Section: 9 Table Number: _______ **Score: ______/20**

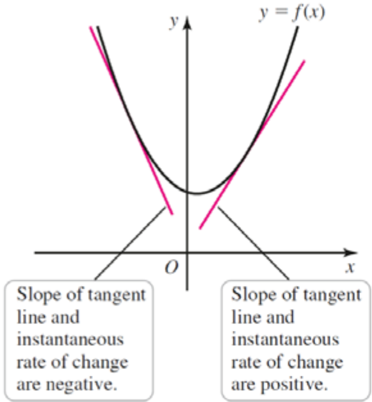
Names of Group Members PRESENT: ___________________________________

Credit is only given for group work to those present on all days L&LA is worked in class and who are also present the day it is turned in. ___________________________________

____

We have spent a good amount of time looking at the tangent line problem – how to find the slope of the line tangent to the given function **at a <u>specific</u> point P** and the answers have been numbers – slopes of the tangent lines.

By examining a function at various selections for the point, it should be easy to see that the value of the derivative varies at different points.



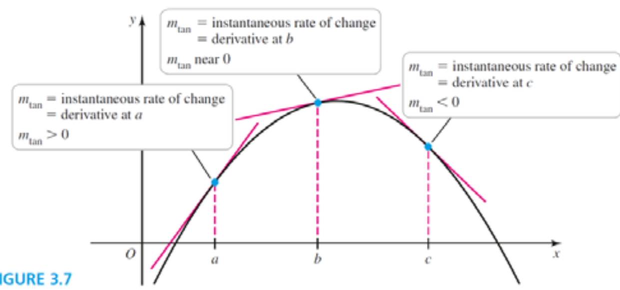


FIGURE 3.7

Let's look at the Wolfram Demonstration on page 156.

So, in general we should be able to find a **function** to describe the derivative.

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

There is an assortment of notations for the derivative (p.157)

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$$

The symbols D and d/dx are called **differentiation operators** because they indicate the operation of **differentiation**, which is the process of calculating a derivative.

Note

The notation $\frac{dy}{dx}$ is read *the derivative of y with respect to x* or *dy dx*. It does not mean dy divided by dx, but it is a reminder of the limit of $\Delta y/\Delta x$.

If we want to indicate the value of a derivative dy/dx in Leibniz notation at a specific number a, we use the notation

$$\left.\frac{dy}{dx}\right|_{x=a} \quad \text{or} \quad \left.\frac{dy}{dx}\right]_{x=a}$$

which is a synonym for $f'(a)$.

There is a video lecture about derivative notation on p 157 of electronic text.

1. a. Find the derivative function $f'(x)$ for the function $f(x)=5x^2-6x+1$.

b. Graph the function and the derivative function together. Do you see anything noteworthy? (more info to come)

c. Find an equation of the line tangent to the graph of $f(x)=5x^2-6x+1$ at $a=2$.

d. Graph the function and the tangent line.

2. a. Find the derivative function $f'(x)$ for the function $f(x)=\sqrt{3x+1}$.

b. Find an equation of the line tangent to the graph of $f(x)=\sqrt{3x+1}$ at $a=8$.

3. a. Find the derivative function $f'(x)$ for the function $f(x)=\dfrac{2}{3x+1}$.

b. Find an equation of the line tangent to the graph of $f(x)=\dfrac{2}{3x+1}$ at $a=-1$.

4. a. Find the derivative function $k'(x)$ for the function $k(x)=\dfrac{1-x}{2x}$.

b. Find an equation of the line tangent to the graph of $k(x)=\dfrac{1-x}{2x}$ at $a=\sqrt{2}$.

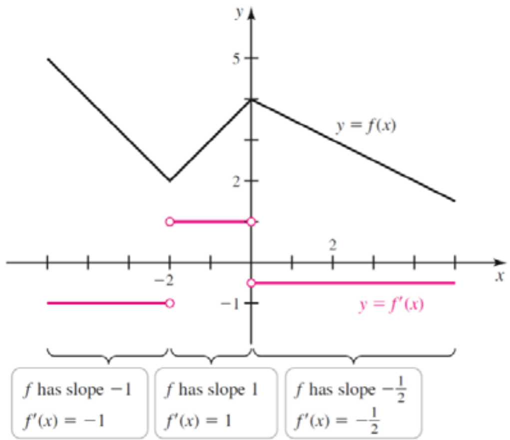
5. Find the derivative of $F(x) = (x-1)^2 + 1$

6. a. Use the definition of the derivative to determine $\frac{d}{dx}\left(\sqrt{ax+b}\right)$, where *a* and *b* are constants.

