Section: ___ Group Number: _______ **Score: ______/10**

Names of Group Members PRESENT: ___________________________________

Credit is only given for group work to those present on all days L&LA is worked in class and who are also present the day it is turned in. ___________________________________

____

1 Definition Suppose $f(x)$ is defined when x is near the number a. (This means that f is defined on some open interval that contains a, except possibly at a itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

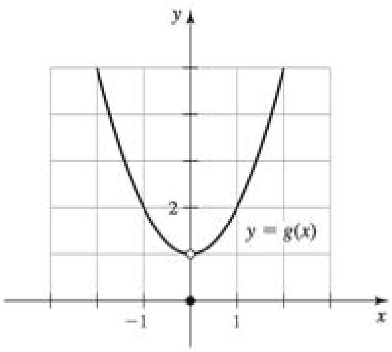
and say "the limit of $f(x)$, as x approaches a, equals L"

if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a.

Limits Graphically:

1. a. Consider $y = x^2$ b. Consider $f(x) = x^2 \quad x \neq 2$ c. Consider $f(x) = \begin{cases} x^2 & x \neq 2 \\ 3 & x = 2 \end{cases}$

2. Use the graph of $g(x)$ in the figure to find the following values, if they exist.



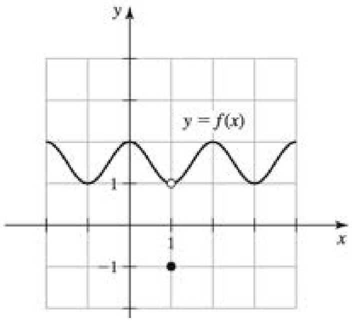
a. $g(0)$

b. $\lim_{x \to 0} g(x)$

c. $g(1)$

d. $\lim_{x \to 1} g(x)$

3. Use the graph of $f(x)$ in the figure to find the following values, if they exist.



a. $f(1)$

b. $\lim_{x \to 1} f(x)$

c. $f(0)$

d. $\lim_{x \to 0} f(x)$

Summary: If $f(x)$ is continuous (more about that later), $\lim_{x\to a} f(x) = f(a)$

Hole at $x = a$ does not affect $\lim_{x\to a} f(x) = L$

Alternate $f(a)$ along with a hole does not affect $\lim_{x\to a} f(x) = L$

Limits Numerically:

4. Let $f(x) = \dfrac{x^3 - 1}{x - 1}$.

a. Calculate $f(x)$ for each value of x in the following table.

x	0.9	0.99	0.999	0.9999
$f(x) = \dfrac{x^3-1}{x-1}$				
x	1.1	1.01	1.001	1.0001
$f(x) = \dfrac{x^3-1}{x-1}$				

b. Make a conjecture about the value of $\lim_{x\to 1} \dfrac{x^3-1}{x-1}$

5. Let $f(x) = (1+x)^{\frac{1}{x}}$

a. Make two tables, one showing the values of f for $x = 0.01,\ 0.001,\ 0.0001,\ 0.00001$ and one showing values of f for $x = -0.01,\ -0.001,\ -0.0001,\ -0.00001$. Round your answers to five digits.

b. Estimate the value of $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$.

c. What mathematical constant does $\lim_{x\to 0}(1+x)^{\frac{1}{x}}$ appear to equal?

6. In the electronic text though WebAssign, Video example 3 on page 90 is an important result we will use in the future. Take the time to look at it later.

DEFINITION One-Sided Limits

1. Right-hand limit Suppose f is defined for all x near a with $x > a$. If $f(x)$ is arbitrarily close to L for all x sufficiently close to a with $x > a$, we write

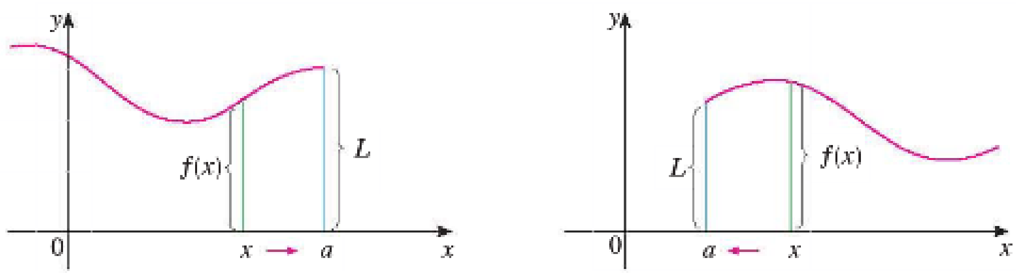
$$\lim_{x \to a^+} f(x) = L$$

and say the limit of $f(x)$ as x approaches a from the right equals L.

