-20}{1}$$
$$= \frac{25.1}{1} = 25.1$$

d. $[0,h]$, where $h>0$ is a real number

$$V_{avg} = \frac{S(h)-S(0)}{h-0} = \frac{-4.9h^2+30h+20-20}{h}$$
$$= \frac{-4.9h^2+30h}{h} = -4.9h+30$$

e. If $h = 3,2,1$ in the answer to part d do you get the same answers?

$h=3$: $V_{avg} = -4.9\times3+30 = 15.3$ Yes
$h=2$: $V_{avg} = -4.9\times2+30 = 20.2$
$h=1$: $V_{avg} = -4.9\times1+30 = 25.1$

5. Suppose $s(t)$ is the position of an object moving along a line at time $t \geq 0$. What is the average velocity between the times $t=a$ and $t=b$?

$$V_{avg} = \frac{S(b)-S(a)}{b-a}$$

NOTE: This introductory section is "just the tip of the iceberg" as the saying goes. If you feel a little iffy about this right now, it should get better later as we develop limit ideas and the definition of the derivative.

HW 2.1: #7, 8

#7 see the video link in the ebook

#8 see the next page

HW 2.1

71 see the video solution in your ebook.

$s(1) = 3\sin\pi + 3\cos\pi = 0 + 3(-1) = -3$

81 (i) $V_{ave} = \dfrac{s(2) - s(1)}{2-1} = \dfrac{2\sin 2\pi + 3\cos 2\pi - 2\sin\pi - 3\cos\pi}{1}$

$= \dfrac{0 + 3 - 0 + 3}{1} = 6 \text{ cm/s}$

(ii) $V_{ave} = \dfrac{s(1.1) - s(1)}{1.1 - 1} = \dfrac{2\sin(1.1\pi) + 3\cos(1.1\pi) + 3}{0.1}$

$= -4.71 \text{ cm/s}$

(iii) $V_{ave} = \dfrac{s(1.01) - s(1)}{1.01 - 1} = \dfrac{2\sin(1.01\pi) + 3\cos(1.01\pi) + 3}{0.01}$

$= -6.13 \text{ cm/s}$

(iv) $V_{ave} = \dfrac{s(1.001) - s(1)}{1.001 - 1} = \dfrac{2\sin(1.001\pi) + 3\cos(1.001\pi) + 3}{0.001}$

$= -6.268 \text{ cm/s}$

$\approx -2\pi \text{ cm/s}$