Homework 4 (2.3)

9| $\lim_{x\to 2} \sqrt{\dfrac{2x^2+1}{3x-2}} = \sqrt{\dfrac{2\cdot 2^2+1}{3\cdot 2-2}} = \sqrt{\dfrac{8+1}{6-2}} = \sqrt{\dfrac{9}{4}} = \dfrac{3}{2}$

12| $\lim_{x\to 4} \dfrac{x^2-4x}{x^2-3x-4}$ $\left[\dfrac{16-16}{16-12-4} = \dfrac{0}{0}\right]$

$= \lim_{x\to 4} \dfrac{x(x-4)}{x^2-4x+x-4} = \lim_{x\to 4} \dfrac{x(x-4)}{x(x-4)+(x-4)} = \lim_{x\to 4} \dfrac{x(x-4)}{(x+1)(x-4)}$

$= \lim_{x\to 4} \dfrac{x}{x+1} = \dfrac{4}{4+1} = \dfrac{4}{5}$

14| $\lim_{x\to -1} \dfrac{x^2-4x}{x^2-3x-4}$ $\left[\dfrac{(-1)^2-4(-1)}{(-1)^2-3(-1)-4} = \dfrac{1+4}{1+3-4} = \dfrac{5}{0}\right]$

DNE

$= \lim_{x\to -1} \dfrac{x(x-4)}{(x+1)(x-4)} = \lim_{x\to -1} \dfrac{x}{x+1}$ DNE

16| $\lim_{x\to -1} \dfrac{2x^2+3x+1}{x^2-2x-3}$ $\left[\dfrac{2-3+1}{1+2-3} = \dfrac{0}{0}\right]$

$= \lim_{x\to -1} \dfrac{2x^2+2x+x+1}{x^2-3x+x-3} = \lim_{x\to -1} \dfrac{2x(x+1)+1(x+1)}{x(x-3)+(x-3)}$

$= \lim_{x\to -1} \dfrac{(x+1)(2x+1)}{(x-3)(x+1)} = \lim_{x\to -1} \dfrac{2x+1}{x-3} = \dfrac{-2+1}{-1-3} = \dfrac{-1}{-4} = \dfrac{1}{4}$

22| $\lim_{u\to 2} \dfrac{\sqrt{4u+1}-3}{u-2}$ $\left[\dfrac{0}{0}\right]$

$= \lim_{u\to 2} \dfrac{\sqrt{4u+1}-3}{u-2} \times \dfrac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3}$

$$= \lim_{u \to 2} \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)} = \lim_{u \to 2} \frac{4u-8}{(u-2)(\sqrt{4u+1}+3)}$$

$$= \lim_{u \to 2} \frac{4(u-2)}{(u-2)(\sqrt{4u+1}+3)} = \lim_{u \to 2} \frac{4}{(\sqrt{4u+1}+3)}$$

$$= \frac{4}{\sqrt{8+1}+3} = \frac{4}{3+3} = \frac{4}{6} = \frac{2}{3}$$

24| $$\lim_{x \to -1} \frac{x^2+2x+1}{x^4-1} = \frac{0}{0}$$

$$= \lim_{x \to -1} \frac{(x+1)^2}{(x^2)^2-1^2} = \lim_{x \to -1} \frac{(x+1)^2}{(x^2-1)(x^2+1)}$$

$$= \lim_{x \to -1} \frac{(x+1)^2}{(x+1)(x-1)(x^2+1)} = \lim_{x \to -1} \frac{x+1}{(x-1)(x^2+1)}$$

$$= \frac{-1+1}{(-1-1)(1+1)} = \frac{0}{-4} = 0$$

25| $$\lim_{t \to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} = \frac{0}{0}$$

$$= \lim_{t \to 0} \frac{\sqrt{1+t}-\sqrt{1-t}}{t} \times \frac{\sqrt{1+t}+\sqrt{1-t}}{\sqrt{1+t}+\sqrt{1-t}}$$

$$= \lim_{t \to 0} \frac{1+t-(1-t)}{t(\sqrt{1+t}+\sqrt{1-t})} = \lim_{t \to 0} \frac{2t}{t(\sqrt{1+t}+\sqrt{1-t})}$$

$$= \lim_{t \to 0} \frac{2}{\sqrt{1+t}+\sqrt{1-t}} = \frac{2}{\sqrt{1+0}+\sqrt{1-0}} = \frac{2}{1+1} = 1$$