2. Left-hand limit Suppose f is defined for all x near a with $x < a$. If $f(x)$ is arbitrarily close to L for all x sufficiently close to a with $x < a$, we write

$$\lim_{x \to a^-} f(x) = L$$

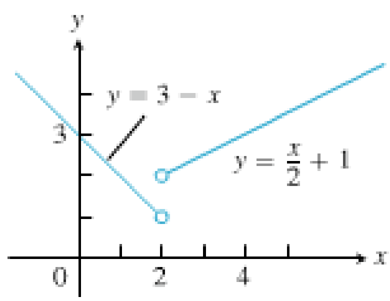
and say the limit of $f(x)$ as x approaches a from the left equals L.



(a) $\lim_{x \to a^-} f(x) = L$ (b) $\lim_{x \to a^+} f(x) = L$

In electronic text, there is a video lecture and video example 7 about one sided limits.

7. Let $f(x) = \begin{cases} 3 - x, & x < 2 \\ \dfrac{x}{2} + 1, & x > 2 \end{cases}$



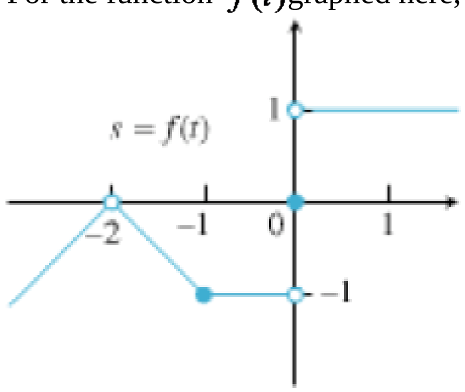
$f(2) =$

$\lim_{x \to 2} f(x)$

But $\lim_{x \to 2^+} f(x) =$

and $\lim_{x \to 2^-} f(x) =$

8. For the function $f(t)$ graphed here, find the following limits.



a) $\lim_{t \to -2} f(t)$

b) $\lim_{t \to -1.5} f(t)$

c) $\lim_{t \to -1} f(t)$

d) $\lim_{t \to 0} f(t)$

e) $\lim_{t \to \frac{1}{2}} f(t)$

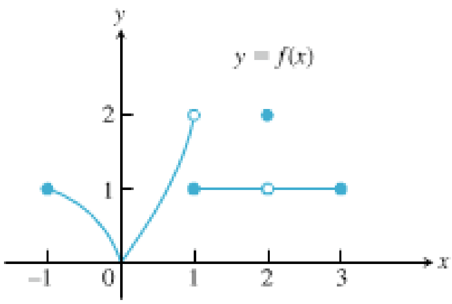
3 $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L$ and $\lim_{x \to a^+} f(x) = L$

Note

As with two-sided limits, the value of a one-sided limit (if it exists) depends on the values of $f(x)$ near $x = a$ but not on the value of $f(a)$.

"Two-sided" limits do not exist when left-hand limit does not equal right hand limits.
"Two-sided" limits do not exist if one of the one-sided limits dne.

9.



$\lim_{x \to 0^-} f(x)$ $\lim_{x \to 0^+} f(x)$ $\lim_{x \to 0} f(x)$

$\lim_{x \to 1^-} f(x)$ $\lim_{x \to 1^+} f(x)$ $\lim_{x \to 1} f(x)$

$\lim_{x \to 2^-} f(x)$ $\lim_{x \to 2^+} f(x)$ $\lim_{x \to 2} f(x)$

$\lim_{x \to 3^-} f(x)$

$\lim_{x \to -1^+} f(x)$

10. **A step function** Let $f(x) = \dfrac{|x|}{x}$, for $x \neq 0$.

a. Sketch a graph of f on the interval $[-2,2]$.

b. Does $\lim_{x \to 0} f(x)$ exist? Explain your reasoning after first examining $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$.

11. Electronic text page 91, video example 6 is interesting about the Heaviside function.

12. Sketch the graph of a function with the given properties. You do not need to find a formula for the function.

$f(1) = 0, f(2) = 4, f(3) = 6,\ \lim_{x \to 2^-} f(x) = -3,\ \lim_{x \to 2^+} f(x) = 5$

13. Sketch the graph of a function with the given properties. You do not need to find a formula for the function.

$$g(1)=0,\ g(2)=1,\ g(3)=-2,\ \lim_{x\to 2} g(x)=0,\ \lim_{x\to 3^-} g(x)=-1,\ \lim_{x\to 3^+} g(x)=-2$$

Infinite limits

4 Definition Let f be a function defined on both sides of a, except possibly at a itself. Then

$$\lim_{x\to a} f(x)=\infty$$

means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to a, but not equal to a.

5 Definition Let f be defined on both sides of a, except possibly at a itself. Then

$$\lim_{x\to a} f(x)=-\infty$$

means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to a, but not equal to a.

