Section: 9 LA-19 **Score: ______/20**

Names of Group Members PRESENT: ________________________________

Credit is only given for group work to those present on all days L&LA is worked in class and who are also present the day it is turned in.

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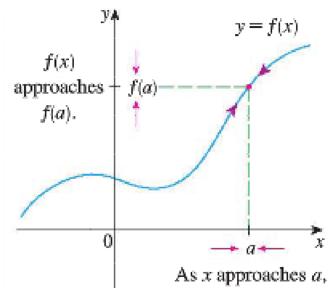


1 Definition A function f is **continuous at a number a** if

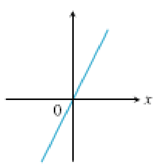
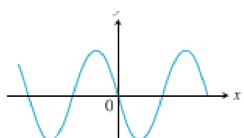
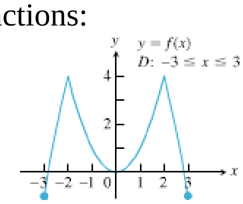
$$\lim_{x \to a} f(x) = f(a)$$

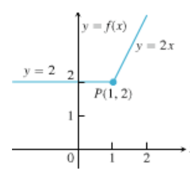
Notice that Definition 1 implicitly requires three things if f is continuous at a:

1. $f(a)$ is defined (that is, a is in the domain of f)
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

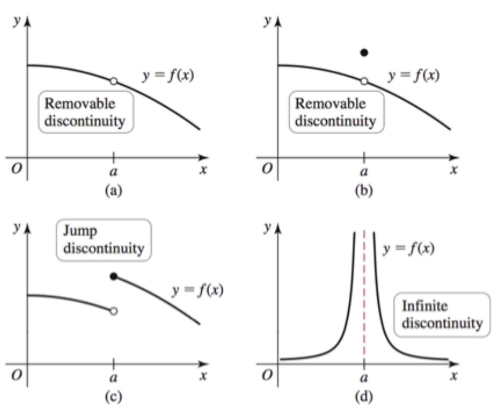
Graphically:
Basic idea: A function is continuous if you can draw it without picking up your pencil.

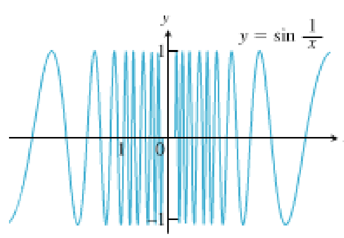
These are examples of continuous functions:



Discontinuities fall into certain classes.



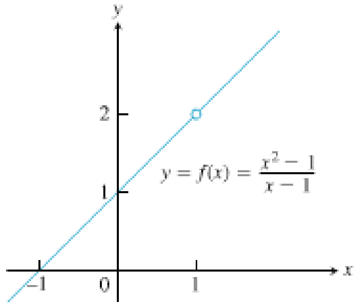


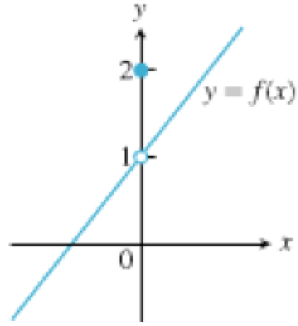
Oscillating discontinuity

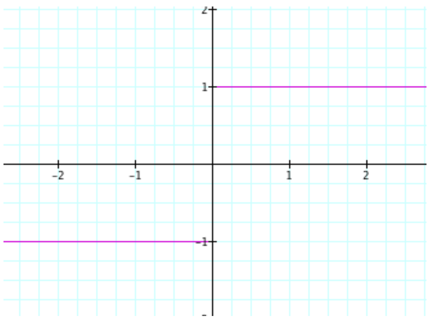
Classifying discontinuities *The discontinuities in graphs (a) and (b) are* removable discontinuities *because they disappear if we define or redefine f at a so that $f(a) = \lim_{x \to a} f(x)$. The function in graph (c) has a* jump discontinuity *because left and right limits exist at a but are unequal. The discontinuity in graph (d) is an* infinite discontinuity *because the function has a vertical asymptote at a.*

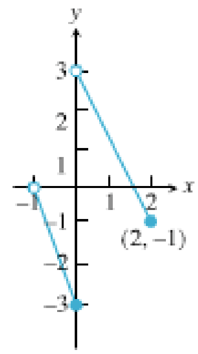
Consider video example 2 page 119 in electronic text.