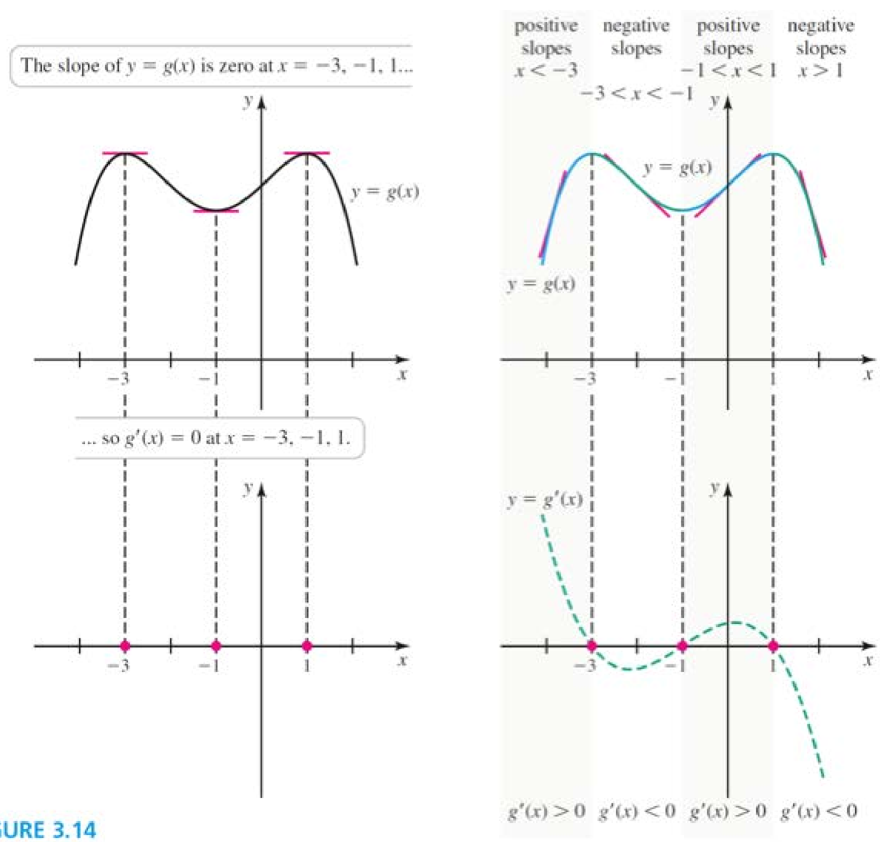
b. Use the result of part (a) to find $\frac{d}{dx}\left(\sqrt{5x+9}\right)$

Graphically we should be able to sketch the derivative **function** based on the idea that the derivative at any given point P is the slope of the tangent line at point P.

Consider these examples



Notice that the slopes of the tangent lines change abruptly at $x = -2$ and $x = 0$. As a result, $f'(-2)$ and $f'(0)$ are undefined and the graph of the derivative is discontinuous at these points.



IGURE 3.14

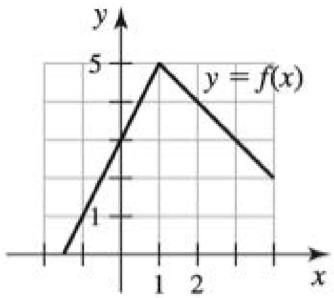
Let's look again at the Wolfram Demonstration on page 156.

Let’s summarize the properties that help sketch the derivative graph:

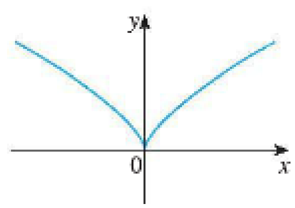
Function	Derivative
Increasing	
Decreasing	
Smooth Maximum or Minimum	
Constant	
Linear	
Quadratic	

7. Sketch the derivative graph of each of these functions.

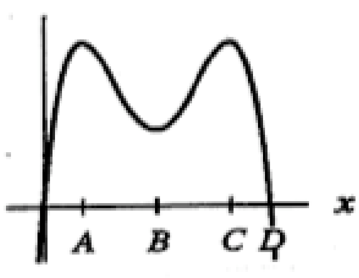
a.