26| $$\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t} \right) = \lim_{t \to 0} \frac{t^2+t-t}{t(t^2+t)} \cancel{\lim_{t \to 0}}$$

$$= \lim_{t \to 0} \frac{t^2}{t(t^2+t)} = \lim_{t \to 0} \frac{t^2}{t^2(t+1)} = \lim_{t \to 0} \frac{1}{t+1}$$

$$= \frac{1}{0+1} = 1$$

27| $$\lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 16} \frac{4-\sqrt{x}}{16x-x^2} \times \frac{4+\sqrt{x}}{4+\sqrt{x}} = \lim_{x \to 16} \frac{(16-x)}{x(16-x)(4+\sqrt{x})}$$

$$= \lim_{x \to 16} \frac{1}{x(4+\sqrt{x})} = \frac{1}{16(4+\sqrt{16})} = \frac{1}{16(4+4)} = \cancel{\frac{1}{256}}$$

$$= \frac{1}{128}$$

28| $$\lim_{h \to 0} \frac{(3+h)^{-1} - 3^{-1}}{h} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{h \to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \lim_{h \to 0} \frac{\frac{3-3-h}{3(3+h)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{3(3+h)} \div h = \lim_{h \to 0} \frac{-h}{3(3+h)} \times \frac{1}{h} = \lim_{h \to 0} \frac{-1}{3(3+h)}$$

$$= \frac{-1}{3(3+0)} = -\frac{1}{9}$$

32|
$$\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\lim_{h\to 0}\frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h}$$

$$=\lim_{h\to 0}\frac{x^2-(x^2+2hx+h^2)}{hx^2(x+h)2}=\lim_{h\to 0}\frac{-2hx-h^2}{hx^2(x+h)2}$$

$$=\lim_{h\to 0}\frac{-2x-h}{x^2(x+h)^2}=\frac{-2x}{x^2(x+0)^2}=\frac{-2x}{x4}$$

$$=\frac{-2}{x3}$$

38|

$$2x\le g(x)\le x^4-x^2+2$$

$$\lim_{x\to 1}(2x)\le\lim_{x\to 1}g(x)\le\lim_{x\to 1}(x^4-x^2+2)$$

$$\Rightarrow\quad 2\le\lim_{x\to 1}g(x)\le 1-1+2=2$$

By Sandwitch/Squeeze Theorem, we have

$$\lim_{x\to 1}g(x)=2$$

39| Prove that
$$\lim_{x\to 0}x^4\cos\frac{2}{x}=0$$

Proof:
$$-1\le\cos\frac{2}{x}\le 1$$

$$-x^4\le x^4\cos\frac{2}{x}\le x4$$

Alternative:

$$\lim_{x\to 0}x^4\cos\frac{2}{x}$$

$= 0^4\times$ [any number between -1 and $+1$]

$= 0$

$$\lim_{x\to 0}(-x^4)\le\lim_{x\to 0}\left(x^4\cos\frac{2}{x}\right)\le\lim_{x\to 0}(x^4)$$

$$\Rightarrow\quad 0\le\lim_{x\to 0}\left(x^4\cos\frac{2}{x}\right)\le 0$$

By Sandwitch Theorem, $\boxed{\lim_{x\to 0}x^4\cos\frac{2}{x}=0}$

44)

$$\lim_{x\to -2} \frac{2-|x|}{2+x} = \lim_{x\to -2} \frac{2-(-x)}{2+x} = \lim_{x\to -2} \frac{2+x}{2+x}$$

$$= \lim_{x\to -2} 1 = 1$$

45)

$$\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right) = \lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{-x}\right) = \lim_{x\to 0^-} \left(\frac{1}{x} + \frac{1}{x}\right)$$

$$= \lim_{x\to 0^-} \frac{2}{x} = -\infty$$