1. Determine the points at which the following functions have discontinuities. For each point(s), classify the discontinuity and state the conditions of the continuity checklist that are violated.



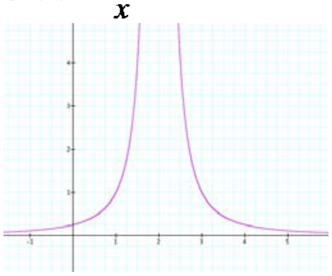




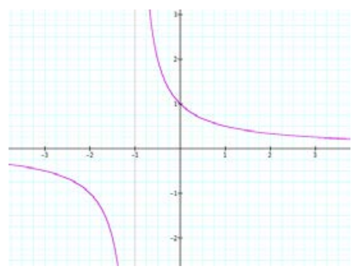


step function

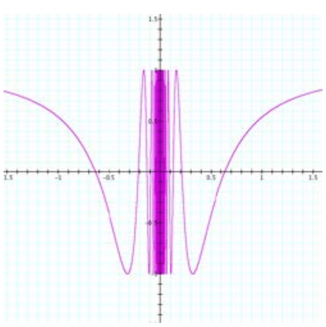
$f(x) = \dfrac{|x|}{x}$, for $x \neq 0$.



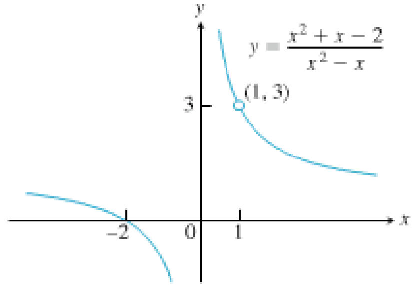
$f(x) = \dfrac{1}{(x-2)^2}$

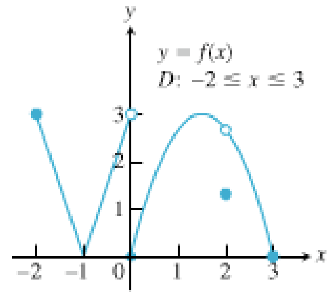


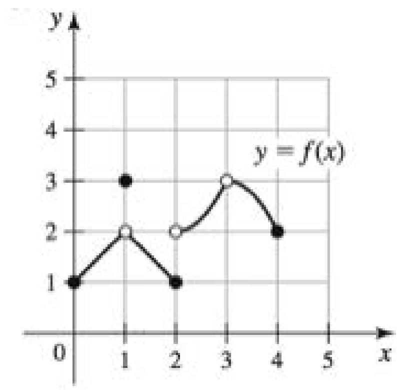
$f(x) = \dfrac{1}{x+1}$

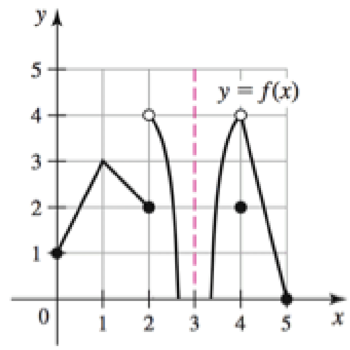


$f(x) = \cos\left(\dfrac{1}{x}\right)$







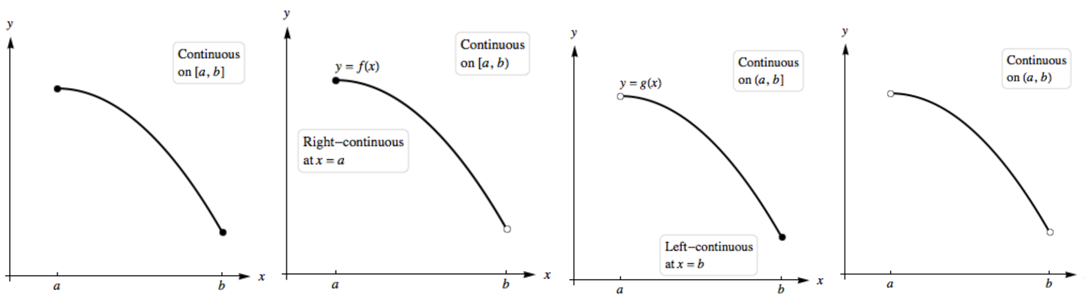


2 Definition A function f is **continuous from the right at a number a** if

$$\lim_{x \to a^+} f(x) = f(a)$$

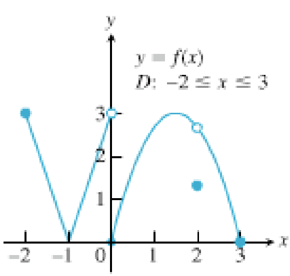
and f is **continuous from the left at a** if

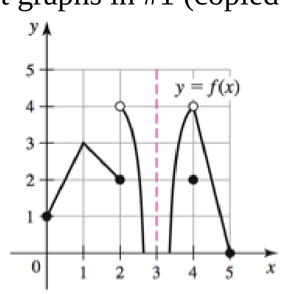
$$\lim_{x \to a^-} f(x) = f(a)$$



3 Definition A function f is **continuous on an interval** if it is continuous at every number in the interval. (If f is defined only on one side of an endpoint of the interval, we understand *continuous* at the endpoint to mean *continuous from the right* or *continuous from the left*.)

2. Determine the intervals of continuity for the last graphs in #1 (copied here).





Algebraically:
We need to be precise in our presentation of why a function is not continuous at a point. We will use the continuity definition breakdown.

Notice that Definition 1 implicitly requires three things if f is continuous at a:

1. $f(a)$ is defined (that is, a is in the domain of f)
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

3. Determine whether the following functions are continuous at a. Use the continuity checklist to justify your answer.

a. $f(x) = x^2 + \sqrt{7 - x}$ at $a = 4$

b. $g(x) = \dfrac{1}{x - 3}$ at $a = 3$

c. $f(x) = \begin{cases} \dfrac{x^2 - x}{x + 1} & \text{if } x \neq -1 \\ 0 & \text{if } x = -1 \end{cases}$ at $a = -1$

d. **Absolute Value** $|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

