**Section: 9 Group Number: _____** **Score: _____/20**
**LA-18**
**Names of Group Members PRESENT: _____**

Credit is only given for group work to those present on all days L&LA is worked in class and who are also present the day it is turned in.

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Informal Definition of Limit

**1 Definition** Suppose $f(x)$ is defined when $x$ is near the number $a$. (This means that $f$ is defined on some open interval that contains $a$, except possibly at $a$ itself.) Then we write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$, as $x$ approaches $a$, equals $L$"

if we can make the values of $f(x)$ arbitrarily close to $L$ (as close to $L$ as we like) by taking $x$ to be sufficiently close to $a$ (on either side of $a$) but not equal to $a$.

FORMAL Definition of Limit (see video lectures on p 109 and 110 in electronic text)

**2 Definition** Let $f$ be a function defined on some open interval that contains the number $a$, except possibly at $a$ itself. Then we say that the **limit of $f(x)$ as $x$ approaches $a$ is $L$**, and we write

$$\lim_{x \to a} f(x) = L$$

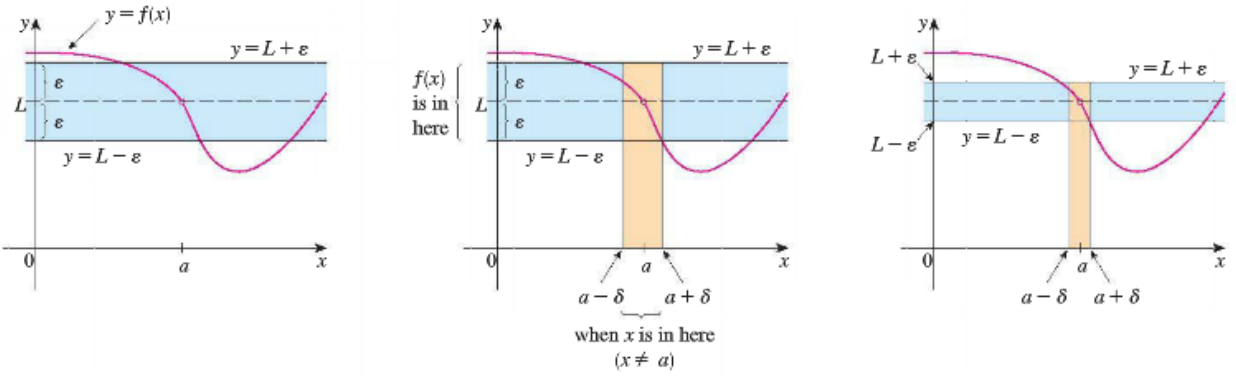
if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if } 0 < |x - a| < \delta \quad \text{then} \quad |f(x) - L| < \varepsilon$$

Geometric Interpretation:

If $\varepsilon > 0$ is given, draw the horizontal lines $L + \varepsilon$ and $L - \varepsilon$ on the graph of $f$.

Then find a number $\delta > 0$ such that if we restrict $x$ to lie in the interval $(a - \delta, a + \delta)$ and take $x \neq a$, then the curve lies between the lines $y = L + \varepsilon$ and $y = L - \varepsilon$




The process must work for every positive number $\varepsilon$, so that if a smaller $\varepsilon$ is chosen, then a smaller $\delta$ may be required.

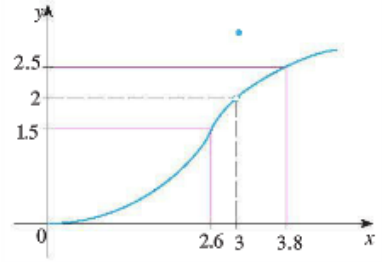
Wolfram demonstration on page 111 and interactive figure on page 112 in electronic text.

DMS

1. (2.4:#2)

**2.** Use the given graph of $f$ to find a number $\delta$ such that

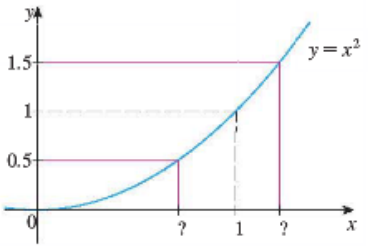
if $0 < |x - 3| < \delta$ then $|f(x) - 2| < 0.5$




2. (2.4:#4)

**4.** Use the given graph of $f(x) = x^2$ to find a number $\delta$ such that

if $|x - 1| < \delta$ then $|x^2 - 1| < \frac{1}{2}$




To PROVE the limits precisely, there are TWO steps:

- Finding $\delta$.
- Proof (showing that $\delta$ works)

Consider video example 2 page 112 in electronic text

3. (2.4:#16)

**15–18** Prove the statement using the $\varepsilon, \delta$ definition of a limit and illustrate with a diagram like Figure 9.

$$\lim_{x \to 4}(2x - 5) = 3$$

4. (2.4:#18)

**15–18** Prove the statement using the $\varepsilon, \delta$ definition of a limit and illustrate with a diagram like Figure 9.

$\lim_{x \to -2}(3x+5) = -1$

5. (2.4:#20) $\lim_{x \to 10}\left(3 - \frac{4}{5}x\right) = -5$

6. (2.4:#21) $\lim_{x \to 2}\frac{x^2 + x - 6}{x - 2} = 5$

7. (2.4:#23) $\lim_{x \to a} x = a$

8. (2.4:#25) $\lim_{x \to 0} x^2 = 0$

9. (2.4:#29) $\lim_{x \to 2} \left( x^2 - 4x + 5 \right) = 1$

10. $\lim_{x \to 2} \left( 4x + 2 \right) = 10$

11. Let $f(x) = 2\sqrt{x}$. Find a value of $\delta$ such that if $|x - 1| < \delta$, then $|f(x) - 2| < \frac{1}{2}$.

DMS

Special situations:

One Sided limits

**3 Definition of Left-Hand Limit**

$$\lim_{x \to a^-} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad a - \delta < x < a \quad \text{then} \quad |f(x) - L| < \varepsilon$$

**4 Definition of Right-Hand Limit**

$$\lim_{x \to a^+} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \quad a < x < a + \delta \quad \text{then} \quad |f(x) - L| < \varepsilon$$

Consider video example 2 page 113 in electronic text

Infinite Limits

**6 Definition** Let $f$ be a function defined on some open interval that contains the number $a$, except possibly at $a$ itself. Then

$$\lim_{x \to a} f(x) = \infty$$

means that for every positive number $M$ there is a positive number $\delta$ such that

$$\text{if} \quad 0 < |x - a| < \delta \quad \text{then} \quad f(x) > M$$

**7 Definition** Let $f$ be a function defined on some open interval that contains the number $a$, except possibly at $a$ itself. Then

$$\lim_{x \to a} f(x) = -\infty$$

means that for every negative number $N$ there is a positive number $\delta$ such that

$$\text{if} \quad 0 < |x - a| < \delta \quad \text{then} \quad f(x) < N$$

Consider video example 5 page 116 in electronic text

HW 2.4:#17, 22, 24, 28,

13 & 14 do these last, OK to use calculator