MATH 1040/1070 DMS — Lecture and Learning Activity — 2.7

Section: 9 LA-21 **Score: ______/20**

Names of Group Members PRESENT: ______________________________________

Credit is only given for group work to those present on all days L&LA is worked in class and who are also present the day it is turned in. ______________________________________

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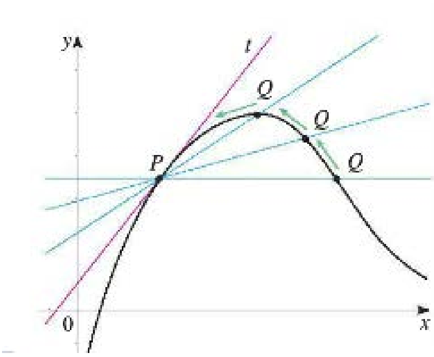
1. Consider the function $f(x) = 3 - 2x$. Find and simplify the expressions $\dfrac{f(x) - f(a)}{x - a}$ and $\dfrac{f(x+h) - f(x)}{h}$.

We have discussed in 2.1 that given a function, $f(x)$, a big question in Calculus is how to find the slope of the tangent line to a specific point P on the function.
(There is a video lecture on page 143 of the electronic text)

From our previous discussion about secant lines limiting at the tangent line at a point $x = a$, the following definitions result.

> **1 Definition** The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope
>
> $$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$
>
> provided that this limit exists.



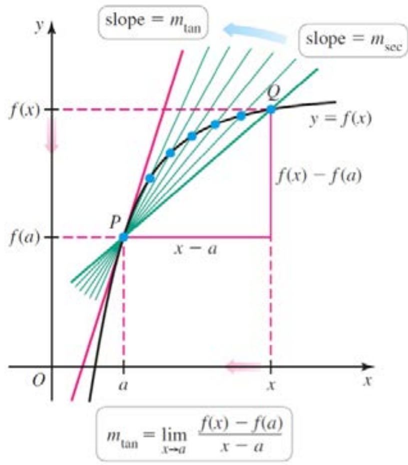


FIGURE 3.3

1.a. Use definition (1) to find the slope of the line tangent to the graph of $f(x) = -5x + 1$ at $P(1,-4)$

b. Determine an equation of the tangent line at $P(1,-4)$.

c. Graph the function and the tangent line.

2. a. Use definition (1) to find the slope of the line tangent to the graph of $f(x) = x^2 - 5$ at $P(3,4)$.

b. Determine an equation of the tangent line at $P(3,4)$.

c. Graph the function and the tangent line.

3. a. Use definition (1) to find the slope of the line tangent to the graph of $f(x) = 5$ at $P(1,5)$.

b. Determine an equation of the tangent line at $P(1,5)$.

c. Graph the function and the tangent line.

4. a. Use definition (1) to find the slope of the line tangent to the graph of $f(x)=3-x^2$ at $P(-1,2)$.

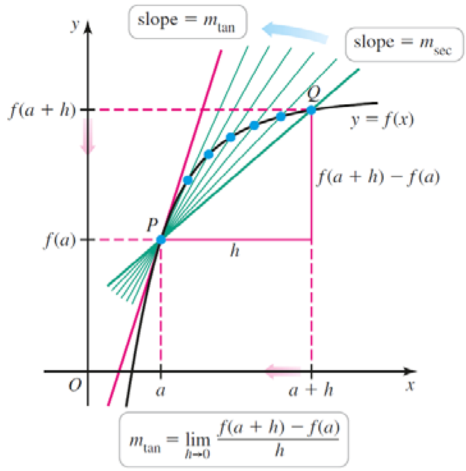
b. Determine an equation of the tangent line at $P(-1,2)$.

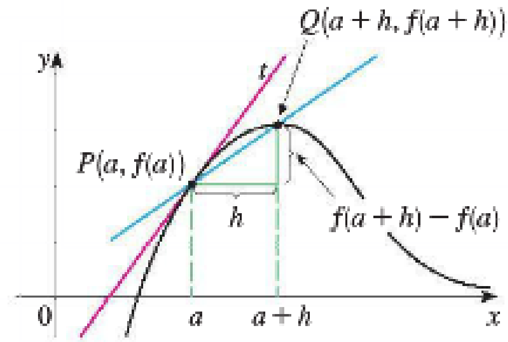
There is another expression for the slope of a tangent line that is sometimes easier to use. If $h = x - a$, then $x = a + h$ and so the slope of the secant line PQ is

$$m_{PQ} = \frac{f(a+h) - f(a)}{h}$$

(See Figure 3 where the case $h > 0$ is illustrated and Q is to the right of P. If it happened that $h < 0$, however, Q would be to the left of P.)

Notice that as x approaches a, h approaches 0 (because $h = x - a$) and so the expression for the slope of the tangent line in Definition 1 becomes





$$\boxed{m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

5. a. Use definition (2) to find the slope of the line tangent to the graph of $f(x) = 3 - x^2$ at $P(-1,2)$.
Note: this is the same function in #4

b. Determine an equation of the tangent line at $P(-1,2)$.