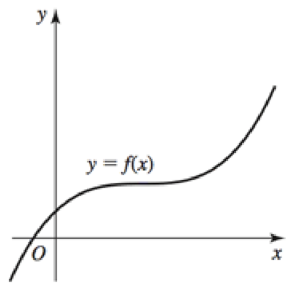
b.



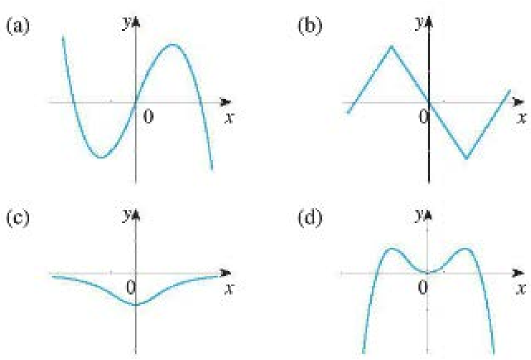
c.

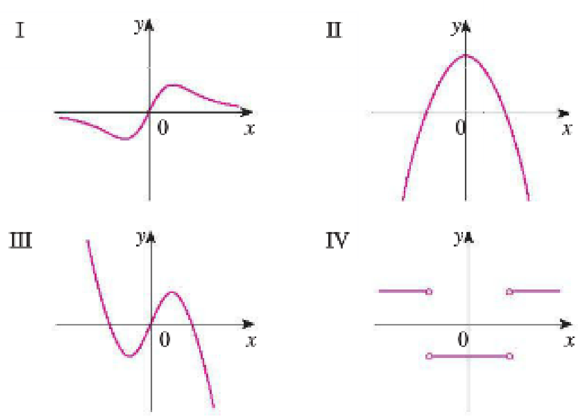


d.



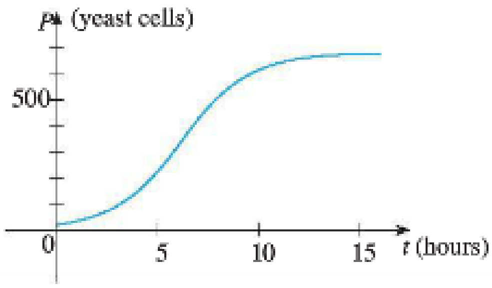
8. (2.8:#3) Match the graph of each function in (a)-(d) with the graph of its derivative in I-IV.





9. (2.8:#12)

Shown is the graph of the population function $P(t)$ for yeast cells in a laboratory culture. Use the method of Example 1 to



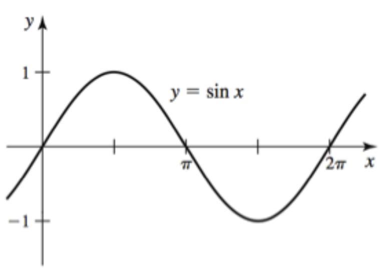
graph the derivative $P'(t)$. What does the graph of P' tell us about the yeast population?

10.

Graph of the derivative of the sine curve

a. Use the graph of $y = \sin x$ (see figure) to sketch the graph of the derivative of the sine function.

b. Based upon your graph in part (a), what function equals $\frac{d}{dx}(\sin x)$?



11. Look one more time at Wolfram Demonstration on page 156.

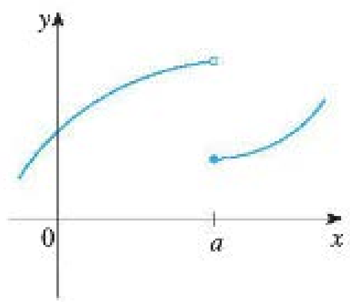
Look at the tab for $f(x) = e^x$. What do you think is the equation of f' for $f(x) = e^x$?

Question: **When is a function differentiable?** i.e. when does the derivative exist?
Easier to answer “when is a function not differentiable?”

3 Definition A function f is **differentiable at** a if $f'(a)$ exists. It is **differentiable on an open interval** (a, b) [or (a, ∞) or $(-\infty, a)$ or $(-\infty, \infty)$] if it is differentiable at every number in the interval.