Similar definitions can be given for the one-sided infinite limits

$$\lim_{x\to a^-} f(x)=\infty \qquad \lim_{x\to a^+} f(x)=\infty$$

$$\lim_{x\to a^-} f(x)=-\infty \qquad \lim_{x\to a^+} f(x)=-\infty$$

14. a. Consider $f(x)=\dfrac{1}{(x-2)^2}$

$\lim_{x\to 2} f(x)=$

b. Consider $f(x)=\dfrac{1}{x+1}$

$\lim_{x\to -1} f(x)=$

c. Consider $f(x)=-\dfrac{1}{(x+3)^4}$

$\lim_{x\to -3} f(x)=$

6 Definition The line $x = a$ is called a **vertical asymptote** of the curve $y = f(x)$ if at least one of the following statements is true:

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

15a. Consider $f(x) = \tan x$

$$\lim_{x \to \frac{\pi}{2}} \tan x$$

b. Consider $f(x) = \ln x$

$$\lim_{x \to 0^+} \ln x$$

Infinite Limits Analytically

When finding a limit the first thing to try is direct substitution.

If you get $\frac{0}{0}$, you have more work to do. (We will discuss this in section 2.3)

If you get $\frac{\text{constant}}{0}$, the limit is infinite. The text says "The fraction $\frac{a}{b}$ grows arbitrarily large in magnitude if b approaches 0 while a remains nonzero and relatively constant."

It must be considered whether the denominator is approaching 0 through negative values or whether the denominator is approaching 0 through positive values.

Then apply the traditional positive/negative rules. $\frac{+}{-} = -$, $\frac{+}{+} = +$, $\frac{-}{+} = -$, $\frac{-}{-} = +$

The text uses writing above and below the function to describe what is happening.

For example:

$$\lim_{x \to 3^+} \frac{\overbrace{2 - 5x}^{\text{approaches } -13}}{\underbrace{x - 3}_{\text{negative and approaches } 0}} = -\infty.$$

To simplify the writing, the 1040/1070 and 1060 instructors have chosen to use a "small –" or "small +" notation or "sp" "sn"

This presentation will be required of you on your HW and test answers to get full credit.

For example:

$$\lim_{x \to 3^+} \frac{2 - 5x}{x - 3} = \frac{-13}{\text{small} +} = -\infty$$

16. Evaluate a. $\lim\limits_{x\to 3^-} \dfrac{2}{(x-3)^3}$ b. $\lim\limits_{x\to 3^+} \dfrac{2}{(x-3)^3}$ c. $\lim\limits_{x\to 3} \dfrac{2}{(x-3)^3}$

17. Evaluate the limits

a. $\lim\limits_{x\to -8^+} \dfrac{2x}{x+8}$ b. $\lim\limits_{x\to -8^-} \dfrac{2x}{x+8}$ c. $\lim\limits_{x\to -8} \dfrac{2x}{x+8}$

18. a. Evaluate $\lim\limits_{x\to 7} \dfrac{4}{(x-7)^2}$

b. Why was $\lim\limits_{x\to 7} \dfrac{4}{(x-7)^2}$ not presented as a one sided limit?

19. (2.2: #30-36 even) Note: #36 looks ahead a bit

30. $\lim\limits_{x\to -3^-} \dfrac{x+2}{x+3}$

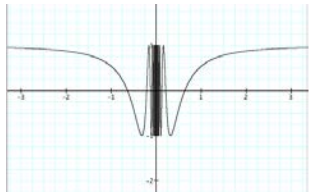
32. $\lim\limits_{x\to 5^-} \dfrac{e^x}{(x-5)^3}$

34. $\lim\limits_{x\to \pi^-} \cot x$

36. $\lim\limits_{x\to 2^-} \dfrac{x^2-2x}{x^2-4x+4}$

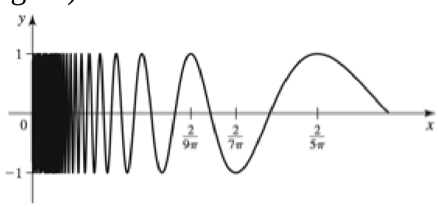
Oscillations There are some functions with "**Strange Behavior**"

20. Find $\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$



21.a. Create a table of values of $\sin\left(\frac{1}{x}\right)$ for $x = \frac{2}{\pi}, \frac{2}{3\pi}, \frac{2}{5\pi}, \frac{2}{7\pi}, \frac{2}{9\pi}$, and $\frac{2}{11\pi}$. Describe the pattern of values you observe.

b. Why does a graphing utility have difficulty plotting the graph of $y = \sin\left(\frac{1}{x}\right)$ near $x = 0$ (see figure)?



c. What do you conclude about $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$?

Summary examples of ways limits fail to exist:

1. The graph <u>jumps</u> at the specific input.
2. The function approaches positive or negative infinity at the specific input. It <u>grows to large</u> to have a limit. (This only needs to happen on one side for the limit to fail to exit)
3. The function <u>oscillates</u> too much to have a limit. (This only needs to happen on one side for the limit to fail to exist.)

Note on #2: Technically, the "infinite limits" do not exist. But we choose to indicate that they approach infinity in our notation.

HW 2.2: #1, 3, 5, 6, 8, 17, 21, 37