e. $f(x) = \begin{cases} \dfrac{x^2 + x - 6}{x^2 - 2x} & \text{if } x \neq 2 \\ -1 & \text{if } x = 2 \end{cases}$

If f is continuous at a, then $\lim_{x \to a} f(x) = f(a)$, and direct substitution may be used to evaluate $\lim_{x \to a} f(x)$.

The text has a good discussion on pages 121-125 on which types of functions are continuous (or continuous except at a few important points). I would suggest that you read these pages in detail. In summary

4 Theorem If f and g are continuous at a and c is a constant, then the following functions are also continuous at a:

1. $f + g$
2. $f - g$
3. cf
4. fg
5. $\dfrac{f}{g}$ if $g(a) \neq 0$

5 Theorem

(a) Any polynomial is continuous everywhere; that is, it is continuous on $\mathbb{R} = (-\infty, \infty)$.

(b) Any rational function is continuous wherever it is defined; that is, it is continuous on its domain.

7 Theorem The following types of functions are continuous at every number in their domains:

polynomials, rational functions, root functions

trigonometric functions, inverse trigonometric functions

exponential functions, logarithmic functions

8 Theorem If f is continuous at b and $\lim_{x\to a} g(x) = b$, then $\lim_{x\to a} f(g(x)) = f(b)$. In other words,

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

9 Theorem If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

4. **25–32** Explain, using Theorems 4, 5, 7, and 9, why the function is continuous at every number in its domain. State the domain.

a. **26.** $G(x) = \dfrac{x^2 + 1}{2x^2 - x - 1}$

b. **29.** $A(t) = \arcsin(1 + 2t)$

5. (2.5:#40) Show that *f* is continuous on $\left(-\infty,\infty\right)$

$$f(x)=\begin{cases}\sin x & \text{if } x<\dfrac{\pi}{4}\\ \cos x & \text{if } x\geq\dfrac{\pi}{4}\end{cases}$$

6. Determine the intervals on which the following functions are continuous. Use proper brackets/parentheses to specify left or right continuity or not.

a. $f(x)=\sqrt{2x+3}$

b. $g(x)=\dfrac{3x^2-6x+7}{x^2+x+1}$

c. $s(x)=\dfrac{x^2-4x+3}{x^2-1}$

d. $g(x)=\sqrt{x^4-1}$

7. a. Sketch the graph of a function that is not continuous at 1, but it is defined at 1.
b. Sketch the graph of a function that is not continuous at 1, but has a limit at 1.