6. a. Use definition (2) to find the slope of the line tangent to the graph of $f(x) = 3x^2 - 4x$ at $P(1,-1)$.

b. Determine an equation of the tangent line at $P(1,-1)$.

7. a. Use definition (2) to find the slope of the line tangent to the graph of $f(x)=\dfrac{8}{x^2}$ at $\mathbf{P}(2,2)$.

b. Determine an equation of the tangent line at $\mathbf{P}(2,2)$.

8. a. Use definition (2) to find the slope of the line tangent to the graph of $f(x)=\dfrac{1}{3-2x}$ at $\mathbf{P}\left(-1,\dfrac{1}{5}\right)$.

b. Determine an equation of the tangent line at $\mathbf{P}\left(-1,\dfrac{1}{5}\right)$.

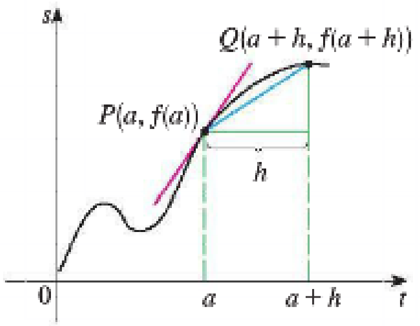
9. a. Find $f'(a)$ for $f(x)=\sqrt{x+1}$ for $a=8$.

b. Determine an equation of the line tangent to the graph of $f(x)$ at $\left(a,f\left(a\right)\right)$ for the given value of a.

10. a. Find $f'(a)$ for $f(x)=\dfrac{x-1}{x+1}$ for $a=0$.

b. Determine an equation of the line tangent to the graph of $f(x)$ at $\left(a,f\left(a\right)\right)$ for the given value of a

In 2.1, we also discussed average velocity and instantaneous velocity.



$$m_{PQ} = \frac{f(a+h) - f(a)}{h} = \text{average velocity}$$

$$\text{average velocity} = \frac{\text{displacement}}{\text{time}} = \frac{f(a+h) - f(a)}{h}$$

$$\boxed{v(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

In more general terms, Calculus study is interested in average rate of change and instantaneous rate of change.

11. (2.7:#14)

14. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.86t^2$.

(a) Find the velocity of the rock after one second.

(b) Find the velocity of the rock when $t = a$.

(c) When will the rock hit the surface?

(d) With what velocity will the rock hit the surface?

12. (2.7:#15)

15. The displacement (in meters) of a particle moving in a straight line is given by the equation of motion $s = 1/t^2$, where t is measured in seconds. Find the velocity of the particle at times $t = a$, $t = 1$, $t = 2$, and $t = 3$.

13. (2.7:#39)

39–40 A particle moves along a straight line with equation of motion $s = f(t)$, where s is measured in meters and t in seconds. Find the velocity and the speed when $t = 5$.

39. $f(t) = 100 + 50t - 4.9t^2$

We have seen that the same type of limit arises in finding the slope of a tangent line (Equation 2) or the velocity of an object (Equation 3). In fact, limits of the form

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

arise whenever we calculate a rate of change in any of the sciences or engineering, such as a rate of reaction in chemistry or a marginal cost in economics. Since this type of limit occurs so widely, it is given a special name and notation.

4 Definition The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

If we write $x = a + h$, then we have $h = x - a$ and h approaches 0 if and only if x approaches a. Therefore an equivalent way of stating the definition of the derivative, as we saw in finding tangent lines, is

5 $$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

14. (2.7:#27) Find $f'(a)$ for $f(x) = 3x^2 - 4x + 1$

15. Find $g'(a)$ for $g(x) = 1 + \sqrt{x}$

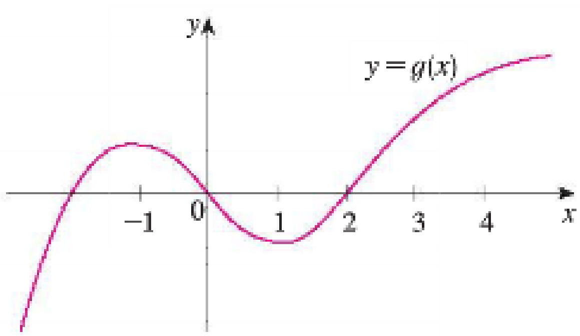
a. with derivative at a point definition 4

b. with derivative at a point definition 5

16. (2.7:#17)

17. For the function g whose graph is given, arrange the following numbers in increasing order and explain your reasoning:

$0 \qquad g'(-2) \qquad g'(0) \qquad g'(2) \qquad g'(4)$



17. (2.7:#19)

19. If an equation of the tangent line to the curve $y = f(x)$ at the point where $a = 2$ is $y = 4x - 5$, find $f(2)$ and $f'(2)$.

HW 2.7: #1, 4, 5, 7, 18, 20, 30