1) Not differentiable if not continuous. –

removable, jump, infinite, oscillating
aka: hole, break, vertical asymptote, oscillation
“a disconnect in the graph”



2) not differentiable at a **corner**

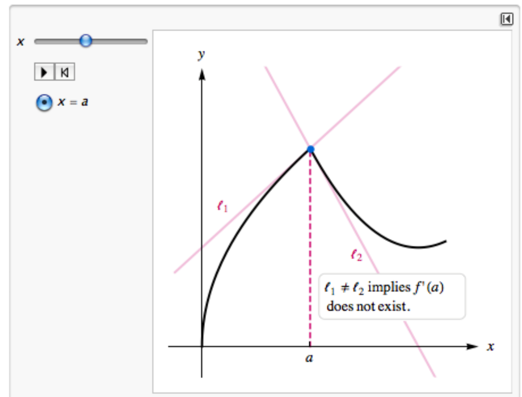
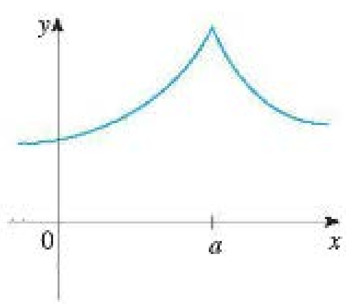
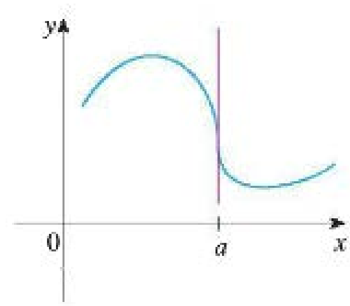


FIGURE 3.16

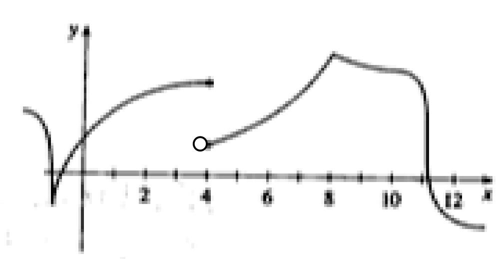
3) not differentiable at a **cusp**

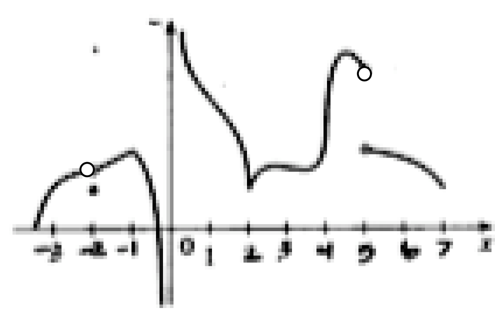


4) not differentiable at a **vertical tangent.**



12. For each function state
   a. Where the function is continuous?
   b. Where the function is differentiable?
   c. Where the function is continuous but not differentiable?
   d. Where the function neither continuous nor differentiable?





4 Theorem If f is differentiable at a, then f is continuous at a.

i.e. Differentiability implies continuity.
Proof on page 158.

WARNING: A function that is continuous is not guaranteed to be differentiable.
(Continuity does not imply differentiability)

For example: $f(x)=|x|$ is continuous at $x=0$ but is not differentiable at $x=0$. There is a good video example 5 on page 157 of electronic text concerning lack of differentiability of $f(x)=|x|$ at $x=0$.

We have seen an assortment of notations for the derivative.

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x)$$

f(x) is a differentiable function, then it has a derivative $f'(x)$.

The derivative $f'(x)$ is also a function.

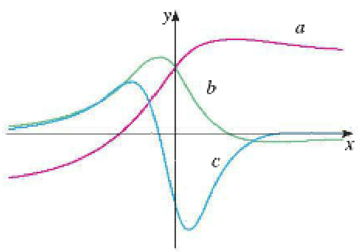
If $f'(x)$ is also differentiable, then it has a derivative $f''(x)$. This third function is called the **second derivative**.

$$f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = y''$$

Very interesting TEC interactive figure on page 160 of electronic text. Let's look at it a little together.

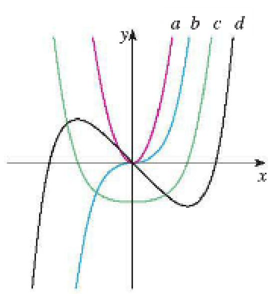
13. (2.8:#43)

The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



14. (2.8:#44)

The figure shows graphs of f, f', f'', and f'''. Identify each curve, and explain your choices.



HW 2.8: #4, 6, 7, 10, 22, 24, 25, 26, 38, 40