8. Determine the value of the constant a for which the function $f(x)=\begin{cases}\dfrac{x^3-1}{x^2-1} & \text{if } x\neq 1\\ a & \text{if } x=1\end{cases}$ is continuous at $x=1$.

9. Determine the value of the constant b for which the function $g(x)=\begin{cases}x & \text{if } x<-2\\ bx^2 & \text{if } x\geq -2\end{cases}$ is continuous at $x=-2$.

10. $f(x)=\dfrac{x^2+x-6}{x-2}$ has a removable discontinuity. Redefine the function so as to remove the discontinuity. Use limit(s) to support your answer.

11. (2.5:#47b) $f(x)=\dfrac{x^3-x^2-2x}{x-2}$ has a removable discontinuity at $x=2$. Redefine the function so as to remove the discontinuity. Use limit(s) to support your answer.

12. Let $f(x)=\begin{cases} x^3+4x+1 & \text{if } x\le 0 \\ 2x^3 & \text{if } x>0 \end{cases}$

a. Show that the function if not continuous at 0.

b. Is the function continuous from the left or right at 0?

13. What types of discontinuities do the following functions have? Support your answers with limits.

a. $y=\dfrac{x^2-3x+2}{x^3-2x^2}$

b. $f(x)=\dfrac{|x-2|}{x-2}$

c. $h(x)=\dfrac{x^3-4x^2+4x}{x^2-x}$

e. Conclusions:
A function has a removable discontinuity when

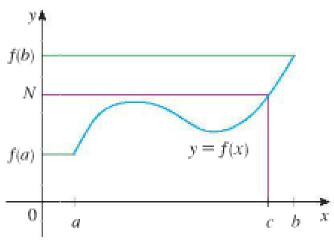
The discontinuity can be removed with

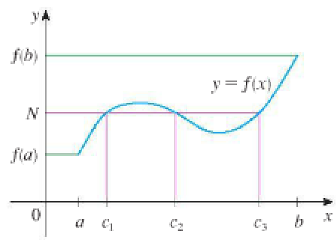
A function has an infinite discontinuity when

A function has a jump discontinuity when

INTERMEDIATE VALUE THEOREM

10 The Intermediate Value Theorem Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.





It is important that the function be continuous.

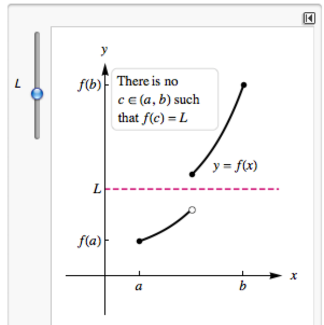


FIGURE 2.49

The IVT lets you show that a function has roots (solutions, zeros)

• Find an “a” value that gives a negative output
• Find a “b” value that gives a positive output
• If the function is continuous it has to have passed through the output of zero.

EXAMPLE:
Show that $f(x) = x^3 + 4x + 4$ has a solution.

14. Use the IVT to show that the equation $\sqrt{x^4 + 25x^3 + 10} = 5$ on the interval $(0,1)$.

15. Use the IVT to show that the equation $-x^5 - 4x^2 + 2\sqrt{x} + 5 = 0$ on the interval $(0,3)$.

16.

Investment problem Assume you invest \$250 at the end of each year for 10 years at an annual interest rate of r. The amount of money in your account after 10 years is $A = \dfrac{250\left[(1+r)^{10} - 1\right]}{r}$. Assume your goal is to have \$3500 in your account after 10 years.

a. Use the Intermediate Value Theorem to show that there is an interest rate r in the interval $(0.01,\ 0.10)$—between 1% and 10%—that allows you to reach your financial goal.

b. Use a calculator to estimate the interest rate required to reach your financial goal.

HW 2.5:# 3, 6, 10a, 10e, 14, 17, 21, 24, 27, 30, 39, 41, 46, 53, 55