Section: 9 **Score: ______/20**
LA-17
Names of Group Members PRESENT: ___________________________________

Credit is only given for group work to those present on all days L&LA is worked in class and who are also present the day it is turned in.
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Limits are fairly easy to read graphically. There is more work to be done when presented with an algebraic limit.
There are limit laws. Knowing their names is not as important as being able to use them.

Limit Laws Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
4. $\lim_{x \to a} [f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$

These five laws can be stated verbally as follows:

Sum Law	1. The limit of a sum is the sum of the limits.
Difference Law	2. The limit of a difference is the difference of the limits.
Constant Multiple Law	3. The limit of a constant times a function is the constant times the limit of the function.
Product Law	4. The limit of a product is the product of the limits.
Quotient Law	5. The limit of a quotient is the quotient of the limits (provided that the limit of the denominator is not 0).

6. $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where n is a positive integer

7. $\lim_{x \to a} c = c$ 8. $\lim_{x \to a} x = a$

9. $\lim_{x \to a} x^n = a^n$ where n is a positive integer

11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where n is a positive integer

[If n is even, we assume that $\lim_{x \to a} f(x) > 0$.]

1. Assume $\lim_{x\to 1} f(x)=8$, $\lim_{x\to 1} g(x)=3$, and $\lim_{x\to 1} h(x)=2$. Compute the following limits.

a. $\lim_{x\to 1}\left[\dfrac{f(x)}{g(x)-h(x)}\right]$

b. $\lim_{x\to 1}\left[h(x)\right]^5$

c. $\lim_{x\to 1}\sqrt[3]{f(x)g(x)+3}$

2. If $\lim_{x\to 2} f(x)=-8$, find $\lim_{x\to 2}\left[f(x)\right]^{\frac{2}{3}}$.

Direct Substitution Property If f is a polynomial or a rational function and a is in the domain of f, then

$$\lim_{x\to a} f(x)=f(a)$$

Direct substitution is always the first thing to try when presented with an algebraic limit.

3. Find the following limits

a. $\lim_{x\to 6}(4)$

b. $\lim_{x\to -7}(3x-2)$

c. $\lim_{x\to 2}\left(x^3-2x^2+1\right)$

d. $\lim_{x\to 0}\left(\sin^2 x+\sec x\right)$

e. $\lim_{x\to 1}\left(\dfrac{1+3x}{1+4x+3x^4}\right)^3$

f. $\lim_{t\to 3}\sqrt[3]{t^2-10}$

When direct substitution does not yield a valid answer, you need to try some other limit technique:

Factoring **Common denominator**
Conjugation or a combination of techniques

NOTE: $\frac{0}{0}$ is NOT a valid answer, but it is also NOT equivalent to "undefined" or "dne"

$\frac{0}{0}$ **means you have more work to do!!!**

Factoring

4. Find the following limits:

a. $\lim_{x\to 1}\frac{x^2-1}{x-1}$

b. $\lim_{x\to 2}\frac{x^2-7x+10}{x-2}$

c. $\lim_{y\to 0}\frac{5y^3+8y^2}{3y^4-16y^2}$

d. $\lim_{w\to -k}\frac{w^2+5kw+4k^2}{w^2+kw}$

Conjugates

5. Find the following limits:

a. $\lim_{x\to 7}\frac{\sqrt{x+2}-3}{x-7}$

b. $\lim_{h\to 0}\frac{\sqrt{16+h}-4}{h}$

c. $\lim_{x\to 2}\frac{x-2}{\sqrt{x^2+5}-3}$

Common Denominator

6. $\lim_{x\to 0} \dfrac{\dfrac{1}{2+x}-\dfrac{1}{2}}{x}$

Expand

7. $\lim_{h\to 0} \dfrac{(2+h)^3-8}{h}$

Variety/Combination

8. a. $\lim_{x\to 4} \dfrac{4x-x^2}{2-\sqrt{x}}$

b. $\lim_{t\to 2} \dfrac{3t^2-7t+2}{2-t}$

c. (2.3:#20) $\lim_{t\to 1} \dfrac{t^4-1}{t^3-1}$

d. $\lim_{x\to 1} \dfrac{x-1}{\sqrt{4x+5}-3}$

9. **Absolute Value**

Recall that $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

a. Evaluate $\lim\limits_{x \to 0^+} |x|$

b. Evaluate $\lim\limits_{x \to 0^-} |x|$

c. Therefore $\lim\limits_{x \to 0} |x|$

10. a. (2.3:#42) $\lim\limits_{x \to -6^-} \dfrac{2x+12}{|x+6|}$

b. (2.3:#43) $\lim\limits_{x \to \frac{1}{2}^-} \dfrac{2x-1}{|2x^3 - x^2|}$

11. **One-sided limits**

Let $f(x) = \begin{cases} 0 & \text{if } x \leq -5 \\ \sqrt{25 - x^2} & \text{if } -5 \leq x < 5 \\ 3x & \text{if } x \geq 5 \end{cases}$. Compute the following limits.

a. $\lim\limits_{x \to -5^-} f(x)$

b. $\lim\limits_{x \to -5^+} f(x)$

c. $\lim\limits_{x \to -5} f(x)$

d. $\lim\limits_{x \to 5^-} f(x)$

e. $\lim\limits_{x \to 5^+} f(x)$

f. $\lim\limits_{x \to 5} f(x)$

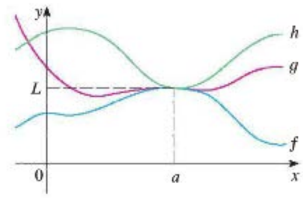
Squeeze Theorem aka Sandwich Theorem aka Pinching Theorem

3 The Squeeze Theorem If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$



Translated:

If $g(x) \leq f(x) \leq h(x)$ and $\lim_{x \to c} g(x) = L$ and $\lim_{x \to c} h(x) = L$

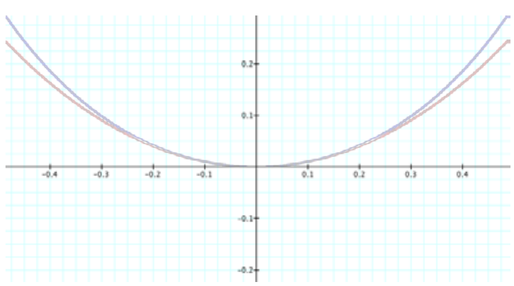
Limit the three sides of the inequality $\lim_{x \to c} g(x) \leq \lim_{x \to c} f(x) \leq \lim_{x \to c} h(x)$

Then $L \leq \lim_{x \to c} f(x) \leq L$ forces $\lim_{x \to c} f(x) = L$

The key to getting credit for these problems is writing the math down well.

12. If $2 - x^2 \leq g(x) \leq 2\cos x$ for all x, find $\lim_{x \to 0} g(x)$.

13. It can be shown that $0 \leq x^2 \sec x^2 \leq \left(x^4 + x^2\right)$ for x near 0. (See graph). Use the Squeeze Theorem to determine $\lim_{x \to 0} x^2 \sec x^2$.



14. (2.3:#40) Prove that $\lim_{x \to 0^+} \sqrt{x} \cdot e^{\sin\left(\frac{\pi}{x}\right)} = 0$.

Home work 3

The following problems are for home work 3. The home work should be submitted separately(not with learning activity) and individually and its due is on 10/07/2015

HW 2.3: #9, 12, 14, 16, 22, 24, 25, 26, 27, 28, 32, 38, 39, 44, 45